directorship at Berlin and consequent removal to the capital, he was sent to a gymnasium there, and in due course completed his education at the university of Frankfort-on-the-Oder. He is said to have shown neither liking nor aptitude for intellectual pursuits till after his attendance on the lectures of Kant at Königsberg, in his twentieth or twenty-first year, when, suddenly lighted up as by inspiration, he set to work in right earnest, mastered the Greek and Latin languages, acquired as perfect a knowledge of French as could well be attained by one who was not a Frenchman, and a sufficient familiarity with English to enable him to translate from it with clearness and fluency. He also managed to gain an intimate acquaintance with English commerce and finance, which he afterwards turned to good account. The extent of his acquirements was rendered more remarkable by his confirmed habits of dissipation; for from the commencement to the conclusion of his career he was remarkable for the manner in which, in the midst of the gravest occupations, he indulged his fondness for female society and a ruinous passion for play. In 1786 he was appointed private secretary to the royal general directory, and was soon afterwards promoted to the rank of *Kriegsrath* (war-councillor). Like Mackintosh, he was fascinated by the French Revolution at its dawn, and, like Mackintosh, was converted to a sounder estimate of its then pending results by Burke. He broke ground in literature in 1794, by a translation of the celebrated *Essay on the French Revolution*, followed in 1794 and 1795 by translations from Mallet du Pan and Mounier. In 1795 he founded and edited a monthly journal which soon came to an untimely end. In November 1797 he published a pamphlet under the title of a *Sendscreiben* or *Missive* addressed to Frederick William III. of Prussia on his accession, pointing out the duties of the new sovereign and especially recommending the complete freedom of the press. In the course of the next three years he contributed to the *Historisches Journal* a series of articles "On the Origin and Character of the War against the French Revolution," with express reference to Great Britain. These led to his visiting England, where he formed intimate relations with Mackintosh, Lord Grenville, Pitt, and other eminent men, which proved lasting, flattering, and remunerative. The first entries in his published diary, beginning April 14, 1800, and continued (with breaks) to the end of 1828, run thus:—

"On the 14th of April, an agreeable surprise. The Jew elder, Hirsch, brought me 50 thalers for drawing up I know not what representation (*Vorstellung*). *May* 28.—Received through Baron Krüdener a watch set with (small) brilliants, a present from the emperor of Russia. *June* 1.—Received through Garlicke a letter from Lord Grenville, together with a donation of £500, the first of its kind."

The last entry for this year, 1800, is:—"At the end of the year great pecuniary embarrassment. Received £100 from Garlicke and negotiated with Carysfort."

The diary for 1801 begins:—"*February.*—Very remarkable that on the one side Lord Carysfort charged me with the translation into French of the English Notes against Prussia, and shortly afterwards Count Haugwitz with the translation into German of the Prussian Notes against England."

Frequently recurring entries of this kind illustrate his position through life. He was to all intents and purposes a mercenary of the pen, but he was so openly and avowedly, and he was never so much as suspected by those who knew him best of writing contrary to his own convictions at the time. This is why he never lost the esteem or confidence of his employers;—of Prince Metternich, for example, who, when he was officially attached to the Austrian Government, was kept regularly informed of the sources from which the greater part of his income was derived. Embarrassments of all sorts, ties and temptations from which he was irresistibly impelled to tear himself, led to his change of country; and an entry for May 1802 runs:—"On the 15th I take leave of my wife, and at three in the morning of the 20th I leave Berlin with Adam Müller, never to see it again." It does not appear that he ever saw his wife again either; and his intimacies with other women, mostly of the highest rank, are puzzling from their multiplicity. He professes himself unable to explain the precise history of his settlement in Vienna. All he remembers is that he was received with signs of jealousy and distrust, and that the emperor, to whom he was presented by Count Colloredo, showed no desire to secure his services. Many years were to elapse before the formation of the connexion with Metternich, the most prominent feature and crowning point of his career.

Before entering into any kind of engagements with the Austrian Government he applied to the king of Prussia for a formal discharge, which was granted with an assurance that his Majesty, "in reference to his merits as a writer, coincided in the general approbation which he had so honourably acquired." A decisive proof of the confidence placed in him was his being invited by Count Haugwitz to the Prussian headquarters shortly before the battle of Jena, and commissioned to draw up the Prussian manifesto and the king's letter to Napoleon. It was in noticing this letter that Napoleon spoke of the known and avowed writer as "a wretched scribe named Gentz, one of those men without honour who sell themselves for money." In the course of 1806, he published *War between Spain and England*, and *Fragments upon the Balance of Power in Europe*, on receiving which (at Bombay) Mackintosh wrote:—"I assent to all you say, sympathize with all you feel, and admire equally your reason and your eloquence throughout your masterly fragment." The bond of union between him and Metternich was formed in 1840. This was one reason, joined to his general reputation, for his being named first secretary to the congress of Vienna in 1814, where, besides his regular duties, he seems to have made himself useful to several of the plenipotentiaries, as he notes in his diary that he received 22,000 florins in the name of Louis XVIII. from Talleyrand, and £600 from Lord Castlereagh, accompanied by "*les plus folles promesses.*" He acted in the same capacity at the congress or conference of Paris in 1815, of Aix in 1818, Karlsbad and Vienna in 1819, Troppau and Laybach in 1820 and 1821, and Verona in 1822. The following entry in his diary for December 14, 1819, has exposed him to much obloquy as the interested advocate of reactionary doctrines:—"About eleven, at Prince Metternich's: attended the last and most important sitting of the commission to settle the 13th article of the Bundes-Akt, and had my share in one of the greatest and worthiest results of the transactions of our time. A day more important than that of Leipsic." The 13th article provides that in all states of the Bund the constitutional government shall be by estates instead of by a representative body in a single chamber: "in allen Bundestaaten wird eine landständische Verfassung stattfinden." Remembering what ensued in France from the absorption of the other estates in the Tiers État, it would have been strange if Gentz had not supported this 13th article. He was far from a consistent politician, but he was always a sound Conservative at heart; and his reputation rests on his foreign policy, especially on the courage, eloquence, and efficiency with which he made head against the Napoleonic system till it was struck down.

The most remarkable phase of Gentz's declining years was his passion, in his sixty-seventh year, for Fanny Elssler, the celebrated *danseuse*, which forms the subject of some very remarkable letters to his attached friend Rahel (the wife of Varnhagen von Ense) in 1830 and 1831. He died June 9, 1832. There is no complete edition of his works. The late Baron von Prokesch was engaged in preparing one when the Austrian Government interfered, and the design was perforce abandoned. (A. H.)

GEODESY

GEODESY ($\gamma\tilde{\eta}$, the earth, $\delta\alpha\acute{\iota}\omega$, to divide) is the science of surveying extended to large tracts of country, having in view not only the production of a system of maps of very great accuracy, but the determination of the curvature of the surface of the earth, and eventually of the figure and dimensions of the earth. This last, indeed, may be the sole object in view, as was the case in the operations conducted in Peru and in Lapland by the celebrated French astronomers Bouguer, La Condamine, Maupertuis, Clairaut, and others; and the measurement of the meridian arc of France by Mechain and Delambre had for its end the determination of the true length of the "metre" which was to be the legal standard of length of France.

The basis of every extensive survey is an accurate triangulation, and the operations of geodesy consist in—the measurement, by theodolites, of the angles of the triangles; the measurement of one or more sides of these triangles on the ground; the determination by astronomical observations of the azimuth of the whole network of triangles; the determination of the actual position of the same on the surface of the earth by observations, first for latitude at some of the stations, and secondly for longitude.

To determine by actual measurement on the ground the length of a side of one of the triangles, wherefrom to infer the lengths of all the other sides in the triangulation, is not the least difficult operation of a trigonometrical survey. When the problem is stated thus—To determine the number of times that a certain standard or unit of length is contained between two finely marked points on the surface of the earth at a distance of some miles asunder, so that the error of the result may be pronounced to lie between certain very narrow limits,—then the question demands very serious consideration. The representation of the unit of length by means of the distance between two fine lines on the surface of a bar of metal at a certain temperature is never itself free from uncertainty and probable error, owing to the difficulty of knowing at any moment the precise temperature of the bar; and the transference of this unit, or a multiple of it, to a measuring bar, will be affected not only with errors of observation, but with errors arising from uncertainty of temperature of both bars. If the measuring bar be not self-compensating for temperature, its expansion must be determined by very careful experiments. The thermometers required for this purpose must be very carefully studied, and their errors of division and index error determined.

The base apparatus of Bessel and that of Colby have been described in FIGURE OF THE EARTH (vol. vii. p. 598). The average probable error of a single measurement of a base line by the Colby apparatus is, according to the very elaborate investigations of Colonel Walker, C.B., R.E., the Surveyor-General of India, $\pm 1{\cdot}5\mu$ (μ meaning "one millionth"). W. Struve gives $\pm 0{\cdot}8\mu$ as the probable error of a base line measured with his apparatus, being the mean of the probable errors of seven bases measured by him in Russia; but this estimate is probably too small. Struve's apparatus is simple: there are four wrought iron bars, each two toises (rather more than 13 feet) long; one end of each bar is terminated in a small steel cylinder presenting a slightly convex surface for contact, the other end carries a contact lever rigidly connected with the bar. The shorter arm of the lever terminates below in a polished hemisphere, the upper and longer arm traversing a vertical divided arc. In measuring, the plane end of one bar is brought into contact with the short arm of the contact lever (pushed forward by a weak spring) of the next bar. Each bar has two thermometers, and a level for determining the inclination of the bar in measuring. The manner of transferring the end of a bar to the ground is simply this: under the end of the bar a stake is driven very firmly into the ground, carrying on its upper surface a disk, capable of movement in the direction of the measured line by means of slow-motion screws. A fine mark on this disk is brought vertically under the end of the bar by means of a theodolite which is planted at a distance of 25 feet from the stake in a direction perpendicular to the base. Struve investigates for each base the probable errors of the measurement arising from each of these seven causes:—alignment, inclination, comparisons with standards, readings of index, personal errors, uncertainties of temperature, and the probable errors of adopted rates of expansion.

The apparatus used in the United States Coast Survey consists of two measuring bars, each 6 metres in length, supported on two massive tripod stands placed at one quarter length from each end, and provided, as in Colby's apparatus, with the necessary mechanism for longitudinal, transverse, and vertical adjustment. Each measuring rod is a compensating combination of an iron and a brass bar, supported parallel to one another and firmly connected at one end, the medium of connexion between the free ends being a lever of compensation so adjusted as to indicate a constant length independent of temperature or changes of temperature. The bars are protected from external influences by double tubes of tinned sheet iron, within which they are movable on rollers by a screw movement which allows of contacts being made within $\frac{1}{10000}$ of an inch. The abutting piece acts upon the contact lever which is attached to the fixed end of the compound bar, and carries a very sensitive level, the horizontal position of which defines the length of the bar. It is impossible here to give a full description of this complicated apparatus, and we must refer for details to the account given in full in the United States Coast Survey Report for 1854. This apparatus is doubtless a very perfect one, and the manipulation of it must offer great facilities, for it appears to be possible, under favourable circumstances, to measure a mile in one day, 1·06 mile having been measured on one occasion in eight and a half hours. In order to test to the utmost the apparatus, the base at Atlanta, Georgia, was measured twice in winter and once in summer 1872–73, at temperatures 51°, 45°, 90° F.; the difference of the first and second measurements was $+0{\cdot}30$ in., of the second and third $+0{\cdot}34$ in.,—the actual length and computed probable error expressed in metres being $9338{\cdot}4763 \pm 0{\cdot}0166$. It is to be noted that in the account of a base recently measured in the United States Lake Survey, some doubt is expressed as to the perfection of the particular apparatus of this description there used, on account of a liability to permanent changes of length.

The last base line measured in India with Colby's compensation apparatus had a length of 8912 feet only, and in consequence of some doubts which had arisen as to the accuracy of this compensation apparatus, the measurement was repeated four times, the operations being conducted in such a manner as to indicate as far as possible the actual magnitudes of the probable errors to which such measures are liable. The direction of the line (which is at Cape Comorin) is north and south, and in two of the measurements the brass component was to the west, in the other two it was to the east. The differences between the individual measurements and the mean of the four are $+{\cdot}0017$, $-{\cdot}0049$, $-{\cdot}0015$, $+{\cdot}0045$ in feet. The measurements occupied from seven to ten days each,—the average rate of such work in India being about a mile in five days.

The method of M. Porro, adopted in Spain, and by the French in Algiers, is essentially different from those just described. The measuring rod, for there is only one, is a thermometric combination of two bars, one of platinum and one of brass, in length 4 metres, furnished with three levels and four thermometers. Suppose A, B, C three micrometer microscopes very firmly supported at intervals of 4 metres with their axes vertical, and aligned in the plane of the base line by means of a transit instrument, their micrometer screws being in the line of measurement. The measuring bar is brought under say A and B, and those micrometers read; the bar is then shifted and brought under B and C. By repetition of this process, the reading of a micrometer indicating the end of each position of the bar, the measurement is made. The probable error of the central base of Madridejos, which has a length of 14664·500 metres, is estimated at $\pm 0{\cdot}17\mu$. This is the longest base line in Spain; there are seven others, six of which are under 2500 metres in length; of these one is in Majorca, another in Minorca, and a third in Iviça. The last base just measured in the province of Barcelona has a length of 2483·5381 metres according to the first measurement, and 2483·5383 according to the second.

The total number of base lines measured in Europe up to the present time is about eighty, fifteen of which do not exceed in length 2500 metres, or about a mile and a half, and two—one in France, the other in Bavaria—exceed 19,000 metres. The question has been frequently discussed whether or not the advantage of a long base is sufficiently great to warrant the expenditure of time that it requires, or whether as much precision is not obtainable in the end by careful triangulation from a short base. But the answer cannot be given generally; it must depend on the circumstances of each particular case.

It is necessary that the altitude above the level of the sea of every part of a base line be ascertained by spirit levelling, in order that the measured length may be reduced to what it would have been had the measurement been made on the surface of the sea, produced in imagination. Thus if l be the length of a measuring bar, h its height at any given position in the measurement, r the radius of the earth, then the length radially projected on to the level of the sea is $l - \frac{h}{r}l$. In the Salisbury Plain base line the reduction to the level of the sea is $-0{\cdot}6294$ feet.

In working away from a base line ab, stations c, d, e, f are carefully selected so as to obtain from well-shaped triangles gradually increasing sides. Before, however, finally leaving the base line it is usual to verify it by triangulation thus: during the measurement two or more points, as p, q (fig. 1), are marked in the base in positions such that the lengths of the different segments of the line are known; then, taking suitable external stations, as h, k, the angles of the triangles bhp, phq, hqk, kqa are measured. From these angles can be computed the ratios of the segments, which must agree, if all operations are correctly performed, with the ratios resulting from the measures. Leaving the base line, the sides increase up to ten, thirty, or fifty miles, occasionally, but seldom, reaching a hundred miles. The triangulation points may either

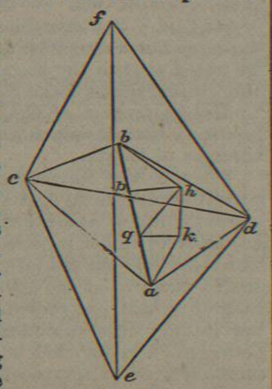

Fig. 1.

be natural objects presenting themselves in suitable positions, such as church towers; or they may be objects specially constructed in stone or wood on mountain tops or other prominent ground. In every case it is necessary that the precise centre of the station be marked by some permanent mark. In India no expense is spared in making permanent the principal trigonometrical stations—costly towers in masonry being erected. It is essential that every trigonometrical station shall present a fine object for observation from surrounding stations.

Horizontal Angles.

In placing the theodolite over a station to be observed from, the first point to be attended to is that it shall rest upon a perfectly solid foundation. The method of obtaining this desideratum must depend entirely on the nature of the ground; the instrument must if possible be supported on rock, or if that be impossible a solid foundation must be obtained by digging. When the theodolite is required to be raised above the surface of the ground in order to command particular points, it is necessary to build two scaffolds,—the outer one to carry the observatory, the inner one to carry the instrument,—and these two edifices must have no point of contact. Many cases of high scaffolding have occurred on the English Ordnance Survey, as for instance at Thaxted Church, where the tower, 80 feet high, is surmounted by a spire of 90 feet. The scaffold for the observatory was carried from the base to the top of the spire; that for the instrument was raised from a point of the spire 140 feet above the ground, having its bearing upon timbers passing through the spire at that height. Thus the instrument, at a height of 178 feet above the ground, was insulated, and not affected by the action of the wind on the observatory.

At every station it is necessary to examine and correct the adjustments of the theodolite, which are these:—the line of collimation of the telescope must be perpendicular to its axis of rotation; this axis perpendicular to the vertical axis of the instrument; and the latter perpendicular to the plane of the horizon. The micrometer microscopes must also measure correct quantities on the divided circle or circles. The method of observing is this. Let A, B, C . . . . be the stations to be observed taken in order of azimuth; the telescope is first directed to A and the cross-hairs of the telescope made to bisect the object presented by A, then the microscopes or verniers of the horizontal circle (also of the vertical circle if necessary) are read and recorded. The telescope is then turned to B, which is observed in the same manner; then C and the other stations. Coming round by continuous motion to A, it is again observed, and the agreement of this second reading with the first is some test of the stability of the instrument. In taking this round of angles—or "arc," as it is called on the Ordnance Survey—it is desirable that the interval of time between the first and second observations of A should be as small as may be consistent with due care. Before taking the next arc the horizontal circle is moved through 20° or 30°; thus a different set of divisions of the circle is used in each arc, which tends to eliminate the errors of division.

It is very desirable that all arcs at a station should contain one point in common, to which all angular measurements are thus referred,—the observations on each arc commencing and ending with this point, which is on the Ordnance Survey called the "referring object." It is usual for this purpose to select, from among the points which have to be observed, that one which affords the best object for precise observation. For mountain tops a "referring object" is constructed of two rectangular plates of metal in the same vertical plane, their edges parallel and placed at such a distance apart that the light of the sky seen through

appears as a vertical line about 10″ in width. The best distance for this object is from one to two miles.

It is clear that no correction is required to the angles measured by a theodolite on account of its height above the sea-level; for its axis of rotation coincides with the normal to the surface of the earth, and the angles measured between distant points are those contained between the vertical planes passing through the axis of the instrument and those points.

The theodolites used in geodesy vary in pattern and in size—the horizontal circles ranging from 10 inches to 36 inches in diameter. In Ramsden's 36-inch theodolite the telescope has a focal length of 36 inches and an aperture of 2·5 inches, the ordinarily used magnifying power being 54; this last, however, can of course be changed at the requirements of the observer or of the weather. The probable error of a single observation of a fine object with this theodolite is about 0″·2.

Fig. 2 represents an altazimuth theodolite of an improved pattern now used on the Ordnance Survey. The

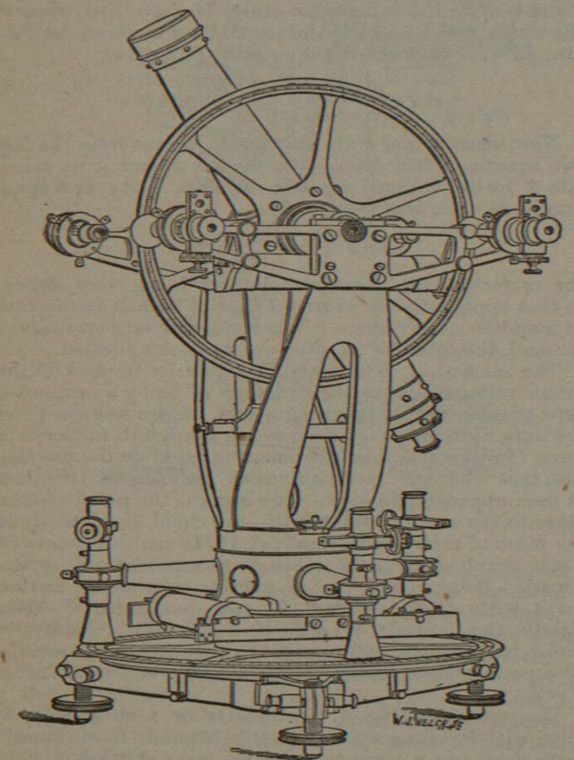

Fig. 2.—Altazimuth Theodolite.

horizontal circle of 14 inches diameter is read by three micrometer microscopes; the vertical circle has a diameter of 12 inches, and is read by two microscopes.

In the Great Trigonometrical Survey of India the theodolites used in the more important parts of the work have been of 2 and 3 feet diameter,—the circle read by five equidistant microscopes. Every angle is measured twice in each position of the zero of the horizontal circle, of which there are generally ten; the entire number of measures of an angle is never less than 20. An examination of 1407 angles showed that the probable error of an observed angle is on the average $\pm$ 0·28.

For the observations of very distant stations it is usual to employ a heliostat. In its simplest form this is a plane mirror 4, 6, or 8 inches in diameter, capable of rotation round a horizontal and a vertical axis. This mirror is placed at the station to be observed, and in fine weather it is kept so directed that the rays of the sun reflected by it strike the distant observing telescope. To the observer the heliostat presents the appearance of a star of the first or second magnitude, and is generally a pleasant object for observing.

Astronomical Observations.

The direction of the meridian is determined either by a theodolite or a portable transit instrument. In the former case the operation consists in observing the angle between a terrestrial object—generally a mark specially erected and capable of illumination at night—and a close circumpolar star at its greatest eastern or western azimuth, or, at any rate, when very near that position. If the observation be made t minutes of time before or after the time of greatest azimuth, the azimuth then will differ from its maximum value by

$$(450t)^2 \sin 1'' \frac{\sin 2\delta}{\sin z} \pm \ldots$$

in seconds of angle, omitting smaller terms. Here the symbol δ is the star's declination, z its zenith distance. The collimation and level errors are very carefully determined before and after these observations, and it is usual to arrange the observations by the reversal of the telescope so that collimation error shall disappear. If b, c be the level and collimation errors, the correction to the circle reading is $b \cot z \pm c \operatorname{cosec} z$, b being positive when the west end of the axis is high. It is clear that any uncertainty as to the real state of the level will produce a corresponding uncertainty in the resulting value of the azimuth,—an uncertainty which increases with the latitude, and is very large in high latitudes. This may be partly remedied by observing in connexion with the star its reflexion in mercury. In determining the value of "one division" of a level tube, it is necessary to bear in mind that in some the value varies considerably with the temperature. By experiments on the level of Ramsden's 3-foot theodolite, it was found that though at the ordinary temperature of 66° the value of a division was about one second, yet at 32° it was about five seconds.

The portable transit in its ordinary form hardly needs description. In a very excellent instrument of this kind used on the Ordnance Survey, the uprights carrying the telescope are constructed of mahogany, each upright being built of several pieces glued and screwed together; the base, which is a solid and heavy plate of iron, carries a reversing apparatus for lifting the telescope out of its bearings, reversing it, and letting it down again. Thus is avoided the change of temperature which the telescope would incur by being lifted by the hands of the observer. Another form of transit is the German diagonal form, in which the rays of light after passing through the object glass are turned by a total reflexion prism through one of the transverse arms of the telescope, at the extremity of which arm is the eye-piece. The unused half of the ordinary telescope being cut away is replaced by a counterpoise. In this instrument there is the advantage that the observer without moving the position of his eye commands the whole meridian, and that the level may remain on the pivots whatever be the elevation of the telescope. But there is the disadvantage that the flexure of the transverse axis causes a variable collimation error depending on the zenith distance of the star to which it is directed; and moreover it has been found that in some cases the personal error of an observer is not the same in the two positions of the telescope.

To determine the direction of the meridian, it is well to erect two marks at nearly equal angular distances on either side of the north meridian line, so that the pole star crosses the vertical of each mark a short time before and after attaining its greatest eastern and western azimuths.

If now the instrument, perfectly levelled, is adjusted to have its centre wire on one of the marks, then when elevated to the star, the star will traverse the wire, and its exact position in the field at any moment can be measured by the micrometer wire. Alternate observations of the star and the terrestrial mark, combined with careful level readings and reversals of the instrument, will enable one, even with only one mark, to determine the direction of the meridian in the course of an hour with a probable error of less than a second. The second mark enables one to complete the station more rapidly, and gives a check upon the work. As an instance, at Findlay Seat, in latitude 57° 35′, the resulting azimuths of the two marks were 177° 45′ 37″·29 ± 0″·20 and 182° 17′ 15″·61 ± 0″·13, while the angle between the two marks directly measured by a theodolite was found to be 4° 31′ 37″·43 ± 0″·23.

We now come to the consideration of the determination of time with the transit instrument. Let fig. 3 represent the sphere stereographically projected on the plane of the horizon,—*ns* being the meridian, *we* the prime vertical, Z, P the zenith and the pole. Let p be the point in which the production of the axis of the instrument meets the celestial sphere, S the position of a star when observed on a wire whose distance from the collimation centre is c. Let a be the azimuthal deviation, namely, the angle wZp, b the level error so that $Zp = 90° - b$. Let also the hour angle corresponding to p be $90° - n$, and the declination of the same $= m$, the star's declination being δ, and the latitude ϕ. Then to find the hour angle $ZPS = \tau$ of the star when observed, in the triangles vPS. vPZ we have. since $pPS = 90 + \tau - n$,

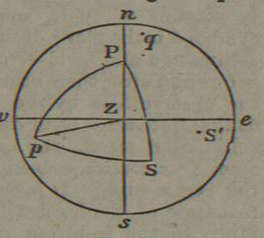

Fig. 3.

$$-\text{Sin } c = \sin m \sin \delta + \cos m \cos \delta \sin (n - \tau),$$
$$\text{Sin } m = \sin b \sin \phi - \cos b \cos \phi \sin a,$$
$$\text{Cos } m \sin n = \sin b \cos \phi + \cos b \sin \phi \sin a.$$

And these equations solve the problem, however large be the errors of the instrument. Supposing, as usual, a, b, m, n to be small, we have at once $\tau = n + c \sec \delta + m \tan \delta$, which is the correction to the observed time of transit. Or, eliminating m and n by means of the second and third equations, and putting z for the zenith distance of the star, t for the observed time of transit, the corrected time is

$$t + \frac{a \sin z + b \cos z + c}{\cos \delta}$$

Another very convenient form for stars near the zenith is this—

$$\tau = b \sec \phi + c \sec \delta + n (\tan \delta - \tan \phi).$$

Suppose that in commencing to observe at a station the error of the chronometer is not known; then having secured for the instrument a very solid foundation, removed as far as possible level and collimation errors, and placed it by estimation nearly in the meridian, let two stars differing considerably in declination be observed—the instrument not being reversed between them. From these two stars, neither of which should be a close circumpolar star, a good approximation to the chronometer error can be obtained; thus let ϵ_1, ϵ_2 be the apparent clock errors given by these stars, if δ_1, δ_2 be their declinations the real error is

$$\epsilon = \epsilon_1 + (\epsilon_1 - \epsilon_2)\frac{\tan \phi - \tan \delta_1}{\tan \delta_1 - \tan \delta_2}.$$

Of course this is still only approximative, but it will enable the observer (who by the help of a table of natural tangents can compute ϵ in a few minutes) to find the meridian by placing at the proper time, which he now knows approximately, the centre wire of his instrument on the first star that passes—not near the zenith.

The transit instrument is always reversed at least once in the course of an evening's observing, the level being frequently read and recorded. It is necessary in most instruments to add a correction for the difference in size of the pivots.

The transit instrument is also used in the prime vertical for the determination of latitudes. In the preceding figure let q be the point in which the northern extremity of the axis of the instrument produced meets the celestial sphere. Let nZq be the azimuthal deviation $= a$, and b being the level error, $Zq = 90° - b$; let also $nPq = \tau$ and $Pq = \psi$. Let S′ be the position of a star when observed on a wire whose distance from the collimation centre is c, positive when to the south, and let h be the observed hour angle of the star, viz., ZPS′. Then the triangles qPS′, qPZ give

$$\text{Sin } c = \sin \delta \cos \psi - \cos \delta \sin \psi \cos (h + \tau),$$
$$\text{Cos } \psi = \sin b \sin \phi + \cos b \cos \phi \cos a,$$
$$\text{Sin } \psi \sin \tau = \cos b \sin a.$$

Now when a and b are very small, we see from the last two equations that $\psi = \phi - b$, $a = \tau \sin \psi$, and if we calculate ϕ' by the formula $\cot \phi' = \cot \delta \cos h$, the first equation leads us to this result—

$$\phi = \phi' + \frac{a \sin z + b \cos z + c}{\cos z}.$$

the correction for instrumental error being very similar to that applied to the observed time of transit in the case of meridian observations. When a is not very small and z is small, the formulæ required are more complicated.

The method of determining latitude by transits in the prime vertical has the disadvantage of being a somewhat slow process, and of requiring a very precise knowledge of the time, a disadvantage from which the zenith telescope is free. In principle this instrument is based on the proposition that when the meridian zenith distances of two stars at their upper culminations—one being to the north and the other to the south of the zenith—are equal, the latitude is the mean of their declinations; or, if the zenith distance of a star culminating to the south of the zenith be Z, its declination being δ, and that of another culminating to the north with zenith distance Z′ and declination δ', then clearly the latitude is $\frac{1}{2}(\delta + \delta') + \frac{1}{2}(Z - Z')$. Now the zenith telescope does away with the divided circle, and substitutes the measurement micrometrically of the quantity $Z' - Z$.

The instrument (fig. 4) is supported on a strong tripod, fitted with levelling screws; to this tripod is fixed the azimuth circle and a long vertical steel axis. Fitting on this axis is a hollow axis which carries on its upper end a short transverse horizontal axis. This latter carries the telescope, which, supported at the centre of its length, is free to rotate in a vertical plane. The telescope is thus mounted excentrically with respect to the vertical axis around which it revolves. An extremely sensitive level is attached to the telescope, which latter carries a micrometer in its eyepiece, with a screw of long range for measuring differences of zenith distance. For this instrument stars are selected in pairs, passing north and south of the zenith, culminating within a few minutes of time and within about twenty minutes (angular) of zenith distance of each other. When a pair of stars is to be observed, the telescope is set to the mean of the zenith distances and in the plane of the

meridian. The first star on passing the central meridional wire is bisected by the micrometer; then the telescope is rotated very carefully through 180° round the vertical axis, and the second star on passing through the field is bisected

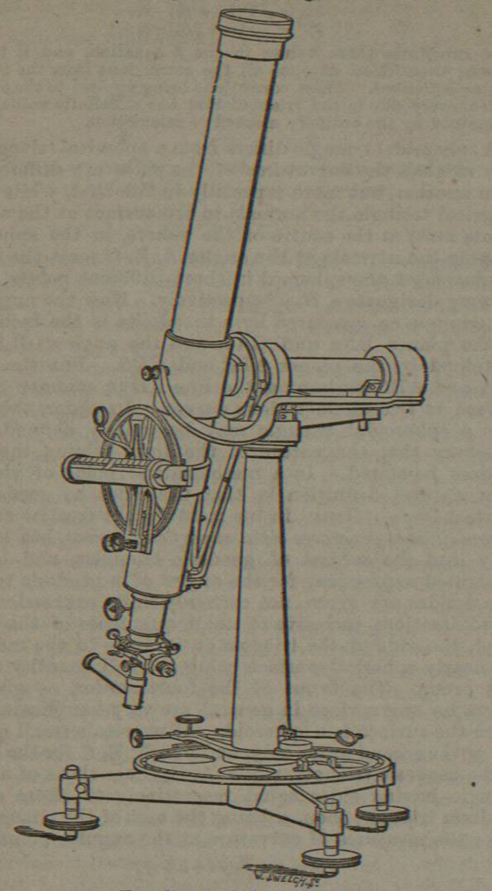
FIG. 4.—Zenith Telescope.

by the micrometer on the centre wire. The micrometer has thus measured the difference of the zenith distances, and the calculation to get the latitude is most simple. Of course it is necessary to read the level, and the observations are not necessarily confined to the centre wire. In fact if n, s be the north and south readings of the level for the south star, n', s' the same for the north star, l the value of one division of the level, m the value of one division of the micrometer, r, r' the refraction corrections, μ, μ' the micrometer readings of the south and north star, the micrometer being supposed to read *from* the zenith, then, supposing the observation made on the centre wire,—

$$\phi = \tfrac{1}{2}(\delta + \delta') + \tfrac{1}{2}(\mu - \mu')m + \tfrac{1}{4}(n + n' - s - s')l + \tfrac{1}{2}(r - r').$$

It is of course of the highest importance that the value m of the screw be well determined. This is done most effectually by observing the vertical movement of a close circumpolar star when at its greatest azimuth.

In a single night with this instrument a very accurate result, say with a probable error of about 0″·3 or 0″·4, could be obtained for latitude from, say, twenty pair of stars; but when the latitude is required to be obtained with the highest possible precision, four or five fine nights are necessary. The weak point of the zenith telescope lies in the circumstance that its requirements prevent the selection of stars whose positions are well fixed; very frequently it is necessary to have the declinations of the stars selected for this instrument specially observed at fixed observatories. The zenith telescope is made in various sizes from 30 to 54 inches in focal length; a 30-inch telescope is sufficient for the highest purposes, and is very portable. The zenith telescope is a particularly pleasant instrument to work with, and an observer has been known (a sergeant of Royal Engineers, on one occasion) to take every star in his list during eleven hours on a stretch, namely, from 6 o'clock P.M. until 5 A.M., and this on a very cold November night on one of the highest points of the Grampians. Observers accustomed to geodetic operations attain considerable powers of endurance. Shortly after the commencement of the observations on one of the hills in the Isle of Skye a storm carried away the wooden houses of the men and left the observatory roofless. Three observatory roofs were subsequently demolished, and for some time the observatory was used without a roof, being filled with snow every night and emptied every morning. Quite different, however, was the experience of the same party when on the top of Ben Nevis, 4406 feet high. For about a fortnight the state of the atmosphere was unusually calm, so much so, that a lighted candle could often be carried between the tents of the men and the observatory, whilst at the foot of the hill the weather was wild and stormy.

Calculation of Triangulation.

The surface of Great Britain and Ireland is uniformly covered by triangulation, of which the sides are of various lengths from 10 to 111 miles. The largest triangle has one angle at Snowdon in Wales, another on Slieve Donard in Ireland, and a third at Scaw Fell in Cumberland; each side is over a hundred miles, and the spherical excess is 64″.

The more ordinary method of triangulation is, however, that of chains of triangles, in the direction of the meridian and perpendicular thereto. The principal triangulations of France, Spain, Austria, and India are so arranged. Oblique chains of triangles are formed in Italy, Sweden, and Norway, also in Germany and Russia, and in the United States. Chains are composed sometimes merely of consecutive plain triangles; sometimes, and more frequently in India, of combinations of triangles forming consecutive polygonal figures. In this method of triangulating, the sides of the triangles are generally from 20 to 30 miles in length—seldom exceeding 40.

The inevitable errors of observation, which are inseparable from all angular as well as other measurements, introduce a great difficulty into the calculation of the sides of a triangulation. Starting from a given base in order to get a required distance, it may generally be obtained in several different ways—that is, by using different sets of triangles. The results will certainly differ one from another, and probably no two will agree. The experience of the computer will then come to his aid, and enable him to say which is the most trustworthy result; but no experience or ability will carry him through a large network of triangles with anything like assurance. The only way to obtain trustworthy results is to employ the method of least squares, an explanation of which will be found in FIGURE OF THE EARTH (vol. vii. p. 605). We cannot here give any illustration of this method as applied to general triangulation, for it is most laborious, even for the simplest cases. We may, however, take the case of a simple chain—commencing with the consideration of a single triangle in which all three angles have been observed.

Suppose that the sum of the observed angles exceeds the proper amount by a small quantity ϵ: it is required to assign proper corrections to the angles, so as to cause this error to disappear. To