reached the summits of the passes leading to Tibet and Yarkand. Our relations with Afghanistan led to further exploration. In 1840 Lieutenants Abbott, Conolly, and Shakespear visited Khiva, and in 1841 Colonels Stoddart and Conolly were murdered at Bokhara, while Eldred Pottinger gallantly defended Herat. Sir Alexander Burnes had previously made his remarkable journey from Cabul to Bokhara and back through Persia, and in 1838 Lieutenant Wood of the Indian Navy discovered the source of the Oxus. Butakoff and other Russian officers, in 1848 and subsequent years, surveyed the sea of Aral, and Middendorf made extensive explorations and discoveries in Siberia. After the Afghan war it was long before any progress was made in the exploration of Central Asia, but through the opening of the treaty ports in China and the navigation of the Yangtsze a considerable increase was made in our knowledge of the Celestial Empire. In 1869 Mr R. B. Shaw and Mr Haywood reached the cities of Yarkand and Kashgar, and Mr Shaw published a most graphic account of the physical aspects of Eastern Turkestan. In the previous year Mr Ney Elias surveyed the Yellow River of China, and afterwards made a journey through a previously unknown portion of western Mongolia; and during 1866–68 the distinguished French geographer Lieutenant Garnier surveyed the course of the great Cambodian river. The Russians, meanwhile, in their advance into Central Asia, had enabled scientific travellers like Fedchenko and others to explore Khokand and the northern part of the Pamir, and the more adventurous Prjewalski made important journeys through Mongolia and to the frontiers of Tibet. Colonels Walker and Montgomerie, of the great Trigonometrical Survey of India, organized a system of training native explorers, who made journeys across the Pamir and to the upper waters of the Oxus, as well as through the previously unknown parts of Tibet. In the last mission of Sir Douglas Forsyth to Kashgar, Captain Trotter of the Trigonometrical Survey of India formed one of the staff. He did much valuable exploring work on the Pamir table-land, and verified the work of Lieutenant Wood at the source of the Oxus. In 1845 MM. Huc and Gabet travelled through Tibet; and in western China the French missionaries have since done useful geographical work. English diplomatic officers have found their way from the south-western provinces of China into Burmah, and Baron Richthofen has made very extensive exploring journeys through the Chinese empire. The most important journey across Arabia in the present century was made by Mr W. Gifford Palgrave in 1863.

Africa Geographical discoverers of the 19th century have had a great work to do in Africa. D'Anville and his successors cleared off all that was uncertain on the map, all that had come from the information given by Duarte Lopez to Pigafetta, and from Leo Africanus, and left a great blank. James Bruce and Mungo Park, Clapperton and Tuckey, merely touched the edges or penetrated in single lines across the vast unknown area. But they have been followed by many others, and now great progress has been made. In 1831 Monteiro and Gamitta were sent by the Portuguese Government, in the footsteps of La Cerda, to the capital of Cazembe; while, in 1849 and 1843–47, Ladislaus Magyar and Graça explored some of the southern affluents of the Congo. Rüppell (1838), Harris (1843), and Dr Beke (1840), Lefebvre and Dillon (1839–43), Ferret and Galinier (1847) improved the existing knowledge of Abyssinia, to which a further important contribution was made by the expeditionary field force sent in 1867–68 to enforce the release of English captives; and progress was made, under the auspices of the Egyptian Government, in exploring the White Nile above Khartoum. In 1849 the discoveries of Denham and Clapperton were followed up by Richardson, Overweg, and Barth, who, like their predecessors, went from Tripoli to Mourzouk, the capital of Fezzan. The two first died in Africa, but Dr Barth returned home with a rich harvest of results. He reached Kouka the capital of Bornou, on Lake Tchad, and in 1851 he visited the south side of that lake, and advanced some distance to the eastward. In 1852 he was at Saccatoo, where Clapperton died, whence he crossed the Niger and eventually reached Timbuctoo. After a stay of some months Dr Barth left Timbuctoo in March 1854, and got back to Tripoli in the end of 1855, being the sole survivor of his party. Dr Vogel, in 1853–57, followed up the discoveries in the direction of Lake Tchad, and fell a victim to science; and the researches of Dr Baikie in 1854 supplemented the work of the Landers in the lower part of the course of the Niger. Dr Baikie also explored 250 miles of the river Chadda or Benue.

On the eastern coast of Africa, the missionaries Rebmann and Krapf ascertained the existence of the snowy peaks of Kenia and Kilimanjaro near the equator, and collected reports touching the equatorial lakes in the interior. This led to the expedition of Captain Burton in 1857, who, accompanied by Captain Speke, landed opposite to Zanzibar, and, advancing westward, discovered Lake Tanganyika. Captain Burton's admirable description of the region between the coast and the great lake he had discovered is one of the most valuable contributions to African descriptive geography. His companion, Captain Speke, made an excursion northwards to the southern coast of a lake which he judged to be a main source of the Nile. In this belief he again set out in 1860 to attempt the achievement of a journey from Bagamoyo, opposite Zanzibar, to the Nile. This great enterprise was crowned with success. Speke traced out the western shore, and visited the northern outlet, of the Victoria Nyanza, the main reservoir of the White Nile. He then marched northwards to Gondokoro and descended the Nile. He had heard of a second great Nile reservoir, which Sir Samuel Baker discovered in 1864, and named the Albert Nyanza. The Bahr el Ghazal and other western feeders of the Nile were visited by Consul Petherick, and explored in 1868–71 by Dr Schweinfurth, whose work ranks with that of Burton as a record of African discovery.

Livin stone The travels of Dr Livingstone in Southern Africa also added considerably to our knowledge of the geography of that continent. In 1848 he started from Cape Colony, visited Lake Ngami in 1849, and eventually reached the Portuguese town of St Paul Loanda in 1855. Thence he marched across the continent, discovering the great falls and a considerable part of the course of the Zambesi. In his second expedition he proceeded up the Zambesi and its tributary the Shire, and discovered the Lake Nyassa. On his third and last expedition he landed on the east coast at the mouth of the Rovuma, and made his way thence to Lake Nyassa. The great traveller then followed in the footsteps of Dr Lacerda and Monteiro to the Cazembe's capital, and thence to Lake Tanganyika. From Ujiji, on that lake, he made his way westward to the river Lualaba (the upper course of the Congo), and returning in a destitute condition to Ujiji, he was there succoured by Mr Stanley. Finally he once more started, and died in the midst of his discoveries among the remoter sources of the Congo. Lieutenant Cameron's expedition in 1873 had for its main object the succour of Livingstone, but the news of the great traveller's death was received at Unyanyembe. Cameron then continued his march by a new route to Ujiji, and completed the survey of the southern half of Lake Tanganyika, discovering the Lukuga outlet. Thence he advanced westward across the Manyuema country to Livingstone's furthest point at Nyangwe, crossed the Lualaba, and traversed the whole width of the African continent

reaching St Paul Loanda on the west coast. Mr Stanley followed in 1874. He circumnavigated and fixed the outline of the Victoria Nyanza, followed Cameron across Lake Tanganyika to Nyangwe, and then descended the great River Congo, discovering its course, and connecting the work of Livingstone with that of Tuckey. Mr Young has since completed the survey of Lake Nyassa; Nachtigal has supplemented the work of Barth and Vogel in the Tchad region; while Duveyrier and other French explorers have examined the region of the Sahara. In the far south the Limpopo basin, and the country intervening between the Limpopo and Zambesi, have been made known to us by St Vincent Erskine and Elton, Carl Mauch and Baines. Thus the extent of the unknown parts of Africa has been rapidly curtailed, while our knowledge has been widened during the last half century.

American surveys. On the American continent scientific progress has been made in the United States and the dominion of Canada, where, within the last half century, boundary commissions and surveys have fixed positions and described previously unknown regions of great extent. **South America.** In South America there are vast unexplored regions to the eastward of the Andes, and in the basins of the great rivers. Sir Robert Schomburgk did much valuable work in Guiana, and explored the delta of the Orinoco in 1841; while Spix and Martius, Poeppig and Castelnau, Maw and Smyth, Herndon and Gibbon, Spruce and Bates, Wallace and Chandless, and others, explored the basin of the Amazon. The labours of Pissis in Chili, of Raimondi and Werthermann in Peru, of Codazzi in Colombia and Venezuela, and of Morales and others in the Argentine Republic, have been most valuable to geographical science. In Patagonia, Fitz Roy and King explored the Santa Cruz river, Cox and Morales have since added to our knowledge, and Commander Musters, R.N., was the first traveller who traversed the whole of Patagonia from south to north, 960 miles of latitude, of which 780 were previously unknown to Europeans.

Australia. The difficulty of exploring the interior of the Australian continent was caused by the scarcity of water, and the immense distances it was necessary to cross without supplies of any kind. Hence the work of exploration has required and called forth high and noble qualities in a degree quite equal to any that have been recorded in any other part of the world. The names of Sturt and Leichhardt, of Eyre and Grey, of Macdouall Stewart and Burke, of Gregory, of Forrest and Warburton, will be handed down as those of intrepid and courageous explorers who laid open the secrets of the interior of Australia.

The Pacific Ocean was explored by numerous expeditions during the 18th and early part of the 19th centuries. Still much remained to be done in the way of verification and more complete survey. From 1826 to 1836 Captain Fitzroy, with the naturalist Darwin, surveyed Magellan's Strait and the west coast of South America; and further important surveys in the Pacific were afterwards executed by Captain Wilkes of the United States Navy, and by Belcher, Kellett, and Denham.

But the great geographical work of the present century must be the extension of discovery in the Arctic and Antarctic regions. Progress has been made in both directions, and in both much remains to be done. It is this polar work which calls forth the highest qualities of an explorer; it is here that the greatest difficulties must be overcome; and it is here that the most valuable scientific results are to be obtained.

Between the years 1830 and 1843 much was done in the Antarctic regions. In 1830–32 Mr John Biscoe, R.N., made a voyage in a brig belonging to Messrs Enderby, and discovered "Enderby Land" and "Graham Land" in 67° S.; and from 1837 to 1840 Dumont d'Urville discovered "La Terre Adèle" and "Coté Clarie," going as far south as 66° 30′. Auckland Island was discovered by Bristow in 1806. In 1839 Balleny, in another vessel belonging to Messrs Enderby, discovered the Balleny Islands in 66° 44′ S., and Sabrina Island in 65° 10′ S. The Antarctic expedition of Sir James Ross sailed from England in 1839. In 1840 Sir James explored Kerguelen Island, and wintered at Hobart Town. He then visited the Auckland Islands, and, crossing the Antarctic Circle, reached the great icy barrier, and discovered Victoria Land, with its lofty volcanoes, in January 1841. He gained the latitude of 78° 4′ S. in 187° E., and established the continuity of the southern continent from 70° to 79° S. In 1841 Ross again wintered at Van Diemen's Land, and in January 1842 crossed the Antarctic circle in 156° 28′ W. He was once more stopped by the great icy barrier in 78° 10′ S., after having penetrated through ice floes of more than 1000 miles in width. Extraordinary dangers were encountered in the ice; many valuable observations were taken, and in 1842 the expedition wintered at the Falkland Islands. In the following season another exploring voyage was made beyond the Antarctic Circle, and in September 1843 this most important expedition returned to England.

On the return of Sir James Ross attention was once more turned to the Arctic regions; and in the spring of 1845 Sir John Franklin's Arctic expedition, consisting of the **Franklin** "Erebus" and "Terror," sailed from Woolwich. His instructions were to make the North-West Passage, but the main object of the expedition was the advancement of science, and to secure it the most accomplished officers in the navy were appointed, as well as the eminent naturalist Dr Goodsir. It is now known that, in the first and second seasons, the expedition was very successful. In 1845 Sir John Franklin made a remarkable run up Wellington Channel to 77° N.; in 1846, proceeding south, he had almost achieved the North-West Passage when his ships were permanently beset to the north of King William Island in 70° 5′ N. and 98° 23′ W. Here the veteran explorer died on June 11, 1847; and all his companions perished in the attempt to reach one of the Hudson's Bay Company's settlements in the summer of 1848. Those among them who reached Cape Herschel, and it is certain that some did reach that point, undoubtedly discovered the North-West Passage.

The expeditions which were sent out in search of Sir John Franklin's ships did much important geographical work; but their principal use was the establishment, through their means, of the true method of extensive Arctic exploration. The grand object of the officers and men employed on this service was the relief of their missing countrymen, and their utmost efforts were devoted to the examination of the largest possible extent of coast-line. Hence the discovery of the modern system of Arctic sledge travelling, the only efficient means of exploring the icy regions around the North Pole. In 1848–49 Sir James Ross discovered the western side of North Somerset, and Sir Leopold M'Clintock served his first apprenticeship in the ice under that veteran explorer. Austin's expedition sailed in 1850, and wintered nearly in the centre of the region discovered by Parry during his first voyage. It was then that M'Clintock developed and put in practice the system of Arctic sledge-travelling which has since achieved such grand results; and Captain Ommanney, M'Clintock, and his colleagues Sherard Osborn, Frederick Mecham, Robert Aldrich, and Vesey Hamilton made what were then unparalleled journeys in various directions. In December 1849, also, Captains Collinson and M'Clure went out to conduct further search by way of Behring Strait. The former made the most remarkable voyage on record along the north coast of America, while M'Clure took his ship between the west coast of Banks Island and the tremendous polar pack, until he was within sight of the position attained by Parry in his first voyage from Baffin's Bay. Here M'Clure's ship was finally iced up in the Bay of God's Mercy. On the return of Austin's expedition, the same ships were again sent out under Captains Belcher and Kellett by Baffin's Bay; and M'Clintock, Osborn, Mecham, and Hamilton, who were once more in the front rank of searchers, surpassed even their former efforts. Mecham discovered a record left by M'Clure on Melville Island which revealed his position, and thus he and his officers and crew, by marching from their abandoned ship to the "Resolute" and returning to England with the expedition of Belcher and Kellett, were enabled to make the North-West Passage partly by ship and partly sledging over the ice. They all returned in 1854. But the concluding search was made by Sir Leopold M'Clintock in the "Fox" from 1857 to 1859, when he found the record on King William Island, and thus discovered the fate of Franklin. These search expeditions added immensely to our knowledge of the Arctic regions, and established the true method of exploration. Sea voyages in the summer season are useful for reconnaissances, but efficient polar work can only be achieved by wintering at a point beyond any previously reached, and sending out extended sledge parties in the spring.

After the return of M'Clintock, England neglected the great work of Arctic exploration for fifteen years; but a deep interest was taken in the discovery of the unknown polar regions by other nations, and numerous efforts to explore them were made in the interval. In 1853–55 Dr Kane, with the American brig "Advance," wintered just within the entrance of Smith Sound, and sent an exploring party for some distance up the east side of the channel; and in 1860–61 Dr Hayes wintered near the same spot, and made a sledge journey up the west side. Ten years afterwards Captain Hall, accompanied by Dr Bessels, a German scientific explorer, sailed in the "Polaris" in August 1871, and succeeded in making his way up the channels leading north from Smith Sound for 250 miles, wintering in 81° 38′ N. Captain Hall unfortunately died in the autumn of 1871, and his comrades returned after suffering great hardships. The "Polaris" was abandoned, but she had attained the highest latitude ever reached by any vessel up to that date. In the direction of Spitzbergen and Novaya Zemlya the Norwegian walrus hunters made many daring voyages. They circumnavigated both those masses of Arctic land, and yearly frequented the hitherto closed Sea of Kara. The Swedes, under the lead of the accomplished and indefatigable Nordenskiöld, have made voyage after voyage to Spitzbergen, and afterwards to the north-east. The first Swedish expedition to Spitzbergen was in 1857, the second in 1861, the third in 1864, the fourth in 1868, consisting of the steamer "Sophia," which reached the highest latitude ever attained by a vessel trying the Spitzbergen route, namely, 81° 42′ N. In 1872 a fifth expedition started, and Nordenskiöld then passed his first winter in the Arctic regions, and gained experience of sledge-travelling in the spring, exploring a large area of North-East Land. Experience also proved that the Spitzbergen route was not one by which large results could be secured, although the scientific researches of the Swedes in Spitzbergen itself were most valuable. In 1875 therefore Professor Nordenskiöld made his first attempt towards the north-east, reaching the mouth of the Yenisei; and in 1876 he made an equally successful voyage in the same direction. The Germans also entered the field of Arctic enterprise. In 1868 Captain Koldewey made a summer voyage to Spitzbergen; and in 1869–70 he went in the "Germania" to the east coast of Greenland, accompanied by Lieutenant Payer, wintered at Pendulum Island, discovered by Clavering in 1823,

Spitzbergen explored.

whence they made a sledge journey to the northward as far as 77°, and explored a deep fjord in about 73° 15′ N. during the navigable season. English yachtsmen, notably Lamont and Leigh Smith, were also in the field; and the latter made important corrections of the charts of North-East Land. But by far the most important and successful voyage in this period was that of Lieutenants Weyprecht and Payer in the Austrian steamer "Tegethoff." Sailing in 1872, they were beset in the ice to the north of Novaya Zemlya during the winter of 1872–73, and were drifted northwards until, on August 31, 1873, they sighted a previously unknown country. It proved to be very extensive, and was named Franz Josef Land. In March 1874 Lieutenant Payer started on an extended sledge journey, in the equipment of which he closely followed M'Clintock's system. He discovered a great extent of coast-line, and attained a latitude of 82° 5′ N. at Cape Fligely. The Austrian explorers were eventually obliged to abandon the "Tegethoff," reaching Norway in September 1874; but their expedition was a great success, and they added an extensive region to the map of the known world.

Austrian expedition.

In England the very important branch of geographical research relating to the Arctic regions was neglected by the Government during this interval of fifteen years, while Americans, Swedes, Norwegians, Germans, Austrians, and English yachtsmen were making praiseworthy efforts with more or less success. The resumption of English Arctic research on an adequate scale is due to the exertions and arguments of Admiral Sherard Osborn from 1865 until 1875. He set forth the valuable results to be obtained, and the means of success. Basing his arguments on long experience, he showed that it was necessary for success that an expedition should follow a coast-line, that it should pass beyond any point previously reached and there winter, and that the work should be completed by extended sledge parties in the spring. At length an expedition was fitted out on these principles, the Smith Sound route was selected, and in May 1875 the "Alert" and "Discovery" sailed from Portsmouth under the command of Captain Nares. As regards the ice navigation the success of the expedition was complete. Captain Nares, in the face of unparalleled difficulties, brought the ships to a point farther north than any vessel of any nation had ever reached before, wintered the "Alert" in 82° 27′ N., and, in the face of still greater difficulties, brought both vessels safely home again. The extended sledge-travelling called forth an amount of heroic devotion to duty, and of resolute perseverance in spite of greater obstacles than had ever been encountered before, which add a proud page to the history of English naval enterprise. The exploring parties were led by Commander Markham and Lieutenants Aldrich and Beaumont. Advancing over the great frozen Polar Sea, Markham reached 83° 20′ 26″ N., the highest latitude ever attained by any human being. He thus won the blue ribbon of Arctic discovery. Aldrich discovered 200 miles of coast to the westward, while Beaumont added to our knowledge of the north coast of Greenland. The results of the Arctic expedition of 1875–76 were the creation of a young generation of experienced Arctic officers, the discovery of 300 miles of new coast-line and of a large section of the Polar Ocean, the attainment of the highest latitude ever reached by man, a year's magnetic and meteorological observations at two stations both further north than any before taken, tidal observations, the examination of the geology of a vast region and the discovery of a fossil forest in 82° N., and large natural history collections representing the fauna and flora of a new region.

English Arctic expedition.

Markham's highest

The return of this memorable expedition again incited our neighbours to further efforts. In the summer of 1878 the Dutch entered the field, and the schooner "William

Barents," under Lieutenants de Bruyne and Koolemans Beynen, made a useful reconnaissance of the Barent's Sea; while Professor Nordenskiöld left Sweden in July 1878, in the well-equipped steamer "Vega," to achieve the North-East Passage. In August he rounded Cape Chelyuskin, the most northern point of the Old World, and reached the mouth of the Lena. But much work remains to be done in the polar regions, in order to complete the connexion between Aldrich's furthest in 1876 and M'Clintock's in 1854, to complete the discovery of the north side of Greenland, to explore the northern bounds of Franz Josef Land, and to discover lands north of Siberia.

There is one great branch of physical geography which has only been effectively studied within the last thirty years, namely, the physical geography of the sea. Mathew Fontaine Maury, by his wind and current charts, by his trade wind, storm, rain, and whale charts, and above all by his charming work *The Physical Geography of the Sea*, gave the first impulse to this study. It was Captain Maury who organized the first deep-sea soundings in the North Atlantic, which up to that time was deemed to be unfathomable; and when his work was published, the illustrious Humboldt declared Maury to be the founder of a new and important science—the meteorology of the sea. He first took charge of the Washington Observatory in 1842; he resigned that post under a deep sense of duty in April 1861, after a career of great usefulness; and he ended a noble and well-spent life in 1872. The investigations into the physical geography of the sea, which were combined into a system by Maury, have since been ably and zealously continued by others, among whom the names of Dr Carpenter, Sir Wyville Thomson, and Professor Mohn of Christiania are pre-eminent. The voyage of the "Challenger" from 1873–1876, under Captains Nares and Thomson, with Sir Wyville Thomson as chief of the scientific staff, was organized with the object of examining and mapping the bottom of the ocean, of describing the fauna of the great depths, of ascertaining the temperatures at various depths, and of solving questions relating to oceanic circulation. The area thus explored in the Atlantic, Antarctic, Pacific, and Indian Oceans is of vast extent, and the researches, ably and zealously conducted, have resulted in an important addition to geographical knowledge.

In this rapid sketch of the history of geographical discovery, the labours of numerous explorers during many generations have been enumerated; but its perusal will show that, notwithstanding all this work, there is much remaining to be done. Vast areas round both poles, and in the interior of Asia, Africa, South America, and New Guinea, are still unknown, even more extensive regions have only been partially explored, and millions of square miles remain to be surveyed, before the work of geographers is complete. (C. R. M.)

II. Mathematical Geography.

All our knowledge of the planet on which we live, whether obtained from the explorations of travellers, the voyages of navigators, or the discoveries of astronomy in modern times, goes to confirm the doctrine held and taught by philosophers in a remote antiquity that the earth is spherical. What is spherical, however, is not the actual surface of the earth, but rather that of the sea produced in imagination to pass through the continents. That the surface of the sea is convex any one may—at a seaside station where there is a high cliff—convince himself, by noting with a telescope at the top of the cliff the exact appearance of a ship in, or slightly beyond, the horizon, and then, immediately after, repeating at the foot of the cliff the same observation on the same ship. By a more precise observation of the sea horizon from a known altitude one may even calculate the radius of the earth.

Let m (fig. 1) be a point on the top of a mountain; hnk a portion of the earth's surface; mnv a line drawn from m towards the centre of the earth; mh a tangent from m to the spherical surface; and ml a horizontal line through m, that is, ml is perpendicular to mv. Then by the mere measure of the angle lmh, or the depression of the sea horizon, one can, knowing mn, calculate very simply the radius of the earth.

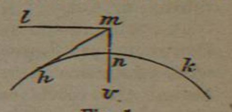

Fig. 1.

Let the height $mn=h$, the angle $lmh=\delta$, and the radius of the earth $=r$; then since the angle subtended at the earth's centre by hn is δ, it is clear that $(h+r)\cos\delta=r$, which gives r in terms of h and δ, known quantities. In fact, since h and δ are both small, $r=\frac{1}{2}h\div\sin^2\frac{1}{2}\delta$. But here we have assumed that the ray of light proceeding from h to m takes a rectilinear course; this is not true however, for the path is curved, its concavity being turned towards the earth—a consequence of terrestrial refraction. From the laws of terrestrial refraction, which have been very minutely studied, we know that the formula last written down should be $r=\cdot 422h\div\sin^2\frac{1}{2}\delta$. Now to take an actual case—the depression of the sea horizon at the top of Ben Nevis is 64′ 48″ (this is the mean of several observations, taken with special precautions for the express purpose of this experimental calculation), and the height of the hill is 4406 feet, or ·8345 of a mile. The formula gives at once $r=3965$ miles, which is remarkably near the truth. But this method is not capable of precision on account of the variableness of terrestrial refraction. In connexion with the appearance of the sea horizon from a height the following formulæ are useful:—h being the height in feet, δ the depression or *dip* of the horizon in minutes, s the distance of the horizon in miles, then

$$\delta=\left(1-\frac{1}{40}\right)\sqrt{h}\,; \qquad s=\frac{4}{3}\sqrt{h}.$$

Thus, for instance, to a spectator on the top of Snowdon, which is 3590 feet in height, the distance of the sea horizon is about 80 miles.

The first great fact in the description of the earth being that it is spherical (or at any rate so nearly so that, were a perfect model of it constructed, no one could, by unaided vision, discover that it is not spherical), the next points to be noted are,—secondly, that the earth rotates uniformly round an axis passing through its centre, and fixed, or very nearly fixed as to direction, in space; and thirdly, that its figure is not spherical but spheroidal, the surface being that found by the revolution of an ellipse round its minor axis, the axis of figure corresponding with the axis of diurnal rotation. The spheroidal figure is a necessary consequence of the rotation. The rotation of the earth once in 24 hours, although made evident by the rising and setting of the heavenly bodies, is rendered perhaps more distinctly visible by Foucault's pendulum experiment. Let a heavy ball be suspended by a fine thread, free from tension, from a fixed point. Let it be drawn aside from the position of equilibrium and then dropped so that it commences to oscillate in a vertical plane passing through the point of suspension. Then a careful observation of the pendulum will show that its plane of oscillation is not fixed, but has a uniform rotation in a direction opposite to that of the earth's rotation. Suppose, for instance, that the pendulum were suspended at the north pole and that it were set oscillating in a plane passing through any one fixed star, then it will continue to oscillate in that same plane notwithstanding the earth's rotation. Consequently, to the observer there the plane of the pendulum's oscillation will appear to rotate through 360° in 24 hours. At the equator, since there is no component of rotation there, the pendulum would continue to move in one and the same plane. At intermediate stations the rate of rotation is easily calculated; and observations confirm the calculations, and have made the earth's rotation actually visible.

The poles of the earth are the points in which the axis of rotation, or of figure, meet the surface; and the equator is the circle in which the surface is intersected by a plane through the earth's centre, perpendicular to the axis of rotation Every point of the equator is therefore equidistant from the poles.

To determine the position of a point in space three co-ordinates or measurements are necessary; they may be three lines, or two lines and one angle, or two angles and one line. Thus, to define the precise position of a point on the earth's surface, we express it by latitude, longitude, and altitude; the first two are angular measures, the third a linear magnitude. namely the height above the surface of the sea.

The line in which the surface of the earth is intersected by a plane through the axis of rotation is called a meridian, and all meridians are evidently similar curves. A line perpendicular to the surface at any point is called a vertical line; it corresponds with the direction of gravity there; being produced outwards, that is, away from the earth's centre it meets the heavens in the *zenith*; and produced downwards it intersects the axis of revolution; it would of course pass through the earth's centre were it a sphere; as it is, it passes *near* the earth's centre.

The angle between the meridian planes of two stations as A and B is called the difference of longitude of A and B, or the longitude of B with reference to A. In British maps the longitudes of all places are expressed with reference to the Royal Observatory of Greenwich.

The latitude of any point is the angle made by the vertical line there with the plane of the equator, or the co-latitude is the angle between the vertical line and the axis of rotation. The surface of the earth being one of revolution, any intersecting plane parallel to the equator cuts it in a circle. If we imagine the vertical lines drawn at any two points, as P and Q, in such a circle it is evident from the symmetry of the surface that these verticals make the same angle with the equator; in other words, the latitudes of all points on this circle are equal. Such circles are called parallels; they intersect meridians at right angles.

If we suppose that at any point Q of the surface the meridian, or a small bit of it, is actually traced on the surface, and also a portion of the parallel through the same point, then these lines, crossing at right angles in Q, mark there the directions which we call north and south, east and west—the meridian lying north and south, the parallel east and west. Planes containing the vertical line at Q are vertical planes there. A vertical plane is defined by its azimuth, which is the angle it makes with the meridian plane; the azimuth at Q of any object (or point) celestial or terrestrial is the angle which the vertical plane passing through the object makes with the meridian. The south meridian is generally taken as the zero of azimuth. The plane touching the surface at Q is the visible horizon there—a plane parallel to this through the centre of the earth being called the rational horizon. The altitude at Q of a heavenly body, as a star, is the angle which the line drawn from Q to the star makes with the plane of the horizon,—the zenith distance of the same star being the angle between its direction and the vertical at Q.

By a degree of the meridian is meant this: if E, F are points on the same meridian such that the directions of their verticals make with each other an angle of one degree—a ninetieth part of a right angle—then the distance between E and F measured along the meridian is a degree of the meridian. As the radius of curvature of an ellipse is variable, increasing from the extremity of the major axis to the extremity of the minor axis, so on the earth's surface a degree of the meridian is found by geodetic measurement to increase from the equator to the poles.

The actual length of a degree of the meridian at the equator is 362746·4 feet; at either pole it is 366479·8 feet. The length of one degree of the equatorial circle is 365231·1 feet.

With regard to the figure of the earth as a whole, the polar radius is 3949·79 miles, and the radius of the equator 3963·3C miles; the difference of these, called the ellipticity, is $\frac{1}{293}$ of the mean radius. A spheroid with these semiaxes is equivalent in volume to a sphere having a radius of 3958·79 miles. Without referring further here to the spheroidal figure, we shall now, having given the precise dimensions, regard the earth as a sphere whose radius is 3959 miles. On such a sphere one degree is 69·09 miles. From the definitions given above it appears that the radius of the parallel which corresponds to all points whose latitude is ϕ is $3959 \cos\phi$; and that one degree of this circle, *i.e.*, one degree of longitude in the latitude ϕ is $69{\cdot}09 \cos\phi$ expressed in miles.

In the representation of the spherical earth (fig. 2) P is the pole, QQ the equator, E, F any two points on the surface, PE*e*, PF*f* the meridians of those points intersecting the equator in *e* and *f* Join EF by a great circle; then in the spherical triangle PEF the angle at P is the difference of longitude of E and F, PE is the co-latitude of E, and PF the co-latitude of F, the latitudes being *e*E and *f*F respectively. The angle at E, being that contained between the meridian there and a vertical plane passing through F, is the azimuth of F (measured in this case from the north), while the angle at F is the azimuth of E. If, then, there be given the latitudes and longitudes of two places, to find their distance apart, and their relative bearings, it becomes necessary to calculate a spherical triangle (PEF) in which two sides and the included angle are given,—the calculation bringing out the third side, which is the required distance, with the adjacent azimuthal angles.

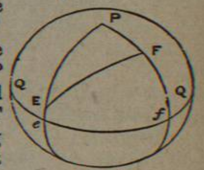

Fig. 2.

The latitudes and longitudes of places on the earth's surface are determined by observations of the stars, of the sun, and of the moon. As the earth rotates, the zenith of any place (not being on the equator) traces out among the stars a small circle having for centre that point in which the axis of rotation meets the heavens. If there were a star at this last point it would be apparently motionless, having always the same altitude and azimuth. The pole star, though very conveniently near the north pole of the heavens, and without perceptible motion to the unaided eye, is in reality moving in a very small circle. The zenith of a point on the equator traces out in the heavens a great circle, namely, the celestial equator.

As the positions of points on the earth are defined with reference to the equator and a certain fixed meridian, so the positions of stars are defined by their angular distance from the celestial equator, called in this case declination, and by their right ascension, which corresponds to terrestrial longitude. Stars which are on the same meridian plane (extended to the heavens) have the same right ascension. Right ascension is expressed in time from 0^h to 24^h. A sidereal clock, going truly, indicates 24^h for every revolution of the earth: at every observatory, the sidereal clock there shows, at each moment, the right ascension of the stars which at that moment are on the meridian. Thus the right ascension of the zenith is the sidereal time.

In the left hand circle of the diagram (fig. 3) two

concentric small circles are drawn such that the sum of their radii is a right angle or 90°. Let the inner circle be that traced among the stars by the zenith of any given place, say Q, then the outer circle encloses all those stars

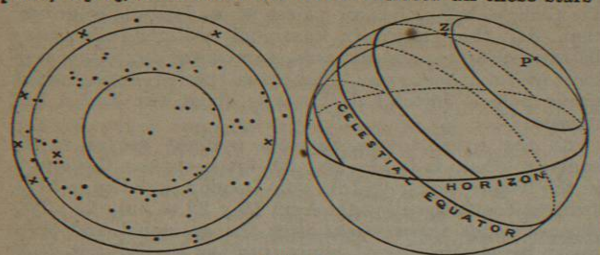

Fig. 3.

which are circumpolar at Q, that is, whose entire course is performed above that horizon; for clearly the zenith distance of none of these can exceed 90° at Q. Or if the outer circle be that described by the zenith of Q, then the inner circle encloses all those stars which are circumpolar at Q. The second circle in the diagram shows the diurnal paths of stars with reference to the horizon.

If we consider in the first circle the changes of distance between any one star and the zenith of Q as the latter traces out its path in the heavens, we see that the distance becomes alternately a maximum and a minimum every twelve hours, namely, when the meridian of Q passes through the star. This is called the star's culmination or meridian transit. It will be clear from an inspection of the figure that, if for instance the star culminate to the south of the zenith, the star's declination plus its zenith distance at culmination is equal to the latitude of the zenith, that is, of Q. A corresponding rule is easily made for a northern transit. Thus the simplest manner of determining the latitude is to measure the zenith distance of a known star at its meridian transit.

The position of the zenith at any moment may be determined by simultaneous observation of the zenith distance of two known stars. For these distances clearly determine a point in the heavens (two points rather, which however need not be confounded) whose declination and right ascension can be computed by spherical trigonometry. Thus, at the same time, are obtained both the time and the latitude. For the success of this method, which is suitable for travellers exploring an unknown country, it is desirable that the stars should differ in azimuth by about a right angle.

If the path of the zenith, that is, the latitude, be known, then clearly a single observation of the zenith distance of a known star, which should be towards the east or west, not towards the north or south, will fix the place or right ascension of the zenith, that is, the sidereal time, at the moment of observation. Here the pole, the zenith, and the star are the angular points of a spherical triangle, of which the three sides are known: the angle at the pole, being computed, is the difference of right ascension of the star and the zenith. Thus the sidereal time is found.

The determination of the difference of longitude of the two stations AB on the earth's surface requires that the true time be kept at each. All that is necessary is a comparison of these times at any instant. For instance, the time at B may, by the transport of chronometers, be brought to A, and thus the difference of the local times be ascertained, or the indications of the clock at A may be conducted by electro-telegraphy to B. The difference of the local times at A and B is the time a star takes to pass from the meridian of the one to that of the other; and this is the difference of longitude which may be converted into angle at the rate of 360° to 24^h.

But the traveller in unknown lands, who seeks to fix astronomically his position, has no telegraph to count on and his expectations for longitude depend chiefly on observations of the moon. In the *Nautical Almanac* are published the angular distances of the moon from certain stars in its path for every three hours of Greenwich time. Therefore, by actually observing the distance of the moon from one of these stars, one can infer the corresponding Greenwich time at the moment of observation. The comparison of this with the local time gives the longitude.

Observations on the sun have shown that it traces out amongst the stars in the course of a year a great circle, inclined to the equator at an angle of $23\frac{1}{2}°$; at midsummer it attains a maximum northern declination of $23\frac{1}{2}°$, and at midwinter a maximum southern declination of the same amount. Hence it is inferred that the earth moves round the sun in a plane, completing one orbital revolution yearly, the axis of the earth's diurnal rotation being inclined to this plane at an angle of $66\frac{1}{2}°$. Upon this angle of inclination depend the seasons, and in great measure the climates of the different portions of the earth's surface.

It is usual to draw on globes and in maps a circle or parallel at the distance of $23\frac{1}{2}°$ from the equator on either side; of these circles the northern is called the Tropic of Cancer, the southern is the Tropic of Capricorn. A circle drawn with a radius of $23\frac{1}{2}°$ from the North Pole as centre is the Arctic Circle; a similar and equal circle round the South Pole is the Antarctic Circle.

When the sun is in the equator—which it crosses from north to south in September, and from south to north in March—it is in the horizon of either pole. When the sun has northern declination, the North Pole is in constant daylight and the South Pole in darkness. When the sun has southern declination the North Pole on the contrary is in constant darkness while the South Pole is illuminated by sunshine. At midsummer in the northern hemisphere the whole region within the Arctic Circle is in constant daylight, and that within the Antarctic Circle is in darkness; at midwinter this state of things is exactly reversed. The portion of the globe lying between the Tropic of Cancer and the Arctic Circle is called the North Temperate Zone; that between the Tropic of Capricorn and the Antarctic Circle is the South Temperate Zone. In the former the sun is always to the south of the zenith; in the latter it is always to the north.

In the Torrid Zone, which lies between the Tropics, the sun, at any given place, passes the meridian to the north of the zenith for part of the year, and to the south for the remainder.

When the sun is to the north of the equator the days are longer than the nights in the northern hemisphere, while in the southern hemisphere the nights are longer than the days; when the sun has southern declination this condition is reversed. As the sun increases his north declination from 0° to $23\frac{1}{2}°$, not only do the days increase in length in the northern hemisphere, but the rays of the sun—in the Temperate and Arctic regions—impinge more perpendicularly on the surface; hence the warmth of summer. Even in summer the rays of the sun in the Arctic regions strike the surface very obliquely; this, combined with the protracted season of darkness, produces excessive cold. Summer in the northern hemisphere is thus contemporaneous with winter in the southern; while winter in the northern hemisphere is simultaneous with summer in the southern.

The length of the day at any place at any season of the year is easily ascertained from the following considerations. Let *ns* (fig. 4) be the axis of rotation, *eq* the equator orthographically projected on a meridian plane, *ab* the parallel of the given place; draw the diameter *fg* making the angle *nog* equal to the sun's declination, which we suppose to be north, then the hemisphere *gnaef* is in sunshine, while the hemisphere *gbqsf* is in darkness. As the earth rotates, a point which is at *a* at midday is carried from *a* towards *b*, which it reaches at midnight; *h* is reached at 6 o'clock P.M. and *k* at sunset. Now if ϕ be the latitude of the place and δ the sun's declination $hk = \sin\phi\tan\delta$; this in the parallel whose radius is $\cos\phi$ corresponds to an angle whose sine is $\tan\phi\tan\delta$. Call this angle η; the time taken to rotate through it is $\frac{1}{15}\eta$; hence the length of the daylight is $12^h + \frac{2}{15}\eta$, and the length of night $12^h - \frac{2}{15}\eta$.

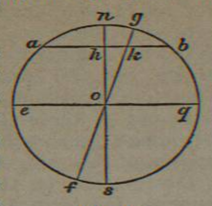

Fig. 4.

Now η vanishes when either ϕ or δ is zero; that is, at the equator the nights and days are equal in length throughout the year; and again when the sun is in the equator, that is, at the equinox, the nights and days are equal in all latitudes. When the sun's declination is equal to the co-latitude, η is a right angle, and the sun does not actually set; this can only happen at places within the polar circle. The longest day at Gibraltar is $14^h\ 27^m$, at Falmouth $16^h\ 11^m$, and in Shetland $18^h\ 14^m$; while in Iceland it is 20^h on the south coast and 24^h on the north. At Washington the longest day is $14^h\ 44^m$, and at Quebec $15^h\ 40^m$.

All this, however, is on the supposition that day ends with sunset; but the length of apparent day is increased by atmospheric refraction and reflection. When the disk of the setting sun first seems to touch the horizon it is in reality wholly below it and is only seen by refraction. After the sun has wholly set at any given place his light still continues to illuminate the upper portion of the atmosphere there, so that, instead of ending abruptly, daylight gradually fades away until the sun is 18° below the horizon.

In a diagram (fig. 5) similar to the last draw *mi* parallel to *gf*, and at a distance from it equal to the sine of 18°; then *gbf* being the hemisphere unenlightened by the direct rays of the sun, *gmif* will represent the twilight zone. A point in the latitude of *a* describing the parallel *ab* loses sight of the sun at *k*, and is in twilight until it reaches the small circle *mi*, when the sun's zenith distance is 108°. The duration of twilight corresponds then to the portion *kl* of *ab*, the angle rotated through being

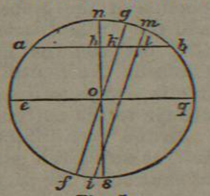

Fig. 5.

$$\sin^{-1}(hl : hb) - \sin^{-1}(hk : hb);$$

this converted into time gives the duration of twilight. Here

$$hk = \sin\phi\tan\delta\ ;\ kl = \sin 18°\sec\delta.$$

At any given latitude the twilight is shortest when the great circle passing through *k* and *l* passes also through the sun. Expressed algebraically, if τ be the duration of the shortest twilight in angular measure and δ the sun's declination at the time, then

$$-\sin\delta = \sin\phi\tan 9°$$
$$\sin\tfrac{1}{2}\tau = \sec\phi\sin 9°.$$

Suppose in the last diagram the sun to be at his greatest northern declination, then $ng = 23\tfrac{1}{2}°$, $gm = 18°$, and $mq = 48\tfrac{1}{2}°$. Hence a place whose latitude is $48\tfrac{1}{2}°$ N. has, at midsummer, twilight lasting from sunset to midnight and continuing from midnight to sunrise, that is, for a few days there is no absolute darkness. A little further south this twilight is interrupted by a short period of darkness. Since $is = 23\tfrac{1}{2}° - 18° = 5\tfrac{1}{2}°$, we see from the diagram that the South Pole is at this time in total darkness, which extends to all places within $5\tfrac{1}{2}°$ of it. When the sun's declination is 9° south, the North Pole is in the centre of the twilight belt; thus all places whose latitude is greater than 81° then move in continual twilight, alternating between clearness and dimness, never attaining either daylight or total darkness. The actual period during which either pole is in total darkness is about two and a half months.

At the equator, the shortest twilight occurs at the equinox, when it is $1^h\ 12^m$; the longest when the sun is in the tropics, being $1^h\ 18^m$. At London, in latitude $51\tfrac{1}{2}°$, twilight continues all night from May 22 to July 21; it is shortest about three weeks after the autumnal and three weeks before the vernal equinox, when its duration is $1^h\ 50^m$. At Washington the shortest twilight (being $1^h\ 33^m$) occurs on the 6th of March and 7th October; at Quebec the shortest is $1^h\ 46^m$, falling on the 3d March and 10th October.

At page 205, fig. 19 is a perspective representation of the earth—of more than a hemisphere, in fact—namely, the segment *mgnafi* in fig. 5. It exhibits all those regions of the earth which at Greenwich apparent noon at midsummer are in sunshine *and* twilight. It is very remarkable how Asia and America, but especially the former, just escape going into darkness.

Construction of Maps.

In the construction of maps, one has to consider how a portion of spherical surface, or a configuration traced on a sphere, can be represented on a plane. If the area to be represented bear a very small ratio to the whole surface of the sphere, the matter is easy: thus, for instance, there is no difficulty in making a map of a parish, for in such cases the curvature of the surface does not make itself evident. If the district is larger and reaches the size of a county, as Yorkshire for instance, then the curvature begins to be sensible, and one requires to consider how it is to be dealt with. The sphere not being a developable surface cannot be opened out into a plane like the cone or cylinder, consequently in a plane representation of configurations on a sphere it is impossible to retain the desired proportions of lines or areas or equality of angles. But though one cannot fulfil all the requirements of the case, we may fulfil some by sacrificing others; that is to say, we may, for instance, have in the representation exact similarity to all very small portions of the original, but at the expense of the areas, which will be quite misrepresented. Or we may retain equality of areas if we give up the idea of similarity. It is therefore usual, excepting in special cases, to steer a middle course, and, by making compromises, endeavour to obtain a representation which shall not offend the eye.

A globe gives a perfect representation of the surface of the earth; but practically, the necessary limits to its size make it impossible to represent in this manner the details of countries. A globe of the ordinary dimensions serves scarcely any other purpose than to convey a clear conception of the earth's surface as a whole, exhibiting the figure, extent, position, and general features of the continents and islands, with the intervening oceans and seas; and for this purpose it is indeed absolutely essential and cannot be replaced by any kind of map.

The construction of a map virtually resolves itself into the drawing of two sets of lines, one set to represent meridians, the other to represent parallels. These being drawn, the filling in of the outlines of countries presents no difficulty. The first and most natural idea that occurs to one as to the manner of drawing the circles of latitude and longitude is to draw them according to the laws of

perspective. But, as Lagrange has remarked, one may regard geographical maps from a more general point of view as representations of the surface of the globe, for which purpose we have but to draw meridians and parallels according to any given law; then any place we have to fix must take that position with reference to these lines that it has on the sphere with reference to the circles of latitude and longitude. Let the law which connects latitude and longitude, ϕ and ω, with the rectangular coordinates x and y in the representation be such that $dx = md\phi + nd\omega$, and $dy = m'd\phi + n'd\omega$. In fig. 6 let the lines intersecting in the parallelogram PQRS be the representations of the meridians *rp*, *sq* and parallels *rs*, *pq* intersecting in the indefinitely small rectangle *pqrs* on the surface of the sphere. The coordinates of P being x and y, while those of p are ϕ and ω the coordinates of the other points will stand thus

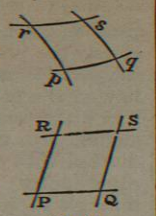

Fig. 6.

q . . .	ϕ	$\omega + d\omega$
r . . .	$\phi + d\phi$	ω
s . . .	$\phi + d\phi$	$\omega + d\omega$
Q . . .	$x + nd\omega$	$y + n'd\omega$
R . . .	$x + md\phi$	$y + m'd\phi$
S . . .	$x + md\phi + nd\omega$	$y + m'd\phi + n'd\omega$.

Thus we easily see that $PR = (m^2 + m'^2)^{\frac{1}{2}} d\phi$; and $PQ = (n^2 + n'^2)^{\frac{1}{2}} d\omega$; also the area of the parallelogram PQRS is equal to $(m'n - mn')d\phi d\omega$. If $90° \pm \psi$ are the angles of the parallelogram, then

$$\tan\psi = \frac{mn + m'n'}{m'n - mn'}.$$

If the lines of latitude and of longitude intersect at right angles, then $mn + m'n' = 0$. Since the length of pr is $= d\phi$, its representation PR is too great in the proportion of $(m^2 + m'^2)^{\frac{1}{2}} : 1$; and pq being in length $\cos\phi\, d\omega$, its representation PQ is too great in the ratio of $(n^2 + n'^2)^{\frac{1}{2}} : \cos\phi$. Hence the condition that the rectangle PQRS is similar to the rectangle *pqrs* is $(m^2 + m'^2)\cos^2\phi = n^2 + n'^2$, together with $mn + m'n' = 0$; or, which is the same, the condition of similarity is expressed by

$$-n' = m\cos\phi\ ;\ n = m'\cos\phi.$$

Since the area of the rectangle *pqrs* is $\cos\phi\, d\phi\, d\omega$, the exaggeration of area in the representation will be expressed by $m'n - mn' : \cos\phi$. Thus when the nature of the lines representing the circles of latitude and longitude is defined we can at once calculate the error or exaggeration of scale at any part of the map, whether measured in the direction of a meridian or of a parallel: and also the misrepresentation of angles.

The lines representing in a map the meridians and parallels on the sphere are constructed either on the principles of true perspective or by artificial systems of developments. The perspective drawings are indeed included as a particular case of development in which, with reference to a certain point selected as the centre of the portion of spherical surface to be represented, all the other points are represented in their true azimuths,—the rectilinear distances from the centre of the drawing being a certain function of the corresponding true distances on the spherical surface. For simplicity we shall first apply this method to the projection or development of parallels and meridians when the pole is the centre. According to what has been said above, the meridians are now straight lines diverging from the pole, dividing the 360° into equal angles; and the parallels are represented by circles having the pole as centre, the radius of the parallel whose co-latitude is u being ρ, a certain function of u. The particular function selected determines the nature of the development.

Let P*pq*, P*rs* (fig. 7) be two contiguous meridians crossed by parallels *rp*, *sq*, and O*p'q'*, O*r's'* the straight lines representing these meridians. If the angle at P is $d\mu$, this also is the value of the angle at O. Let the co-latitude

$$Pp = u,\ Pq = u + du\ ;\ Op' = \rho,\ Oq' = \rho + d\rho,$$

the circular arcs $p'r'$, $q's'$ representing the parallels pr, qs. If the radius of the sphere be unity,

$$p'q' = d\rho\ ;\quad p'r' = \rho d\mu,$$
$$pq = du\ ;\quad vr = \sin u\, d\mu.$$

Put

$$\sigma = \frac{d\rho}{du}\ ;\ \sigma' = \frac{\rho}{\sin u},$$

then $p'q' = \sigma pq$ and $p'r' = \sigma' pr$. That is to say, σ, σ' may be regarded as the relative scales, at co-latitude u, of the representation, σ applying to meridional measurements, σ' to measurements perpendicular to the meridian. A small square situated in co-latitude u, having one side in the direction of the meridian—the length of its side being i—is represented by a rectangle whose sides are $i\sigma$ and $i\sigma'$; its area consequently is $i^2\sigma\sigma'$.

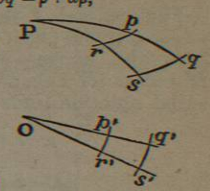

Fig. 7.

If it were possible to make a perfect representation, then we should have $\sigma = 1$, $\sigma' = 1$ throughout. This, however, is impossible. We may make $\sigma = 1$ throughout by taking $\rho = u$. This is known as the *Equidistant Projection*, a very simple and effective method of representation.

Or we may make $\sigma' = 1$ throughout. This gives $\rho = \sin u$, a perspective projection, namely, the *Orthographic*. Or we may require that areas be strictly represented in the development. This will be effected by making $\sigma\sigma' = 1$, or $\rho d\rho = \sin u du$, the integral of which is $\rho = 2\sin\frac{1}{2}u$, which is the *Equivalent Projection* of Lambert, sometimes referred to as *Lorgna's Projection*. In this system there is misrepresentation of form, but no misrepresentation of areas. Or we may require a projection in which all small parts are to be represented in their true forms. For instance, a small square on the spherical surface is to be represented as a small square in the development. This condition will be attained by making $\sigma = \sigma'$, or $\frac{d\rho}{\rho} = \frac{du}{\sin u}$, the integral of which is, c being an arbitrary constant, $\rho = c\tan\frac{1}{2}u$. This, again, is a perspective projection, namely, the *Stereographic*. In this, though all small parts of the surface are represented in their correct shapes, yet, the scale varying from one part of the map to another, the *whole* is not a similar representation of the original. The scale $\sigma = \frac{1}{2}c\sec^2\frac{1}{2}u$, at any point, applies to all directions round that point.

These two last projections are, as it were, at the extremes of the scale; each, perfect in its own way, is in other respects very objectionable. We may avoid both extremes by the following considerations. Although we cannot make $\sigma = 1$ and $\sigma' = 1$, so as to have a perfect picture of the spherical surface, yet considering $\sigma - 1$ and $\sigma' - 1$ as the local errors of the representation, we may make $(\sigma - 1)^2 + (\sigma' - 1)^2$ a minimum over the whole surface to be represented. To effect this we must multiply this expression by the element of surface to which it applies, viz., $\sin u\, du\, d\mu$, and then integrate from the centre to the (circular) limits of the map. Let β be the spherical radius of the segment to be represented, then the total misrepresentation is to be taken as

$$\int_0^\beta \left\{ \left(\frac{d\rho}{du} - 1\right)^2 + \left(\frac{\rho}{\sin u} - 1\right)^2 \right\} \sin u\, du,$$

which is to be made a minimum. Putting $\rho = u + y$, and giving to y only a variation subject to the condition $\delta y = 0$ when $u = 0$, the equations of solution—using the ordinary notation of the calculus of variations—are

$$N - \frac{d(P)}{du} = 0\ ;\ P_\beta = 0,$$

P_β being the value of $2p\sin u$ when $u = \beta$. This gives