and there are certain advantages in considering them at least first by the former method.

We consider first, as the more essential, the relative lengths of the axes, and, secondly, the angular inclination of these.

1. In the cubic system the axes are all equal, and all intersect at right angles. Here is the most perfect simplicity, and the most perfect regularity.

2. In the tetragonal system two only of the axes are equal; but all still intersect at right angles. Here is a departure from simplicity as regards the length of one axis, but no departure as regards the angular inclination.

3. In the right prismatic system none of the axes are equal, but all still intersect at right angles. Here is total loss of regularity in the first particular, but still none in the second.

4. In the oblique prismatic system none of the axes are equal, and only two intersect at right angles. Here there is again a total loss of simplicity in the first particular, and a certain amount of departure from it in the second.

5. In the anorthic system none of the axes are equal, and none of them intersect at right angles,—so that here, as expressed by the name, there is a total departure from regularity in both particulars.

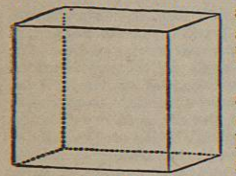

Fig. 26.

6. The hexagonal system is anomalous in relation to this mode of consideration. It is regarded as having four axes, three of which lie in one plane, parallel to the base, and intersect each other at equal angles

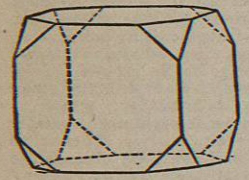

Fig. 27.

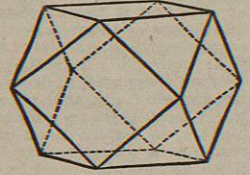

Fig. 28.

(necessarily angles of 60°). The fourth axis intersects these at right angles, and may be longer, shorter, or equal to them. This system is generally considered after the tetragonal system, as having one axis which differs in length from the others, and only one which cuts the others at right angles. By some a rhombohedron is considered as the primary of this system; it then comes to have three axes, all equal, but none intersecting at right angles.

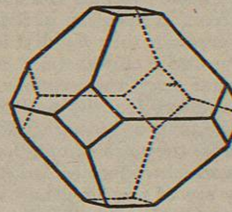

Fig. 29.

In considering these systems, or in describing the form of a crystal, the vertical or erect axis is named the principal axis of the figure, and that axis is chosen as the vertical which is the only one of its kind. In the cubic system there is no such axis, so that any one may be chosen as the vertical.

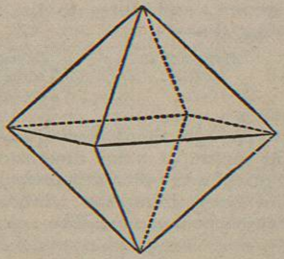

Fig. 30.

It will be convenient, before proceeding to the consideration of the laws of crystallography and the combinations of forms,—especially in view of the terminology that must be employed in illustrating those general aspects of the subject,—to give an outline of one of the six systems here. For this preliminary description the cubic system, as the simplest and most regular, naturally suggests itself as the most suitable.

I. *The Cubic System.*—Here the axes are all equal, and all intersect at right angles. The "cube" (fig. 26), "octahedron" (fig. 30), and "rhombic dodecahedron" (fig. 33), which are here included, are alike in their perfect symmetry; the height, length, and breadth are equal; and their axes are equal, and are rectangular in their intersections.

In the cube (fig. 5) these axes connect the centres of opposite faces; in the octahedron (fig. 15) the apices of opposite solid angles; in the dodecahedron (fig. 18) the apices of opposite acute solid angles. The relation of these forms to each other, and the correspondence in their axes, will be made manifest through a consideration of the transition between the forms. If a cube be projected with the axes in the above position, or if a model of it in any sectile material be employed, and if the eight angles are sliced off evenly, keeping the planes thus formed equally inclined to the original faces, we first obtain the form in fig. 27, then that in fig. 28 and fig. 29, and finally a regular octahedron (fig. 30); and the last disappearing point of each face of the cube is the apex of each solid angle of the octahedron. Hence the axes of the former, being in no way displaced, necessarily connect the apices of the solid angles of the latter. By cutting off as evenly the twelve edges of another cube, the knife being equally inclined to the faces, we have the form in fig. 31, then fig. 32, and finally the rhombic dodecahedron (fig. 33), with the axes of the cube connecting the acute angles of the new form. These forms are thus mutually derivable. Moreover, they are often presented by the same mineral species, as is exemplified in galena, pyrites, and the diamond.

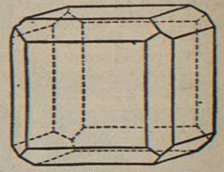

Fig. 31.

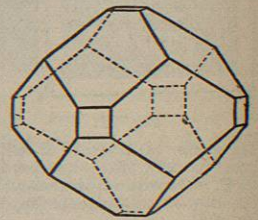

Fig. 32.

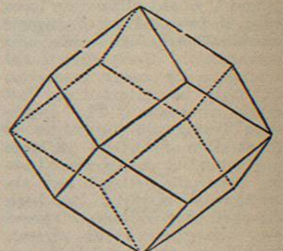

Fig. 33.

The process may be reversed, and the cube made from the octahedron, as will be readily understood from a comparison, in reverse order, of figs. 26 to 30. Or the cube may be similarly derived from the dodecahedron, as seen by inspecting figs. 33, 32, 31, 26. The octahedron also is changed to a rhombic dodecahedron by removing its twelve edges (figs. 34, 35), and continuing the removal till the original faces are obliterated, thus producing the dodecahedron.

It will be observed that throughout all these changes the position of the axes, as determinants of dimensions, need not be altered,—that, in fact, one set of axes has served for all the forms.

The relationships of the principal forms of this system being thus disclosed, the forms themselves have next to be considered.

The cube (fig. 26) is bounded by six equal squares, has twelve edges formed by faces meeting at 90°, and eight solid trigonal angles. The axes are taken as joining the centres of each two opposite faces. Examples are hallite, galena, and fluor.

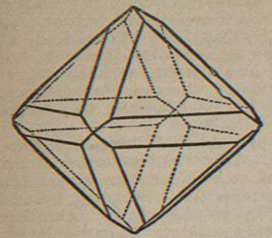

Fig. 34.

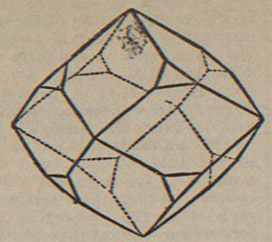

Fig. 35.

The octahedron (fig. 30), bounded by eight equilateral triangles, has twelve equal edges with planes meeting at 109° 28′ 16″, and six tetragonal angles. The principal axes join the opposite solid angles. Examples: magnetite, gold, cuprite.

Dodecahedron. The rhombic dodecahedron (fig. 33) is bounded by twelve equal and similar rhombi, has twenty-four equal edges of 120°, and has six tetragonal and eight trigonal angles. Each of the principal axes joins two opposite tetragonal angles. Examples: garnet, cuprite, blende.

The tetrakishexahedrons (figs. 36, 37, 38, varieties of icositetrahedron) are bounded by twenty-four isosceles triangles, placed so as to form four-sided pyramids on the faces of the cube, arranged in six groups of four each. They have twelve longer edges, which correspond to those of the primitive or inscribed cube, and twenty-four shorter edges placed over each of its faces. The angles are eight hexagonal and six tetragonal, the latter joined two and two by the principal axes. Examples: fluorite, gold. This form varies much in general aspect. The four-sided pyramid which rests on the edges of each face of the cube may be so low as almost to fall into it (fig. 36); or it may rise so high that each side forms a level surface with that which is adjacent to it upon the nearest cubic face (fig. 38). In the latter case the form has become the rhombic dodecahedron; so that the more or less acute varieties of the form are but stages of a passage of the cube into the latter figure, through an increasing accretion of matter in the lines of the axes of the cube. This is termed a "transition by increment."

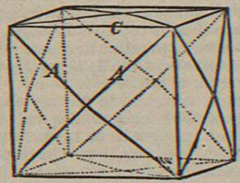

Fig. 36.

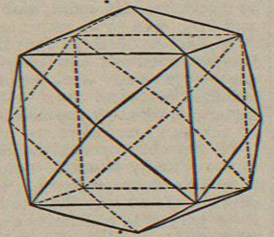

Fig. 37.

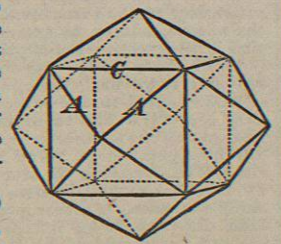

Fig. 38.

The triakisoctahedrons, fig. 39 (variety of icositetrahedron, fig. 40), are bounded by twenty-four isosceles triangles, in eight groups of three, arranged as pyramids on the edges of the faces of the octahedron. Like the previous form they vary in general aspect, the variation here being from the octahedron on one side to the rhombic dodecahedron on the other; while the increased accretion here is in the direction of lines joining the centres of the faces of the octahedron or the solid angles of the cube. The passage of the forms is similar to that illustrated in the last-con-

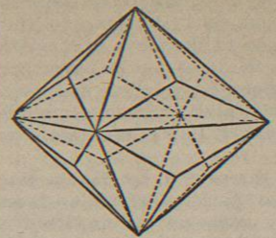

Fig. 39.

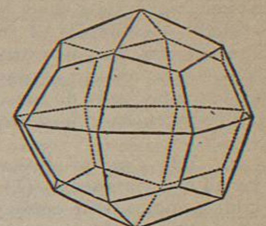

Fig. 40.

sidered form. The edges are twelve longer, corresponding with those of the inscribed octahedron, and twenty-four shorter, three and three over each of the faces. The angles are eight trigonal and six ditetragonal (formed by eight faces), the latter angles joined two and two by the principal axes. Examples: galena, diamond.

The icositetrahedrons (fig. 40) are bounded by twenty-four deltoids. This form varies from the octahedron to the cube, sometimes approaching the former and sometimes the latter in general aspect. A four-sided pyramid rests on the angles of the faces of the cube. When increased accretion takes place along the cubic axes, an octahedron results. When it is along lines joining the solid angles of the cube, that form itself results. The edges are twenty-four longer and twenty-four shorter. The solid angles are six tetragonal joined by the principal axes, eight trigonal, and twelve rhombic or tetragonal with unequal angles. Examples: analcime, garnet. Icositetrahedron.

The hexakisoctahedrons (fig. 41), bounded by forty-eight scalene triangles, vary much in general aspect, approaching more or less to all the preceding forms, into all of which they may pass; but most frequently they have the faces arranged either in six groups of eight on the faces of the cube, or eight of six on the faces of the octahedron, or twelve of four on the faces of the dodecahedron. There are twenty-four long edges, often corresponding to those of the rhombic dodecahedron or bisecting the long diagonal of the trapezohedron, twenty-four intermediate edges lying in pairs over each edge of the inscribed octahedron, and twenty-four short edges in pairs over the edges of the inscribed cube. There are six ditetragonal angles joined by the principal axes, eight hexagonal, and twelve rhombic angles. Examples: diamond, fluorite. Hexakisoctahedron.

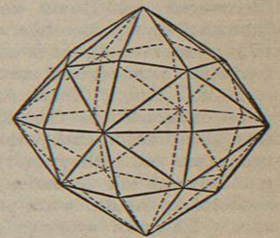

Fig. 41.

General Laws of Crystallography.—The seven forms of crystals now described are related to each other in the most intimate manner. This will appear more distinctly from the account which is to follow of the mode of derivation of the forms, with which is conjoined an explanation of the crystallographic signs or symbols by which they are designated. These symbols were introduced by Naumann, in the belief that they not only mark the forms in a greatly abbreviated manner, but also exhibit the relations of the forms and combinations in a way which words could hardly accomplish. In order to follow out this derivation of forms, it is necessary to state briefly the following laws, which have been established in crystallography. It is to be remembered that these laws apply, not merely to the cubic system just described, but to all the systems.

1. *The Law of the Invariability of the Angles of Crystals*, which was established by Romé de l'Isle, may be thus stated :—the angles of inclination of the faces of a crystal are constant, however unequally the faces may be developed. The corresponding angles of different crystalline specimens of the same body do not, however, always absolutely agree. Differences have been found, amounting sometimes even to 10′.

2. *The Law of Symmetry*, discovered by Haüy, may be thus expressed :—(1) similar parts of crystals—faces, edges, angles, and consequently axes—are all modified in the same manner, and dissimilar parts are modified separately or differently; (2) the modifications produce the same effect on the faces or edges which form the modified part, when they are equal; when they are not equal, they produce a different effect. That is, if an edge be truncated or bevelled, every similar edge will be similarly truncated or bevelled; if an angle be truncated or acuminated, every similar angle will be similarly truncated or acuminated; and consequently every similar axis will be equally affected by the modifications. Thus the cube has eight similar angles and twelve similar edges. In the physical production of the cube, if one of the angles or edges be modified, all will be similarly modified. This, which is the most important law of crystallography, is, however, subject to an exception which was fully formulated by Weiss. The law was—all the similar parts of crystals, faces, edges, angles, and consequently axes, are modified at the same time and in the same manner; the forms resulting from this law are termed "holohedral." The exception is that half of them or one-fourth of them only may be similarly modified. When only half of the similar parts are modified, we get the "hemihedral" forms; when one-fourth only are modified, which occurs only rarely, we get "tetartohedral" forms.

3. *The Law of the Parallelism of the Faces of a Crystal*, discovered by Romé de l'Isle, may be expressed as follows :—every face of a crystal has a similar face parallel to it; or every figure is bounded by pairs of parallel faces (with the exception of certain hemihedral forms).

4. *The Law of Zones*, first established by Weiss, may be thus enunciated :—the lines in which several faces of a crystal intersect each other (or would do so if they were produced until they met) frequently form a system of parallels. Such a series of faces is termed a "zone." Sometimes the zones are parallel to one of the symmetrical axes. Thus, in every prism, the faces of the prism constitute a zone which encircles the axis of the prism. Faces may be in a zone although they do not actually intersect on the form.

5. *The Law of the Rationality of the Parameters* of the faces of crystalline series, first indicated by Malus, is that the position of planes may be assigned by numbers bearing some simple ratio to the relative lengths of the axes of the crystal. This law was the outcome of investigations into the relationship of forms glanced at in commencing the consideration of the cubic system, and was arrived at through the study of the mode of derivation of forms.

The derivation of forms is that process by which, from one form chosen for the purpose, and considered as the type,—the fundamental or primary form,—all the other forms of a system may be produced, according to fixed principles or general laws. In order to understand this process or method of derivation, it must be noted that the position of any plane is fixed when the position of any three points in it, not all in one straight line, is known. To determine the position, therefore, of the face of a crystal, it is only necessary to know the distance of three points in it from the centre of the crystal, which is the point in which the axes intersect each other. As the planes of all crystals are referred to their axes, the points in which the face (or its supposed extension) meets the three axes of the crystal are chosen. and the portions of the axes between these points and the centre are named parameters of the face: and the position of the face is sufficiently known when the relative length or proportion of these parameters is ascertained. When the position of one face of a simple form is thus fixed or described, all the other faces of the form are in like manner fixed in accordance with law 2, since they are all equal and similar, and have equal parameters—that is, intersect the axis in the same proportions. Hence the expression which marks or describes one face marks and describes the whole figure, with all its faces.

The octahedron is adopted as the primary or fundamental form of the cubic system, and distinguished by the first letter of the name, O. Its faces cut the half-axes at equal distances from the centre; so that these semiaxes, the parameters of the faces, have to each other the proportion $1:1:1$. In order to derive the other forms from the octahedron, the following construction is employed.

Proportion of parameters express by symbols Notation of Naumann.

Suppose a plane to be laid down perpendicular to one axis, and consequently parallel to the two other axes (or to cut them at an infinite distance, expressed by ∞, the sign of infinity); then the hexahedron or cube is produced, designated by the crystallographic sign $\infty O \infty$,—expressing the proportion of the parameters of its faces, or $\infty : 1 : \infty$. If a plane is supposed placed on each edge, parallel to one axis, and cutting the two other axes at equal distances, the resulting figure is the rhombic dodecahedron, designated by the sign ∞O, the proportion of the parameters of its faces being $\infty : 1 : 1$. The triakisoctahedron arises when, on each edge of the octahedron, planes are placed cutting the axis not belonging to that edge at a distance from the centre m, which is a rational number greater than 1. The proportion of its parameters is therefore $m:1:1$, and its sign mO; the most common varieties are $\frac{3}{2}O$, $2O$, and $3O$, seen in diamond and fluorite. When, on the other hand, from a similar distance m in each two semiaxes prolonged a plane is drawn to the other semiaxis, or to each angle, an icositetrahedron is formed; the parameters of its faces have consequently the proportion $m:m:1$, and its sign is mOm; the most common varieties are $2O2$ and $3O3$,—the former very frequent in leucite, analcime, and garnet, the latter in gold and amalgam. When, again, planes are drawn from each angle, or the end of one semiaxis of the octahedron, parallel to a second axis, and cutting the third at a distance n, greater than 1, then the tetrakishexahedron is formed; the parameter of its faces is $\infty : n : 1$; its sign is ∞On; and the most common varieties in nature are $\infty O\frac{3}{2}$, $\infty O2$, and $\infty O3$. Finally, if in each semiaxis of the octahedron two distances m and n be taken, each greater than 1, and m also greater than n, and planes be drawn from each angle to these points, so that the two planes lying over each edge cut the second semiaxis belonging to that edge at the smaller distance n, and the third axis at the greater distance m, then the hexakisoctahedron is produced; the parameters are $m:n:1$, its sign mOn, and the most common varieties $3O\frac{3}{2}$, $4O2$, and $5O\frac{5}{3}$, seen in diamond and fluorite.

It must be observed that the numbers in the above signs refer to the parameters of the faces,—not to the axes of the crystal, which are always equal. One parameter also has always been, in the above, assumed $=1$, and then, either one only of the two other parameters, marked by the number before O, or both of them, marked by the numbers before and after O, have been changed.

In the above consideration of the mode of derivation of these forms actually found in nature, which belong to the cubic system, it will be observed (though the illustrations were limited) that the value of m and n in these indicated, by the precision of the proportions $\frac{3}{2}$, 2, or 3, a definite numerical relationship. This at once led up to the extended observations which established the law above stated of proportionality in the modification of crystals, or the rationality of the parameters, which gives a mathematical basis to the science, adding to symmetry of arrangement a numerical relation in the position of the planes.

To illustrate this in a general form (and not merely with special reference to the mode of notation or expression of Naumann, which is that adopted in the subsequent descriptions), let AOA′, BOB′, COC′ (fig. 42) be the three axes of a crystal, drawn in perspective, and cutting one another in the centre O. The semiaxes OA, OB, OC are three parameters. Now in the line OA take $Oa_2 = \frac{1}{2}OA$, and $Oa_3 = \frac{1}{3}OA$,—making as many points as may be necessary between OA, *rational fractions* of OA. Subdivide OB and OC in a similar manner. Further produce OA, OB, OC to A*o*, B*o*, C*o*, in each direction to an infinite distance, or to a supposed infinite distance, as expressed by the arrow-head; and suppose these extended axes to be divided in a manner similar to the subdivisions of the parameters, by rational multiples of OA, OB, and OC. *All the planes of a crystal will be parallel to one or other of the planes which pass through three of the points thus determined.*

First, in order to apprehend the relationship of faces to these axes, or to the half axes,—the parameters of the faces,—let us suppose one

plane of a crystal to be so situated as to cut the three parameters OA, OB, OC at their extremities A, B, C, which it must be remembered are points equidistant from the centre; or let it be supposed that a glass plate rests upon three intersecting wires at such points. It is evident that such a plane or plate will have a definite inclination or slope. Suppose further a second plane or plate to exist, which cuts the three semiaxes in the points a_2, b_2, c_2, which have been measured off (along with a_1, b_1, c_1) as equidistant from O. It will be evident that such a plane, though smaller, will be parallel to the first, seeing that, like it, it cuts the three parameters at equal distances from O.

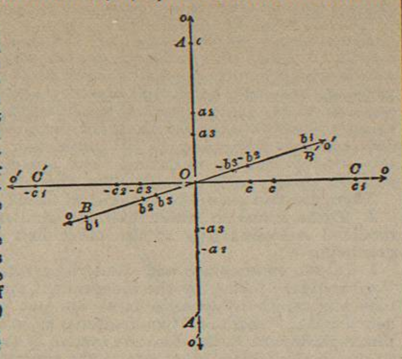

Fig. 42.

A little consideration will show that, whatever the absolute distances from the centre may be, so long as the supporting subdivisions are equal, no new slope of the glass plates or planes is possible; planes so situated must be parallel and similar. Any sign which may be adopted to express the slope of one of such planes must be applicable to all. A plane, however, cutting the points a_1, b_1, c_2 will have quite a different slope.

Let us now suppose a plane to cut a different set of the semiaxes, namely, OA′, OB′, OC′, in the points $-a_1$, $-b_1$, $-c_1$. Such a plane would be parallel to one cutting the points $-a_2$, $-b_2$, $-c_2$, and also to the set of planes first described, but on the opposite side of the centre of the crystal. If again, however, we had a plane cutting the semiaxes OA′ and OB′ in $-a_1$, $-b_1$, but the semiaxis OC′ in the point $-c_2$, it is clear that the slope of this plane would be quite different from that of the planes just described, but it would be parallel to the plane cutting the points a_1, b_1, c_2. This slope, like the other, evidently depends, not on the absolute lengths of the portions of OA′, OB′, OC′ cut off, but upon their proportions or ratios; and such is the case with all the planes which are referred to the same axes.

As there are three axes, and each or all of them may be cut at any points and at any ratios, it is evident that the number of planes which is possible is infinite; and it must be also evident that the inclinations of all are fixed or determinate if we know the ratios. While, however, the possible number of planes is infinite, the actual number occurring among minerals is either small or moderate, in virtue of the fact that the ratios of subdivision of the axes are always simple, and not numerous.

Naumann's symbols for the notation or individualizing of planes have been glanced at. A simpler method is that of employing as indices the denominators (if simple fractions) of the fractional parts of the axis cut. Thus 111 is used for any plane parallel to that cutting the axes in a_1, b_1, c_1; 122 for those parallel to a_1, b_2, c_2; 313 for a_3, b_1, c_3; and so on.

When any of the points referred to have negative signs, the corresponding indices have negative signs placed over them. Thus $\bar{1}22$ is the index for a plane parallel to $a'_1 b_2 c_2$. $\bar{1}0\bar{3}$ is the index of the plane a'_1, $b\infty$, c'_3. ∞ here indicates infinity; that is, the plane never would cut the axis B however far it were extended; in other words, it is parallel to it. The necessity for elongating the axes is brought about by the occurrence of highly acuminating planes, which in many cases would not meet the axes at all unless these were prolonged.

If the axes are unequal, as in the trimetric forms, then the ratio is of the same character, except that the relative lengths of the axes come into consideration; but here, as in the regular system, irrational values cannot occur, and in even the most complex crystals they seldom exceed seven, either as aliquot parts or multiples.

It will thus be seen that in crystals there is no haphazard scattering of faces, but a complete subserviency to law, a law which may be said to be the linear equivalent to the law of multiple proportions by weight, and Gay Lussac's law of multiple proportions in combination by volume.

In abbreviation of all the systematic modes of notation, letters of the Latin and Greek alphabets are frequently employed in a more or less arbitrary manner, and with advantage in the case of highly complex forms.

6. *The Law of Symmetry of Crystalline Combination* is the consequence of the law of symmetry and the law of the rationality of the parameters, and has been partially stated in enunciating these laws. It is thus expressed :—(1) a substance can only crystallize in forms, whether simple or compound, which have the same relative symmetry, that is, belong to the same crystalline system, and the parameters of the faces of which bear a simple relation to each other, that is, belong to the same axis; (2) a form cannot be modified by faces belonging to a different system, or a different series.

Certain exceptions to the first part of this law occur. The element carbon occurs as the diamond, which is cubic, and as graphite, which is hexagonal. Sulphur occurs near volcanoes in needle crystals belonging to the oblique prismatic system, and also in caves (deposited apparently from solution) in crystals belonging to the right prismatic system. Titanic acid is tetragonal in rutile, and right prismatic in brookite. Carbonate of lime is hexagonal in calcite, and prismatic in aragonite. These are probably only apparent exceptions. The elementary substances which go to form them occur in different allotropic states, with different amounts of specific heat; and it is probable that in these different states they go to form the above modifications, which are therefore, in every respect, except in their chemical composition, different mineral bodies. The physical differences between diamond and graphite may suffice as an illustration. The diamond is transparent, colourless, brittle, and extremely hard; graphite is opaque, black, tough, and so soft as to be utilized as a lubricant.

Spheres of Projection.—The foregoing scheme for the development of the relation which subsists between faces of crystals and their axes affords but slight aid in displaying the position of the faces, or their mutual relationships. The delineation even of a considerable series of crystal forms does not indeed go far in effecting this,—on account, first, of very unequal development in the size of the faces of crystals, and, secondly, on account of the *habit* of development of these faces not only differing largely, but being special to certain localities,—as in the entire absence of some faces, and in the preponderance of others.

Maps of the whole domain occupied by the forms of each mineral have been happily projected for such display. The projection is laid down as on a globe, in accordance with stereographic projection. and admitting of calculation according to the laws of spherical trigonometry. These globe maps are called "spheres of projection." The centre O is the common centre of the crystal and of the sphere in which the axes intersect. The three axes will of course meet the circumference of the sphere in six points, called the "poles of the axes." From the centre radii are supposed to be drawn. meeting each plane perpendicularly. It is evident that such radii will have fixed inclinations to each other. They are called "normals" to the planes, and the points in which when produced they meet the circumference of the sphere of projection are called the "poles" of the corresponding faces. A face and its pole thus call for only one symbol. The angle included by any two normals is the supplement of that included by the two corresponding faces.

It is thus easy to determine the angles of any two normals when that of the corresponding faces is known, or *vice versa*. Thus, if the angle between two faces is 125°, that of the normals will be 55°. The spheres of projection are specially adapted to enable us to avail ourselves of the aid to calculation afforded by the forenoted fact that sets of faces lie parallel to each other, forming zones; for, when projected on such a sphere, the normals of the parallel faces will all lie in one plane, and the poles, all cutting its surface in the direction of one line, may be connected, and so form a great circle on the sphere. This is called the "zone circle." A line drawn through the centre of the zone plane, cutting it at right angles, is the "zone axis"; it is parallel to all the faces, and intersections of the faces (if they are extended enough to intersect), of the zone. A face may be common to two or more zones; its normals will then coincide with the intersections of the several zone planes.

In the absence of actual spheres upon which to detail the facts which go to form the "sphere of projection" of each substance, the hemisphere is represented on a plane surface. This has of necessity the disadvantage, except as regards the circumferential zone, of introducing spherical distance-distortion—foreshortening of all parts lying near the circumference; but the eye soon gets accustomed to this. Fig. 43 presents the principal zones of the cubic system, and

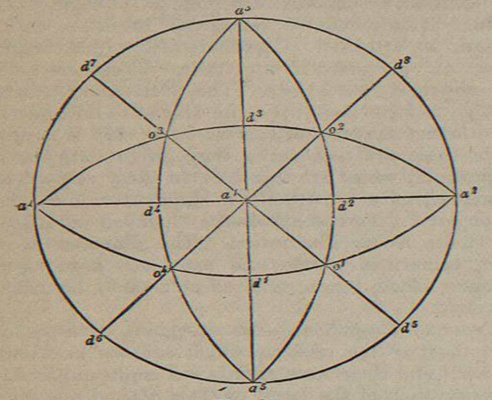

Fig. 43.

shows the position of the poles of the faces of the cube, the octahedron, and the rhombic dodecahedron. o_1, o_2, o_3, &c., are the poles of the octahedral faces; a_1, a_2, a_3, &c., those of the faces of the cube; and d_1, d_2, d_3, &c., those of the rhombic dodecahedron. It will be observed that the faces of the cube fall into the zone circles of the octahedron and dodecahedron, while those of the octahedron fall into those

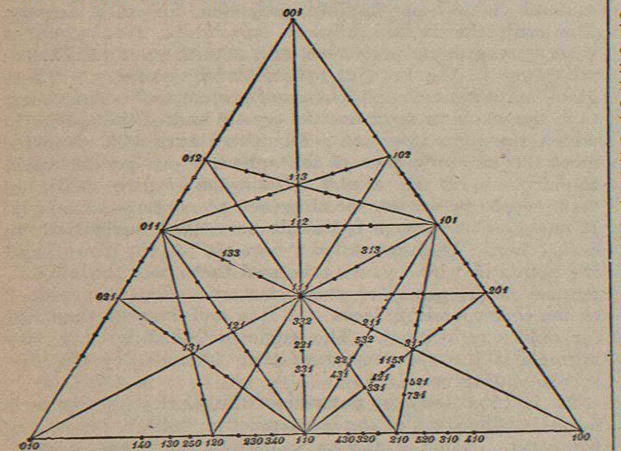

Fig. 44.—Principal Poles of Cubic System in Octant of Sphere.

of the rhombic dodecahedron. Considering this as a delineation of a globe, these zone circles come to represent latitude and longitude; and, as almost all the faces in this system fall into some zone circle, it is clear that the latitude and longitude of all normals may be readily laid down, and their relations at once determined by spherical trigonometry. Fig. 44 shows the arrangement of the poles of all the forms belonging to the cubic system noticed above, or referred to in the present article,—delineated on an octant of the sphere of projection. It displays the perfect regularity of the system.

Hemihedral forms.

Hemihedral and Tetartohedral Forms.—The exception to the second law (that of symmetry), which was formulated by Weiss, was to the effect that one-half or even one-fourth only of the faces which go to form a holohedral crystal may be present. When but one-half of the faces present themselves, the form is termed hemihedral; when only one-fourth, it is tetartohedral. These restrained developments have now to be considered. In hemihedral forms the development is restrained, but symmetry is not deranged; half the similar parts are still alike, though unlike the other half.

There are two classes of hemihedral forms:—

I. Those forms in which half the similar angles or edges are modified independently of the other half ("hemi-holohedral"), producing—

1. In the monometric and dimetric systems "tetrahedral" and "sphenoidal" forms, by the independent replacement of the alternate angles; their opposite faces are not parallel, and they are hence called "inclined" hemihedrons; as in chalcopyrite, boracite.[1] The replacement in the dimetric system of two opposite basal edges at one base and the other two at the opposite base is of the same kind; as in edingtonite.

2. In the trimetric system "monoclinic" forms, by the replacement of half the similar parts of one base and the diagonally opposite of the other, unlike the other half; as in datholite, humite.

3. In the trimetric and hexagonal systems "hemimorphic" forms, by independent replacements at the opposite extremities of the crystal; as in topaz, calamine, tourmaline.

4. In the rhombohedral system, by the replacements of the alternate basal edges or angles of the rhombohedron, forms usually called "tetartohedral" or quarter forms, on the ground that mathematically the rhombohedron is a hemihedral form derived from the hexagonal prism, which is the type of the hexagonal system. Rock crystal is usually developed according to this law.

II. Those forms in which all the similar angles or edges are modified, but by half the full or normal number of planes ("holo-hemihedral"), producing—

1. In the monometric system "pyritohedral" forms, by a replacement of the edges or angles; as in pyrites. Such forms have opposite faces parallel, and are often called parallel hemihedrons.

2. In the dimetric system "pyramidal" and "scalenoidal" forms, by a replacement of the eight solid angles of the primary prism, according to two methods.

3. In the hexagonal system "pyramoidal" and "gyroidal" forms, by a replacement of the solid angles of the hexagonal prism, or of the six lateral angles of the rhombohedron, according to two methods; as in quartz and apatite.

The above illustrations show that hemihedrism is not only divided into two classes, but is of various kinds, and these have been systematized as follows:—"holomorphic," in which the occurring planes pertain equally to the upper and lower (or opposite) ranges of sectants, as in ordinary hemihedrons; and (2) "hemimorphic," in which each set of planes pertains to either the upper or the lower range, but not to both. As to the relative position of the sectants which contain the planes, the forms may be vertically direct, as in baryte; vertically alternate, as in the tetrahedron, the rhombohedron, and the plagihedral faces of quartz; and vertically oblique, as in many forms of chondrodite.

Hemimorphic forms.

In hemimorphic forms symmetry is deranged; the crystals are bounded at the opposite ends of their main axes by faces belonging to distinct forms or modifications,—always, however, of the same system; hence only the upper or the under half of each crystal can be regarded as complete, as regards the form there seen; and so for each end it is half formed.

Fig. 45 represents a crystal of tourmaline, which is bounded on the upper end by the planes of the rhombohedrons R (P) and $-2R$ (o), and on the lower end by the basal pinacoid (k'). In fig. 46 of smithsonite the upper extremity shows the base k, two brachydomes o and p, and two macrodomes m and l;

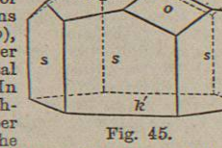

Fig. 45.

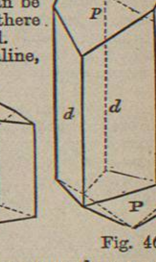

Fig. 46.

[1] As the parts of either half are alternate, there still results a symmetrical solid. As either one or other half may be the one thus modified, there may result two such symmetric solids, which stand in an inverse position to one another. When the modifications affect the upper right-hand solid angle, the resulting form is called +; when the upper left-hand angle it is −

whilst on the lower end it is bounded by the faces P of the primary alone.

It has been found that all hemimorphic crystals become electrically polar when heated, that is, exhibit opposite kinds of electricity at opposite ends of the crystal. The subject will be more fully considered under the electricity of minerals.

The hemihedral forms of the cubic system are the following:—

1. The tetrahedron (fig. 47), hemihedral of the octahedron, is bounded by four equilateral triangles. It has six equal edges with faces meeting at 70° 32′, and four trigonal angles. The principal axes join the middle points of each two opposite edges. Examples: fahlore, boracite, and helvine.

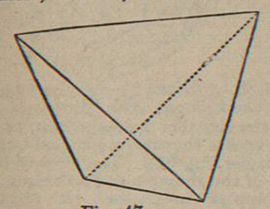
Fig. 47.

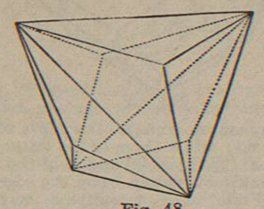
Fig. 48.

2. The trigonal dodecahedrons (fig. 48), hemihedral of the icositetrahedron, are bounded by twelve isosceles triangles, and vary in general form from the tetrahedron to the cube. There are six longer edges corresponding to those of the inscribed tetrahedron, and twelve shorter, placed three and three over each of its faces, and four hexagonal and four trigonal angles. Example: tetrahedrite.

3. The deltoid dodecahedrons (fig. 49), hemihedral of the triakisoctahedron, are bounded by twelve deltoids, and vary in general form from the tetrahedron on the one hand to the rhombic dodecahedron on the other. They have twelve longer edges lying in pairs over the edges of the inscribed tetrahedron, and twelve shorter edges, three and three over each of its faces. There are six tetragonal (rhombic), four acute trigonal, and four obtuse trigonal angles. The principal axes join two and two, opposite rhombic angles. Example: tetrahedrite.

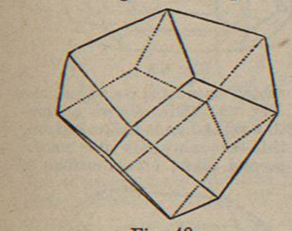
Fig. 49.

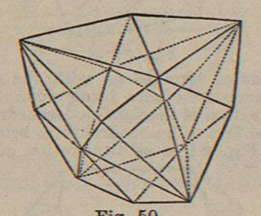
Fig. 50.

4. The hexakistetrahedrons (fig. 50), hemihedral of the hexakisoctahedron, are bounded by twenty-four scalene triangles, and most commonly have their faces grouped in four systems of six each. The edges are twelve shorter and twelve longer, lying in groups of three over each face of the inscribed tetrahedron, and twelve intermediate in pairs over its edges. The angles are six rhombic, joined in pairs by the principal axes, and four acuter and four obtuser hexagonal angles. Example: diamond.

In these forms, often named "tetrahedral," the faces are oblique to each other. Their derivation and signs are as follows. The tetrahedron arises when four alternate faces of the octahedron, two opposite above and two intermediate below, are enlarged so as to obliterate the other four; and its sign is hence $\frac{O}{2}$. But, as either four faces may be thus enlarged or obliterated, two tetrahedrons can be formed, similar in all respects except in position, and together making up the octahedron. These are distinguished by the signs + and −, added to the above symbol, but only the latter in general expressed, thus $-\frac{O}{2}$. In all hemihedric systems two forms similarly related occur, which may thus be named complementary forms. The trigonal dodecahedron is derived from the icositetrahedron by the expansion of the alternate trigonal groups of faces. Its sign is $\frac{mOm}{2}$, the most common variety being $\frac{2O2}{2}$. The deltoid dodecahedron is in like manner the result of the increase of the alternate trigonal groups of faces of the triakisoctahedron, and its sign is $\frac{mO}{2}$. Lastly, the hexakistetrahedron arises in the development of alternate hexagonal groups of faces in the hexakisoctahedron, and its sign is $\frac{mOn}{2}$.

Two semitesseral forms with parallel faces occur. (1) The pentagonal dodecahedrons (fig. 51), bounded by twelve *symmetrical* pentagons, vary in general aspect between the cube and the rhombic dodecahedron. They have six regular (and in general longer) edges, lying over the faces of the inscribed cube, and twenty-

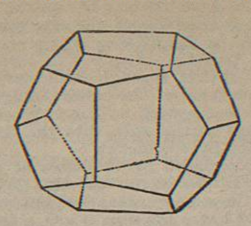
Fig. 51.

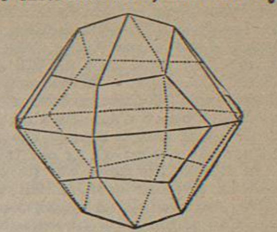
Fig. 52.

four, generally shorter (seldom longer), edges, usually lying in pairs over its edges. The solid angles are eight of three equal interfacial angles, and twelve of three interfacial angles, of which only two are equal. Each principal axis unites two opposite regular edges. This form is derived from the tetrakishexahedron, and its sign is $\frac{\infty On}{2}$. It is found frequently in iron pyrites and cobaltine.

(2) The dyakisdodecahedron (fig. 52), bounded by twenty-four trapezoids with two sides equal, has twelve short, twelve long, and twenty-four intermediate edges. The angles are six equiangular rhombic, united in pairs by the principal axes, eight trigonal, and twenty-four irregular tetragonal angles. It is derived from the hexakisoctahedron, and its sign is $\left[\frac{mOn}{2}\right]$, the brackets being used to distinguish it from the hexakistetrahedron, also derived from the same primary form. It occurs in iron pyrites and cobaltine. The two other semitesseral forms, the pentagonal

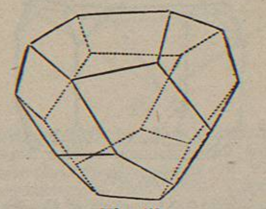
Fig. 53.

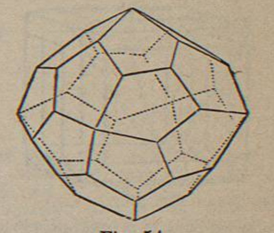
Fig. 54.

dodecahedron (fig. 53), and the pentagonal icositetrahedron (fig. 54), both bounded by *irregular* pentagons, have not yet been observed in nature.

Combinations.

Combinations.—The above-mentioned forms of the tesseral system (and this is true also of the five other systems of crystallization) not only occur singly, but often two, three, or more occur united in the same crystal, forming what are named combinations.

In this case it is evident that no one of the individual forms can be complete, because the faces of one form must interfere with, by diminishing, the faces of other forms. A combination therefore implies that the faces of one form shall appear symmetrically disposed between the faces of other forms, and consequently take the place of certain of their edges and angles. These edges and angles are thus, as it were, cut off, and a greater number of new ones produced in their place, which properly belong neither to the one form nor the other, but are angles of combination. These new faces are hence termed modifications, and the original or primary or simple form is said to be modified. Usually one form predominates more than the others, or has more influence on the general aspect of the crystal, and hence is distinguished as the predominant form, the others being considered subordinate.

The sign of the combination consists of those of its constituent forms, written in the order of their influence or importance in the combination, with a point between each pair.

It will be readily seen that such combinations may be exceedingly numerous, or rather infinite; and only a few of the more common