

entered Aberdeen, where he thoroughly succeeded in effecting his object, on the second occasion carrying off the head of the Gordons, the marquis of Huntly, as a prisoner to Edinburgh.

In July 1639, after the signature of the treaty of Berwick, Montrose was one of the Covenanter leaders who visited Charles upon the borders. This change of policy on his part is frequently ascribed to the fascination of the king's conversation. In reality it arose from the nature of his own convictions. He wished to get rid of the bishops without making presbyters masters of the state. His was essentially a layman's view of the situation. Taking no account of the real forces of the time, he aimed at an ideal form of society in which the clergy should confine themselves to their spiritual duties, and in which the king, after being enlightened by open communication with the Scottish nation, should maintain law and order without respect of persons. In the Scottish parliament which met in September, Montrose attempted to carry out this policy, and found himself in opposition to Argyll, who had placed himself at the head of the Presbyterian and national party, which, by an alteration of the rules that had hitherto regulated the selection of the Lords of the Articles, gave supremacy in parliament to the representatives of the middle classes. Montrose, on the other hand, wished to bring the king's authority to bear upon parliament to defeat this object, and offered him the support of a great number of the nobles, who were by this time as much opposed to the predominance of the Presbyterian clergy acting upon the middle classes as they had before been opposed to the predominance of the bishops. He failed, because Charles could not even then consent to abandon the bishops, and because no Scottish party of any weight could be formed unless Presbyterianism were established ecclesiastically.

Rather than give way, Charles prepared in 1640 to invade Scotland. As usual, he prepared difficulties for those who wished to support him. Montrose was of necessity driven to play something of a double part. In August 1640 he signed the Bond of Cumbernauld as a protest against the particular and direct practising of a few—in other words, against the ambition of Argyll. But he took his place amongst the defenders of his country, and in the same month he was the first to wade across the Tweed at the head of the invaders of England. After the invasion had been crowned with success, Montrose still continued to cherish his now hopeless policy. On 27th May 1641 he was summoned before the Committee of Estates charged with intrigues against Argyll, and on 11th June he was imprisoned in Edinburgh Castle. When Charles visited Scotland to give his formal assent to the abolition of Episcopacy, Montrose communicated to him his belief that Hamilton was a traitor. It has indeed been alleged, on Clarendon's authority, that he proposed to murder Hamilton and Argyll; but this is in all probability only one of Clarendon's many blunders. His letters to Charles, however, must be taken in connexion with this so-called incident. During the progress of the investigation of this plot, Montrose remained in custody, and upon the king's return to England he shared in the amnesty which was tacitly accorded to all Charles's partisans.

For a time Montrose retired, not voluntarily, from public life. After the Civil War in England began he constantly pressed Charles to allow him to make a diversion on Scotland. At last in 1644, when the Scottish army entered England to take part against the king, Montrose, now created a marquis, was allowed to try what he could do. He set out to invade Scotland with about 1000 men. But his followers deserted, and his condition appeared hopeless. Genius, however, inspired him with courage. Disguised

as a groom, he, with only two gentlemen, started on 18th August to make his way to the Highlands. No enterprise might seem rasher. Highlanders had never before been known to combine together, but Montrose knew that most of the clans detested Argyll, not because they were royalist but because Argyll, as the head of the Campbells, was the chief of an aggressive and unscrupulous tribe. Montrose did not miscalculate his chances. The clans rallied to his summons. About 2000 Irish had crossed the sea to assist him. He won battle after battle. He defeated the Covenanters at Tippermuir on 1st September, and at the Bridge of Dee on 12th September. Rapidity of movement was the distinguishing feature of his generalship. He crossed the mountains deep with a winter's snow into the country of Argyll, burning and destroying as he rested for a time from more active operations. On 2d February 1645 he crushed the Campbells at Inverlochy, whilst the head of the house, who was no warrior, looked on at the disaster from a boat. The Scottish parliament declared Montrose to have forfeited his life and estate as a traitor, but it could not reach him to execute the sentence. On 19th February he captured Elgin, through March he was ravaging Aberdeenshire and Kincardineshire, on 3d April he stormed Dundee, then on 9th May came the victory of Auldearn, on 2d July the victory of Alford, and on 15th August the great victory of Kilsyth. Never till after this battle had Montrose ventured far from the Highland hills. The Highlanders had the habit of running home after a victory to secure their booty. Now, however, Montrose found himself apparently master of Scotland. In the name of the king, who now appointed him lord-lieutenant and captain-general of Scotland, he summoned a parliament to meet at Glasgow on 20th October, in which he no doubt hoped to reconcile loyal obedience to the king with the establishment of a non-political Presbyterian clergy. That parliament never met. In England Charles was in evil case. He had been defeated at Naseby on 14th June, and Montrose must come to his help if there was to be still a king to proclaim. He never had a chance of knowing what Montrose could do against the "new model" army. David Leslie, the best of the Scottish generals, was despatched against Montrose to anticipate the invasion. On 12th September he came upon Montrose, deserted by his Highlanders and guarded only by a little group of followers, at Philiphaugh. He won an easy victory. Montrose cut his way through to the Highlands; but he failed to reorganize an army. On 3d September 1646 he embarked for Norway.

Montrose was to appear once more on the stage of Scottish history. In June 1649 he was restored by the exiled Charles II. to his nominal lieutenant-governorship of Scotland. In March 1650 he landed in the Orkneys to take the command of a small force which he had sent on before him. Crossing to the mainland, he tried to raise the clans, but the clans would not rise, and on 27th April he was surprised and captured at Corbiesdale in Ross-shire. On 18th May he entered Edinburgh as a prisoner. On the 20th he was sentenced to death by the parliament, and he was hanged on the 21st, with Wishart's laudatory biography of him put round his neck. To the last he protested that he was a real Covenanter and a loyal subject. "The Covenant which I took," he said, "I own it and adhere to it. Bishops, I care not for them; I never intended to advance their interest." Something, at least, of Montrose's dream, so impossible to realize at that time, has been realized in Scotland. Scotland has remained ecclesiastically Presbyterian. The political legality which Montrose wished to uphold against factions by means of the king has been upheld by means of the political ripeness of the Scottish nation itself. (S. R. G.)

MONT ST MICHEL, a curious rocky islet, consisting

of a mass of granite about 3000 feet in compass and 165 feet in height, rises at a distance of nearly a mile from the shore in the bay of St Michel, near the mouth of the Couësnon, at the vertex of the angle formed by the coasts of Brittany and Normandy. The quicksands by which it is surrounded, and which stretch far to seaward, are exposed at low water, and highly dangerous to those who venture on them without a guide. Recently efforts at reclamation have been made, and amongst other works a causeway has been constructed connecting Mont St Michel with the nearest point of the mainland (near Moidrey); an unfortunate consequence of these operations has been sapped by the altered tidal currents. The fortress-abbey, to which the rock owes its fame, stands upon the more precipitous side towards the north and west; the sloping portion towards the east and south is occupied by dwelling-houses. The strong machicolated and turreted wall by which the whole is surrounded is pierced only by a single gateway. The northward wall of the abbey (La Merveille), dating from the 13th century, is of remarkable boldness; it is 246 feet in length and 108 feet in height, is supported by twenty buttresses, and is pierced by a variety of openings. The single street of the island, leading from the one gateway up to the donjon of the fortress, is lined with houses, most of them used as lodging-houses by visitors and pilgrims; it contains an old parish church, and the house of Du Guesclin is also pointed out. The abbey consists principally of two parallel buildings of three stories each, that on the east containing hospitiary, refectory, and dormitory, and that on the west the cellar, knights' hall, and cloister. The knights' hall is a superb piece of Gothic architecture, measuring 85 feet by 59, with three rows of richly-ornamented pillars. The cloister is one of the purest and most graceful works of the 13th century (1228). The church has a number of imperfect turrets, and is surmounted by a square tower of the 17th century, with a statue of St Michael, which was crowned in 1877. The nave, which dates from the 11th century, is Norman; but the choir, which collapsed in 1421, has been rebuilt in the flamboyant style. Beneath is a fine crypt.

Mont St Michel was a sacred place even in the time of the Druids. It became a seat of Christian worship in the 8th century, when a monastery was founded upon it (with the usual miraculous accompaniments) by St Aubert, bishop of Avranches. It soon became a favourite resort of pilgrims, not only from all parts of France, but also from Great Britain and Ireland, and even from Italy. It was plundered by the Normans; but Rollo, on his conversion, made restitution. At the time of the Conquest it supplied William of Normandy with six ships, and received a considerable share of the English spoils. About this time the monks began to give themselves to learning and to collect a large library, and in the 12th century the establishment reached its highest prosperity. It was burnt by the troops of Philip Augustus, who afterwards furnished large sums for its restoration (La Merveille). St Louis (Louis IX.) made a pilgrimage to Mont St Michel, and was afterwards very liberal to it. During the hundred years' war it offered a memorable resistance to the English; and here, on 1st August 1469, Louis XI. instituted the order of St Michel, and held a brilliant chapter. A similar celebration was held by Francis I. During the religious wars the Huguenots made repeated unsuccessful attempts to seize the fortress; it opened its gates to Henry IV. after his abjuration. About 1615 the Benedictine monks of Mont St Michel were replaced by monks of the Congregation of St Maur; after the Revolution the abbey was used as a prison for political offenders. It is now an historical monument; it contains an orphanage, and is undergoing repairs.

MONTSERRAT, one of the Leeward Islands in the West Indies, situated 16° 45' N. lat. and 62° 7' W. long., is 12 miles long and 8 broad in its widest part, and has an area of 32 square miles. The uneven and rugged surface suggests possibly volcanic origin. Its general appearance is very picturesque, the most interesting natural feature being the Soufrière. The island was discovered by Columbus in 1493, and received its name either because

of its broken appearance or after the mountain in Spain. It was colonized by the English under Sir Thomas Warner in 1632, and was taken by the French in 1664. Restored to the English in 1668, it capitulated to the French in 1782, but was again restored in 1784. It is now a presidency under the general government at Antigua, and has a legislative council, composed of officials and crown nominees. The climate is the most healthy in the West Indies. The population (10,087) consists principally of negroes, with several hundred whites. The revenue and expenditure average £5600 per annum. Sugar exports range from 1200 to occasionally 2000 tons. An important industry is the cultivation of limes and the manufacture of juice. About 700 puncheons of raw lime juice, 300 hogsheads of concentrated juice, and an increasing quantity of fresh green limes are exported annually. For the three years ending 1880 the average value of imports was £26,390, of exports £32,963. The principal town is Plymouth, lying midway along the south-west coast.

MONTSERRAT. Thirty miles to the north-west of Barcelona in Spain there rises a very remarkable mountain of grey conglomerate, 24 miles in circumference, and at its loftiest point (San Geronimo) a little more than 4000 feet in height. From the comparative lowness of the surrounding district, and from its extraordinary configuration, it is a conspicuous object for many miles around. The mountain consists of jagged pinnacles and spires rising abruptly from the base of the mass, which is cloven with many clefts, and abounds with steep precipices. It is the *Mons Serratus* of the Romans, the *Monte Serrado* of the Spaniards, and is thus named either in allusion to its jagged appearance, like the teeth of a saw, or because the eastern face is split, as if sawn,—which occurred, say the Spanish legends, at the time of the crucifixion, when the rocks were rent. The arms of the monastery represent a mountain with a saw resting upon it and penetrating some distance into its mass. Its pinnacles and pyramids and sharp angular masses resemble a mountain of hard crystalline volcanic tuff which occurs between Akureyri and Kalmanstunga in Iceland. The effect of Montserrat may be realized faintly if we place ourselves upon the roof of Milan cathedral, and imagine the forest of spires magnified a thousandfold. The central spire will represent San Geronimo. The result of this varied contour in the case of Montserrat is to make it one of the most picturesque places in Europe. Paths wind along the faces of the precipices, ascending to bare grey summits, descending to sheltered valleys filled with evergreens and flowers. The Pyrenees are seen in one direction, the sea in another, while the Llobregat winds at the foot of the mountain through the village of Monistrol. Manresa and other villages are seen scattered over the plain; and hills covered with a warm red soil alternate with rich valleys. Street says of Montserrat,—“After much experience of mountains, it strikes me more each time that I see it as among the very noblest of rocks.”

The monastery, a great pile of buildings, stands upon a narrow platform on the edge of a vast chasm in the eastern face of the mountain. It owes its existence to an image of the Virgin, said to have been carved by St Luke, and brought to Barcelona by St Peter in 30 A.D. When the Moors invaded the province in 717, the image was taken to Montserrat and hidden in a cave. In 880 Gondemar, bishop of Vich, was attracted to the cave by sweet sounds and smells, and there found the image, which he determined to take to Manresa. But at a certain spot on the mountain the image refused to proceed farther; there it was consequently deposited, and a chapel was erected to contain it. A stone cross near the walls of the monastery still marks the spot where the image refused to move. Round the chapel a nunnery was built, and in 976 this was enlarged

and converted into a Benedictine monastery. Philip II. built the present church. In 1835 the monastery was suppressed and despoiled of the vast treasures which had accumulated during the Middle Ages. But the buildings were allowed to remain, as well as a few of the fathers to take charge of the Virgin's shrine. At present they number 19; a hundred years ago there were 76 monks, 28 lay-brothers, 25 singing boys, together with surgeon, physician, and servants. The possessions then consisted of numerous hamlets, besides great quantities of plate and jewels, including 85 silver lamps.

Nuestra Señora de Montserrat, Patrona de Cataluña, is one of the most celebrated images in Spain, and her church is visited annually by more than 80,000 pilgrims. It is a small carved wooden image, "regularly handsome, but the colour of a negro woman," and possesses magnificent robes and jewels. It has been visited by numbers of sovereigns and high ecclesiastics, and by millions of Catalonians. In September 1831 it was solemnly crowned by Leo XIII., who sent a crown from Rome for that purpose. Quantities of *ex votos* are offered at the shrine: wax models of injured or diseased limbs, models of ships, pictures and clothes, jewels and silver hearts. As the celebrity and sanctity of Montserrat increased, so did the number of devotees. Ignatius Loyola laid his sword upon the altar of the Virgin, and, placing himself under her protection, started from Montserrat to commence his new life. Many eminent Spaniards, weary of the world, have retired to this monastery to end their days. Some preferred solitary hermitages perched among the rocks. Of these there were fifteen, eleven of which once formed a *via sacra*, ending at the summit of San Geronimo. They were destroyed by the French, but the ruins of some of them still remain. From all the view is magnificent; some are indeed placed on the edges of precipices in almost inaccessible places. There are also caves in the mountain, some of which were formerly occupied by monks. The most celebrated of these are the cave of the Virgin, in which the *Santa Imagen* remained hidden until found by Gondemar, and the cave of Fray Juan Garin, a notorious sinner, who ended his days in the practice of revolting penances at Montserrat. At Collbato, on the south-east side of the mountain, near the base, there are also some very curious caves.

MONTUCLA, JEAN-ÉTIENNE (1725-1799), a learned mathematician, was the son of a merchant, and was born at Lyons in 1725. He attended the college of the Jesuits in his native city, and was early distinguished for his tenacious memory and his aptitude for mathematics. At the age of sixteen he removed to Toulouse to prosecute the study of law; and after taking the usual degrees he repaired to Paris. There his conversational powers, his solid information, and his acquirements as a linguist soon introduced him to the notice of the learned. In the society of D'Alembert and Lalande his taste for mathematical studies was confirmed and stimulated. After publishing two anonymous treatises on the *Quadrature of the Circle* and on the *Duplication of the Cube*, he gave to the world in 1758 the first part of his great work, *The History of Mathematics*. Not long after this his merits were recognized by the Government, and he was promoted to several important offices. He was appointed intendant-secretary at Grenoble in 1758, secretary to the expedition for colonizing Cayenne in 1764, and "premier commis des bâtiments" and censor-royal for mathematical books in 1765. During the next twenty-five years his time was divided between his official duties and the study of his favourite science. The Revolution then ensuing deprived him of his income, and left him in great destitution. The offer in 1795 of a mathematical chair in one of the schools of Paris was declined on account of his infirm health, and he was still in straitened circumstances in 1798 when he published a second edition of the first part of his *History*. He also enlarged Ozanam's *Mathematical Recreations*, afterwards published in English by Dr Hutton (4 vols., Lond., 1803). About four months before his death (December 1799) a pension of 2400 francs was conferred upon him. His *History of Mathematics* was completed by Lalande, and published at Paris in 1799-1802 (4 vols. 4to).

Montucla's work was the first history of mathematics worthy of the

name. It is characterized alike by elegance of style and by breadth of treatment. Montucla rarely fails in candour, and never in breadth of sympathy; he lived at a time when it would have been pardonable to treat mathematics "as a French science," yet he cannot with justice be accused of Chauvinism. The study of the history of mathematics has greatly revived of late years, especially in Germany, and numerous monographs on special departments have appeared, in which, as was to be expected, many defects and some positive errors in Montucla's work have been pointed out, but, taken as a whole, it stands as yet unsurpassed, unrivalled, fit, as to its admirable style and enduring quality, to be compared with Gibbon's *Decline and Fall of the Roman Empire*.

MONZA (locally Monscia), a city of Italy in the province of Milan, at the branching of the railway for Lecco and Como, lies on the Lambro, a tributary of the Po, mainly on the right bank, in a healthy and attractive situation. Of the mediæval fortifications little remains save the Porta d'Agate. The cathedral of St John Baptist is the principal object of interest: Theodelinda's basilica was enlarged at the close of the 13th century by throwing the atrium into the main building, and the present marble façade was erected about the middle of the 14th by Matteo da Campione. On the left-hand side of the front rises an incongruous brick-built tower, 278 feet high, erected by Peregrini. Within the church are the iron crown of Lombardy (removed by Austria in 1859, and since restored) and the relics of Theodelinda, comprising her crown, fan and comb of gold, and the golden hen and seven chickens, representing Lombardy and her seven provinces. Next to the cathedral in artistic importance come the church of Santa Maria in Istrada, and the broletto or old palace of the commune, usually styled the Arengario: the former (founded in 1357) has a rich Bramantesque façade, reckoned one of the best pieces of terra-cotta work in Lombardy, and the latter is raised on a system of pointed arches, and has a tall square tower terminating in machicolations surrounding a sharp central cone. San Michele was the scene of the coronation of Conrad III. in 1128, and San Gerardo (formerly Sant' Ambrogio) is named after the patron saint of Monza, Gerardo de' Tintori, who founded the first local hospital in 1174. The royal palace of Monza (1777), with its extensive gardens and parks, lies not far from the town on the banks of the Lambro. Cotton goods and felt hats are the staple products of Monza industry; then dyeing, organ-building, and a publishing trade. The population of the city was 15,450 in 1871, and that of the commune increased from 24,661 in 1861 to 28,012 in 1881.

Local antiquaries claim for Monza (Modicia or Modetia) the rank of a Roman colony, but it cannot have been a place of consequence till it attracted the discerning eye of Theodorici; and, though it was a favourite residence with his immediate successors, its first important associations are with Theodelinda (see vol. xiv. p. 815). During the period of the republics Monza was sometimes independent, sometimes subject to Milan. The Visconti, who ultimately became masters of the city, built a castle in 1325 on the site now occupied by the Palazzo Durini. In the course of its history Monza has stood thirty-two sieges, and been repeatedly plundered, notably by the forces of Charles V. The countship (1499-1796) was purchased in 1546 by the wealthy banker Durini, and remained in his family till the Revolution.

MOOLTAN. See MÜLTÁN.

MOON, THE. The subject of the moon divides itself into two separate branches, the one concerned with the constitution of the lunar globe, the other with its motions. For the first subject the reader is referred to the article ASTRONOMY (vol. ii. p. 801 sq.); the present article is confined to the second, which is commonly called the *Lunar Theory*.

The lunar theory does not yet form a well-defined body of reasoning and doctrine, like other branches of mathematical science, but consists only of a series of researches, extending through twenty-centuries or more, and incapable of being welded into a consistent whole.

This state of things arises from the inherent difficulties and complexities of the subject, and from the fact that no one method or system has yet been discovered by which all the difficulties can be surmounted and all the complexities disentangled. Hence each investigator, when he has desired to make any substantial advance beyond his predecessors, has been obliged to take up the subject from a new point of view, and to devise such method as might seem to him most suitable to the special object in hand. The historical treatment is therefore that best adapted to give a clear idea of the results of these investigations. The ancient and modern histories of the subject are quite distinct, the modern epoch commencing with Newton. The great epoch made by Copernicus did not extend to the case of the moon at all, because in every investigation of the moon's motion, modern as well as ancient, the motion is referred to the earth as a centre. Hence the heliocentric system introduced no new conception of this motion, except that of taking place round a moving earth instead of round a fixed one. This change did not affect the consideration of the relative motion of the earth and moon, with which alone the lunar theory is concerned. The two stages of the lunar theory are therefore—(1) that in which the treatment was purely empirical, (2) that in which it was founded rationally on the law of gravitation.

It is in the investigation of the moon's motion that the merits of ancient astronomy are seen to the best advantage. In the hands of Hipparchus (see ASTRONOMY, vol. ii. p. 749) the theory was brought to a degree of precision which is really marvellous when we compare it, either with other branches of physical science in that age, or with the remarks and speculations of contemporary non-scientific writers. Whether this was wholly the work of Hipparchus, or whether he simply perfected a system already devised by his predecessors, it is now impossible to say; but, so far as certain knowledge extends, the works of his predecessors did not embrace more than the determination of the mean motion of the moon and its nodes. Although the general fact of a varying motion may have been ascertained, the circumstances of the variation had probably never been thoroughly investigated. The discoveries of Hipparchus were:—

1. *The Eccentricity of the Moon's Orbit*.—He found that the moon moved most rapidly near a certain point of its orbit, and most slowly near the opposite point. The law of this motion was such that the phenomena could be represented by supposing the motion to be actually circular and uniform, the apparent variations being explained by the hypothesis that the earth was not situated in the centre of the orbit, but was displaced by an amount about equal to one-twentieth of the radius of the orbit. Then, by a well-known law of kinematics, the angular motion round the earth would be most rapid at the point nearest the earth—that is, at *perigee*—and slowest at the point most distant from the earth—that is, at *apogee*. Thus the apogee and perigee became two definite points of the orbit, indicated by the variations in the angular motion of the moon.

2. *The Motion of the Perigee and Apogee*.—As already defined, the perigee and apogee are at the ends of that diameter of the orbit which passes through the eccentrically situated earth, or, in other words, they are on that line which passes through the centre of the earth and the centre of the orbit. This line was called the *line of apsides*. On comparing observations made at different times, it was found that the line of apsides was not fixed, but made a complete revolution in the heavens, in the order of the signs of the zodiac, in about nine years.

3. *The Numerical Determination of the Elements of the Moon's Motion*.—In order that the two capital discoveries just mentioned should have the highest scientific value it

was essential that the numerical values of the elements involved in these complicated motions should be fixed with precision. This Hipparchus was enabled to do by lunar eclipses. Each eclipse gave a moment at which the longitude of the moon was 180° different from that of the sun, and the latter admitted of ready calculation. Assuming the mean motion of the moon to be known and the perigee to be fixed, three eclipses observed in different points of the orbit would give as many true longitudes of the moon, which longitudes could be employed to determine three unknown quantities—the mean longitude at a given epoch, the eccentricity, and the position of the perigee. By taking three eclipses separated at short intervals, both the mean motion and the motion of the perigee would be known beforehand, from other data, with sufficient accuracy to reduce all the observations to the same epoch, and thus to leave only the three elements already mentioned unknown. In the hands of a modern calculator the problem would be a very simple one, requiring little more than the solution of a system of three equations with as many unknown quantities. But without algebra the solution was long and troublesome, and not entirely satisfactory. Still, it was probably correct within the necessary limits of the errors of the observations. The same three elements being again determined from a second triplet of eclipses at as remote an epoch as possible, the difference in the longitude of the perigee at the two epochs gave the annual motion of that element, and the difference of mean longitudes gave the mean motion. Such was the method of determining the elements of the moon's motion down to the time of Copernicus.

The determination of the eccentricity from eclipses, as above described, leads to an important error in the resulting value of the eccentricity, owing to the effect of the neglected evection. We know from our modern theory that the two principal inequalities in the moon's true longitude are—

$$\begin{aligned} &6^{\circ} \cdot 29 \sin g && \text{(Equation of centre),} \\ &+1^{\circ} \cdot 27 \sin (2D - g) && \text{(Evection),} \end{aligned}$$

where g = mean anomaly, and D = mean angular distance of the moon from the sun. Now during a lunar eclipse we always have $D = 180^{\circ}$ very nearly, and $2D = 360^{\circ}$. Hence the evection is then $-1^{\circ} \cdot 27 \sin g$, and so has the same argument, g , as the equation of centre, and so is confounded with it. The value of the equation of centre derived from eclipses is thus $(6^{\circ} \cdot 29 - 1^{\circ} \cdot 27) \sin g$. Therefore the eccentricity found by Hipparchus and Ptolemy was only 5°, and was more than a degree less than its true value.

The next important step in advance was the discovery of the "evection," which is described by Ptolemy (see ASTRONOMY, vol. ii. p. 750) as if made by himself. In view of the bad habit which Ptolemy had of making his own observations verify results previously arrived at, which were sometimes in error, we must view such a discovery by him as quite exceptional, and as best explainable by the large magnitude of the outstanding error. Although, as just shown, the erroneous eccentricity found by Hipparchus would always represent eclipses, so that the error could never be detected by eclipses, the case was entirely different when the moon was in quadratures. Comparing the inequalities already written with that found by Hipparchus, we see that the latter required the correction—

$$\begin{aligned} &1^{\circ} \cdot 27 \{ \sin g + \sin (2D - g) \} = \\ &1^{\circ} \cdot 27 \{ (1 - \cos 2D) \sin g + \sin 2D \cos g \}. \end{aligned}$$

At quadratures we have $D = \pm 90^{\circ}$, $2D = 180^{\circ}$, and hence $\cos 2D = -1$ and $\sin 2D = 0$. The omitted inequalities at these points of the orbit have therefore the value $2^{\circ} \cdot 54 \sin g$, a quantity so large that it could not fail to be detected by careful observations with the astrolabe. Such an inequality as this, superposed upon the eccentric motion of the moon, was very troublesome to astronomers who had no way of representing the celestial motions except by geometrical construction. The construction proposed by Ptolemy was so different from those employed for the