increase in boiler pressure—until now there are few large land engines and scarcely any marine engines that do not employ it. In marine practice, where economy of fuel is a much more important factor in determining the design than it is on land, the principle of compound expansion has lately been greatly extended by the introduction of triple and even quadruple expansion engines, in which the steam is made to expand successively in three or in four cylinders. Even in the building of locomotive engines, where other considerations are of more moment than the saving of coal, the system of compound expansion is beginning to find a place.

The growth of compound expansion has been referred to at some length, because it forms the most distinctive improvement which the steam-engine has undergone since the time of Watt. For the rest, the progress of the steam-engine has consisted in its adaptation to particular uses, in the invention of features of mechanical detail, in the recognition and application of thermodynamical principles, and in improved methods of engineering construction by which it has profited in common with all other machines. These have in particular made possible the use of steam of eight or ten times the pressure of that employed by Watt.

Application to locomotives.

20. The adaptation of the steam-engine to railways, begun by Trevithick, became a success in the hands of George Stephenson, whose engine the "Rocket," when tried along with others on the Stockton and Darlington road in 1829, not only distanced its competitors but settled once and for all the question whether horse traction or steam traction was to be used on railways. The principal features of the "Rocket" were an improved steam-blast for urging the combustion of coal and a boiler (suggested by Booth, the secretary of the railway) in which a large heating surface was given by the use of many small tubes through which the hot gases passed. Further, the cylinders, instead of being vertical as in earlier locomotives, were set in at a slope, which was afterwards altered to a position more nearly horizontal. To these features there was added later the "link motion," a contrivance which enabled the engine to be easily reversed and the amount of expansion to be readily varied. In the hands of George Stephenson and his son Robert the locomotive took a form which has been in all essentials maintained by the far heavier locomotives of to-day.

Application to steam-boats.

21. The first practical steamboat was the tug "Charlotte Dundas," built by William Symmington, and tried in the Forth and Clyde Canal in 1802. A Watt double-acting condensing engine, placed horizontally, acted directly by a connecting-rod on the crank of a shaft at the stern, which carried a revolving paddle-wheel. The trial was successful, but steam towing was abandoned for fear of injuring the banks of the canal. Ten years later Henry Bell built the "Comet," with side paddle-wheels, which ran as a passenger steamer on the Clyde; but an earlier inventor to follow up Symmington's success was the American Robert Fulton, who, after unsuccessful experiments on the Seine, fitted a steamer on the Hudson in 1807 with engines made to his designs by Boulton and Watt, and brought steam navigation for the first time to commercial success.

22. The early inventors had little in the way of theory to guide them. Watt had the advantage, which he acknowledges, of a knowledge of Black's doctrine of latent heat; but there was no philosophy of the relation of work to heat until long after the inventions of Watt were complete. The theory of the steam-engine as a heat-engine dates from 1824, when Carnot published his *Reflexions sur la Puissance Motrice du Feu*, and showed that heat does work only by being let down from a higher to a lower temperature. But Carnot had no idea that any of the heat disappears in the process, and it was not until the doctrine of the conservation of energy was established in 1843 by the experiments of Joule that the theory of heat-engines began a vigorous growth. From 1849 onwards the science of thermodynamics was developed with extraordinary rapidity by Clausius, Rankine, and Thomson, and was applied, especially by Rankine, to practical problems in the use of steam. The publication in 1859 of Rankine's *Manual of the Steam Engine* formed an epoch in the history of the subject by giving inventors a new basis, outside of mere empiricism, from which they could push on the development of the steam-engine. Unfortunately, however, for its bearing on practice, the theory of the steam-engine was to a great extent founded on certain simplifying assumptions which experience has now shown to be far from correct. It was assumed that the cylinder and piston might be treated as behaving to the steam like non-conducting bodies,—that the transfer of heat between the steam and the metal was negligibly small. Rankine's calculations of steam-consumption, work, and thermodynamic efficiency involve this assumption, except in the case of steam-jacketed cylinders, where he estimates that the steam in its passage through the cylinder takes just enough heat from the jacket to prevent a small amount of condensation which would otherwise occur as the process of expansion goes on. If the transfer of heat from steam to metal could be overlooked, the steam which enters the cylinder would remain during admission as dry as it was before it entered, and the volume of steam consumed per stroke would correspond with the volume of the cylinder up to the point of cut-off. It is here that the actual behaviour of steam in the cylinder diverges most widely from the behaviour which the theory assumes. When steam enters the cylinder it finds the metal chilled by the previous exhaust, and a portion of it is at once condensed. This has the effect of increasing, often very largely, the volume of boiler steam required per stroke. As expansion goes on the water that was condensed during admission begins to be re-evaporated from the sides of the cylinder, and this action is often prolonged into the exhaust. In a later chapter the effect which this exchange of heat between the metal of the cylinder and the working fluid produces on the economy of the engine will be discussed, and an account will be given of experimental means by which we may examine the amount of steam that is initially condensed and trace its subsequent re-evaporation. It is now recognized that any theory which fails to take account of these exchanges of heat fails also to yield even comparatively correct results in calculating the relative efficiency of various steam pressures or various ranges of expansion. But the exchanges of heat are so complex that there seems little prospect of submitting them to any comprehensive theoretical treatment, and we must rather look for help in the future development of engines to the scientific analysis of experiments with actual machines. Much careful work of this kind has already been done by Hirn and others, and there is room for much more. Questions relating to the influence (on heat-engine economy) of speed, of pressure, of ratio of expansion, of jacketing, of compound expansion, or of superheating must in the main be settled by an appeal to experiment,—experiment guided and interpreted at every step by reference to the principles of thermodynamics and the theory of steam.

References.—Stuart, *Descriptive History of the Steam-Engine*, 1825; Farey, *Treatise on the Steam-Engine*, 1827; Tredgold, *The Steam-Engine*, 1838; Muirhead's *Mechanical Inventions of James Watt* and *Life of Watt*; Galloway, *The Steam-Engine and its Inventors*; Thurston, *History of the Growth of the Steam-Engine*; Cowper on the Steam-Engine (*Heat Lectures Inst. C.E.*, 1884). Tait, *Sketch of Thermodynamics*.

II. THEORY OF HEAT-ENGINES.

23. A heat-engine acts by taking in heat, converting a part of the heat received into mechanical energy, which appears as the work done by the engine, and rejecting the remainder, still in the form of heat. The theory of heat-engines comprises the study of the amount of work done, in its relation to the heat supplied and to the heat rejected. The theory is based on the two laws of thermodynamics, which may be stated here as follows:—

LAW 1. *When mechanical energy is produced from heat, 1 thermal unit of heat goes out of existence for every 772 foot-pounds of work done; and, conversely, when heat is produced by the expenditure of mechanical energy, 1 thermal unit of heat comes into existence for every 772 foot-pounds of work spent.*

The "thermal unit" is the heat required to raise the temperature of 1 ℔ of water 1 degree Fahr. when at its temperature of maximum density. The equivalent quantity of work, 772 foot-pounds, was determined by the experiments of Joule, and is called Joule's equivalent. Later researches by Joule and others have indicated that this number is probably too small; it should perhaps be as much as 774 foot-pounds. Joule's original value is still generally used by engineers: and as it enters into many published tables it may conveniently be adhered to until its accuracy is more definitely disproved. Since a definite number of foot-pounds are equivalent to 1 thermal unit, we may, if we please, express quantities of work in thermal units, or quantities of heat in foot-pounds; the latter practice will frequently be found useful.

LAW 2. *It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature.*

This is the form in which the second law has been stated by Clausius. Another statement of it, different in form but similar in effect, has been given by Thomson. Its force may not be immediately obvious, but it will be shown below that it introduces a most important limitation of the power which any engine has of converting heat into work. So far as the first law shows, there is nothing to prevent the whole heat taken in by the engine from changing into mechanical energy. In consequence of the second law, however, no heat-engine converts, or can convert, more than a small fraction of the heat supplied to it into work; a large part is necessarily rejected as heat. The ratio

$$\frac{\text{Heat converted into work}}{\text{Heat taken in by the engine}}$$

is a fraction always much less than unity. This ratio is called the *efficiency* of the engine considered as a heat-engine.

Working substance.

24. In every heat-engine there is a *working substance* which takes in and rejects heat, thereby suffering changes of form, or more commonly of volume, and does work by overcoming resistance to these changes of form or volume. The working substance may be gaseous, liquid, or solid. We can, for example, imagine a heat-engine in which the working substance is a long metallic rod, arranged to act as the pawl of a ratchet-wheel with fine teeth. Let the rod be heated so that it elongates sufficiently to drive the wheel forward through the space of one tooth. Then let the rod be cooled (say by applying cold water), the wheel being meanwhile held from returning by a separate click or detent. The rod, on cooling, will retract so as to engage itself with the next succeeding tooth, which may then be driven forward by heating the rod again, and so on. To make it evident that such an engine would do work, we have only to suppose that the ratchet-wheel carries round with it a drum by which a weight is wound up. We have, then, a complete heat-engine, in which the working substance is a solid rod, which receives heat by being brought into contact with some source of heat at a comparatively high temperature, transforms a small part of this heat into work, and rejects the remainder to what we may call a receiver of heat, at a comparatively low temperature. The greater part of the heat may be said simply to pass through the engine, from the source to the receiver, *becoming degraded as regards temperature* as it goes. We shall see presently that this is typical of the action of all heat-engines; when they are doing work, the heat which they reject is rejected at a temperature lower than that at which it is taken in. They convert some heat into work only by letting down a much larger quantity of heat from a high to a relatively low temperature. The action is analogous to that of a water-wheel, which does work by letting down water from a high to a lower level, but with this important difference that in the transfer which occurs in heat-engines an amount of heat disappears which is equivalent to the work done.

25. In almost all actual heat-engines the working substance is a fluid. In some it is air, in some a mixture of several gases. In the steam-engine the working fluid is a mixture (in varying proportions) of water and steam. With a fluid for working substance, work is done by changes of volume only: its amount depends solely on the relation of pressure to volume during the change, and not at all on the form of the vessels in which the change takes place. Let a diagram be drawn (fig. 9) in which the relation of the intensity of pressure to the volume of any supposed working substance is graphically exhibited by the line ABC, where AM, CN are pressures and AP, CQ are volumes, then the work done by the substance in expanding from A to C is the area of the figure MABCN. And similarly, if the substance be compressed from C back to its original volume in such a manner that the line CDA represents the relation of pressure and volume during compression, the work done *upon* the substance is the figure NCDAM. Taking the two operations together, we find that the substance has done a net amount of work equal to the area of the shaded figure ABCDA, or $\int P\,dV$. This is an example and a generalization of the method of representing work which Watt introduced by his invention of the indicator; the figure ABCDA may be called the indicator diagram of the supposed action.

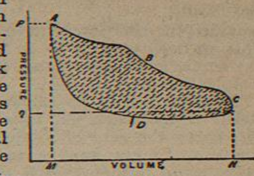

Fig. 9.

26. Generally in heat-engines the working substance returns periodically to the same state of temperature, pressure, volume, and physical condition. When this has occurred the substance is said to have passed through a complete cycle of operations. For example, in a condensing steam-engine, water taken from the hot-well is pumped into the boiler; it then passes into the cylinder as steam, passes thence into the condenser, and thence again into the hot-well; it completes the cycle by returning to the same condition as at first. In other less obvious cases, as in that of the non-condensing steam-engine, a little consideration will show that the cycle is completed, not indeed by the same portion of working substance being returned to the boiler, but by an equal quantity of water being fed to it, while the steam which has been discharged into the atmosphere cools to the temperature of the feed. In the theory of heat-engines it is of the first importance to consider (as was first done by Carnot in 1824) the cycle of operations performed by the working substance as a complete whole. If we stop short of the completion of a cycle matters are complicated by the fact that the substance is in a state different from its initial state, and may therefore have changed its stock of internal energy. After a complete cycle, on the other hand, we know at once that, since the condition is the same, the internal energy of the substance is the same as at first, and therefore—

Heat taken in = work done + heat rejected.

Engine working with a perfect gas.

27. It will serve our purpose best to approach the theory of heat-engines by considering, in the first instance, the action of an engine in which the working substance is any one of the so-called permanent gases, or a mixture of them, such as air. The word permanent, as applied to a gas, can now[1] be understood only as meaning that the gas is liquefied with difficulty—either by the use of extremely low temperature or extremely high pressure or both. So long as gases are under conditions of pressure and temperature widely different from those which produce liquefaction, they conform very approximately to certain simple laws—laws which may be regarded as *rigorously* applicable to ideal substances called *perfect* gases. After stating these laws briefly we shall examine the efficiency of a heat-engine using a gas in a certain manner as working substance, and then show that the conclusion so derived has a general application to all heat-engines whatsoever. In this procedure there is no sacrifice of generality, and a part of the process is of independent service in the discussion of actual air-engines.

28. The laws of the permanent gases are the following:—

LAW 1 (Boyle). *The volume of a given mass of gas varies inversely as the pressure, the temperature being kept constant.*

Thus, if V be the volume of 1 ℔ of a gas in cubic feet, and P the pressure in pounds per square foot, so long as the temperature is unchanged—

$$P \propto V^{-1}, \text{ or } PV = \text{constant}.$$

For air the value of the constant is 26220 when the temperature is 32° F.

29. LAW 2 (Charles). *Under constant pressure equal volumes of different gases increase equally for the same increment of temperature. Also, if a gas be heated under constant pressure, equal increments of its volume correspond very nearly to equal intervals of temperature as determined by the scale of a mercury thermometer.*

Thus, let us take, say, 493 cubic inches of hydrogen, also of oxygen, of air, &c., all at 32° F., and, keeping each at a constant pressure (not necessarily the same for all), heat all so that their temperature rises 1° F. We shall find that each has expanded by sensibly the same amount and now occupies 494 cubic inches. And further, if we heat any one through another 1° F. to 34° F., we shall find that its volume is now 495 cubic inches, and so on. Thus for any gas, kept at constant pressure, if the volume was

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[1] Since the liquefaction of hydrogen and other gases by MM. Cailletet and Pictet.

it would be 493 at 32° F.,
492 at 31° F.,
⋮
461 at 0° F.,
⋮
and finally 0 at $-461°$ F.,

provided the same law were to hold at indefinitely low temperatures. This we may assume to be the case with a perfect gas, although any actual gas would change its physical state long before so low a temperature were reached.

This result may be concisely expressed by saying that if we reckon temperature, not from the ordinary zero but from a point 461° below the zero of Fahrenheit's scale, the volume of a given quantity of a gas, kept at constant pressure, is proportional to the temperature reckoned from that zero. Temperatures so reckoned are called absolute temperatures, and the point $-461°$ F. is called the absolute zero of temperature. Denoting any temperature according to the ordinary scale by t, and the corresponding absolute temperature by τ, we have

$$\tau=t+461 \text{ on the Fahrenheit scale,}$$

and

$$\tau=t+274 \text{ on the Centigrade scale.}$$

Charles's law shows that if temperatures be measured by thermometers in which the expanding substance is air, hydrogen, oxygen, or any other permanent gas, and, if those intervals of temperature be called equal which correspond to equal amounts of expansion, then the indications of these thermometers always agree very closely with each other, and also agree, though less closely, with the indications of a mercury thermometer. We shall see later that the theory of heat-engines affords a means of forming a thermometric scale which is independent of the properties, as to expansion, of any substance, and that this scale coincides with the scale of a perfect gas thermometer, a fact which justifies the use of the term absolute, as applied to temperatures measured by the expansion of a gas.

30. Combining laws 1 and 2, we have, for a given mass of any gas,

$$PV=c\tau,$$

where c is a constant depending on the specific density of the gas and on the units in which P and V are measured. In what follows we shall assume that P is measured in pounds per square foot, that V is the volume of 1 ℔ in cubic feet, and that τ is the absolute temperature in Fahrenheit degrees. In air, with these units,

$$PV=53{\cdot}18\tau.$$

31. LAW 3 (Regnault). *The specific heat at constant pressure is constant for any gas.*

By specific heat at constant pressure is meant the heat taken in by 1 ℔ of a substance when its temperature rises 1° F., while the pressure remains unchanged—the volume of course increasing. The law states that this quantity is the same for any one gas, no matter what be the temperature, or what the constant pressure, at which the process of heating takes place.

32. Another important quantity in the theory of heat-engines is the specific heat at constant volume, that is, the heat taken in by 1 ℔ of the substance when its temperature rises 1° F. while the volume remains unchanged—the pressure of course increasing. We shall denote specific heat at constant pressure by K_p and specific heat at constant volume by K_v. Let 1 ℔ of a gas be heated at constant pressure P from temperature τ_1 to temperature τ_2 (absolute). Let V_1 be the volume at τ_1 and V_2 the volume at τ_2. Heat is taken in, and external work is done by the expansion of the gas, namely—

$$\text{Heat taken in}=K_p(\tau_2-\tau_1).$$
$$\text{Work done}=P(V_2-V_1)=c(\tau_2-\tau_1).$$

The difference between these quantities, or $(K_p-c)(\tau_2-\tau_1)$, is the amount by which the stock of internal energy possessed by the gas has increased during the process. We shall see presently that this gain of internal energy would have been the same had the gas passed in any other manner from τ_1 to τ_2.

33. LAW 4 (Joule). *When a gas expands without doing external work, and without taking in or giving out heat, its temperature does not change.*

To prove this, Joule connected a vessel containing compressed gas with another vessel that was empty, by means of a pipe with a closed stop-cock. Both vessels were immersed in a tub of water and were allowed to assume a uniform temperature. Then the stop-cock was opened, the gas expanded without doing external work, and finally the temperature of the water in the tub was found to have undergone no change. The temperature of the gas was unaltered, and no heat had been taken in or given out by it.

34. Since the gas had neither gained nor lost heat, and had done no work, its internal energy was the same at the end as at the beginning of the experiment. The pressure and volume had changed, but the temperature had not. We must therefore conclude that the internal energy of a given mass of a gas depends only on its temperature, and not upon its pressure or volume; in other words, a change of pressure and volume not associated with a change of temperature does not alter the internal energy. Hence in any change of temperature the change of internal energy is independent of the relation of pressure to volume throughout the operation. Now we have seen above that the quantity

$$(K_p-c)(\tau_2-\tau_1)$$

measures the gain of internal energy when 1 ℔ of a gas has its temperature changed from τ_1 to τ_2 in one particular way, namely, at constant pressure. Hence this same quantity also measures the gain of internal energy when 1 ℔ of a gas has its temperature changed from τ_1 to τ_2 in any manner whatsoever.

35. Next consider the case of 1 ℔ of a gas heated from τ_1 to τ_2 at constant volume. The heat taken in is

$$K_v(\tau_2-\tau_1).$$

Since no work is done, this is all gain of internal energy, and is therefore (§ 34) equal to

$$(K_p-c)(\tau_2-\tau_1).$$

Hence in any gas

$$K_p-c=K_v.$$

The ratio K_p/K_v will be denoted by γ; obviously $K_v=c/(\gamma-1)$. The following table of values of K_p, K_v, c, and γ will be found useful in dealing with air and gas engines.

TABLE I.—*Properties of Gases.*

	K_p.	K_v.	c.	γ.
	Foot-lbs.	Foot-lbs.	Foot-lbs.	
Dry air	183·4	130·2	53·2	1·409
Oxygen	167·9	119·8	48·1	1·402
Nitrogen	188·2	133·4	54·8	1·411
Hydrogen	2632	1864	768	1·412
Carbonic oxide	189·1	133·4	55·7	1·418
Carbonic acid	167·4	132·6	34·8	1·263
Marsh gas	457·7	363·1	94·6	1·261
Olefiant gas	311·9	257·7	54·2	1·213
Steam gas, or highly superheated steam	371	285·5	85·5	1·30

Work done in expansion.

36. We shall now return to the consideration of diagrams like that of § 25, which exhibit the action of a working substance by curves showing the relation of P to V during expansion or compression. In most of the instances which occur in the theory of heat-engines such curves may be exactly or approximately represented by equations of the form

$$PV^n=\text{constant},$$

where the index n has various numerical values. Let AB, fig. 10, be a curve of expansion (of any substance) in which PV^n is constant, from pressure P_1 and volume V_1 at A to pressure P_2 and volume V_2 at B. We have, by assumption,

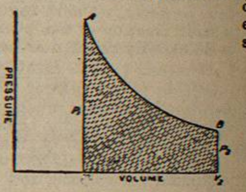

Fig. 10.

$$P_1V_1^n=P_2V_2^n.$$

The work done is

$$\int_{V_1}^{V_2}PdV=P_1V_1^n\int_{V_1}^{V_2}\frac{dV}{V^n}=\frac{P_1V_1^n(V_2^{1-n}-V_1^{1-n})}{1-n}.$$

This may also be written

$$\frac{P_1V_1(1-r^{1-n})}{n-1},$$

where r is the ratio V_2/V_1, which may be called the ratio of expansion.

Still another form of the above expression for the work done is

$$\frac{P_1V_1-P_2V_2}{n-1}.$$

37. Applying this result to the case of an expanding gas, we have

$$\text{Work done}=c(\tau_1-\tau_2)/(n-1).$$

The loss of internal energy during expansion is, by § 34,

$$K_v(\tau_1-\tau_2), \text{ or } c(\tau_1-\tau_2)/(\gamma-1), \text{ by § 35.}$$

Suppose now that the mode of expansion is such that the loss of internal energy is equal to the external work done, then

$$\frac{c(\tau_1-\tau_2)}{n-1}=\frac{c(\tau_1-\tau_2)}{\gamma-1}, \text{ or } n=\gamma,$$

and the law of expansion is

$$PV^\gamma=\text{constant}.$$

The same formula applies when a gas is being compressed in such a manner that the work spent upon the gas is equal to the gain of internal energy by the gas.

38. This mode of expansion (or compression) is termed *adiabatic.* It occurs when the working substance is neither gaining nor losing heat by conduction or radiation or internal chemical action. It would be realized if we had a substance expanding or being compressed, without chemical change, in a cylinder which (with the piston) was a perfect non-conductor of heat. In adiabatic expansion the external work is done entirely at the expense of the substance's stock of internal energy. Hence in the adiabatic expansion of a gas the temperature falls, and in adiabatic compression it rises. To find the change of temperature in a gas when expanded or compressed adiabatically we have only to combine the equations

$$P_1V_1^\gamma=P_2V_2^\gamma \text{ and } \frac{P_2V_2}{P_1V_1}=\frac{\tau_2}{\tau_1},$$

and we find

$$\tau_2=\tau_1\,(V_1/V_2)^{\gamma-1}.$$

It is clear from the above that if, during expansion, n is less than γ the fluid is taking in heat, and if n is greater than γ the fluid is rejecting heat.

39. Another very important mode of expansion or compression is that called *isothermal*, in which the temperature of the working substance is kept constant during the process.

In the case of a gas the curve of isothermal expansion is a rectangular hyperbola, having the equation

$$PV=\text{constant}=c\tau.$$

When a gas expands (or is compressed) isothermally at temperature τ from V_1 to V_2 the work done by (or upon) it (per ℔) is

$$\int_{V_1}^{V_2}PdV=P_1\int_{V_1}^{V_2}\frac{dV}{V}=PV\log_\epsilon r=c\tau\log_\epsilon r,$$

where r is the ratio V_2/V_1 as before.[1]

During isothermal expansion or compression a gas suffers no change of internal energy (by § 34, since τ is constant). Hence during isothermal expansion a gas must take in an amount of heat just equal to the work it does, and during isothermal compression it must reject an amount of heat just equal to the work spent upon it. The expression $c\tau\log_\epsilon r$ consequently measures, not only the work done by or upon the gas, but also the heat taken in during isothermal expansion or given out during isothermal compression. In the diagram, fig. 11, the line AB is an example of a curve of isothermal expansion for a perfect gas, called for brevity an isothermal line, while AC is an adiabatic line starting from the same point A.

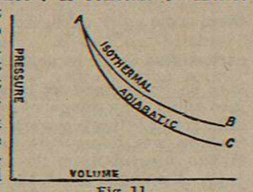

Fig. 11.

Carnot's cycle of operations.

40. We shall now consider the action of an ideal engine in which the working substance is a perfect gas, and is caused to pass through a cycle of changes each of which is either isothermal or adiabatic. The cycle to be described was first examined by Carnot, and is spoken of as Carnot's cycle of operations. Imagine a cylinder and piston composed of a perfectly non-conducting material, except as regards the bottom of the cylinder, which is a conductor. Imagine also a hot body or indefinitely capacious source of heat A, kept always at a temperature τ_1, a perfectly non-conducting cover B, and a cold body or indefinitely capacious receiver of heat C, kept always at a temperature τ_2, which is lower than τ_1. It is supposed that A, B, or C can be applied to the bottom of the cylinder. Let the cylinder contain 1 ℔ of a perfect gas, at temperature τ_1, volume V_a, and pressure P_a to begin with. The suffixes refer to the points on the indicator diagram, fig. 12.

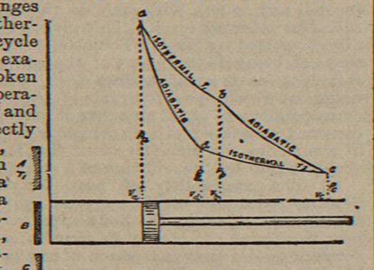

FIG. 12.—Carnot's Cycle, with a gas for working substance.

(1) Apply A, and allow the piston to rise. The gas expands isothermally at τ_1, taking heat from A and doing work. The pressure changes to P_b and the volume to V_b.

(2) Remove A and apply B. Allow the piston to go on rising. The gas expands adiabatically, doing work at the expense of its internal energy, and the temperature falls. Let this go on until the temperature is τ_2. The pressure is then P_c, and the volume V_c.

(3) Remove B and apply C. Force the piston down. The gas is compressed isothermally at τ_2, since the smallest increase of temperature above τ_2 causes heat to pass into C. Work is spent upon the gas, and heat is rejected to C. Let this be continued until a certain point d (fig. 12) is reached, such that the fourth operation will complete the cycle.

(4) Remove C and apply B. Continue the compression, which is now adiabatic. The pressure and temperature rise, and if the point d has been properly chosen, when the pressure is restored to its original value P_a, the temperature will also have risen to its original value τ_1. [In other words, the third operation must be stopped when a point d is reached such that an adiabatic line drawn through d will pass through a.] This completes the cycle.

To find the proper place at which to stop the third operation, we have, by § 38, $\tau_1/\tau_2=(V_c/V_b)^{\gamma-1}$ in the second operation, and again $\tau_1/\tau_2=(V_d/V_a)^{\gamma-1}$ in the fourth operation. Hence $V_c/V_b=V_d/V_a$, and V_b/V_a, the ratio of isothermal expansion, is equal to V_c/V_d, the ratio of isothermal compression. For brevity we shall denote either of these last ratios by r.

41. The following are the transfers of heat to and from the working fluid, in successive stages of the cycle :—

(1) Heat taken in from $A=c\tau_1\log_\epsilon r$ (by § 39).
(2) No heat taken in or rejected.
(3) Heat rejected to $C=c\tau_2\log_\epsilon r$ (by § 39).
(4) No heat taken in or rejected.

Hence, by the first law of thermodynamics, the net external work done by the gas is

$$c(\tau_1-\tau_2)\log_\epsilon r;$$

and the efficiency of the engine (§ 23) is

$$\frac{c(\tau_1-\tau_2)\log_\epsilon r}{c\tau_1\log_\epsilon r}=\frac{\tau_1-\tau_2}{\tau_1}$$

This is the fraction of the whole heat given to it which an engine following Carnot's cycle converts into work. The engine takes in an amount of heat, at the temperature of the source, proportional to τ_1; it rejects an amount of heat, at the temperature of the receiver, proportional to τ_2. It works within a range of temperature extending from τ_1 to τ_2, by letting down heat from τ_1 to τ_2 (§ 24), and in the process it converts into work a fraction of that heat, which fraction will be greater the lower the temperature τ_2 at which heat is rejected is below the temperature τ_1 at which heat is received.

42. Next let us consider what will happen if we reverse Carnot's cycle, that is to say, if we force this engine to act so that the same indicator diagram as before is traced out, but in the direction opposite to that followed in § 40. Starting as before from the point a and with the gas at τ_1, we shall require the following four operations :—

(1) Apply B and allow the piston to rise. The gas expands adiabatically, the curve traced is ad, and when d is reached the temperature has fallen to τ_2.

(2) Remove B and apply C. Allow the piston to go on rising. The gas expands isothermally at τ_2, taking heat from C, and the curve dc is traced.

(3) Remove C and apply B. Compress the gas. The process is adiabatic. The curve traced is cb, and when b is reached the temperature has risen to τ_1.

(4) Remove B and apply A. Continue the compression, which is now isothermal, at τ_1. Heat is now rejected to A, and the cycle is completed by the curve ba.

In this process the engine is not doing work; on the contrary, work is spent upon it equal to the area of the diagram, or $c(\tau_1-\tau_2)\log_\epsilon r$. Heat is taken in from C in the first operation, to the amount $c\tau_2\log_\epsilon r$. Heat is rejected to A in the fourth operation, to the amount $c\tau_1\log_\epsilon r$. In the first and third operations there is no transfer of heat.

The action is now in every respect the reverse of what it was before. The same work is now spent upon the engine as was formerly done by it. The same amount of heat is now given to the hot body A as was formerly taken from it. The same amount of heat is now taken from the cold body C as was formerly given to it, as will be seen by the following scheme :—

Carnot's Cycle, Direct.

Work done by the engine $=c(\tau_1-\tau_2)\log_\epsilon r$;
Heat taken from $A=c\tau_1\log_\epsilon r$;
Heat rejected to $C=c\tau_2\log_\epsilon r$.

Carnot's Cycle, Reversed.

Work spent upon the engine $=c(\tau_1-\tau_2)\log_\epsilon r$;
Heat rejected to $A=c\tau_1\log_\epsilon r$;
Heat taken from $C=c\tau_2\log_\epsilon r$.

The reversal of the work has been accompanied by an exact reversal of each of the transfers of heat.

43. An engine in which this is possible is called, from the thermodynamic point of view, a *reversible* engine. In other words, a reversible heat-engine is one which, if forced to trace out its indicator diagram reversed in direction, so that the work which would be done by the engine, when running direct, is actually spent upon it, will reject to the source of heat the same quantity of heat as, when running direct, it would take from the source, and will take from the receiver of heat the same quantity as, when running direct, it would reject to the receiver. By "the source of heat" is meant the hot body which acts as source, and by "the receiver" is meant the cold body which acts as receiver, when the

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[1] In calculations where this expression is involved it is convenient to remember that $\log_\epsilon$, the hyperbolic logarithm, of any number is 2·3026 times the common logarithm of the number.

engine is running direct. Carnot's engine is one example of a reversible engine. The idea of thermodynamic reversibility is highly important, for the reason that no heat-engine can be more efficient than a reversible engine, if both take in and reject heat at the same pair of temperatures.

44. To prove this, let it be supposed that we have two engines M and N, of which N is reversible in the above sense, and that we have a hot body A capable of acting as a source of heat, and a cold body C capable of acting as a receiver of heat. The engine M is set to work as a heat-engine, taking heat from A and rejecting heat to C. To prove that M cannot be more efficient than the reversible engine N, we shall assume that it *is* more efficient, and trace the consequences of that assumption.

Let M, working direct, be coupled so as to work N reversed; if we suppose that the engines are without mechanical friction, and can be coupled up without loss of power, the work represented by the indicator diagram of M is spent on N, and N will therefore reject to A an amount of heat which we will call Q_A and take from B an amount of heat which we will call Q_B. Now, since N is reversible, if it worked direct, taking Q_A from A, it would do the same amount of work as, in the supposed circumstances, is spent upon it. Hence, if M is more efficient than N it is taking from A an amount of heat *less* than Q_A, and consequently also is giving to B an amount of heat correspondingly less than Q_B. The joint effect, therefore, of M working direct and N working reversed is that the heat taken from A by M is less than the heat given to A by N, while the heat given to B by M is less (to an equal extent) than the heat taken from B by N. The consequence is that the hot body A is gaining heat on the whole, and the cold body B is losing an equal amount of heat; in other words, with the continued action of the double system heat passes, in indefinitely large quantity, from a cold body to a hot body, by means of an agency which, it is to be observed, is purely self-acting, for if we suppose there is no mechanical friction the system requires no help from without. Now this result is, by the second law of thermodynamics (§ 23), contrary to all experience; and we are forced to conclude that the assumption that M is more efficient than the reversible engine N, when both take in and reject heat at the same two temperatures, is false. Hence, with given temperatures of source and receiver of heat no engine is more efficient than a reversible engine.

Next, let M and N both be reversible and both work between the same limits, but be different in any other respect. Then by the foregoing argument M cannot be more efficient than N, neither can N be more efficient than M. Hence all reversible heat-engines taking in and rejecting heat at the same temperatures are equally efficient.

45. These results imply that reversibility, in the thermodynamic sense, is the criterion of what may be called perfection in a heat-engine. A reversible engine is perfect in the sense that it cannot be improved on as regards efficiency: no other engine, taking in and rejecting heat at the same temperatures, will convert into work a greater fraction of the heat which it takes in. Moreover, if this criterion be satisfied, it is as regards efficiency a matter of complete indifference what is the nature of the working substance, or what, in other respects, is the mode of the engine's action.

Efficiency of a perfect heat-engine.

46. Further, since all engines that are reversible are equally efficient, provided they work between the same temperatures, an expression for the efficiency of one will apply equally to all. Now, the engine whose efficiency we have found in § 41 is one example of a reversible engine. Hence its efficiency

$$(\tau_1 - \tau_2)/\tau_1$$

is the efficiency of any reversible heat-engine whatsoever taking in heat at τ_1 and rejecting heat at τ_2. And, as no engine can be more efficient than one that is reversible, this expression is the measure of *perfect efficiency*. We have thus arrived at the immensely important conclusion that no heat-engine can convert into work a greater fraction of the heat which it receives than is expressed by the excess of the temperature of reception above that of rejection divided by the absolute temperature of reception.

47. Briefly recapitulated, the steps of the argument by which this result has been reached are as follows. After stating the experimental laws to which gases conform, we examined the action of a heat-engine in which the working substance took in heat when at the temperature of the source and rejected heat when at the temperature of the receiver, the change of temperature from one to the other of these limits being accomplished by adiabatic expansion and adiabatic compression. Taking a special case in which this engine had for its working substance a perfect gas, we found that its efficiency was $(\tau_1 - \tau_2)/\tau_1$ (§ 41). We also observed that it was, in the thermodynamic sense, a reversible engine (§ 43). Then we found, by an application of the second law of thermodynamics, that no heat-engine can have a higher efficiency than a reversible engine, when taking in and giving out heat at the same two temperatures τ_1 and τ_2; this was shown by the fact that a contrary assumption leads to a violation of the second law (§ 44). Hence, we concluded that all reversible heat-engines receiving and rejecting heat at the same temperatures τ_1 and τ_2 respectively are equally efficient, and hence that the efficiency $(\tau_1 - \tau_2)/\tau_1$, already determined for one particular reversible engine, measures the efficiency of any reversible engine, and is a limit of efficiency which no engine whatever can exceed.

48. The second law of thermodynamics, on which (along with the first law) this conclusion rests has been given in many different forms. The statement of it in § 23 is that of Clausius, and is very similar to that of Sir W. Thomson. Rankine, to whom with Thomson and Clausius is due the development of the theory of heat-engines from the point at which Carnot left it, has stated the second law in a form which is neither easy to understand, nor obvious, as an experimental result, when understood. His statement runs:—

"If the absolute temperature of any uniformly hot substance be divided into any number of equal parts, the effects of those parts in causing work to be performed are equal."[1]

To make this intelligible we may suppose that any quantity q of heat from a source at temperature τ_1 is taken by the first of a series of perfect heat-engines, and that this engine rejects heat at a temperature τ_2 less than τ_1 by a certain interval $\Delta\tau$. Let the heat so rejected by the first engine form the heat supply of a second perfect engine working from τ_2 to τ_3 through an equal interval $\Delta\tau$; let the heat which it in turn rejects form the heat-supply of a third perfect engine working again through an equal interval from τ_3 to τ_4; and so on. The efficiencies of the several engines are (by § 46) $\frac{\Delta\tau}{\tau_1}$, $\frac{\Delta\tau}{\tau_2}$, $\frac{\Delta\tau}{\tau_3}$, &c. The amounts of heat supplied to them are q, $q\frac{\tau_2}{\tau_1}$, $q\frac{\tau_3}{\tau_2}$, &c. Hence the amount of work done by each engine is the same, namely, $q\frac{\Delta\tau}{\tau_1}$. Thus Rankine's statement is to be understood as meaning that each of the equal intervals into which any range of temperature may be divided is equally effective in allowing work to be produced from heat when heat is made to pass, doing work in the most efficient possible way, through all the intervals from the top to the bottom of the range.

Thermodynamic absolute measurement of temperature.

49. A point of much theoretical interest may be noted in passing. In place of measuring temperature, as we have done, by the expansion of a perfect gas, a scale of temperature might be formed thus. Starting from any one temperature, let a series of intervals be taken such that a series of reversible engines, each working with one of the intervals for its range, in the manner described in § 48 (so that the heat rejected by the first forms the supply of the second, and so on), will each do the same amount of work; then *call these intervals equal*. This gives a scale of temperature (originally suggested by Sir W. Thomson) which is truly absolute in the sense of being independent of the properties of any substance; it coincides, as is evident from § 48, with the scale we have been using, in which equal intervals of temperature are defined as those corresponding to equal amounts of expansion of a perfect gas under constant pressure; and it coincides approximately with the scale of a mercury thermometer when that is graduated to read from the absolute zero by the addition of a suitable constant (§ 29).

Conditions of maximum efficiency.

50. The availability of heat for transformation into work depends essentially on the range of temperature through which the heat is let down from the hot source to the cold body into which heat is rejected; it is only in virtue of a difference of temperature between bodies that conversion of any part of their heat into work becomes possible. If τ_1 and τ_2 are the highest and lowest temperatures of the range through which a heat-engine works, it is clear that the maximum of efficiency can be reached only when the engine takes in all its heat at τ_1 and rejects at τ_2 all that is rejected. With respect to every portion of heat taken in and rejected the greatest ideal efficiency is

$$= \frac{\text{Temperature of reception} - \text{temperature of rejection}}{\text{Temperature of reception}}.$$

Any heat taken in at a temperature below τ_1 or rejected at a temperature above τ_2 will have less availability for conversion into work than if taken in at τ_1 and rejected at τ_2, and hence, with a given pair of limiting temperatures, it is essential to maximum efficiency that no heat be taken in by the engine except at the top of the range, and no heat rejected except at the bottom of the range. Further, as we have seen in § 45, when the temperatures at which heat is received and rejected are assigned, an engine attains the maximum of efficiency if it be reversible.

51. It is therefore important to inquire more particularly what kinds of action are reversible in the thermodynamic sense. A little consideration will show that a transfer of heat from the source or to the receiver is reversible only when the working substance is at sensibly the same temperature as the source or the receiver, as the case may be, and an expansion is reversible only when it occurs by the gradual displacement of some part of the containing envelope in such a manner that the expanding fluid does external work on the envelope, and does not waste energy to any sensible extent in setting itself in motion. This excludes what may be termed free expansion, such as that of the gas in Joule's experiment, § 33, and it excludes also what may be called imperfectly-resisted expansion, such as would occur if the fluid were allowed to expand into a closed chamber in which the pressure was less than that of the fluid, or if the piston in a cylinder rose so fast as to cause, through the inertia of the expanding fluid, local variations of pressure throughout the cylinder.

To make a heat-engine, working within given limits of temperature, as efficient as possible we must therefore strive—(1) to take in no heat except at the highest temperature, and to reject no heat except at the lowest temperature; (2) to secure that the working substance shall, when receiving heat, be at the temperature of the body from which the heat comes, and that it shall, when giving up heat, be at the temperature of the body to which heat is given up; (3) to avoid free or imperfectly-resisted expansion. If these conditions are fulfilled the engine is a perfect heat-engine.

The first and second of these conditions are satisfied if in the action of the engine the working substance changes its temperature from τ_1 to τ_2 by adiabatic expansion, and from τ_2 to τ_1 by adiabatic compression, thereby being enabled to take in and reject heat at the ends of the range without taking in or rejecting any by the way. This is the action in Carnot's engine (§ 40).

52. But if we can cause the working substance to deposit heat in some body within the engine while passing from τ_1 to τ_2, in such a manner that the transfer of heat from the substance to this body is reversible (satisfying the second condition above), then when we wish the working substance to pass from τ_2 to τ_1 we may reverse this transfer and so recover the heat that was deposited in this body. This alternate storing and restoring of heat may then take the place of adiabatic expansion and compression, in causing the temperature of the working substance to pass from τ_1 to τ_2 and from τ_2 to τ_1 respectively. The alternate storing and restoring is an action occurring wholly within the engine, and is therefore distinct from the taking in and rejecting of heat by the engine.

53. In 1827 Robert Stirling designed an apparatus, called a *regenerator*, by which this process of alternate storing and restoring of heat could be actually performed. For the present purpose it will suffice to describe the regenerator as a passage through which the working fluid can travel in either direction, whose walls have a very large capacity for heat, so that the amount alternately given to or taken from them by the working fluid causes no more than an insensible rise or fall in their temperature. The temperature of the walls at one end of the passage is τ_1, and this tapers continuously down to τ_2 at the other end. When the working fluid at temperature τ_1 enters the hot end and passes through, it comes out at the cold end at temperature τ_2, having stored in the walls of the regenerator an amount of heat which it will pick up again when passed through in the opposite direction. During the return journey its temperature rises from τ_2 to τ_1. The process is strictly reversible, or rather would be so if the regenerator had an unlimited capacity for heat, if no conduction of heat took place along its walls from the hot to the cold end, and if no loss took place by conduction or radiation from its external surface.

54. Using air as the working substance, and employing his regenerator, Stirling made an engine (to be described later) which, allowing for practical imperfections, is the earliest example of a truly reversible engine. The cycle of operations in Stirling's engine is substantially this:—

(1) Air (which has been heated to τ_1 by passing through the regenerator) is allowed to expand isothermally through a ratio r, taking in heat from a furnace and raising a piston. Heat taken in (per lb of air) $= c\tau_1 \log_{\epsilon} r$.

(2) The air is caused to pass through the regenerator from the hot to the cold end, depositing heat and having its temperature lowered to τ_2, without change of volume. Heat stored in regenerator $= K_p(\tau_1 - \tau_2)$. The pressure of course falls.

(3) The air is then compressed isothermally to its original volume at τ_2 in contact with a refrigerator (or receiver of heat). Heat rejected $= c\tau_2 \log_{\epsilon} r$.

(4) The air is again passed through the regenerator from the cold to the hot end, taking up heat and having its temperature raised to τ_1. Heat restored by the regenerator $= K_p(\tau_1 - \tau_2)$.

$$\text{Efficiency} = \frac{c\tau_1 \log_{\epsilon} r - c\tau_2 \log_{\epsilon} r}{c\tau_1 \log_{\epsilon} r} = \frac{\tau_1 - \tau_2}{\tau_1}$$

The indicator diagram of the action is shown in fig. 13, and a diagram of the engine is given in chap. XIV. Stirling's engine is important, not as a present-day heat-engine (though it has recently been revived in small forms after a long interval of disuse), but because it is typical of the only mode, other than Carnot's plan of adiabatic expansion and compression, by which the action of a heat-engine can be made reversible. Valuable as the regenerator has proved in metallurgy and other industrial processes, its actual application to heat-engines has hitherto been very limited. Another way of using it in air-engines was designed by Ericsson, and attempts have been made by C. W. Siemens and F. Jenkin to apply it to steam-engines and to gas-engines. But almost all actual engines, in so far as they can be said to approach the condition of reversibility, do so, not by the use of the regenerative principle, but by more or less nearly adiabatic expansion and compression after the manner of Carnot's ideal engine.

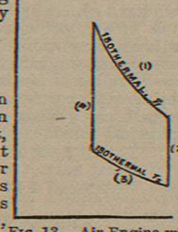

FIG. 13.—Air-Engine with Regenerator (Stirling).

III. PROPERTIES OF STEAM AND THEORY OF THE STEAM-ENGINE.

55. We have now to consider the action of heat-engines in which the working substance is water and water-vapour or steam. The properties of steam are most conveniently stated by referring in the first instance to what happens when steam is formed *under constant pressure*. This is substantially the process which occurs in the boiler of a steam-engine when the engine is at work. To fix the ideas we may suppose that the vessel in which steam is to be formed is a long upright cylinder fitted with a piston which may be loaded so that it exerts a constant pressure on the fluid below. Let there be, to begin with, at the foot of the cylinder a quantity of water (which for convenience of numerical statement we shall take as 1 lb), at any temperature t_0; and let the piston press on the surface of the water with a force of P lb per square foot. Let heat now be applied to the bottom of the cylinder. As it enters the water it will produce the following effects in three stages:—

(1) The temperature of the water rises until a certain temperature t is reached, at which steam begins to be formed. The value of t depends on the particular pressure P which the piston exerts. Until the temperature t is reached there is nothing but water below the piston.

(2) Steam is formed, more heat being taken in. The piston (which is supposed to exert a constant pressure) rises. No further increase of temperature occurs during this stage, which continues until all the water is converted into steam. During this stage the steam which is formed is said to be *saturated*. The volume which the piston encloses at the end of this stage,—the volume, namely, of 1 lb of saturated steam at pressure P (and temperature t),—will be denoted by V in cubic feet.

(3) If after all the water is converted into steam more heat be allowed to enter, the volume will increase and the temperature will rise. The steam is then said to be *superheated*.

Saturated and superheated steam.

56. The difference between saturated and superheated steam may be expressed by saying that if water (at the temperature of the steam) be mixed with steam some of the water will be evaporated if the steam is superheated, but none if the steam is saturated. Any vapour in contact with its liquid and in thermal equilibrium is necessarily saturated. When saturated its properties differ considerably, as a rule, from those of a perfect gas, but when superheated they approach those of a perfect gas more and more closely the farther the process of superheating is carried, that is to say, the more the temperature is raised above t, the temperature of saturation corresponding to the given pressure P.

Relation of pressure and temperature in saturated steam.

57. The temperature t at which steam is formed depends on the value of P. Their relation was determined with great care by Regnault, in a series of classical experiments on which our knowledge of the properties of steam chiefly depends.[1] The pressure of saturated steam rises with the temperature at a rate which increases rapidly in the upper regions of the scale. This will be apparent from the first and second columns of Table II., given on next page, which is compiled from Rankine's reduction of Regnault's results. The first column gives the temperature on the Fahr. scale; the second gives the corresponding pressure in pounds per square inch. Rankine has also expressed the relation of temperature and pressure in saturated steam by the following formula (which is applicable with other constants to other vapours[2]):—

$$\log p = 6{\cdot}1007 - \frac{2732}{\tau} - \frac{396945}{\tau^2}$$

where p is the pressure in pounds per square inch, and τ is the absolute temperature in Fahr. degrees. For most purposes, however, it is more convenient to find the pressure corresponding to a given temperature, or the temperature corresponding to a given pressure, from the table by interpolation.

58. The same table shows the volume V, in cubic feet, occupied by 1 lb of saturated steam at each pressure. This is a quantity the direct experimental measurement of which is of very great difficulty. It may, however, be calculated, from a knowledge of other properties of steam, by a process which will be described later (§ 75). The values of V given in the table were determined by Rankine by means of this process; they agree fairly well with such direct observations of the density of steam as have been hitherto

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[1] *Manual of the Steam Engine and other Prime Movers*, § 243.

[1] *Mem. Inst. France*, 1847, vol. xxi. An account of Regnault's methods of experiment and a statement of his results expressed in British measures will be found in Dixon's *Treatise on Heat*.

[2] *Phil. Mag.*, Dec. 1854, or *Manual of the Steam-Engine*, p. 237.