made.[1] The relation of P to V may be approximately expressed by the formula[2]

$$PV^{\frac{17}{16}} = \text{constant} = 68500 \text{ (nearly)},$$

when P is stated in lb per sq. ft. and V in cub. ft. per lb.

TABLE II.—*Properties of Saturated Steam.*

Temperature.	Pressure.	Volume of 1 lb.	Heat of Formation.	
			H.	h.
Degrees F.	lb per sq. in.	Cub. Ft.	Thermal Units.	Thermal Units.
32	0·085	3390	1091·8	0
41	0·122	2406	1094·5	9·0
50	0·173	1732	1097·3	18·0
59	0·241	1264	1100·0	27·0
68	0·333	935	1102·8	36·0
77	0·452	699	1105·5	45·0
86	0·607	529	1108·2	54·0
95	0·806	405	1111·0	63·0
104	1·06	313	1113·7	72·0
113	1·38	244	1116·5	81·0
122	1·78	192	1119·2	90·1
131	2·27	152·4	1121·9	99·1
140	2·88	122·0	1124·7	108·1
149	3·62	98·45	1127·4	117·1
158	4·51	80·02	1130·2	126·2
167	5·58	65·47	1132·9	135·2
176	6·87	53·92	1135·6	144·3
185	8·38	44·70	1138·4	153·3
194	10·16	37·26	1141·1	162·4
203	12·26	31·26	1143·9	171·4
212	14·70	26·36	1146·6	180·5
221	17·53	22·34	1149·3	189·6
230	20·80	19·03	1152·1	198·7
239	24·54	16·28	1154·8	207·8
248	28·83	14·00	1157·6	216·9
257	33·71	12·09	1160·3	226·0
266	39·25	10·48	1163·1	235·2
275	45·49	9·124	1165·8	244·3
284	52·52	7·973	1168·6	253·5
293	60·40	6·992	1171·3	262·7
302	69·21	6·153	1174·1	271·9
311	79·03	5·433	1176·8	281·1
320	89·86	4·816	1179·5	290·3
329	101·9	4·280	1182·2	299·5
338	115·1	3·814	1185·0	308·7
347	129·8	3·410	1187·7	318·0
356	145·8	3·057	1190·4	327·3
365	163·3	2·748	1193·2	336·6
374	182·4	2·476	1195·9	345·9
383	203·3	2·236	1198·6	352·2
392	225·9	2·025	1201·4	364·5
401	250·3	1·838	1204·1	373·9
410	276·9	1·672	1206·9	383·2
419	305·5	1·525	1209·6	392·6
428	336·3	1·393	1212·4	402·0

Supply of heat in formation of steam under constant pressure.

59. We have next to consider the supply of heat. During the first stage, until the temperature rises from its initial value t_0 to t, the temperature at which steam begins to form under the given pressure, heat is required only to warm the water. Since the specific heat of water is nearly constant, the amount of heat taken in during the first stage is approximately $t - t_0$ thermal units or $J(t - t_0)$ foot-pounds, J being Joule's equivalent (§ 23), and this expression for it will generally serve with sufficient accuracy in practical calculations. More exactly, however, the heat taken in is somewhat greater than this, for Regnault's experiments show that the specific heat of water increases slightly as the temperature rises. In stating the amount of heat required for this first stage, t_0 must be taken as a known temperature; for convenience in numerical statement the temperature 32° F. is usually chosen as an arbitrary starting-point from which the reception of heat is to be reckoned. We shall employ the symbol h to designate the heat required to raise 1 lb of water from 32° F. to the temperature t at which steam begins to form. The value of h in thermal units is given, approximately, by the equation

$$h = t - 32.$$

More exact values, which take account of the variation in the specific heat of water, will be found in the last column of Table II. During the first stage, sensibly all the heat supplied goes to increase the stock of internal energy which the fluid possesses, the amount of external work which is done by the expansion of the fluid being negligible.

60. The heat taken in during the second stage is what is called the *latent heat* of steam, and is denoted by L. Of it a part is spent in doing external work,—namely, P multiplied by the excess of the volume of the steam over the volume of the water,—and the remainder is the difference of internal energy between 1 lb of steam at t and 1 lb of water at t. The volume of 1 lb of water, at such temperatures as are usual in steam-engines, is nearly 0·017 cubic feet. We may therefore write the external work (in foot-pounds) done during the production of 1 lb of steam under constant pressure P,—

$$\text{External work} = P(V - 0{\cdot}017).$$

61. Adding together the heat taken in during the first and second stages we have a quantity designated by H and called the *total heat* of 1 lb of saturated steam:—

$$H = h + L.$$

Regnault's values of H are very accurately expressed (in thermal units) by the formula

$$H = 1082 + 0{\cdot}305t.$$

They are given in the fourth column of Table II. A similar formula gives approximate values of L, exact enough for use in practical calculations,—

$$L = 1114 - 0{\cdot}7t.$$

The total heat of formation of 1 lb of steam, when formed under constant pressure from water at any temperature t_0, is of course $H - h_0$, where h_0 corresponds to t_0.

Internal energy.

62. Of the whole latent heat of steam, L, the part $P(V - 0{\cdot}017)$ is, as has been said above, spent in doing external work. The remainder (in foot-pounds)—

$$JL - P(V - 0{\cdot}017)—$$

is the change of internal energy which the substance undergoes during evaporation. This quantity, for which it is convenient to have a separate symbol, will be denoted by ρ in thermal units, or $J\rho$ in foot-pounds. In dealing with the heat required to produce steam we adopted the state of water at 32° F. as an arbitrary starting-point from which to reckon the reception of heat. In the same way it is convenient to use this arbitrary starting-point in reckoning what may be called the *internal energy* of the substance, which is the excess of the heat taken in over the external work done by the substance during its reception of heat. Thus the internal energy I of 1 lb of saturated steam at pressure P is equal to the total heat H, less that part of the total heat which is spent in doing external work, or (in foot-pounds)

$$JI = JH - P(V - 0{\cdot}017),$$

or

$$I = L + h - P(V - 0{\cdot}017)/J = h + \rho.$$

The notion of internal energy is useful in calculating the heat taken in or rejected by steam during any stage of its expansion or compression in an engine. When a working substance passes from one condition to another, its gain or loss of heat is determined by the equation

Heat taken in = increase of internal energy + external work.

Any of the terms of this equation may be negative; the last term is negative when work is done, not by, but upon the substance.

Heat of formation under [illegible] conditions.

63. The same equation gives the means of finding the amount of heat required to form steam under any assigned conditions, in place of the condition assumed at the beginning of this chapter, where the formation of steam under constant pressure was considered. Whatever be the condition as to pressure under which the process of formation is carried on, the total heat required is the sum of the internal energy of the steam when formed and the work done by the substance during the process. Thus in general

$$\text{Heat of formation} = I + J^{-1}\int P dV,$$

the limits of integration being the final volume of the steam and the original volume of the water. When steam is formed in a closed vessel of constant volume no external work is done; the heat of formation is then equal to the internal energy, and is less than the total heat of formation (H) of steam, when formed at a constant pressure equal to the pressure reached in the vessel, by the quantity $P(V - 0{\cdot}017)$.

64. In calculations which relate to the action of steam in engines we have generally to deal, not with *dry* saturated steam, but with *wet* steam, or steam which either carries in suspension, or is otherwise mixed with, a greater or less proportion of water. In every such mixture the steam and water have the same temperature, and the steam is saturated. The dryness of wet steam is measured by the proportion q of dry steam in each pound of the mixed substance. When that is known it is easy to determine the other physical constants: thus—

Latent heat of 1 lb of wet steam $= qL$;
Total heat of 1 lb of wet steam $= h + qL$;
Volume of 1 lb of wet steam $= qV + (1 - q)0{\cdot}017$.
$= qV$ very nearly,
unless the steam is so wet as to consist mainly of water;
Internal energy of 1 lb of wet steam $= h + q\rho$.

65. Steam is superheated when its temperature is raised, in any manner, above the temperature corresponding to saturation at the actual pressure. When much superheated, steam behaves like a

[1] See Fairbairn and Tate, "On the Density of Steam at Different Temperatures," *Phil. Trans.*, vol. cl., 1860.

[2] This is Rankine's formula. Zeuner gives $PV^{1{\cdot}0646}$=constant.

perfect gas, and may be called "steam gas." It then follows the equation

$$PV = 85{\cdot}5\tau,$$

and the specific heat at constant pressure, K_p, is 371 foot-pounds or 0·48 thermal unit. At very low temperatures steam approximates closely to the condition of a perfect gas when very slightly superheated, and even when saturated; at high temperatures a much greater amount of superheating is necessary to bring about an approach to the perfectly gaseous state. The total heat required for the production of superheated steam under any constant pressure, when the superheating is sufficient to bring the steam to the state of steam gas, may therefore be reckoned by taking the total heat of saturated steam at a low temperature and adding to it the product of K_p into the excess of temperature above that. Thus Rankine treating saturated steam at 32° F. as a gas, gives the formula

$$H' = 1092 + 0{\cdot}48(t' - 32)$$

to express the heat of formation (under any constant pressure) of superheated steam, at any temperature t' which is so much above the temperature of saturation corresponding to the actual pressure that the steam may be treated as a perfect gas. Calculated from its chemical composition, the density of steam gas should be 0·622 times that of air at the same pressure and temperature. The value of γ or K_p/K_v for steam gas is 1·3. These formulas, dealing as they do with steam which is so highly superheated as to be perfectly gaseous, fail to apply to high-pressure steam that is heated but little above its temperature of saturation. The relation of pressure to volume and temperature in the region which lies between the saturated and the perfectly gaseous states has been experimented on by Hirn.[1] Formulas which are applicable with more or less accuracy to steam in either the saturated or superheated condition have been devised by Hirn, Zeuner,[2] Ritter,[3] and others.

Isothermal expansion of steam.

66. The expansion of volume which occurs during the conversion of water into steam under constant pressure—the second stage of the process described in § 55—is isothermal. From what has been already said it is obvious that steam, or any other saturated vapour, can be expanded or compressed isothermally only when wet, and that evaporation (in the one case) or condensation (in the other) must accompany the process. Isothermal lines for a working substance which consists of a liquid and its vapour are straight lines of uniform pressure.

Adiabatic expansion of steam.

67. The form of adiabatic lines for substances of the same class depends not only on the particular fluid, but also on the proportion of liquid to vapour in the mixture. In the case of steam, it has been shown by Rankine and Clausius that if steam initially dry be allowed to expand adiabatically it becomes wet, and if initially wet (unless very wet[4]) it becomes wetter. A part of the steam is condensed by the process of adiabatic expansion, at first in the form of minute particles suspended throughout the mass. The temperature and pressure fall; and, as that part of the substance which remains uncondensed is saturated, the relation of pressure to temperature throughout the expansion is that which holds for saturated steam. The following formula, proved by Rankine[5] and Clausius[6] (see § 75), serves to calculate the extent to which condensation takes place during adiabatic expansion, and so allows the relation of pressure to volume to be determined.

Before expansion, let the initial dryness of the steam be q_1 and its absolute temperature τ_1. Then, if it expand adiabatically until its temperature falls to τ, its dryness after expansion is

$$q = \frac{\tau}{L}\left(\frac{q_1 L_1}{\tau_1} + \log_e \frac{\tau_1}{\tau}\right).$$

L_1 and L are the latent heats (in thermal units) of 1 lb of steam before and after expansion respectively. When the steam is dry to begin with, $q_1 = 1$.

This formula is easily applied to the construction of the adiabatic curve when the initial pressure and the pressure after expansion are given, the corresponding values τ and L being found from the table. It is less convenient if the data are the initial pressure and the initial and final volumes, or the initial pressure and the ratio of expansion r. An approximate formula more appropriate in that case is

$$Pv^n = \text{constant, or } P/P_1 = (v/v_1)^n = r^n.$$

Here v and v_1 denote the volume of 1 lb of the mixture of steam and water before and after expansion respectively, and are to be distinguished from V and V_1, which we have already used to denote the volume of 1 lb of dry saturated steam at pressures P and P_1. The index n has a value which depends on the degree of initial dryness q_1.

According to Zeuner,[7] $n = 1{\cdot}035 + 0{\cdot}1q_1$, so that for

$q_1 = 1$	0·95	0·9	0·85	0·8	0·75	0·7
$n = 1{\cdot}135$	1·130	1·125	1·120	1·115	1·110	1·105

Rankine gave for this index the value $\frac{10}{9}$, which is too small if the steam be initially dry. He determined it by examining the expansion curves of indicator diagrams taken from working steam engines; but, as we shall see later, the expansion of steam in an actual engine is by no means adiabatic, on account of the transfer of heat which goes on between the working fluid and the metal of the cylinder and piston. When it is desired to draw an adiabatic curve for steam, that value of n must be chosen which refers to the degree of dryness at the beginning of the expansion.

68. We are now in a position to study the action of a heat-engine employing steam as the working substance. To simplify the first consideration as far as possible, let it be supposed that we have, as before, a long cylinder composed of non-conducting material except at the base, and fitted with a non-conducting piston; also a source of heat A at some temperature τ_1; a receiver of heat, or, as we may now call it, a condenser C, at a lower temperature τ_2; and a non-conducting cover B (as in § 40). Then we can perform Carnot's cycle of operations as follows. To fix the ideas, suppose that there is 1 lb of water in the cylinder to begin with, at the temperature τ_1:—

(1) Apply A, and allow the piston to rise. The water will take in heat and be converted into steam, expanding isothermally at constant pressure P_1. This part of the operation is shown by the line ab in fig. 14.

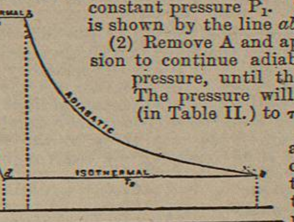

FIG. 14.—Carnot's Cycle with water and steam for working substance.

(2) Remove A and apply B. Allow the expansion to continue adiabatically (bc), with falling pressure, until the temperature falls to τ_2. The pressure will then be P_2, corresponding (in Table II.) to τ_2.

(3) Remove B, apply C, and compress. Steam is condensed by rejecting heat to C. The action is isothermal, and the pressure remains P_2. Let this be continued until a certain point d is reached, after which adiabatic compression will complete the cycle.

(4) Remove C and apply B. Continue the compression, which is now adiabatic. If the point d has been rightly chosen, this will complete the cycle by restoring the working fluid to the state of water at temperature τ_1.

The indicator diagram for the cycle is given in fig. 14, as calculated by the help of the equations in § 67 and of Table II. for a particular example, in which $p_1 = 90$ lb per square inch ($\tau_1 = 781$), and the expansion is continued down to the pressure of the atmosphere, 14·7 lb per square inch ($\tau_2 = 673$). Since the process is reversible, and since heat is taken in only at τ_1 and rejected only at τ_2, the efficiency is $(\tau_1 - \tau_2)/\tau_2$. The heat taken in per lb of the fluid is L_1, and the work done is $L_1(\tau_1 - \tau_2)/\tau_1$, a result which may be used to check the calculation of the diagram.

Efficiency of a perfect steam-engine. Limits of temperature.

69. If the action here described could be realized in practice, we should have a thermodynamically perfect steam-engine using saturated steam. The fraction of the heat supplied to it which such an engine would convert into work would depend simply on the temperature, and therefore on the pressure, at which the steam was produced and condensed. The temperature of condensation is limited by the consideration that there must be an abundant supply of some substance to absorb the rejected heat; water is actually used for this purpose so that τ_2 has for its lower limit the temperature of the available water-supply.

To the higher temperature τ_1 and pressure P_1 no limit can be set except such as is brought about in practice by the mechanical difficulties, with regard to strength and to lubrication, which attend the use of high-pressure steam. By a very special construction of engine and boiler Mr Perkins has been able to use steam with a pressure as high as 500 lb per square inch; with engines of the usual construction the value ranges from 190 lb downwards.

If the temperature of condensation be taken as 60° F., as a lower limit, the efficiency of a perfect steam-engine, using saturated steam, would depend on the value of P_1, the absolute pressure of production of the steam, as follows:—

For perfect steam-engine, with condensation at 60° F.,

P_1 in lb per square inch being	40	80	120	160	200
Highest ideal efficiency =	·284	·326	·350	·368	·381

But it must not be supposed that these values of the efficiency are actually attained, or are even attainable. Many causes conspire to prevent steam-engines from being thermodynamically perfect, and some of the causes of imperfection cannot be removed. These numbers will serve, however, as a standard of comparison in judging of

[1] *Théorie Mécanique de la Chaleur.*

[2] *Ztschr. d. Vereins deutscher Ingenieure*, vol. xi.

[3] *Wied. Ann.*, 1878. For a discussion of several of these formulas, see a paper by H. Dyer, *Trans. Inst. of Engineers and Shipbuilders in Scotland*, 1885.

[4] Prof. Cotterill, in his *Treatise on the Steam-Engine*, § 79, has calculated (using the equation which follows in the text) that, when a mixture of steam and water expands adiabatically, steam condenses if the proportion of steam be, roughly, over 50 per cent., but water is evaporated if the proportion of steam be less than about 50 per cent. The exact proportion depends on the initial pressure.

[5] *Steam-Engine*, § 281.

[6] *Mechanical Theory of Heat* (tr. by W. R. Browne), chap. vi. § 12.

[7] *Grundzüge der Mech. Wärmetheorie*, p 342. See also Grashof, *Resultate aus der Mech. Wärmetheorie*, § 37. In the adiabatic compression of wet steam $n = 1{\cdot}034 + 0{\cdot}11q_1$, where q_1 is the dryness at the beginning of compression.

the performance of actual engines, and as setting forth the advantage of high-pressure steam from the thermodynamic point of view.

70. As a contrast to the ideally perfect steam-engine of § 68 we may next consider a cyclic action such as occurred in the early engines of Newcomen or Leupold, when steam was used non-expansively,—or rather, such an action as would have occurred in engines of this type had the cylinder been a perfect non-conductor of heat. Let the cycle of operations be this :—

(1) Apply A and evaporate the water as before at P_1. Heat taken in $=L_1$.

(2) Remove A and apply C. This at once condenses a part of the steam, and reduces the pressure to P_2.

(3) Compress at P_2, in contact with C, till condensation is complete, and water at τ_2 is left.

(4) Remove B and apply A. This heats the water again to τ_1 and completes the cycle. Heat taken in $=h_1-h_2$.

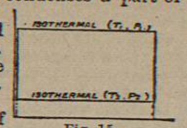

Fig. 15.

The indicator diagram for this series of operations is shown in fig. 15.

Here the action is not reversible. To calculate the efficiency, we have

$$\frac{\text{Work done}}{\text{Heat taken in}}=\frac{(P_1-P_2)(V_1-0{\cdot}017)}{J(L_1+h_1-h_2)}.$$

The values of this will be found to range from 0·067 to 0·072 for the values of P_1 which are stated in § 69, when the temperature of condensation is 60° F.

Engine with separate organs.

71. In the ideal engine represented in fig. 14 the functions of boiler, cylinder, and condenser are combined in a single vessel; but after what has been said in chap. II. it is scarcely necessary to remark that, provided the working substance passes through the same cycle of operations, it is indifferent whether these are performed in several vessels or in one. To approach a little more closely the conditions that hold in practice, we may think of the engine which performs the cycle of § 70 as consisting of a boiler A (fig. 16) kept at τ_1, a non-conducting cylinder and piston B, a surface condenser C kept at τ_2, and a feed-pump D which restores the condensed water to the boiler. Then for every pound of steam supplied and used non-expansively as in § 70, we have work done on the piston $=(P_1-P_2)V_1$; but an amount of work has to be expended in driving the feed-pump $=(P_1-P_2)\,0{\cdot}017$. Deducting this, the net work done per lb of steam is the same as before, and the heat taken in is also the same. An indicator diagram taken from the cylinder would give the area *efgh* (fig. 17), where $oe=P_1$, $ef=V_1$, $oh=P_2$; an indicator diagram taken from the pump would give the negative area *hjie*, where *ei* is the volume of the feed-water, or 0·017 cub. ft. The difference, namely, the shaded area, is the diagram of the complete cycle gone through by each pound of the working substance. In experimental measurements of the work done in steam-engines, only the action which occurs within the cylinder is shown on the indicator diagram. From this the work spent on the feed-pump is to be subtracted in any accurate determination of the thermodynamic efficiency. If the feed-water is at any temperature τ_0 other than that of the condenser as assumed in § 70, it is clear that the heat taken in is H_1-h_0 instead of H_1-h_2.

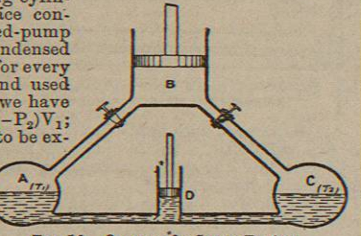

Fig. 16.—Organs of a Steam-Engine.

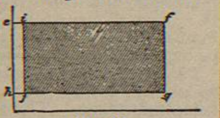

Fig. 17.

72. We have now to inquire how nearly, with the engine of fig. 16 (that is to say, with an engine in which the boiler and condenser are separate from the cylinder), we can approach the reversible cycle of § 68. The first stage of that cycle corresponds to the *admission* of steam from the boiler into the cylinder. Then the point known as the point of *cut-off* is reached, at which admission ceases, and the steam already in the cylinder is allowed to expand, exerting a diminishing pressure on the piston. This is the second stage, or the stage of *expansion*. The process of expansion may be carried on until the pressure falls to that of the condenser, in which case the expansion is said to be complete. At the end of the expansion *release* takes place, that is to say, communication is opened with the condenser. Then the return stroke begins, and a period termed the *exhaust* occurs, that is to say, steam passes out of the cylinder, into the condenser, where it is condensed at pressure P_2, which is felt as a *back pressure* opposing the return of the piston. So far, all has been essentially reversible, and identical with the corresponding parts of Carnot's cycle.

But we cannot complete the cycle as Carnot's cycle was completed. The existence of a separate condenser makes the fourth stage, that of adiabatic compression, impracticable, and the best we can do is to continue the exhaust until condensation is complete, and then return the condensed water to the boiler by means of the feed-pump.

It is true that we may, and in actual practice do, stop the exhaust before the return stroke is complete, and compress that portion of the steam which remains below the piston, but this does not materially affect the thermodynamic efficiency; it is done partly for mechanical reasons, and partly to avoid loss of power through clearance (see chap. IV.). In the present instance it is supposed that there is no clearance, in which case this compression is out of the question. The indicator diagram given by a cylinder in which steam goes through the action described above is shown to scale in fig. 18 for a particular example, in which it is supposed that 1 cubic foot of dry saturated steam is admitted at an absolute pressure of 90 lb per square inch, and is expanded twelve times, or down to a pressure of 5·4 lb per square inch, at which pressure it is discharged to the condenser.

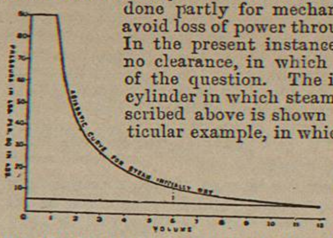

Fig. 18.—Ideal Indicator Diagram for Steam used expansively.

As we have assumed the cylinder to be non-conducting, and the steam to be initially dry, the expansion follows the law $PV^{1{\cdot}135}=$ constant. The advantage of expansion is obvious, that part of the diagram which lies under the curve being so much clear gain.

Efficiency of engine working expansively.

73. To calculate the efficiency, we have

Work done per lb during admission $=P_1V_1$;

„ „ during expansion to volume $rV_1=\dfrac{P_1V_1-P_2rV_1}{n-1}$,

(by § 36), $=(P_1V_1-P_2rV_1)/0{\cdot}135$;

Work spent during return stroke $=P_2rV_1$;

„ „ on the feed-pump $=(P_1-P_2)0{\cdot}017$;

Heat taken in $=H_1-h_0$.

Incomplete expansion.

74. These expressions refer to complete expansion. When the expansion is incomplete, as it generally is, the expression given above for the work done during expansion still applies if we take P_2 to be the pressure at the end of expansion, while the work spent on the steam during the back-stroke is P_brV_1 and that spent on the feed-pump is $(P_1-P_b)0{\cdot}017$, P_b being the back pressure. Incomplete expansion is illustrated by the dotted line in fig. 18.

It is easy, by the aid of §§ 64 and 67 to extend these calculations to cases where the steam, instead of being initially dry, is supposed to have any assigned degree of wetness. The efficiency which is calculated in this way, which for the present purpose may be called the theoretical efficiency corresponding to the assumed conditions of working, is always much less than the ideal efficiency of a perfect engine, since the cycle we are now dealing with is not reversible. But even this theoretical efficiency, short as it falls of the ideal of a perfect engine, is far greater than can be realized in practice when the same boiler and condenser temperatures are used, and the same ratio of expansion. The reasons for this will be briefly considered in the next chapter; at present the fact is mentioned to guard the reader from supposing that the results which the above formulas give apply to actual engines.

Calculation of density of saturated steam.

75. The results of § 68 have been turned to account by Rankine and Clausius for the purpose of deducing the density of steam from other properties which admit of more exact direct measurement. Let the perfect steam-engine there described work through a very small interval of temperature $\Delta\tau$ between two temperatures τ and $\tau-\Delta\tau$. The efficiency is $\Delta\tau/\tau$, and the work done (in foot-lbs.) is $JL\Delta\tau/\tau$. The indicator diagram is now reduced to a long narrow strip, whose length is $V-0{\cdot}017$ and its breadth ΔP, the difference in pressure between steam at temperatures τ and $\tau-\Delta\tau$. Hence the work done is also $\Delta P(V-0{\cdot}017)$, and therefore

$$V-0{\cdot}017=\frac{JL}{\tau}\cdot\frac{\Delta\tau}{\Delta P}.$$

Here $\dfrac{\Delta\tau}{\Delta P}$, or (in the limit) $\dfrac{d\tau}{dP}$, is the rate of increase of temperature with increase of pressure in saturated steam at the particular temperature τ. It may be found roughly from Table II., p. 484, or more exactly by differentiating the equation given in § 57. L is also known, and hence the value of V corresponding to any assigned temperature may be calculated with a degree of accuracy which it would be difficult to reach in direct experiment. The volumes given in the Table are determined in this way.[1]

[1] The result of § 75 may be applied as follows to give the formula of § 67 for the adiabatic expansion of wet steam. For brevity we may write $V-0{\cdot}017=u$. In adiabatic expansion the work done is equal to the loss of internal energy, or

$$Pd(qu)=-JdI=-Jd(h+q\rho).$$

Since $dh=d\tau$, and $\rho=L-Pu/J$, this may be written $Jd\tau+Jd(qL)-qudP=0$.

By § 75, $udP=\dfrac{JL}{\tau}d\tau$; hence $1+\dfrac{d}{d\tau}(qL)-\dfrac{qL}{\tau}=0$;

and by integration,

$$\log_e\tau+qL/\tau=\text{constant}=\log_e\tau_1+q_1L_1/\tau_1,$$

which is the equation of § 67.

IV. Actual Behaviour of Steam in the Cylinder.

76. In fig. 18 we have what may be called a first approximation to the theoretical indicator diagram of a steam-engine. In the action then described it was assumed—(1) that the steam supplied was dry and saturated, and had during admission the full (uniform) pressure of the boiler P_1; (2) that there was no transfer of heat to or from the steam except in the boiler and in the condenser; (3) that after more or less complete expansion all the steam was discharged by the return stroke of the piston, during which the back pressure was the (uniform) pressure in the condenser P_2; (4) that the whole volume of the cylinder was swept through by the piston. It remains to be seen how far these assumptions are untrue in practice, and how far the efficiency is affected in consequence.

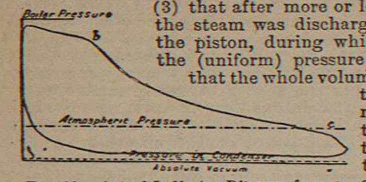

Fig. 19.—Actual Indicator Diagram from a Condensing Steam-Engine.

The actual conditions of working differ from these in the following main respects, some of which are illustrated by the practical indicator diagram of fig. 19, which is taken from an actual engine.

Wire-drawing.

77. Owing to the resistance of the ports and passages, and to the inertia of the steam, the pressure within the cylinder is less than P_1 during admission and greater than P_2 during exhaust.

Moreover P_1 and P_2 are themselves not absolutely uniform, and P_2 is greater than the pressure of steam at the temperature of the condenser, on account of the presence of air in the condenser.

During admission the pressure of steam in the cylinder is less than the boiler pressure by an amount which increases as the piston advances, on account of the increased velocity of the piston's motion and the consequent increased demand for steam. When the ports and passages offer much resistance the steam is expressively said to be throttled or "wire-drawn." Wire-drawing of steam is in fact a case of imperfectly-resisted expansion (§ 51). The steam is dried by the process to a small extent, and if initially dry it becomes superheated. In an indicator diagram wire-drawing causes the line of admission to lie below a line drawn at the boiler pressure, and to slope downwards. In fairly good practical instances the mean absolute pressure during admission is about nine-tenths of the pressure in the boiler.

In the same way, during the exhaust the actual back pressure exceeds the pressure in the condenser (shown by a dotted line in fig. 19) by an amount depending on the freedom with which the steam makes its exit from the cylinder. In condensing engines with a good vacuum the actual back pressure is from 3 to 5 lb per square inch, and in non-condensing engines it is 16 to 18 lb in place of the mere 14·7 lb which is the pressure of the atmosphere. The excess of back pressure may be greatly increased by the presence of water in the cylinder. The effects of wire-drawing do not stop here. The valves open and close more or less slowly; the points of cut-off and release are therefore not absolutely sharp, and the diagram has rounded corners at *b* and *c* in place of the sharp angles which mark those events in fig. 18. For this reason release is allowed in practice to occur a little before the end of the forward stroke, hence the toe of the diagram takes a form like that shown in fig. 19. The sharpness of the cut-off, and to a less extent the sharpness of the release, depends greatly on the kind of valves and valve-gear used; valves of the Corliss type (to be described later), which are noted for the suddenness with which admission of steam is stopped, have the merit amongst others of producing a very sharply defined diagram.

Clearance.

78. When the piston is at either end of its stroke there is a small space left between it and the cylinder cover. This space, together with the volume of the passage or passages leading thence to the steam and exhaust valves, is called the *clearance*. It constitutes a volume through which the piston does not sweep, but which is nevertheless filled with steam when admission occurs, and the steam in the clearance forms a part of the whole steam which expands after the supply from the boiler is cut-off. If AC be the volume swept through by the piston up to release, OA the volume of the clearance, and AB the volume swept through during admission, the apparent ratio of expansion is AC/AB, but the real ratio is (OA + AC)/(OA + AB).

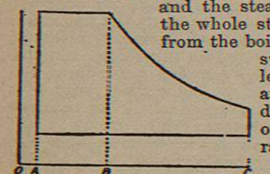

Fig. 20.—Effect of Clearance.

Clearance must obviously be taken account of in any calculation of curves of expansion. It is conveniently allowed for in indicator diagrams by shifting the line of no volume back through a distance corresponding to the clearance (fig. 20). In actual engines OA is from $\frac{1}{10}$ to $\frac{1}{50}$ of the volume of the cylinder.

79. Clearance affects the thermodynamic efficiency of the engine chiefly by altering the consumption of steam per stroke, and its influence depends materially on the *compression* (§ 72). If during the back stroke the process of exhaust is discontinued before the end, and the remaining steam is compressed, this cushion of steam will finally fill the volume of the clearance; and by a proper selection of the point at which compression begins the pressure of the cushion may be made to rise just up to the pressure at which steam is admitted when the valve opens. This may be called complete compression, and when it occurs the existence of clearance has no direct effect on the consumption of steam nor on the efficiency; the whole fluid in the cylinder may then be thought of as consisting of two parts,—a permanent cushion which is alternately expanded and compressed without net gain or loss of work, and the working part proper, which on admission fills the volume AB (fig. 20), and which enters and leaves the cylinder in each stroke. But if compression be incomplete or absent there is, on the opening of the admission valve, an inrush of steam to fill up the clearance space. This increases the consumption to an extent which is only partly counterbalanced by the increased area of the diagram, and the result is that the efficiency is reduced. The action is, in fact, a case of unresisted expansion (§ 51), and consequently tends, so far as its direct effects go, to make the engine less than ever reversible. It is to be noted, however, that by such unresisted expansion the entering steam is dried to some extent, and this helps in a measure to counteract the cause of loss which will be described below. Compression has the mechanical advantage that it obviates the shock which the admission of steam would otherwise cause, and that by giving the piston work to do while its velocity is being rapidly reduced it reduces those stresses in the mechanism which are due to the inertia of the reciprocating parts.

Influence of cylinder walls.

80. The third and generally by far the most important element of difference between the action of a real engine and that of our hypothetical engine is that alluded to at the end of chap. I., the difference which proceeds from the fact that the cylinder and piston are not non-conductors. As the steam fluctuates in temperature there is a complex give-and-take of heat between it and the metal it touches, and the effects of this, though not very conspicuous on the indicator diagram, have an enormous influence in reducing the efficiency by increasing the consumption of steam. Attention was drawn to this action by Mr D. K. Clark as early as 1855 (*Railway Machinery*, or art. Steam-Engine, *Ency. Brit.*, 8th edition[1]), and the results of his experiments on locomotives were confirmed some years later by Mr Isherwood's trials of the engines of the United States steamer "Michigan." Rankine in his classical work on the steam-engine notices the subject only very briefly, and takes no account of the action of the cylinder walls in his calculations. Its importance has now been established beyond dispute, notably by the experiments of Messrs Loring and Emery on the engines of certain revenue steamers of the United States,[2] and by a protracted series of investigations carried out by M. Hallauer and other Alsatian engineers under the direction of Hirn,[3] whose name should be specially associated with the rational analysis of engine tests. In the next chapter some account will be given of how steam-engines are experimentally examined and how (following Hirn) we may deduce the exchanges of heat which occur between the steam and the cylinder throughout the stroke. The following is, in general terms, what experiments with actual engines show to take place.

Initial condensation.

81. When steam is admitted at the beginning of the stroke, it finds the metallic surfaces of the cylinder and piston chilled by having been in contact with low-pressure steam during the exhaust of the previous stroke. A portion of it is therefore liquefied, and, as the piston advances, more and more of the chilled cylinder surface is exposed and more and more of the hot steam is condensed. At the end of the admission, when communication with the boiler is cut off, the cylinder consequently contains a film of water spread over the exposed surface, in addition to saturated steam. The boiler has therefore been drawn upon for a supply greater than that corresponding to the volume of steam in the admission space. The importance of this will be obvious from the fact, demonstrated by experiment, that the steam which is thus condensed during admission frequently amounts to 30 and even 50 per cent. of the whole quantity that comes over from the boiler.

82. Then, as expansion begins, more cold metal is uncovered, and some of the remaining steam is condensed upon it. But the pressure of the steam now falls, and the layer of water which has been previously deposited begins to be re-evaporated as soon as the temperature of the expanding steam falls below that of the liquid layer. On the whole, then, the amount of water present will increase during the earliest part of the expansion, but a stage will soon be reached when the condensation which occurs on the newly exposed metal is balanced by re-evaporation of older portions of the layer. The percentage of water present is then a maximum; and from this point onwards the steam becomes more and more dried by re-evaporation of the layer.

83. If the amount of initial condensation has been small this

[1] See also *Min. Proc. Inst. C.E.*, vol. lxxii. p. 275.

[2] A useful abstract of Messrs Loring and Emery's reports is given in *Engineering*, vols. xix. and xxi., and in Mr Maw's *Recent Practice in Marine Engineering*.

[3] *Bull. Soc. Industr. de Mulhouse*, from 1877. For other references see chap. V.

re-evaporation may be complete before release occurs. Very usually, however, there is still an undried layer at the end of the forward stroke, and the process of re-evaporation continues during the return stroke, while exhaust is taking place. In extreme cases, if the amount of initial condensation has been very great, the cylinder walls may fail to become quite dry even during the exhaust, and a residue of the layer of condensed water may either be carried over as water into the condenser, or, if the exhaust valves are so badly arranged as to prevent its discharge, this unevaporated residue may gather in the cylinder, requiring perhaps the drain-cocks to be left open to allow of its escape. When any water is retained in this way the initial condensation is enormously increased, for the hot steam then meets not only cold metal but cold water. The latter causes much condensation, partly because of its high specific heat, and partly because it is brought into intimate mixture with the entering steam.

84. Apart, however, from this extreme case, whatever water is re-evaporated during expansion and exhaust takes heat from the metal of the cylinder, and so brings it into a state that makes condensation inevitable when steam is next admitted from the boiler. Mere contact with low-pressure steam during the exhaust stroke would cool the metal but little; the cooling which actually occurs is due mainly to the re-evaporation of the condensed water. Thus if an engine were set in action, after being heated beforehand to the boiler temperature, the cylinder would be only slightly cooled during the first exhaust stroke, and little condensation would occur during the next admission. But the metal would be more cooled in the subsequent expansion and exhaust, since it would part with heat in re-evaporating this water. In the third admission more still would be condensed, and so on, until a permanent régime would be established in which condensation and re-evaporation were exactly balanced. The same permanent régime is reached when the engine starts cold.

Wetness of steam during expansion.

85. The wetness of the working fluid to which the action of the walls of the cylinder gives rise is essentially superficial. A film of water forms on the walls, but except for this the body of the steam remains dry, until (by adiabatic or nearly adiabatic expansion) it becomes wet throughout its volume. The water formed by the act of expansion takes form as a mist diffused throughout the steam, and on it the sides of the cylinder exert practically no influence. This latter wetness is in fact increasing while the substance, as a whole, is getting dried by the re-evaporation of the liquid film. During expansion the working substance may be regarded as made up of two parts,—a core of steam, which is expanding adiabatically but is at the same time receiving additions to its amount in the form of saturated steam from the liquid layer, and a liquid layer which is turning into steam.

Waste of heat.

86. From a thermodynamic point of view all initial condensation of the steam is bad, for, however early the film be re-evaporated, this can take place only after its temperature has cooled below that of the boiler. The process consequently involves a misapplication of heat, since the substance, after parting with high temperature heat, takes it up again at a temperature lower than the top of its range. This causes a loss of efficiency (chap. II.), and the loss is greater the later in the stroke re-evaporation occurs. The heat that is drawn from the cylinder by re-evaporation of the condensed film becomes less and less effective for doing work as the end of the expansion is approached, and finally, whatever evaporation continues during the back stroke is an unmitigated source of waste. The heat it takes from the cylinder does no work; its only effect, indeed, is to increase the back pressure by augmenting the volume of steam to be expelled. A small amount of initial condensation reduces the efficiency of the engine but little; a large amount causes a much more than proportionally larger loss.

Effect of jacket.

87. The action of the cylinder walls is increased by any loss of heat which the engine suffers by radiation and conduction from its external surface. The entering steam has then to give up enough heat to provide for this waste, as well as enough to produce the subsequent re-evaporation of the condensed film. The consequence is that more steam is initially condensed. The loss of efficiency due to this cause will therefore be greater in an unprotected cylinder than in one which is well lagged or covered with non-conducting material. On the other hand, if the engine have a steam-jacket the deleterious action of the walls is reduced. The working substance is then on the whole gaining instead of losing heat by conduction during its passage through the cylinder. The jacket accelerates the process of re-evaporation and tends to make it finish at a point in the stroke when the temperature of the steam is still comparatively high. When the process is complete the cylinder walls give up very little additional heat to the steam during the remainder of the expansion and exhaust, for conduction and radiation between dry steam and the metal of the cylinder are incompetent to cause any considerable exchange of heat. The earlier, therefore, that evaporation is complete the less is the metal chilled, and the less is the subsequent condensation. Moreover, after this stage in the stroke has passed, a steam-jacket continues to give heat to the metal during the remainder of the double stroke, and so warms it to a temperature more nearly equal to that of the boiler steam before the next admission takes place.

88. Thus a steam-jacket, though in itself a thermodynamically imperfect contrivance, inasmuch as its object is to supply heat to the working substance at a temperature lower than the source, acts beneficially by counteracting, to some extent, the more serious misapplication of heat which occurs through the alternate cooling and heating of the cylinder walls. The heat which a jacket communicates to working steam often increases the power of an engine to an extent far greater than corresponds to the extra supply of heat which the jacket itself requires. Besides its thermodynamic effect a jacket has the drawback that it increases waste by external radiation, since it both enlarges the area of radiating surface and raises its temperature; notwithstanding this, however, many experiments have shown that in large and especially in slow-running engines, the influence of a steam-jacket on the efficiency is, in general, good; and this is to be ascribed to the fact that it reduces, though it does not entirely remove, the evils of initial condensation. To be effective, however, jackets must be well drained and kept full of "live" steam, instead of being, as many are, traps for condensed water or for air.

89. It is interesting to notice, in general terms, the effects which certain variations of the conditions of working may be expected to produce on the loss that occurs through the action of the cylinder walls. Initial condensation will be increased by anything that augments the range of temperature through which the inner surface of the cylinder fluctuates in each stroke, or that exposes a larger surface of metal to the action of a given quantity of steam, or that prolongs the contacts in which heat is exchanged. The influence of time is specially important; for it must be borne in mind that the whole action depends on the rate at which heat is conducted into the substance of the metal. The changes of temperature which the metal undergoes are in every case mainly superficial; the alternate heating and cooling of the inner surface initiates waves of high and low temperature in the iron whose effects are sensible only to a small depth; and the faster the alternate states succeed each other the more superficial are the effects. In an engine making an indefinitely large number of strokes per minute the cylinder sides would behave like non-conductors and the action of the working substance would be adiabatic. — Influence of speed.

We may conclude, then, that in general an engine running at a high speed will have a higher thermodynamic efficiency than the same engine running at a low speed, all the other conditions of working being the same in both cases.

Again, as regards range of temperature, the influence of the cylinder walls will be greater (other things being equal) with high than with low pressure steam, and in condensing than in non-condensing engines. On the other hand, high pressure has the good effect of reducing the surface of metal exposed to the action of each pound of steam.

In large engines the action of the walls will be less than in small engines, since the proportion of wall surface to cylinder volume is less. This conclusion agrees with the well-known fact that no small engines achieve the economy that is easily reached with larger forms, especially with large marine engines, which eclipse all others in the matter of size. — Influence of size.

Cylinder condensation is increased when the ratio of expansion is increased, all the other circumstances of working being left unaltered. The metal is then brought into more prolonged contact with low-temperature steam. The volume of admission is reduced to a greater extent than the surface that is exposed to the entering steam, since that surface includes two constant quantities, the surface of the cylinder cover and of the piston. For these and perhaps other reasons, we may conclude that with an early cut-off the initial condensation is relatively large; and this conclusion is amply borne out by experiment. An important result is that increase of expansion does not, beyond a certain limit, involve increase of thermodynamic efficiency; when that limit is passed the augmentation of waste through the action of the cylinder walls more than balances the increased economy to which, on general principles, expansion should give rise, and the result is a net loss. With a given engine, boiler pressure, and speed, a certain ratio of expansion will give maximum efficiency. But the conditions on which this maximum depends are too complex to admit of theoretical solution; the best ratio can be determined only by experiment. It may even happen that an engine which is required to work at a specified power will give better results, in point of efficiency, with moderate steam-pressure and moderate expansion, than with high steam-pressure and a very early cut-off. — Influence of variation in cut-off.

90. The effect of increased expansion in augmenting the action of the sides and so reducing the efficiency, when carried beyond a certain moderate grade, is well illustrated by the American and Alsatian experiments alluded to above. The following figures (Table III.), relating to a single-cylinder Corliss engine, are reduced from one of Hallauer's papers:[1]—

[1] *Bull. Soc. Industr. de Mulhouse*, May 26, 1880.

Ratio of Expansion.	Percentage of Water present		Consumption of Steam per Indicated Horse-Power per Hour.
	At End of Admission.	At End of Expansion.	lb
7·3	24·2	17·8	17·8
9·4	30·8	18·6	17·6
15·1	37·5	20·8	17·7

Here a maximum of efficiency lies between the extreme grades of expansion to which the test extends. In the American experiments the best results were obtained with even more moderate ratios of expansion. The compound engines of the United States revenue steamer "Bache," when tested with steam in the jacket of the large cylinder, with the boiler pressure nearly uniform at 80 lb by gauge, or 95 lb per square inch absolute, and the speed not greatly varied, gave the results shown in Table IV. Here the efficiency is very little affected by a large variation in the cut-off, but when the ratio of expansion becomes excessive a distinct loss is incurred.

TABLE IV.

Ratio of Total Expansion.	Consumption of Steam per I.H.P. per Hour.
	lb
4·2	21·2
5·7	20·
7·0	20·3
9·2	20·7
16·8	25·1

Experiments with engines, in the conditions which hold in ordinary practice, show that it is not unusual to find 20 or 30 per cent. of the steam that comes over from the cylinder condensed during admission. In favourable cases the amount is less than this; occasionally, on the other hand, the amount condensed is as much as half, or even more than half, the whole steam supply.

91. The action of the cylinder walls is reduced—(1) by jacketing, (2) by superheating, and (3) by using compound expansion. The advantage of the steam-jacket has been already mentioned. In high-speed engines its beneficial effect is necessarily small, and in certain cases the benefit may be even more than neutralized by the drawbacks which have been alluded to above (§ 88). In general, however, the steam-jacket forms a valuable means of reducing the wasteful action of the cylinder walls, especially when the ratio of expansion is considerable. Experiments made with and without a jacket, on the same engine, have shown that jacketing may increase the efficiency by 20 or 25 per cent. When a jacket is working properly it uses, in a single-cylinder engine, 4 or 5 per cent., and in a compound engine 8 to 12 per cent., of the whole steam supply.

Effect of superheating.

92. Superheating the steam before its admission reduces the amount of initial condensation, by lessening the quantity of steam needed to give up a specified amount of heat, and this in its turn lessens the subsequent cooling by re-evaporation. That it has a marked advantage in this respect has been experimentally demonstrated by Hirn. On general thermodynamic grounds superheating is good, because it extends the range of temperature through which the working substance is carried. In modern practice superheating (to any considerable extent) is seldom attempted. It occurs to a small extent whenever dry steam is throttled, and a slight superheating is occasionally given to steam in its passage from the high-pressure to the low-pressure cylinder of a compound engine. In former years superheated steam was a common feature of marine practice, but serious practical difficulties caused engineers to abandon its use and to seek economy rather by increasing the initial pressure and using compound expansion. In those days, however, the theoretical advantage of superheating was less understood than it is now. The economy of fuel which its employment would probably secure is so great as to warrant a fresh and energetic attempt to overcome the mechanical difficulties of construction and lubrication that have hitherto stood in the way.

93. The most important means of preventing cylinder condensation from becoming excessive is the use of compound expansion. If the vessels were non-conductors of heat it would be, from the thermodynamic point of view, a matter of indifference whether expansion was completed in a single vessel or divided between two or more, provided the passage of steam from one to the other was performed without introducing unresisted expansion (§ 51). But with actual materials the compound system has the important merit that it subjects each cylinder to a greatly reduced range of temperature variation. For this reason the amount initially condensed in the high-pressure cylinder is greatly less than if admission were to take place at once into the low-pressure cylinder and the whole expansion were to be performed there. Further, the steam which is re-evaporated from the first cylinder during its exhaust does work in the second, and it is only the re-evaporation that occurs during the exhaust from the second cylinder that is absolutely wasteful. The exact advantage of this division of range, as compared with expansion (through the same ratio) in a single cylinder, would be hard to calculate; but it is easy to see in a general way that an advantage is to be anticipated, and (though there are isolated instances to the contrary) experience bears out this conclusion. In large engines, working with high pressure, much expansion, and a slow stroke, the fact that compound engines are in general more efficient than single engines cannot be doubted. Additional evidence to the same effect is furnished when a compound engine is tested first with compound expansion and then as a simple engine with the same grade of expansion in the large cylinder alone. Thus in the American experiments the compound engine of the "Bache" when worked as a simple engine used 24 lb of steam per I.H.P. per hour, as compared with about 20 lb when the engine worked compound, with the same boiler pressure, the same total expansion, and steam in the jacket in both cases. The necessity for compounding, if efficiency is to be secured, becomes greater with every increase of boiler pressure. So long as the initial pressure is less than about 100 lb per square inch (absolute) it suffices to reduce the range of temperature into two parts by employing two-cylinder compound engines; with the higher pressures now common in marine practice triple and even quadruple expansion is being introduced.

The action of the cylinder walls would be greatly reduced if it were practicable to use a non-conducting material as an internal lining to the cylinder and to the exposed surfaces of the piston. No cure for the evils of initial condensation would be so effectual as this; and in view of the economy of heat which would result, it is a matter of some surprise that the use of a non-conductir lining has not received more serious attention.

94. The principal reasons have now been named which make the actual results of engine performance differ from the results which would be obtained if the steam conformed in every respect to the simple theory stated in chap. III. It remains to state, very shortly, a few of the results of recent practice as to the actual efficiency of steam-engines considered as heat-engines. — Actual efficiency of steam engines.

The performance of a steam-engine, as regards economy in its consumption of heat, may be stated in a number of ways. In some of these the engine alone is treated as an independent machine; in others the engine, boiler, and furnace are considered as a whole.

The performance of the engine alone is best expressed by stating either (1) the thermodynamic "efficiency" or (2) the number of thermal units used per horse-power per minute. These terms require a short explanation. The "efficiency" of a heat-engine has already been defined as the ratio of the work done to the heat supplied. The "work done" ought in strictness to be reckoned as the net work done by the working substance in passing through a complete cycle of operations; it should therefore be determined by subtracting from the work which the substance does in the cylinder the work which is spent upon the substance in the feed-pump. The latter is a comparatively small quantity, and engineers generally neglect it in their calculations of thermodynamic efficiency. In making comparison, however, between the efficiency which is actually realized and the efficiency of a perfect engine or of an engine working under any assumed conditions, account should be taken of the negative work done in the feed-pump. Account should also in strictness be taken of that part of the work spent in driving the air-pump which is done upon the working substance, as distinguished from the water of injection. The "heat supplied" is the total heat of the steam delivered to the engine, less the heat contained in the corresponding amount of feed-water. This quantity depends on the amount of steam used, on the temperature of the feed, on the boiler pressure, and on the extent to which the boiler "primes." Priming is the delivery by the boiler of water mixed with the steam. Except where there is actual superheating the steam supply is always more or less wet; in a badly designed or overworked boiler large volumes of water may be carried over with the steam, but in a good boiler of adequate size the amount of priming is less (often much less) than 5 per cent. of the whole supply. The effect of priming is, of course, to reduce the supply of heat per lb of the working substance. — Modes of statement.

One horse-power is the mechanical equivalent of 42·75 thermal units per minute. The relation between the above two methods of stating engine performance is therefore expressed by the equation

$$\text{Efficiency} = \frac{42{\cdot}75}{\text{Number of T.U. per I.H.P. per minute}}.$$

Another very common mode is to give the number of pounds of steam supplied per horse-power per hour. This is unsatisfactory, even as a method of stating the comparative economy of different engines, or of one engine in different conditions, for several reasons. It ignores variations in boiler pressure, in feed-water temperature, and in the dryness of the supply, although each of these things affects the amount of heat required for the production of a pound of steam. But the total heat of production of dry steam does not vary greatly within the limits of practical pressures; moreover, since (in condensing engines) feed-water is generally taken from the hot-well, its temperature does not differ much from that of the air-pump discharge, or (say), 100° F. Finally, in many comparative trials the amount of priming is nearly if not quite constant. Hence it happens that this mode of statement often furnishes a fairly accurate test of the economy of engines, and it