has the advantage of putting results in a way that is easy to understand and remember.

95. None of these modes of statement include the efficiency of the boiler and furnace. The performance of a boiler is most usually expressed by giving the number of pounds of water at a stated temperature converted into steam at a stated pressure by the combustion of 1 ℔ of coal. The temperature commonly chosen is 212° F., and the water is supposed to be evaporated under atmospheric pressure; the result may then be stated as so many pounds of water evaporated from and at 212° F. per 1 ℔ of coal. But the term "efficiency" may also be applied to a boiler and furnace (considered as one apparatus) in the sense of the ratio between the heat that is utilized and the potential energy that is contained in the fuel. This ratio is, in good boilers, about 0·7. Thus, for example, 1 ℔ of Welsh coal contains about 15.500 thermal units of potential energy, an amount which is equal to the heat of production (L) of about 16 ℔ of steam from and at 212°. In practice, however, 1 ℔ of coal serves to evaporate only about 11 ℔ of water under these conditions, or about 9·5 ℔ when the feed-water enters at 100° F. and the absolute pressure is 100 ℔ per square inch.

The efficiency of the engine multiplied by that of the furnace and boiler gives a number which expresses the ratio between the heat converted into work and the potential energy of the fuel.—a number which is, in other words, the efficiency of the system of engine, boiler, and furnace considered as a whole. Instead, however, of expressing this idea by the use of the term efficiency, engineers are more usually in the habit of stating the performance of the complete system by giving the number of pounds of coal consumed per horse-power per hour. It must be borne in mind that this quantity depends on the performance of the boiler as much as on that of the engine, and that the difference in thermal value between one kind of coal and another makes it, at the best, a rough way of specifying economy. It is, however, an easy quantity to measure; and to most users of engines the size of the coal-bill is a matter of greater interest than any results of thermodynamic analysis. Still another expression for engine performance, similar to
"Duty." this last, is the now nearly obsolete term "duty," or number of foot-pounds of work done for every 1 cwt. of coal consumed. Its relation to the pounds of coal per horse-power per hour is this—

$$\text{Duty} = \frac{112 \times 33000 \times 60}{\text{Number of ℔s. of coal per I.H.P. per hour}}.$$

A good condensing engine of large size, supplied by good boilers, consumes about 2 ℔ of coal per horse-power per hour; its duty is then about 110 millions.

Results of trials. 96. To illustrate the subject of this chapter more fully the following summary is given of the results of tests of pumping engines by Mr J. G. Mair, described in two excellent papers in *Min. Pro. Inst. Civ. Eng.* (vols. lxx. and lxxix.). The first group (Table V.) refers to single cylinder beam rotative engines, all of the same type, working at about 120 horse-power (in all except the last trial there were steam-jackets in use):—

Boiler Pressure (Abs.).	Total Ratio of Expansion.	Percentage of Water Present at Cut-off.	Lbs. of Dry Steam per I.H.P. per Hour.	Efficiency.
48	6·8	44	22·1	0·099
57	4·3	29	22·1	0·099
59	3·2	22	21·3	0·102
59	1·9	15	23·6	0·093
56	3·8	37	26·5	0·083

In these engines, which ran at the slow speed of about 20 revolutions per minute, the influence of steam jacketing was very marked. In the trials made with jackets in action, the percentage of water present at cut-off, when plotted in relation to the ratio of expansion, gives a diagram which is sensibly a straight line; by drawing this line it may be seen that with an expansion of 3·8 in a similar jacketed cylinder there would be about 25 per cent. of initial condensation instead of the much greater amount (37 per cent.) which the absence of a jacket caused in the last trial.

The next group of tests (Table VI.) refer to compound engines, of the types named (for explanation of the terms see chap. VI.):[1]—

Type.	Boiler Pressure Abs.	Total Ratio of Expansion.	Number of Revolutions per Minute.	Percentage of Water present at Cut-off.	Lbs. of Dry Steam per I. H. P. per Hour.	Efficiency.
Woolf beam, without jackets..	58	9·3	18	51	26·6	0·082
" with jackets......	62	15·8	20	41	17·3	0·126
" without jackets..	89	7·8	34	34	19·2	0·113
" with jackets......	88	9·6	34	38	17·4	0·125
" "	68	11·9	18	25	15·6	0·139
" "	78	16·5	23	31	15·5	0·140
" "	75	13·2	27	29	15·1	0·144
Woolf tandem, without jackets	86	11·5	80	43	21·6	0·101
Receiver beam, with jackets...	76	13·6	24	34	14·8	0·147

[1] For other comparative trials, see Hallauer's papers, especially *Bull. Soc. Ind. de Mulhouse*, Dec. 30, 1878, and May 26, 1880.

V. The Testing of Steam-Engines.

97. Under this head we may include experiments made to determine—(*a*) the horse-power of an engine; (*b*) the thermodynamic efficiency, or some more or less nearly equivalent quantity, such as the relation of power to steam supply or to coal consumption (§ 95); (*c*) the distribution of steam, that is, the relation which the several events of steam-admission, expansion. exhaust, and compression bear to the stroke of the piston; (*d*) the amount of initial condensation. the wetness of the steam throughout the stroke. and the transfer of heat between it and the cylinder walls; (*e*) the efficiency of the mechanism, or the ratio which the work done by the engine on the machinery it drives bears to the work done by the steam in the cylinder.

Tests (*a*) and (*c*) are of common application; test (*b*), in the simple form of a comparison of horse-power with coal burnt per hour. is not unusual. The actual measurement of efficiency, whether thermodynamic (*b*) or mechanical (*e*), and the analysis involved in (*d*) have been carried out in comparatively few instances.

98. In all these operations the taking of indicator diagrams forms a principal part. The indicator, invented by Watt and improved by M'Naught and by Richards, consists of a small steam cylinder, fitted with a piston which slides easily within it and is pressed down by a spiral spring of steel wire. The cylinder of the indicator is connected by a pipe below this piston to one or other end of the cylinder of the engine, so that the piston of the indicator rises and falls in response to the fluctuations of pressure which occur in the engine cylinder. The indicator piston actuates a pencil, which rises and falls with it and traces the diagram on a sheet of paper fixed to a drum that is caused to rotate back and forth through a certain arc, in unison with the motion of the engine piston. In M'Naught's indicator the pencil is directly attached to the indicator piston, in Richards's the pencil is moved by means of a system of links so that it copies the motion of the piston on a magnified scale. This has the advantage that an equally large diagram is drawn with much less movement of the piston, and errors which are caused by the piston's inertia are consequently reduced. In high-speed engines especially it is important to minimize the inertia of the indicator piston and the parts connected with it. In Richards's indicator the linkage employed to multiply the piston's motion is an arrangement similar to the parallel motion introduced by Watt as a means of guiding the piston-rod in beam engines (see § 188). In several recent forms of indicator lighter linkages are adopted, and other changes have been made with the object of fitting the instrument better for high-speed work. One of these modified forms of Richards's indicator (the Crosby) is shown in fig. 21. The pressure of steam in the engine cylinder raises the piston P, compressing the spring S and causing the pencil Q to rise in a nearly straight line through a distance proportional, on a magnified scale, to the compression of the spring and therefore

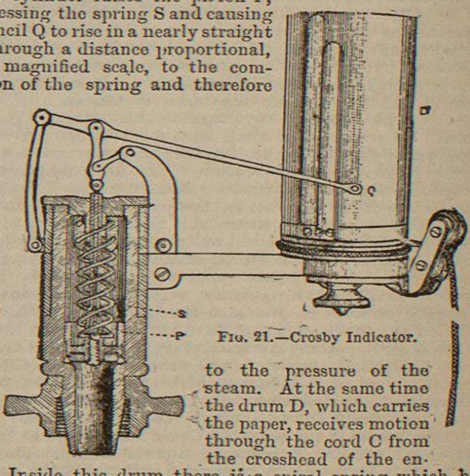

Fig. 21.—Crosby Indicator.

to the pressure of the steam. At the same time the drum D, which carries the paper, receives motion through the cord C from the crosshead of the engine. Inside this drum there is a spiral spring which becomes wound up when the cord is pulled, and serves to turn the drum in the reverse direction during the back stroke. The cap of the indicator cylinder has holes in it which admit air freely to the top of the piston, and the piston has room to descend, extending the spring S, when the pressure of the steam is less than that of the atmosphere. The spring is easily taken out and replaced by a more or less stiff one when higher or lower pressures have to be dealt with.

99. To register correctly, an indicator must satisfy two conditions: (1) the motion of the piston must be proportional to the change of steam pressure in the engine cylinder; and (2) the motion of the drum must be proportional to that of the engine piston.

The first of these requires that the pipe which connects the

indicator with the cylinder should be short and of sufficient bore. and that it should open in the cylinder at a place where the pressure in it will not be affected by the kinetic action of the inrushing steam. Frequently pipes are led from both ends of the cylinder to a central position where the indicator is set. so that diagrams may be taken from either end without shifting the instrument; much better results are obtained, especially when the cylinder is long, by using a pair of indicators, each fixed with the shortest possible connecting pipe, or by taking diagrams successively from the two ends of the cylinder with a single instrument set first at one end and then at the other. The general effect of an insufficiently free connexion between the indicator and the engine cylinder is to make the diagram too small. The first condition is also invalidated to some extent by the friction of the indicator piston, of the joints in the linkage, and of the pencil on the paper. The piston must slide very freely; nothing of the nature of packing is permissible, and any steam that leaks past it must have a free exit through the cover. The pencil pressure must not exceed the minimum which is necessary for clear marking. By careful use of a well-made instrument the error due to friction in the piston and connected parts need not be serious. Another source of disturbance is the inertia of these parts, which tends to set them into oscillation whenever the indicator piston suffers a comparatively sudden displacement. These oscillations, superposed upon the legitimate motions of the piston, give a wavy outline to parts of the diagram, especially when the speed is great and when the last-named source of error (the friction) is small. When they appear on the diagram a continuous curve should be drawn midway between the crests and hollows of the undulations. To keep them within reasonable compass in high-speed work a stiff spring must be used and an indicator with light parts should be selected. Finally, to secure accuracy in the pencil's movement, the strain of the spring must be kept well within the limit of elasticity, so that the strain may be as nearly as possible proportional to the steam pressure. Care must be taken that the spring is graduated to suit the temperature (about 212° F.) to which it is exposed when in use; its stiffness at this temperature is about 3 per cent. less than when cold.

With regard to the motion of the drum, it is, in the first place, necessary to have a reducing mechanism which will give a sufficiently accurate copy, on a small scale, of the engine piston's stroke. Many contrivances are used for this purpose; in some a rigorous geometrical solution of the problem is aimed at, in others a close approximation only. Fig. 22 shows a good form of indicator gear. A pendulum rod AB is pinned at one end to the crosshead A (the end of the piston-rod) of the engine. Its upper end is carried by a pin which is free to turn and slide in the fixed slot B. A cord from an intermediate point C leads over pulleys to the indicator drum. The pendulum rod should be much longer than the piston stroke, and the cord should lead off for a considerable distance in the direction sketched, at right angles to the mean position of the rods. The accuracy of the drum's motion does not, however, depend merely on the geometrical condition

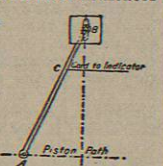

Fig. 22.—Indicator Gear.

of the gear. It depends also on the rigidity of the parts, and especially on the stretching of the cord. The elasticity of the cord will cause error if it is not maintained in a state of uniform tension throughout the double stroke, and this error will be greater the longer and the more extensible the cord is. Hence short cords are to be preferred; and fine wire, which stretches much less, may often be substituted for cord with great advantage. The stretching of the cord is perhaps the most serious and least noticed source of error the indicator is subject to in ordinary practice. The tension of the cord varies for three reasons,—the inertia of the drum, the varying resistance of the drum spring, and the friction of the drum, which has the effect of increasing the tension during the forward stroke and of reducing it during the back stroke. This last cause of variation can be minimized only by good construction and careful use of the instrument; but the other two causes can be made to neutralize one another almost completely. Since the motion is nearly simple harmonic, the acceleration of the drum varies in a nearly uniform manner from end to end of the stroke. The resistance of the drum spring also varies uniformly; and it is therefore only necessary to adjust the stiffness of the drum spring so that the increase in its resistance as the motion of the drum proceeds may balance the decrease in the force that the cord has to exert in setting the drum into motion. This adjustment will secure an almost uniform tension in the cord throughout the whole stroke; it must, of course, be altered to suit different engine speeds. The indicator plays so important a part in the testing of heat-engines, whether for practical or scientific purposes, that no pains should be spared to avoid the numerous and serious sources of error to which it is liable through faulty construction or unintelligent use.[1]

100. To determine the indicated horse-power, the mean effective pressure is found by dividing the area of the diagram by the length of its base. This gives a mean height, which, interpreted on the scale of pressures. is the mean effective pressure in pounds per square inch. This has to be multiplied by the effective area of the piston in square inches and by the length of the piston stroke in feet, to find the work done per stroke in foot-pounds on that side of the piston to which the diagram refers. Let A_1 be the area of the piston on one side and A_2 on the other; p_1 and p_2 the mean effective pressures on the two sides respectively; L the length of the stroke in feet; and n the number of complete double strokes or revolutions per minute. Then the indicated horse-power

$$\text{I.H.P.} = \frac{nL(p_1A_1 + p_2A_2)}{33000}.$$

In finding the mean pressure the area of the diagram may be conveniently measured by a planimeter or calculated by the use of Simpson's rule. A less accurate plan, frequently followed, is to divide the diagram by lines drawn at the middle of strips of equal width, as in figs. 23 and 24, and to take the mean pressure as the average height of these lines.

101. Space admits of no more than a few illustrations of actual indicator diagrams. Fig. 23 is a diagram taken from an antiquated non-condensing engine working without expansion. The line AB has been drawn at a height which represents the boiler pressure, in order to show the loss of pressure in admission. The line CD is drawn at atmospheric pressure by the indicator itself. In this engine admission continues till the end of the forward stroke, and as a result the back pressure is great, especially during the first stage of the exhaust. The diagram shows a slight amount of oscillation produced by the sudden admission of steam. This feature, however, is better illustrated by fig. 24, which is another diagram taken from the same engine, at the same boiler pressure, but with the steam much throttled.

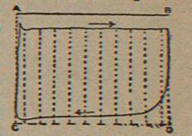

Fig. 23.

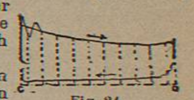

Fig. 24.

Fig. 25 shows a pair of diagrams taken from a condensing engine in which the distribution of steam is effected by a common slide valve (chap. VIII.). The two diagrams refer to opposite ends of the cylinder and are taken on the same paper by the plan already alluded to (§ 99) of fixing the indicator about midway between the ends of the cylinder, with a pipe leading from it to each end. Steam is cut off at a and a', release occurs at b and b', and compression begins at c and c'. The gradual closing of the slide valves throttles the steam considerably before the cut-off is complete. The line of no pressure EF is drawn 14·7 ℔ per square inch below CD, which is the atmospheric line; and the line of no volume AE or BF is drawn (for each end of the cylinder) at a distance (from the end of the diagram) equal to the volume of the clearance.

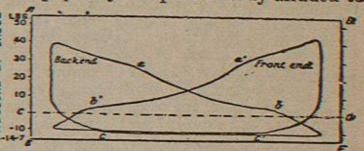

Fig. 25.—Indicator Diagram from Condensing Engine, with Slide-Valve.

Fig. 26 is a diagram taken from a Corliss engine working with a large ratio of expansion. The Corliss valve-gear, which will be described in chap. IX., causes the admission valve to close suddenly, and consequently defines the point of cut-off pretty sharply in the diagram. Through this point a dotted curve has been drawn (by aid of the equation PV^n = const., § 67), which is the curve that would be followed if the expansion were adiabatic. In drawing this curve it has been assumed that at the end of admission the steam contains 25 per cent. of water. The actual curve first falls below and then rises above this adiabatic curve, in consequence of the continued condensation which takes place during the early stages of the expansion and the re-evaporation of condensed water during later stages (§ 82). Fig. 27 is another diagram from a Corliss engine, running light, and with the condenser not in action. Diagrams of this kind are often taken when engines are first erected, for the purpose of testing the setting of the valves. Other indicator diagrams, for compound engines, will be given in chap. VI.

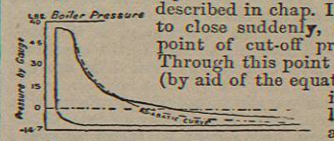

Fig. 26.—Indicator Diagram from Corliss Engine.

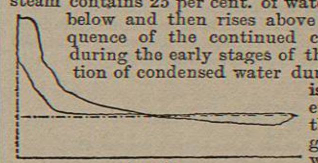
Fig. 27.

[1] A valuable discussion and experimental investigation of the errors of the indicator will be found in papers by Prof. Osborne Reynolds and Mr H. W. Brightmore (*Min. Proc. Inst. C.E.*, vol. lxxxiii., 1886). In the discussion which followed the reading of the papers a description will be found of an ingenious apparatus which the makers of the Crosby indicator employ to test the uniformity of the cord's tension throughout the stroke.

In place of the ordinary indicator an apparatus is occasionally used which integrates the two coordinates which it is the business of the indicator diagram to represent, and exhibits the power developed from stroke to stroke by the progressive movement of an index round a dial.

102. In tests of thermodynamic efficiency we may measure either the heat supplied or the heat rejected, and compare it with the work done. The heat supplied is on the whole capable of more exact measurement, but in any case a determination of the heat rejected furnishes a valuable check on the accuracy of the result. The trial must be continued for a period of some hours at least, during which the engine and boiler are to be kept working as uniformly as possible in all respects. The power is determined by taking indicator diagrams at short intervals. The heat supplied is found by noting the amount of feed-water required to keep the water-level in the boiler constant during the trial, the temperature of the feed, and the pressure of the steam. The only uncertainty which attaches to the measurement of heat-supply is due to priming. Every pound of water that passes over unevaporated to the engine takes less heat by the amount L (§ 60) than if it went over in the state of steam. To measure the degree of wetness in steam is a matter of some difficulty; it may be done by passing the steam into a known quantity of cold water, so as to condense it, and observing the rise of temperature which has taken place when the whole quantity of water present has increased by a measured amount.

If L_1 be the latent heat of steam at the boiler pressure, h_0 the heat in the feed-water per lb, h_1 the heat in the boiler water per lb, and q the dryness of the steam as it leaves the boiler, the heat taken in per lb of the substance supplied to the cylinder is

$$qL_1 + h_1 - h_0.$$

To this must be added, in the case of a jacketed engine, the heat supplied to the jacket, a quantity which depends on the amount of steam condensed there, and also on whether the water that gathers in the jacket is drained back into the boiler or allowed to escape into the hot-well.

The heat rejected by an engine fitted with an injection condenser is made up of the following parts:—(*a*) heat rejected in the condensed water, less the heat returned to the boiler in the feed (if the feed is directly drawn from the hot-well without giving the water time to cool sensibly, this quantity vanishes; in a jacketed engine this item must include the heat rejected in the jacket drains); (*b*) heat used in warming the condenser water from the temperature of injection to the temperature of the air-pump discharge; (*c*) heat rejected in air and vapour from the air-pump; (*d*) heat lost by radiation, conduction to supports, and aerial convection,—or, more properly, the excess of this heat over the heat developed within the engine by the friction of piston, valves, &c. Of these quantities, (*a*) is found without difficulty from a knowledge of the amount of the feed-water, its temperature, the temperature of the air-pump discharge, and amount and temperature of water drained from the jacket; (*b*) is measured by gauging the whole discharge from the pump, deducting from it the amount returned to the boiler as feed-water, and measuring its temperature and that of the injection water; (*c*) does not admit of direct measurement; (*d*) may be approximately estimated for a jacketed engine by filling the jacket with steam while the engine is out of action, and observing the amount of steam condensed in the jacket during a long interval, through radiation, &c., from the external surface.

In calculating the supply of heat by the boiler it is convenient to take the temperature 32° F. as a starting point from which to reckon what may be termed the gross supply, and then to deduct from this the heat which is restored to the boiler in the feed-water. The difference, which may be called the net supply, is the true consumption of heat, and is to be used in calculating the efficiency of the engine. A similar convention may be followed in dealing with the heat rejected.

103. This subject is most easily made intelligible by help of a numerical example. For this purpose the following data of an actual engine-test have been taken from one of Mr Mair's papers[1]; the data have been independently reduced, with results that differ only to a small and unimportant extent from those stated by Mr Mair. The engine under trial was a compound beam engine, steam-jacketed, with an intermediate receiver between the cylinders. The cylinders were 21 inches and 36 inches in diameter, and the stroke $5\frac{1}{2}$ feet. The total ratio of expansion was 13·6.

Data.

Boiler pressure, absolute, 76 lb per sq. in.

Time of trial	6 hours.
Revolutions	8632, or 24·0 per min.
I.H.P.	127·4.
Feed-water	12,032 lb, or 1·394 lb per rev. (M.).
Air-pump discharge	1226 lb per min., or 51·1 lb per rev
Water drained from jackets	1605 lb, or 0·186 lb per rev. (M_j).
Percentage of priming	4.
Temperature of feed, t_0	59°
Temperature of injection, t_2	50°.
Temperature of air-pump discharge, t_3	73°·4.

[1] *Min. Proc. Inst. C. E.* vol. lxx.

Results.

Dryness of boiler steam, $q = 0·96$.

Supply to cylinder, $M_c = M - M_j = 1·028$ lb per rev.

Injection water per rev. $= 51·1 - 1·208 = 49·9$ lb.

$L_1 = 898,\ h_1 = 278,\ h_2 = 18,\ h_3 = 41·4,\ h_0 = 27.$

Gross supply of heat from boiler to cylinder per revolution

$$= M_c(qL_1 + h_1)$$
$$= 1·208\ (0·96 \times 898 + 278) = 1377 \text{ T.U.}$$

Gross supply of heat from boiler to jackets per revolution

$$= M\ (qL_1 + h_1)$$
$$= 0·186\ (0·96 \times 898 + 278) = 212 \text{ T.U.}$$

Total gross supply per revolution $= 1377 + 212 = 1589$ T.U.

Heat restored to boiler per revolution,

By feed water $= Mh_0 = 1·394 \times 27 = 38$ T.U

By jacket drains $= 0$.

Net supply of heat per revolution $= 1589 - 38 = 1551$ T.U.

Heat converted into work, per revolution

$$= \frac{\text{I.H.P.} \times 42·75}{24} = 227 \text{ T.U.}$$

Total heat rejected per revolution $= 1551 - 227 = 1324$ T.U.

The rejected heat is accounted for as follows:—

Net heat rejected in air-pump discharge = Gross heat rejected in air-pump discharge—heat in injection water—heat restored to the boiler by the feed $= 51·1 \times 41·4 - 49·9 \times 18 - 38 = 1179$ T.U.

Heat rejected in jacket drains $= M_j h_1 = 0·186 \times 278 = 52$ T.U.

These two items account for 1231 units of rejected heat and leave a balance of 93 units unaccounted for. The balance is made up of heat rejected in air and vapour by the air-pump, heat lost by radiation, &c., and errors of experiment. In the example considered the loss by radiation was estimated at 45 thermal units, which reduces the discrepancy between the two sides of the account to 48 units, or only about 3 per cent. of the whole heat supplied.

The efficiency of the engine is $\frac{227}{1551}$ or 0·146. The efficiency of a perfect engine working between the same limits of temperature, 308° F. and 50° F., would be 0·335.

Calculation of wetness of the steam.

104. When it is desired to deduce from the test of an engine not only the thermodynamic efficiency but also the amount of initial condensation and the subsequent changes of wetness which the working fluid undergoes during expansion, it is necessary to know, in addition to the above data, the volume of cylinder and clearance, the relation of pressure to volume during the several stages of the stroke, and the whole amount of working substance present in the cylinder. This last is a quantity whose precise value is not easily ascertained. Assuming that the point at which compression begins can be distinguished on the diagram, we have the pressure and the volume of the steam that is afterwards compressed into the clearance space. From its pressure and volume we can infer its amount, if only its degree of dryness be known. The assumption usually made is that at the beginning of compression the steam shut up in the cylinder is dry. This assumption is to a certain extent supported by the fact that re-evaporation has been going on during expansion and exhaust; in good engines it is probably not far from the truth, though there are cases where, owing to excessive initial condensation and to the exhaust ports being badly situated for draining the cylinder, water may accumulate in considerable quantities. Except in extreme cases of this kind, however, the assumption that the steam is dry when compression begins does not introduce an error which can seriously affect the subsequent calculations. Having found the quantity shut up in the clearance, we add to it the quantity delivered from the boiler per single stroke, to find the whole quantity of working substance in the cylinder. The substance is, and continues, a mixture in varying proportions of steam and water. Its volume may practically be taken as the volume of the dry steam it contains, the volume of the water being comparatively small. Taking any point of the stroke, and measuring the pressure and the volume there, we can say how much steam (at that pressure) would be required to fill the volume which the mixture then occupies. This quantity will always be less than the actual amount of the mixture; and the difference between them is the amount of water that is present. This calculation is of special interest at two places in the stroke—the point of cut-off and the point of release.

105. To illustrate it we may continue the numerical example quoted above. In the high-pressure cylinder of the engine to which the test refers the volume at the beginning of compression (including clearance) was 1·52 cubic feet. The pressure, just before compression began, is shown by the indicator diagram (of which fig. 28 is a copy) to have been 14·8 lb per square inch. At this pressure the density (or mass of 1 cubic foot) of steam is 0·038 lb. Hence (on the above assumption that the steam was then dry) the quantity shut up in the clearance was $1·52 \times 0·038 = 0·058$ lb.

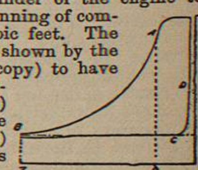

Fig. 28.

The amount delivered to the cylinder per single stroke (or half revolution) was 0·604 lb. The whole quantity of working substance present from the end of the admission to the beginning of the exhaust was therefore 0·662 lb.

At the point of cut-off the pressure is shown by the diagram to have been 64 lb per square inch (absolute), and the volume, including clearance, was 2·92 cubic feet. The density of steam for this

pressure is 0·151 lb per cubic foot. Hence, out of the whole mixture, the amount of steam was $2·92 \times 0·151 = 0·440$ lb. The water present at the point of cut-off was therefore $0·662 - 0·440$ or 0·222 lb. This is 33·5 per cent. of the whole amount of the mixture, and shows (after allowing for the priming water) that about 32 per cent. of the steam admitted was condensed on admission.

Next, to find the amount of water present at the end of the expansion. The diagram shows that at this point the pressure was 15·2 lb per square inch and the volume 13·235 cubic feet. Steam of this pressure has a density of 0·0392 lb per cubic foot. The quantity of steam at release was therefore $13·235 \times 0·0392$, or 0·519 lb, and the quantity of water $0·662 - 0·519 = 0·143$ lb. It appears therefore that re-evaporation from the cylinder walls during expansion reduced the amount of water present by 0·079 lb, so that the percentage of water fell from 33·5 at the point of cut-off to 21·6 at the point of release. The same method of calculation can obviously be applied to any other point in the expansion curve, and can be extended to the low-pressure cylinder of an engine which (like the one in this example) is compound. The amount of dry steam present at the point of release is sometimes spoken of as the "steam accounted for by the indicator diagram."

106. Having completed this analysis of the working-substance, we may proceed to find the quantity of heat which it gives to or takes from the walls of the cylinder during any stage of its action, by considering the changes of internal energy which the working substance undergoes, along with the external work done, from stage to stage. If we write m for the amount of steam and m' for the amount of water present in the cylinder at any one stage, the internal energy of the mixture is (§ 62)

$$(m + m')h + m\rho.$$

Let the value of this quantity be denoted by I_A at any one stage in the expansion or compression of the mixture, such as the point of cut-off A, and by I_B at a later stage, such as the point of release B, the corresponding volumes of the whole mixture being V_A and V_B respectively. Then in passing from the first condition to the second the substance loses $I_A - I_B$ of internal energy. It also does an amount of external work W_{AB} measured by $\int_{V_A}^{V} P\,dV$, or the area of the figure AB*ab*. If W_{AB} is equal to $I_A - I_B$ the process is adiabatic; otherwise the amount of heat taken up (from the cylinder walls) during the process is

$$Q_{AB} = W_{AB} - (I_A - I_B).$$

If A is the point of cut-off and B that of release, the quantity so calculated is the heat taken up from the cylinder walls during the whole process of expansion. The calculation applies equally, however, in determining the heat taken up during any stage of the process. When this has a negative value heat has been given up by the substance to the cylinder walls. In the numerical example which has been cited above the internal energy of the mixture at the beginning of expansion was 540 thermal units. At the end of expansion the internal energy was 584 thermal units. Between these points the indicator diagram (fig. 28) shows that the work done was equivalent to 55 thermal units. $44 + 55 = 99$ units of heat were therefore taken from the cylinder walls during the process of expansion. A similar calculation, applied to the compression curve, shows that in that part of the operation heat was given up to the cylinder walls. During compression W is of course negative, since work is then spent upon the steam.

107. During admission and also during exhaust another item enters into the account,—the amount of the working substance is then undergoing change. To find the heat given up by the steam during admission we have first to calculate (by the method already described) the internal energy of the mixed steam and water that is shut into the clearance space at the end of the previous stroke; this may be called I_D. The steam which then enters brings with it an additional amount of internal energy which we may calculate from a knowledge of the quantity of steam, its pressure at admission, and its dryness. Let I_O denote this additional supply of internal energy. At the end of admission the state of the mixture is known from the indicator diagram; hence its internal energy I_A may be found. The work done during admission, W_{DA}, is also determined from the diagram. Then we have, for the heat given up by the steam during admission,

$$Q_{DA} = I_D + I_O - I_A - W_{DA}.$$

In attempting to apply the same method of calculation to determine the heat taken up from the cylinder walls during exhaust (Q_{BC}), we are met by the difficulty that we do not know the state, as regards dryness, of the mixture during its expulsion from the cylinder. We may, however, estimate the value of Q_{BC} as follows. Let Q_{CD} and Q_{DA} be, as before, the heat given up by the steam to the cylinder walls during compression and admission respectively, and let Q_{AB} be the heat taken from the cylinder walls during exhaust; also let Q_R be the heat which the cylinder loses (per single stroke) by radiation (less the heat produced by piston and valve friction), and Q_J the heat which it gains by condensation of steam in the jacket, if there is one. Then, as the cylinder neither gains nor loses heat on the whole, after a uniform régime has been arrived at, we have

$$Q_{BC} = Q_{CD} + Q_{DA} + Q_J - Q_R.$$

The quantity Q_{BC} may also be calculated directly from a knowledge of the gross heat rejected to the condenser, since the gross heat rejected is

$$I_B + W_{BC} + Q_{BC},$$

I_B being the internal energy of the mixture at release and W_{BC} being the work done upon the steam in expelling it from the cylinder.

108. This heat Q_{BC}, which is taken up by the steam from the cylinder walls during exhaust, is a part of the heat deposited there during admission. It has passed through the cylinder without contributing in the smallest degree to the work of the engine. Probably for this reason it is treated by some writers as a quantity which measures the wasteful influence of the cylinder walls. This, however, is not strictly the case. The magnitude of Q_{BC} is certainly in some sense an index of the extent to which the alternate heating and cooling of the metal causes inefficiency; it is so much heat absolutely lost, and lost by the action of the walls. [In the high-pressure cylinder of a compound engine this loss is, of course, absolute only as regards that cylinder; the heat represented by Q_{BC} assists in the work of the low-pressure cylinder.] But besides this loss there is another which the walls cause by taking heat from the steam on admission and restoring it during the later stages of expansion. That part of the heat abstracted during admission which is restored before the point of release does not appear in Q_{BC}; nevertheless it is a source of inefficiency. With steam that is dry at the end of the expansion the value of Q_{BC} is almost negligible; still the cylinder walls may cause a very sensible loss by abstracting heat from the hot steam as it enters and restoring it as the mixture expands. The quantity which has been denoted here by Q_{BC}—that heat, namely, which the steam takes up from the cylinder walls after release and during exhaust—appears in the writings of Hirn and his followers under the symbol R_c. He terms it "le refroidissement au condenseur," and refers to it, somewhat inexactly, as "l'effet réel des parois."[1] Prof. Cotterill applies the name "exhaust waste" to the sum of the two quantities Q_{BC} and Q_R.[2]

109. It is obvious that the above analysis depends fundamentally on the strict accuracy with which the indicator diagram not only gives a measure of the work done by the engine under test, but shows the relation of pressure to volume at each stage in the process. Engine tests of a complete kind have now been made and discussed by a number of independent observers, working with widely different data. The results are in good general agreement. They demonstrate the influence of the sides beyond question, showing that 30 per cent. is no unusual amount of water to be present in the mixture at the point of cut-off, even in compound engines of the best types; that half of this water, or even more, is frequently found at the end of expansion; and that the heat denoted above by Q_{BC} ranges from about 10 to 20 per cent. of the whole heat supplied.[3]

Efficiency of the mechanism.

110. An engine employed to drive other machinery delivers to it an amount of power less than the indicated power by an amount which is wasted in overcoming the friction of piston and piston-rod, slides, valves, journals, &c. The efficiency of the mechanism is the ratio of the "effective" or "brake" horse-power to the indicated horse-power. It may be tested by measuring the power delivered by the engine when at work, either by using a transmission dynamometer or by substituting an absorption dynamometer for the mechanism usually driven. In the case of a pumping engine the efficiency of the engine and pumps together may be determined by observing the actual work done in raising water or in delivering a measured volume against a known pressure. Attempts are sometimes made to find the amount of power wasted in engine friction by testing the indicated power needed to drive the engine against no other resistance than its own friction. This, however, fails to show the power which will be spent in overcoming friction when the engine runs under ordinary conditions, since the pressures at the slides, the journals, and elsewhere are then widely different from what they are when the engine is running without load. Experiments with large engines show that the efficiency of the mechanism may, in favourable cases, be 0·85 or even 0·9; in small engines, or in large engines running under light loads, it is generally much less than this.

[1] *Bull. Soc. Ind. de Mulhouse*, 1881.

[2] *The Steam Engine considered as a Heat Engine*, p. 54.

[3] In this connexion reference should be made to the data supplied by the American experiments (of Messrs Emery and Loring), some of which are discussed fully by Prof. Cotterill; also to the writings of Hirn and the extensive researches of the Alsatian engineers alluded to in chap. IV.; also to Mr Mair's paper (§ 96) and to Mr M. Longridge's *Reports*, as engineer to the Engine, Boiler, and Employers Liability Association, for 1880, 1881, and 1884. The *Report* for 1884 especially exhibits the results of a test with unique fulness and clearness. The *Journal* of the Franklin Institute for 1885 contains an account of experiments by Messrs Gately and Kletzsch on one engine, under varied conditions of boiler pressure, expansion, and speed; these, so far as they go, confirm the conclusions stated, on general grounds, in chap. IV. For a synopsis of Hirn's method of analysis reference should be made to a paper by M. Poupardin, *Bull. Soc. Ind. de Mulhouse*, March 31, 1875.

VI. Compound Expansion.

111. In the original form of compound engine, invented by Hornblower and revived by Woolf, steam passed directly from the first to the second cylinder; the exhaust from the first and admission to the second went on together throughout the whole of the back stroke. This arrangement is possible only when the high and low pressure pistons begin and end their strokes together, that is to say, when their movements either coincide in phase or differ by half a revolution. Engines of the "tandem" type satisfy this condition—engines, namely, whose high and low pressure cylinders are in one line, with one piston-rod common to both pistons. Engines in which the high and low pressure cylinders are placed side by side, and act either on the same crank or on cranks set at 180° apart, may also discharge steam directly from one to the other cylinder; the same remark applies to beam engines, whether of the class in which both pistons act on one end of the beam, or of the class introduced by M'Naught, in which the high and low pressure cylinders stand on opposite sides of the centre. By a convenient usage which is now pretty general the name "Woolf engine" is restricted to those compound engines which discharge steam directly from the high to the low pressure cylinders without the use of an intermediate receiver.

Receiver engine.

112. An intermediate receiver becomes necessary when the phases of the pistons in a compound engine do not agree. With two cranks at right angles, for example, a portion of the discharge from the high-pressure cylinder occurs at a time when the low-pressure cylinder cannot properly receive steam. The receiver is in some cases an entirely independent vessel connected to the cylinders by pipes; very often, however, a sufficient amount of receiver volume is afforded by the valve casings and the steam-pipe which connects the cylinders. The receiver, when it is a distinct vessel, is frequently jacketed.

The use of a receiver is of course not restricted to engines in which the "Woolf" system of compound working is impracticable. On the contrary, it is frequently applied with advantage to beam and tandem compound engines. Communication need not then be maintained between the high and low pressure cylinders during the whole of the stroke; admission to the low-pressure cylinder is stopped before the stroke is completed; the steam already admitted is allowed to expand independently; and the remainder of the discharge from the high-pressure cylinder is compressed into the intermediate receiver. Each cylinder has then a definite point of cut-off, and by varying these points the distribution of work between the two cylinders may be adjusted at will. In general it is desirable to make both cylinders of a compound engine contribute equal quantities of work. If they act on separate cranks this has the effect of giving the same value to the mean twisting moment on both cranks.

Compound diagrams.

113. Wherever a receiver is used, care should be taken that there is no unresisted expansion into it; in other words, the pressure in the receiver should be equal to that in the high-pressure cylinder at the moment of release. If the receiver pressure is less than this there will be what is termed a "drop" in the steam pressure between the high-pressure cylinder and the receiver, which will show itself in an indicator diagram by a sudden fall at the end of the high-pressure expansion. This "drop" is, from the thermodynamic point of view, irreversible, and therefore wasteful. It can be avoided by selecting a proper point of cut-off in the low-pressure cylinder. When there is no "drop" the expansion that occurs in a compound engine has precisely the same effect in doing work as the same amount of expansion in a simple engine would have, provided the law of expansion be the same in both and the waste of energy which occurs by the friction of ports and passages in the transfer of steam from one to the other cylinder be negligible. The work done in either case depends merely on the relation of pressure to volume throughout the process; and so long as that relation is unchanged it is a matter of indifference whether the expansion be performed in one vessel or in more than one. It has, however, been fully pointed out in chap. IV. that in general a compound engine has a thermodynamic advantage over a simple engine using the same pressure and the same expansion, inasmuch as it reduces the exchange of heat between the working substance and the cylinder walls and so makes the process of expansion more nearly adiabatic. The compound engine has also a mechanical advantage which will be presently described. The ultimate ratio of expansion in any compound engine is the ratio of the volume of the low-pressure cylinder to the volume of steam admitted to the high-pressure cylinder. Fig. 29 illustrates the combined action of the two cylinders in a hypothetical compound engine of the Woolf type, in which for simplicity the effect of clearance is neglected and also the loss of pressure which the steam undergoes in transfer from one to the other cylinder. ABCD is the indicator diagram of the high-pressure cylinder. The exhaust line CD shows a falling pressure in consequence of the increase of volume which the steam is then undergoing through the advance of the low-pressure piston. EFGH is the diagram of the low-pressure cylinder drawn alongside of the other for convenience in the construction which follows. It has no point of cut-off; its admission line is the continuous curve of expansion EF, which is the same as the high-pressure exhaust line CD, but drawn to a different scale of volumes. At any point K, the actual volume of the steam is KL+MN. By drawing OP equal to KL+MN, so that OP represents the whole volume, and repeating the same construction at other points of the diagram, we may set out the curve QPR, the upper part of which is identical with BC, and so complete a single diagram which exhibits the equivalent expansion in a single cylinder.

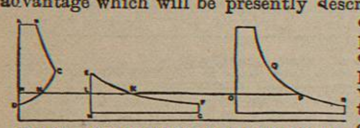

Fig. 29.—Compound Diagrams: Woolf type.

In a tandem compound engine of the receiver type the diagrams resemble those shown in fig. 30. During CD (which corresponds to FG) expansion is taking place into the large or low-pressure cylinder. D and G mark the point of cut-off in the large cylinder, after which GH shows the independent expansion of the steam now shut within the large cylinder, and DE shows the compression of steam by continued discharge from the small cylinder into the receiver. At the end of the stroke the receiver pressure is OE, and this must be the same as the pressure at C, if there is to be no "drop." Diagrams of a similar kind may be sketched without difficulty for the case of a receiver engine with any assigned phase relation between the pistons.[1]

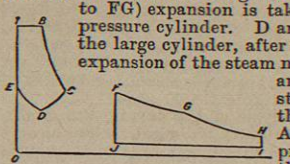

Fig. 30.—Compound Diagrams: Receiver type.

114. By making the cut-off take place earlier in the large cylinder we increase the mean pressure in the receiver; the work done in the small cylinder is consequently diminished. The work done in the large cylinder is correspondingly increased, for the total work (depending as it does on the initial pressure and the total ratio of expansion) is unaffected by the change. The same adjustment serves, in case there is "drop," to remove it. By selecting a suitable ratio of cylinder volumes to one another and to the volume of the receiver, and also by choosing a proper point for the low-pressure cut-off, it is possible to secure absence of drop along with equality in the division of the work between the two cylinders.

To determine that point of cut-off in the low-pressure cylinder which will prevent drop when the ratio of cylinder and receiver volumes is assigned is a problem most easily solved by a graphic process. The process consists in drawing the curve of pressure during admission to the low-pressure cylinder until it meets the curve of expansion which is common to both cylinders.[2] Thus in fig. 31 (where for the sake of simplicity the effects of clearance are neglected) AB represents the admission line and BC the expansion line in the small cylinder. Release occurs at C, and from C to D steam is being taken by the large cylinder. D corresponds to the cut-off in the large cylinder, which is the point to be found. From D to E steam is being compressed into the receiver. To avoid drop the receiver pressure at E is to be the same as the pressure at C. E is therefore known, and may be employed as the starting-point in drawing a curve EF which is the admission line of the low-pressure diagram EFGHI. This line is drawn by considering at each point in the low-pressure piston's stroke what is then the whole volume of the steam. The place at which EF intersects the continuous expansion curve BCG determines the proper point of cut-off. The sketch (fig. 31) refers to the case of a tandem receiver engine; but the process may also be applied to an engine with any assumed phase relation between the cranks. Fig. 32 shows a pair of theoretical indicator diagrams determined in the same way for an engine with cranks at right angles, the high-pressure crank leading. In using the graphic method any form may be assigned to the curve of expansion. Generally this curve may be treated without serious inaccuracy as a common hyperbola, in which the pressure varies inversely as the volume.

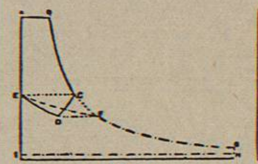

Fig. 31.

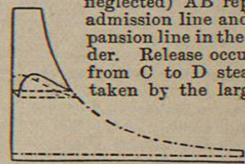

Fig. 32.

Figs. 31 and 32.—Determination of the point of cut-off in the low-pressure cylinder of a compound engine.

115. If this simple relation between pressure and volume be assumed, it is practicable to find algebraically the low-pressure cut-off which will give no drop, with assigned ratios of cylinder and receiver volumes. Taking the simplest case—that of a tandem engine, or of an engine with parallel cylinders whose pistons move together or in opposition—we may proceed thus. Since the point of cut-off to be determined depends on volume ratios we may for brevity treat the volume of the small cylinder as unity. Let R be the ratio to it of the receiver's volume, and L that of the low-pressure cylinder. Let x be the required fraction of the stroke at which cut-off is to occur in the large cylinder; and let p be the pressure at release from the small cylinder. As there is to be no drop, p is also the pressure in the receiver at the beginning of admission to the large cylinder. During that admission the volume changes from $1+R$ to $1-x+R+xL$, and the pressure at cut-off is therefore $\frac{p(1+R)}{1-x+R+xL}$. The steam that remains is now compressed into the receiver, from volume $1-x+R$ to volume R. Its pressure therefore rises to $\frac{p(1+R)}{1-x+R+xL}\cdot\frac{(1-x+R)}{R}$, and this, by assumption, is to be equal to p. We therefore have

$$(1+R)(1-x+R)=R(1-x+R+xL),$$

whence

$$x=(R+1)/(RL+1).$$

Thus, with $R=1$ and $L=3$, cut-off should occur in the large cylinder at half-stroke; with a greater cylinder ratio the cut-off should be earlier.

A similar calculation[1] for a compound engine whose cranks are at right angles, and in which cut-off occurs in the large cylinder before half-stroke, shows that the condition of no drop is secured when

$$2R(xL-1)=1-2\sqrt{x(1-x)}.$$

In some compound engines a pair of high-pressure cylinders discharge into a common receiver; in some a pair of low-pressure cylinders are fed from a receiver which takes steam from one high-pressure cylinder, or in some instances from two. With these arrangements the pressure in the receiver may be kept much more nearly constant than is possible with the ordinary two-cylinder type.

Uniformity of effort in a compound engine.

116. An important mechanical advantage belongs to the compound engine in the fact that it avoids the extreme thrust and pull which would have to be borne by the piston-rod of a single-cylinder engine working at the same power with the same initial pressure and the same ratio of expansion. If all the expansion took place in the low-pressure cylinder, the piston at the beginning of the stroke would be exposed to a thrust much greater than the sum of the thrusts on the two pistons of a compound engine in which a fair proportion of the expansion is performed in the small cylinder. Thus in the tandem engine of fig. 29 the greatest sum of the thrusts will be found to amount to less than two-thirds of the thrust which the large piston would be subjected to if the engine were simple. The mean thrust throughout the stroke is of course not affected by compounding; only the range of variation in the thrust is reduced. The effort on the crank-pin is consequently made more uniform, the strength of the parts may be reduced, and the friction at slides and journals is lessened. The advantage in this respect is obviously much greater when the cylinders are placed side by side, instead of tandem, and work on cranks at right angles. As a set-off to its advantage in giving a more uniform effort, the compound engine has the drawback of requiring more working parts than a simple engine with one cylinder. But in many instances—as in marine engines—two cranks and two cylinders are almost indispensable, to give a tolerably uniform effort and to get over the dead points; and the comparison should then be made between a pair of simple cylinders and a pair of compounded cylinders. Another point in favour of the compound engine is that, although the whole ratio of expansion is great, there need not be a very early cut-off in either cylinder; hence the common slide-valve, which is unsuited to give an early cut-off, may be used in place of a more complex arrangement. The mechanical advantage of the compound engine has long been recognized, and had much to do with its adoption in the early days of high-pressure steam.[2] Its subsequent development has been due in part to this, but probably in much greater part to the thermodynamic advantage which has been discussed above (§ 93).

117. Indicator diagrams taken from compound engines show that the transfer of steam from one cylinder to another is never, under the most favourable conditions, performed without loss of energy. Fig. 33 shows a pair of diagrams from the two cylinders of a tandem Woolf engine, in which the steam passed as directly as possible from the small to the large cylinder. The diagrams are drawn to the same scale of *stroke* and therefore to different scales of *volume*, and the low-pressure diagram is turned round so that it may fit into the high-pressure diagram. There is some drop at the high-pressure release, and, apart from this, there is a loss through friction of the passages, which shows itself by the admission line to the large cylinder lying below the exhaust line from the small one.

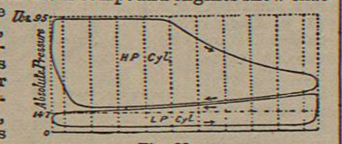

Fig. 33.

118. Fig. 34 is a pair of diagrams taken from a compound tandem receiver engine running at 50 revolutions per minute, with cylinders 30 inches and 52 inches in diameter, and with a 6-feet stroke. The ratio of cylinder volumes is therefore 3 to 1. The capacity of the receiver is nearly $1\frac{1}{2}$ times that of the small cylinder. There is a comparatively early cut-off in both cylinders, and a nearly complete absence of drop. The small cylinder, however, does more work than the large one, in the ratio of nearly 3 to 2.

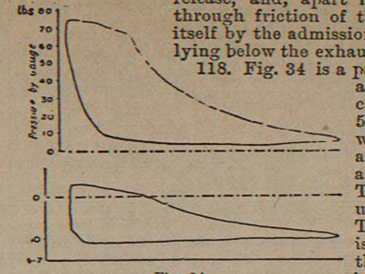

Fig. 34.

Combination of diagrams from compound engines.

Fig. 35 shows the same pair of diagrams combined by drawing both to the same scale of volume and of pressure, and by setting out each by an amount equal to the clearance space from the line of no volume. This makes the expansion curve in each diagram represent correctly the relation of the pressure to the absolute volume of the expanding steam. The broken line is a continuous curve of *adiabatic* expansion, drawn from the point of high-pressure cut-off, on the assumption that the steam then contained about 25 per cent. of condensed water. If the expansion were actually adiabatic, and if there were no loss in the transfer of the steam, the expansion curves for both cylinders would fall into this line.

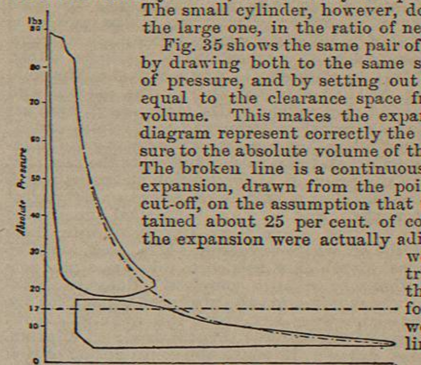

Fig. 35.—Diagrams of fig. 34 combined.

119. Fig. 36 exhibits, in the same manner as fig. 35, a set of diagrams taken by Mr Kirk from the triple expansion engines of the S.S. "Aberdeen." Each diagram is set out from the line of no volume by a distance which represents the clearance in the corresponding cylinder. The boiler pressure is 125 lb per square inch. The cylinders are 32 inches, 46 inches, and 70 inches in diameter, and the stroke is $4\frac{1}{2}$ feet. The cranks make 120° with each other. The means of the diagrams for the two ends of each cylinder have been used in drawing this and the next figure, a practice which should be followed in drawing combined diagrams of the kind here exemplified.

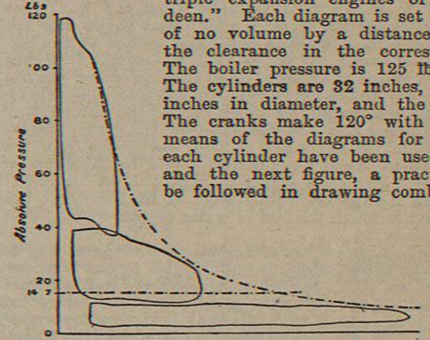

Fig. 36.

120. Fig. 37 shows in the same way a set of diagrams taken by Mr Brock from the *quadruple* expansion engines of the S.S. "Lohara" (by Messrs Denny & Co.). Here the boiler pressure was 154 lb by gauge, or 169 lb absolute, the cylinders were 24 inches, 34 inches, 48 inches, and 68 inches in diameter, the stroke was 4 feet, and the number of revolutions 65 per minute.

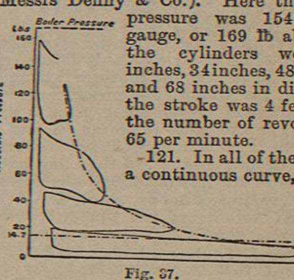

Fig. 37.

121. In all of these cases a continuous curve, shown by a broken line, has been drawn to represent the result of adiabatic expansion, on the same assumption as before—that the steam contains about 25 per cent. of water at the point of cut-off in the

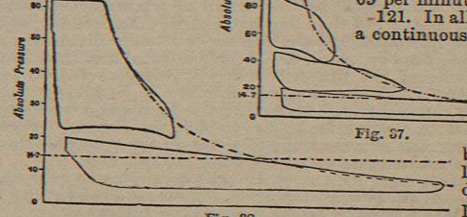

Fig. 38.

[1] An intermediate receiver has the thermodynamic advantage that it reduces the range of temperature in the high-pressure cylinder, and so helps to prevent initial condensation of the steam. This will be made obvious by a comparison of fig. 29 and fig. 30. The lowest temperature reached in the high-pressure cylinder is that corresponding to the pressure at D, and is materially higher in fig. 30 than in fig. 29.

[2] See a paper by Prof. R. H. Smith, "On the Cut-off in the Large Cylinder of Compound Engines," *The Engineer*, November 27, 1885.

[1] Examples of calculations dealing with particular arrangements of two and three cylinder compound engines will be found in an Appendix to Mr R. Sennett's *Treatise on the Marine Steam Engine*.

[2] See a paper by Dr W. Pole, "On the Double Cylinder Expansive Engine," *Proc. Inst. M.E.* 1862.