elasticity, it is hardened, and (in some cases at least) its physical properties go on slowly changing for days or even months. Instances of the hardening effect of permanent set occur when plates or bars are rolled cold, hammered cold, or bent cold, or when wire is drawn. When a hole is punched in a plate the material contiguous to the hole is severely distorted by shear, and is so much hardened in consequence that when a strip containing the punched hole is broken by tensile stress the hardened portion, being unable to extend so much as the rest, receives an undue proportion of the stress, and the strip breaks with a smaller load than it would have borne had the stress been uniformly distributed. This bad effect of punching is especially noticeable in thick plates of mild steel. It disappears when a narrow ring of material surrounding the hole is removed by means of a rimer, so that the material that is left is homogeneous. Another remarkable instance of the same kind of action is seen when a mild-steel plate which is to be tested by bending has a piece cut from its edge by a shearing machine. The result of the shear is that the metal close to the edge is hardened, and, when the plate is bent, this part, being unable to stretch like the rest, starts a crack or tear which quickly spreads across the plate on account of the fact that in the metal at the end of the crack there is an enormously high local intensity of stress (see ELASTICITY, § 72). By the simple expedient of planing off the hardened edge before bending the plate homogeneity is restored, and the plate will then bend without damage.

36. The hardening effect of strain is removed by the process of annealing, that is, by heating to redness and cooling slowly. In iron, very mild steel, and most other metals the rate of cooling is a matter of indifference; but in steel that contains more than about 0·2 per cent. of carbon another kind of hardening is produced if the metal, after being heated to redness, is cooled suddenly. When the proportion of carbon is considerably greater than this, steel may be rendered excessively hard and brittle ("glass-hard") by sudden cooling from a red heat. Further, by being subsequently heated to a moderate temperature, it may be deprived of some of this hardness and rendered elastic through a wide range of strain. This process is called the tempering of steel; its effects depend on the temperature to which the steel is heated after being hardened, and the grade of temper which is acquired is usually specified by the colour (blue, straw, &c.) which appears on a clean surface of the metal during this heating, through the formation of a film of oxide. In the ordinary process of rolling plates or bars of iron or mild steel the metal leaves the rolls at so high a temperature that it is virtually annealed, or pretty nearly so.[1] The case is different with plates and bars that are rolled cold: they, like wire supplied in the hard-drawn state (that is, without being annealed after it leaves the draw-plate), exhibit the higher strength and greatly reduced plasticity which result from permanent set.

37. The extension which occurs when a bar of uniform section is pulled is at first general, and is distributed with some approach to uniformity over the length of the bar. Before the bar breaks, however, a large additional amount of local extension occurs at and near the place of rupture. The material flows in that neighbourhood much more than in other parts of the bar, and the section is much more contracted there than elsewhere. The contraction of area at fracture is frequently stated as one of the results of a test, and is a useful index to the quality of materials. If a flaw is present sufficient to determine the section at which rupture shall occur the contraction of area will in general be distinctly diminished as compared with the contraction in a specimen free from flaws, although little reduction may be noted in the total extension of the piece. Local extension and contraction of area are almost absent in cast-iron and hard steel; on the other hand they are specially prominent in wrought-iron, mild steel, and other metals that combine plasticity with high tensile strength. An example is shown in fig. 16, which is copied from a photograph of a broken test-piece of Whitworth soft fluid-compressed steel.

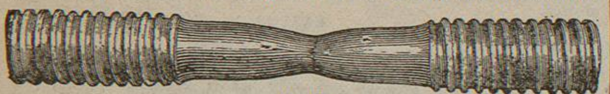
Fig. 16.

38. Experiments with long rods show that the general extension which occurs in parts of the bar not near the break is somewhat irregular;[2] it exhibits here and there incipient local stretching, which has stopped without leading to rupture. This is of course due in the first instance to want of homogeneity. It may be supposed that when local stretching begins at any point in the earlier stages of the test it is checked by the hardening effect of the strain, until, finally, under greater load, a stage is reached in which the extension at one place goes on so fast that the hardening effect cannot keep pace with the increase in intensity of stress which results from diminution of area; the local extension is then unstable, and rupture ensues. Even at this stage a pause in the loading, and an interval of relief from stress, may harden the locally stretched part enough to make rupture occur somewhere else when the loading is continued.

39. Local stretching causes the percentage of elongation which a test-piece exhibits before rupture (an important quantity in engineers' specifications) to vary greatly with the length and section of the piece tested. It is very usual to specify the length which is to exhibit an assigned percentage of elongation. This, however, is not enough; the percentage obviously depends on the relation of the transverse dimensions to the length. A fine wire of iron or steel, say 8 inches long, will stretch little more in proportion to its length than a very long wire of the same quality. An 8-inch bar, say 1 inch in diameter, will show something like twice as much the percentage of elongation as a very long rod. The experiments of M. Barba[3] show that, in material of uniform quality, the percentage of extension is constant for test-pieces of similar form, that is to say, for pieces of various size in which the transverse dimensions are varied in the same proportion as the length. It is to be regretted that in ordinary testing it is not practicable to reduce the pieces to a standard form, with one proportion of transverse dimensions to length, since an arbitrary choice of length and cross-section gives results which are incapable of direct comparison with one another.

Influence on strength.

40. The form chosen for test-pieces in tension tests affects not only the extension but also the ultimate strength. In the first place, if there is a sudden or rapid change in the area of cross section at any part of the length under tension (as at AB, fig. 17), the stress will not be uniformly distributed there. The intensity will be greatest at the edges A and B, and the piece will, in consequence, pass its elastic limit at a less value of the total load than would be the case if the change from the larger to the smaller section were gradual. In a non-ductile material, rupture will for the same reason take place at AB, with a less total load than would otherwise be borne. On the other hand, with a sufficiently ductile material, although the section AB is the first to be permanently deformed, rupture will preferably take place at some section not near AB, because at and near AB the contraction of sectional area which precedes rupture is partly prevented by the presence of the projecting portions C and D. Hence, too, with a ductile material samples such as those of fig. 18, in which the part of smallest section between the shoulders or enlarged ends of the piece is short, will break with a greater load than could be borne by long uniform rods of the same section. In good wrought-iron and mild steel the flow of metal preceding rupture and causing local contraction of section extends over a length six or eight times the width of the piece; and, if the length throughout which the section is uniform be materially less than this, the process of flow will be rendered more difficult and the breaking load of the sample will be raised.[4]

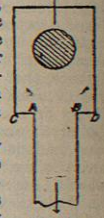

Fig. 17.

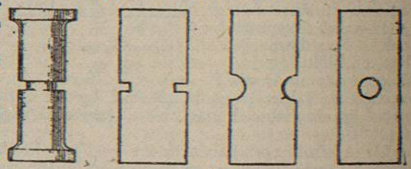
Fig. 18.

These considerations have of course a wider application than to the mere interpretation of special tests. An important practical case is that of riveted joints, in which the metal left between the rivet-holes is subjected to tensile stress. It is found to bear, per square inch, a greater pull than would be borne by a strip of the same plate, if the strip were tested in the usual way with uniform section throughout a length great enough to allow complete freedom of local flow.[5]

41. The tensile strength of long rods is affected by the length in quite a different way. With a perfectly homogeneous material, no difference should be found in the strength of rods of equal sectional area and of different lengths, provided the length of both were great enough to prevent the action described in § 40 from affecting the result. But, since no material is perfectly homogeneous, the longer rod will in general be the weaker, offering as it does more chances of a weak place; and the probable defect of strength in the long rod will depend on the degree of variability of the material. When this has been established by numerous tests of short samples, the strength which a rod of any assigned length may be expected to possess can be calculated by an application of the theory of probabilities. A theory of the strength of long bars has been worked out on this basis by Prof. Chaplin,[1] and has been experimentally confirmed by tests of long and short samples of wire. The theory does not apply when the length is so small that the action of § 40 enters into the case, and the experimental data on which it is based must be taken from tests of samples long enough to exclude that action.

42. In tension tests, rupture may occur, as in fig. 19, by direct separation over a surface which is nearly plane and normal to the line of stress. This is usual in hard steel and other comparatively non-ductile materials. Or it may occur by shearing along an oblique plane, as in fig. 20, which shows the fracture of a piece of steel softer than the specimen of fig. 19. In very ductile samples these two modes of rupture are frequently found in combination, as in fig. 21, where a central core is broken by direct tension while round it is a ring over which separation has taken place by shearing. In this instance the ring is in two parts, one above and one below the surface of rupture of the central flat core. In other instances, such as that of the sample shown in fig. 16, the shorn ring forms a continuous cone or crater round a flat core.


Fig. 19.

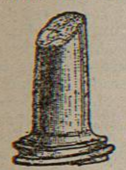
Fig. 20.


Fig. 21.

Fracture by compression.

43. In compression tests of a plastic material, such as mild steel, a process of flow may go on without limit: the piece (which must of course be short, to avoid buckling) shortens and bulges out in the form of a cask. This is illustrated by fig. 22 (from one of Fairbairn's experiments), which shows the compression of a round block of steel (the original height and diameter of which are shown by the dotted lines) by a load equal to 100 tons per square inch of original sectional area. The surface over which the stress is distributed becomes enlarged, and the total load must be increased in a corresponding degree to maintain the process of flow.[2] The bulging often produces longitudinal cracks, as in the figure, especially when the material is fibrous as well as plastic (as in the case of wrought-iron). A brittle material, such as cast-iron brick, or stone, yields by shearing on inclined planes as in figs. 23 and 24, which are taken from Hodgkinson's experiments on cast-iron.[3] The simplest fracture of this kind is exemplified by fig. 23, where a single surface (approximately a plane) of shear divides the compressed block into two wedges. With cast-iron the slope of the plane is such that this simple mode of fracture can take place only if the height of the block is not less than about $\frac{3}{2}$ the width of the base. When the height is less the action is more complex. Shearing must then take place over more than one plane, as in fig. 24, so that cones or wedges are formed by which the surrounding portions of the block are split off. The stress required to crush the block is consequently greater than if the height were sufficient for shearing in a single plane.

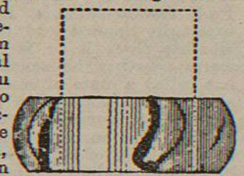
Fig. 22.


Fig. 23.

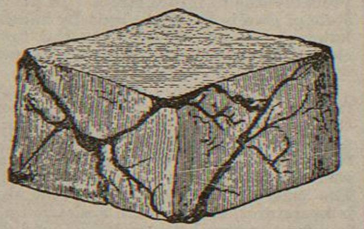
Fig. 24.

44. The inclination of the surfaces of shear, when fracture takes place by shearing under a simple stress of pull or push, is a matter of much interest, throwing some light on the question of how the resistance which a material exerts to stress of one kind is affected by the presence of stress of another kind,—a question scarcely touched by direct experiment. At the shorn surface there is, in the case of tension tests, a normal pull as well as a shearing stress, and in the case of compression tests a normal push as well as shearing stress. If this normal component were absent the material (assuming it to be isotropic) would shear in the surface of greatest shearing stress, which, as we have seen in § 5, is a surface inclined at 45° to the axis. In fact, however, it does not shear on this surface. Hodgkinson's experiments on the compression of cast-iron give surfaces of shear whose normal is inclined at about 55° to the axis of stress,[4] and Kirkaldy's, on the tension of steel, show that when rupture takes place by shear the normal to the surface is inclined at about 25° to the axis.[4] These results show that normal pull diminishes resistance to shearing and normal push increases resistance to shearing. In the case of cast-iron under compression, the material prefers to shear on a section where the intensity of shearing stress is only 0·94 of its value on the surface of maximum shearing stress (inclined at 45°) but where the normal push is reduced to 0·66 of its value on the surface of maximum shearing stress.

45. *Fatigue of Metals.*—A matter of great practical as well as scientific interest is the weakening which materials undergo by repeated changes in their state of stress. It appears that in some if not in all materials a limited amount of stress-variation may be repeated time after time without appreciable deterioration in the strength of the piece; in the balance-spring of a watch, for instance, tension and compression succeed each other some 150 millions of times in a year, and the spring works for years without apparent injury. In such cases the stresses lie well within the elastic limits. On the other hand, the toughest bar breaks after a small number of bendings to and fro, when these pass the elastic limits, although the stress may have a value greatly short of the normal ultimate strength. A laborious research by Wöhler,[5] extending over twelve years, has given much important information regarding the effects on iron and steel of very numerous repeated alternations of stress from positive to negative, or between a higher and a lower value without change of sign. By means of ingeniously contrived machines he submitted test-pieces to direct pull, alternated with complete or partial relaxation from pull, to repeated bending in one direction and also in opposite directions, and to repeated twisting towards one side and towards opposite sides. The results show that a stress greatly less than the ultimate strength (as tested in the usual way by a single application of load continued to rupture) is sufficient to break a piece if it be often enough removed and restored, or even alternated with a less stress of the same kind. In that case, however, the variation of stress being less, the number of repetitions required to produce rupture is greater. In general, the number of repetitions required to produce rupture is increased by reducing the range through which the stress is varied, or by lowering the upper limit of that range. If the greatest stress be chosen small enough, it may be reduced, removed, or even reversed many million times without destroying the piece. Wöhler's results are best shown by quoting a few figures selected from his experiments. The stresses are stated in centners per square zoll;[6] in the case of bars subjected to bending they refer to the top and bottom sides, which are the most stressed parts of the bar.

I. Iron bar in direct tension :—

Stress. Max.	Stress. Min	Number of Applications causing Rupture.	Stress. Max	Stress. Min.	Number of Applications causing Rupture.
480	0	800	320	0	10,141,645
440	0	106,901			
400	0	340,853	440	200	2,373,424
360	0	480,852	440	240	Not broken with 4 millions.

II. Iron bar bent by transverse load :—

Stress. Max.	Stress. Min.	Number of Bendings causing Rupture.	Stress. Max.	Stress. Min.	Number of Bendings causing Rupture.
550	0	169,750	400	0	1,320,000
500	0	420,000	350	0	4,035,400
450	0	481,950	300	0	Not broken with 48 millions.

III. Steel bar bent by transverse load: —

Stress. Max.	Stress. Min.	Number of Bendings causing Rupture.	Stress. Max.	Stress. Min.	Number of Bendings causing Rupture.
900	0	72,450	900	400	225,300
900	200	81,200	900	500	764,900—mean of two trials.
900	300	156,200	900	600	Not broken with 33½ mills.

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[1] In several of Mr Kirkaldy's papers, a comparison is given of the elastic limit, ultimate strength, and ultimate extension of samples which were annealed before testing, and of samples which were tested in the commercial state; in general the annealed samples are distinctly, though not very materially, softer than the others (*On the Relative Properties of Wrought-Iron Plates from Essex and Yorkshire*, London, 1876; also *Experiments on Fagersta Steel*, London, 1873).

[2] See Kirkaldy's *Experiments on Fagersta Steel*, London, 1873; also *Report of the Steel Committee*, part 1.

[3] *Mém. de la Soc. des Ing. Civ.*, 1880; see also a paper by Mr W. Hackney, "On the Adoption of Standard Forms of Test-Pieces," *Min. Proc. Inst. C.E.* 1884.

[4] The greater strength of nicked or grooved specimens seems to have been first remarked by Mr Kirkaldy (*Experiments on Wrought Iron and Steel*, p. 74; also *Experiments on Fagersta Steel*, p. 27). See also a paper by Mr E. Richards on tests of mild steel, *Jour. Iron and Steel Inst.*, 1882.

[5] See Kennedy's "Reports on Rivetted Joints," *Proc. Inst. Mech. Eng.*, 1881-5. In the case of mild steel plates a drilled strip may have as much as 12 per cent. more tensile strength per square inch than an undrilled strip. With punched holes, on the other hand, the remaining metal is much weakened, for the reason referred to in § 35.

[1] *Van Nostrand's Engineering Magazine*, Dec. 1880; *Proc. Engineers' Club of Philadelphia*, March, 1882.

[2] For examples, see Fairbairn's experiments on steel, *Rep. Brit. Ass.*, 1867.

[3] *Report of the Royal Commissioners on the Application of Iron to Railway Structures*, 1849; see also *Brit. Ass. Rep.*, 1837.

[4] *Op. cit.*

[5] *Die Festigkeits-Versuche mit Eisen und Stahl*, Berlin, 1870, or *Zeitschr. für Bauwesen*, 1860-70; see also *Engineering*, vol. xi., 1871. For early experiments by Fairbairn on the same subject, see *Phil. Trans.*, 1864.

[6] According to Bauschinger (*loc. cit.*, p. 44), the centner per square zoll in which Wöhler gives his results is equivalent to 6·837 kilos per square cm., or 0·0434 ton per square inch.

IV. Iron bar bent by supporting at one end, the other end being loaded; alternations of stress from pull to push caused by rotating the bar:—

Stress. From + to −	Number of Rotations causing Rupture.	Stress. From + to −	Number of Rotations causing Rupture.
320	56,430	220	3,632,588
300	99,000	200	4,917,992
280	183,145	180	19,186,791
260	479,490	160	Not broken with 132¼ millions.
240	909,810		

46. From these and other experiments Wöhler concluded that the wrought-iron to which the tests refer could probably bear an indefinite number of stress changes between the limits stated (in round numbers) in the following table (the ultimate tensile strength was about 19½ tons per square inch):—

	Stress in Tons per Sq. Inch.
From pull to push	+7 to −7
From pull to no stress	13 to 0
From pull to less pull	19 to 10½

Hence it appears that the actual strength of this material varies in a ratio which may be roughly given as 3 : 2 : 1 in the three cases of (*a*) steady pull, (*b*) pull alternating with no stress, very many times repeated, and (*c*) pull alternating with push, very many times repeated. Factors of safety applicable to the three cases might therefore rationally stand to one another in the ratio of 1 : 2 : 3. For steel Wöhler obtained results of a generally similar kind. His experiments were repeated by Spangenberg, who extended the inquiry to brass, gun-metal, and phosphor-bronze.[1] On the basis of Wöhler's results formulas have been devised by Launhardt, Weyrauch, and others to express the probable actual strength of metals under assigned variations of stress; these are, of course, of a merely empirical character, and the data are not yet extensive enough to give them much value.[2]

47. Wöhler's experiments, dealing, as all experiments must, with a finite number of stress-changes, leave it an open question whether there are any limits within which a state of stress might be indefinitely often varied without finally destroying the material. It is natural to suppose that a material possessing perfect elasticity would suffer no deterioration from stress-changes lying within limits up to which the elasticity is perfect. But these limits, if they exist at all, are probably very narrow. Indeed, in the case of iron, there is indirect evidence that all alteration of stress whatsoever affects the molecular structure in a way not consistent with the notion of perfect elasticity. When the state of stress in iron is varied, however slowly and however little, the magnetic and thermo-electric qualities of the metal are found to change in an essentially irreversible manner.[3] Every variation leaves its mark on the quality of the piece; the actual quality at any time is a function of all the states of stress in which the piece has previously been placed. It can scarcely be doubted that sufficiently refined methods of experiment would detect a similar want of reversibility in the mechanical effects of stress, even when alterations of stress occur slowly enough to escape the effects of viscosity which have been examined by Sir William Thomson and discussed under ELASTICITY (vol. vii. pp. 802 *sq.*). In any case, the viscosity investigated by Thomson causes such stress-changes as occur rapidly to do work on the material, and the destructive effect of repeated changes may be due in great part to this cause. His experiments show that rapid stress-changes often repeated do produce a cumulative effect in reducing the modulus of elasticity; and it is very probable that this fatigue of elasticity is associated with fatigue of strength.

There are as yet no experiments showing how far fatigue of strength is affected by the frequency, as distinguished from the mere number, of the stress-changes, nor whether a period of rest, after fatigue has been induced, restores strength. That it does so may be conjectured from Thomson's discovery that rest restores elasticity after elastic fatigue. The conjecture is strengthened by Bauschinger's discovery that, after a permanent set has been produced and a period of rest follows, the apparent limit of elasticity (in the strict sense of that term) rises slowly with the lapse of time. Both questions are of obvious practical interest.[4]

48. When a strain is produced within the limits to which Hooke's law applies, the work done in producing it is half the product of the stress into the strain. A load applied to a piece suddenly, but without impact, does an amount of work in straining the piece which is measured by the weight of the load into the distance it sinks in consequence of the strain. Hence, provided this strain falls within the elastic limit, the strain and the stress are twice as great as the same load would produce when in equilibrium. Instances of load applied with complete suddenness, and yet without shock, are rare; but it is a common case for loads to be applied so rapidly that the stress reaches a value intermediate between that due to a static load and the double stress due to the same load applied at once. Thus the Railway Commissioners found that certain bridges were deflected by a train passing at a speed of 50 miles per hour ⅐ more than by the same load at rest.[5] The fact that a "live" load produces greater stress than a dead load is of course to be distinguished from the question Wöhler's experiments deal with—the greater destructiveness of the intermitted or varied stress which a live load causes. In many cases engineers allow in one operation for these quite independent influences of a live load by choosing a higher factor of safety for the live than for the dead part of the whole load on a structure, or (what is the same thing) by multiplying the live load by a coefficient (often 1½), adding the product to the dead load, and treating the sum as if all were dead load.

49. A useful application of diagrams showing the relation of strain to stress is to determine the amount of work done in straining a piece in any assigned way. The term "resilience" is conveniently used to specify the amount of work done when the strain just reaches the corresponding elastic limit. Thus a rod in simple tension or simple compression has a resilience per unit of volume $=f^2/2E$, where f is the greatest elastic pull or push. A blow whose energy exceeds the resilience (reckoned for the kind of stress to which the blow gives rise) must in the most favourable case produce a permanent set; in less favourable cases local permanent set will be produced although the energy of the blow is less than the resilience, in consequence of the strain being unequally distributed. In a plastic material a strain exceeding the limit of elasticity absorbs a relatively large amount of energy, and generally increases the resilience for subsequent strains. Fracture under successive blows, as in the testing of rails by placing them as beams on two supports, and allowing a weight to fall in the middle from a given height, results from the accumulated set which is brought about by the energy of each blow exceeding the resilience

50. In an important paper[6] which is reprinted in the article ELASTICITY, and should be carefully studied in this connexion, Prof. James Thomson has pointed out that the effect of any externally applied load depends, to a very material extent, on whether there is or is not initial internal stress, or, in other words, whether the loaded piece is initially in what Prof. Hearson has called a state of ease. Internal stress, existing without the application of force from without the piece, must satisfy the condition that its resultant vanishes over any complete cross-section. It may exist in consequence of set caused by previously applied forces (a case of which instances are given below), or in consequence of previous temperature changes, as in cast-iron, which is thrown into a state of internal stress by unequally rapid cooling of the mass. Thus in (say) a spherical casting an outside shell solidifies first, and has become partially contracted by cooling by the time the inside has become solid. The inside then contracts, and its contraction is resisted by the shell, which is thereby compressed in a tangential direction, while the metal in the interior is pulled in the direction of the radius. Allusion has already been made to the fact, pointed out by J. Thomson, that the defect of elasticity under small loads which Hodgkinson discovered in cast-iron is probably due to initial stress. In plastic metal a nearly complete state of ease is brought about by annealing; even annealed pieces, however, sometimes show, in the first loading, small defects of elasticity, which are probably due to initial stress, as they disappear when the load is reapplied.

51. Little is exactly known with regard to the effect of temperature on the strength of materials. Some metals, notably iron or steel containing much phosphorus, show a marked increase in brittleness at low temperatures, or "cold-shortness." Experiments on the tensile strength of wrought-iron and steel show in general little variation within the usual atmospheric range of heat and cold. The tensile strength appears to be slightly reduced at very low temperatures, and to reach a maximum when the metal is warmed to a temperature between 100° C. and 200° C. When the temperature exceeds 300° C. the tensile strength begins to fall off rapidly, and at 1000° C. it is less than one-tenth of the normal value.[7] Reference may be made, in this connexion, to the effect which a "blue heat," or temperature short of red heat, is believed to have on the plasticity and strength of iron, and more especially of mild steel. It appears that steel plates and bars bent or otherwise worked at a blue-heat not only run a much more serious risk of fracture in the process than when worked either cold or red-hot, but become deteriorated so that brittleness may afterwards show itself when the metal is cold.[1]

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[1] *Ueber das Verhalten der Metalle bei weiderholten Anstrengungen*, Berlin, 1875.

[2] See Weyrauch, "On the Calculation of Dimensions as depending on the Ultimate Working Strength of Materials," *Min. Proc. Inst. C.E.*, vol. lxiii. p. 275; also a correspondence in *Engineering*, vol. xxix., and Unwin's *Machine Design*, chap. ii.

[3] Ewing, *Phil. Trans.*, 1885, 1886.

[4] For interesting notices of the fatigue of metals in railway axles, bridge ties, &c., and results of experiments showing reduced plasticity in fatigued metal, see Mr B. Baker's address to the Mechanical Section of the British Association, 1885. In most cases where the fatigue of metals occurs in engineering practice the phenomenon is complicated by the occurrence of blows or shocks whose energy is absorbed in producing strains often exceeding the elastic limits, sometimes of a very local character in consequence of the inertia of the strained pieces. Such shocks may cause an accumulation of set which finally leads to rupture in a way that is not to be confused with ordinary fatigue of strength. It appears that the effects of fatigue may be removed by annealing.

[5] *Report of Commissioners on the Application of Iron to Railway Structures*, 1849. A mathematical investigation of this effect of rolling load is given in an appendix to the Report.

[6] *Camb. and Dub. Math. Journ.*, Nov., 1848.

[7] See *Report of a Committee of the Franklin Institute*, 1837; Fairbairn, *Brit. Ass. Rep.*, 1856; Styffe on *Iron and Steel*, trans. by C. P. Sandberg. Notices of these and other experiments will be found in Thurston's *Materials of Engineering*, ii. chap. x., and in papers by J. J. Webster, *Min. Proc. Inst. C.E.*, vol. lx., and A. Martens, *Zeitschr. des Ver. Deutsch. Ing.*, 1883.

52. The following table gives a few representative data regarding the strength of the more important materials used in engineering (the figures are gathered from the writings of Barlow, Hodgkinson, Kirkaldy, Thurston, Rankine, Unwin, Clark, and others):—

	Ultimate Strength. Tons per Square Inch.			Elasticity. Tons. per sq. in.	
	f_t	f_c	f_s	Young's Modulus E.	Mod. of Rigidity C.
Cast-iron	5½ to 10½	25 to 65	9 to 13	5000 to 6000	1300 to 2500
,, average	7	42	11		
,, American (ordnance)	14	36 to 58	...		
,, ,, strengthened by successive fusions	15 to 20	60 to 75	...		
Wrought-iron—					
Finest Lowmoor and Yorkshire plates	27 to 29 along the fibre 24 across the fibre			...	..
Staffordshire	26 along 24 across	,, ,,	18 to 22, or about ⅘ of f_t	12,000 to 13,000	5000
Bridge iron	22 along 19 across	,, ,,			
Bars, finest	27 to 29	...			
Ordinary good	25	20			
Soft Swedish	19 to 24	...			
Wrought-iron wire	25 to 50	...	...	...	...
,, ,, average about	35	...	...	...	...
Steel—					
Mild steel plates (Siemens or Bessemer)	26 to 32 average about 30	...	about ¾ of f_t	12,000 to 13,000	5000 to 5200
Axle and rail steel (do.)	30 to 45	...			
Crucible tool steel	40 to 65	...	...		
Steel castings	about 28	...	...		
Chrome steel	80	...	...		
Tungsten steel	72	...	...		
Whitworth's fluid-compressed steel (mild)	40	...	...		
,, ,, (hard)	48 to 68	...	...		
Steel wire, ordinary, about	70	...	...		
Tempered steel rope wire (highest)	124	...	...	13,000	...
Pianoforte steel wire	150	...	...	13,000	...
Copper, cast	10 to 14	35	10 to 14	7000	...
,, rolled	15 to 16	...	...	...	...
,, wire, hard-drawn	28	...	...	8000	2800
Brass	8 to 13	5	...	5500	1500
,, wire	22	...	...	6400	2200
Muntz metal	22	...	...	...	...
Gun metal	11 to 23	...	...	4500 to 6500	1700
Phosphor bronze	15 to 26	...	...	6000	2400
,, ,, wire	33 to 70	...	...	...	...
Manganese bronze	28 to 32	...	...	...	...
Zinc, cast	2 to 3	...	...	...	...
,, rolled	7 to 10	...	...	5500	...
Tin	2	...	...	...	...
Lead	0·9	3	...	1000	...
Timber—					
Oak	3 to 7	4	1	800	...
White pine	1½ to 3½	2½	...	600	...
Pitch pine	4	...	...	950	...
Riga fir	2½ to 5½	...	...	...	...
Ash	4 to 7	2 to 4	½	750	...
Beech	4 to 6	4	...	...	...
Teak	4 to 7	4	...	950	...
Spanish mahogany	4 to 7	3½	...	650	...
Stone—					
Granite	...	2½ to 5	...	...	...
Sandstone	...	1½ to 2½	...	...	...
Limestone	...	2 to 2½	...	...	...
Slate	...	1½ to 3	...	7000	...
Brick	...	¼ to ¾	...	...	...

53. Space admits of no more than a short and elementary account of some of the more simple straining actions that occur in machines and engineering structures.

The stress which acts on any plane surface AB (fig. 25), such as an imaginary cross-section of a strained piece, may be represented by a figure formed by setting up ordinates A*a*, B*b*, &c., from points on the surface, the length of these being made proportional to the intensity of stress at each point. This gives an ideal solid, which may be called the stress figure, whose height shows the distribution of stress over the surface which forms its base. A line drawn from *g*, the centre of gravity of the stress figure, parallel to the ordinates A*a*, &c., determines the point C, which is called the centre of stress, and is the point through which the resultant of the distributed stress acts. In the case of a uniformly distributed stress, *ab* is a plane surface parallel to AB, and C is the centre of gravity of the surface AB. When a bar is subjected to simple pull applied axially—that is to say, so that the resultant stress passes through the centre of gravity of every cross-section,—the stress may be taken as (sensibly) uniformly distributed over any section not near a place where the form of the cross-section changes, provided the bar is initially in a state of ease and the stress is within the limits of elasticity.

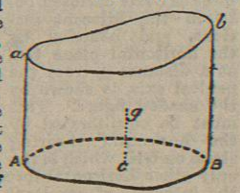

Fig. 25.

54. Uniformly varying stress is illustrated by fig. 26. It occurs (in each case for stresses within the elastic limit) in a bent beam, in a tie subjected to non-axial pull, and in a long strut or column where buckling makes the stress become non-axial. In uniformly varying stress the intensity p at any point P is proportional to the distance of P from a line MN, called the neutral axis, which lies in the plane of the stressed surface and at right angles to the direction AB, which is assumed to be that in which the intensity of stress varies most rapidly. There is no variation of stress along lines parallel to MN. If MN passes through C, the centre of gravity of the surface, as in fig. 27, it may easily be shown that the total pull stress on one side of the neutral axis is equal to the total push stress on the other side, whatever be the form of the surface AB. The resultant of the whole stress on AB is in that case a couple, whose moment may be found as follows. Let dS be an indefinitely small part of the surface at a distance x from the neutral axis through C, and let p be the intensity of stress on dS. The moment of the stress on dS is $xpdS$. But $p=p_1x/x_1=p_2x/x_2$ (see fig. 27). The whole moment of the stress on AB is $\int xpdS=(p_1/x_1)\int x^2dS=p_1I/x_1$ or p_2I/x_2, where I is the moment of inertia of the surface AB about the neutral axis through C.

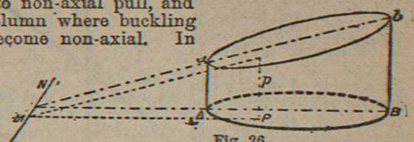

Fig. 26

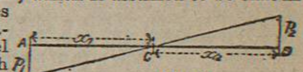

Fig. 27.

55. A stress such as that shown in fig. 26 or fig. 28 may be regarded as a uniformly distributed stress of intensity p_0 (which is the intensity at the centre of gravity of the surface C) and a stress of the kind shown in fig. 27. The resultant is p_0S, where S is the whole area of the surface, and it acts at a distance CD from C such that the moment $p_0\text{S}\,.\,\text{CD}=(p_2-p_0)\text{I}/x_1=(p_1+p_0)\text{I}/x_2$. Hence $p_2=p_0(1+x_2\text{S}\,.\,\text{CD}/\text{I})$, and $p_1=p_0(1-x_1\text{S}.\,\text{CD}/\text{I})$.

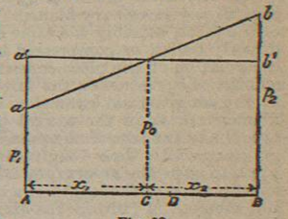

Fig. 28.

56. Simple bending occurs when a beam is in equilibrium under equal and opposite couples in the plane of the beam. Thus if a beam (fig. 29), supported at its ends, be loaded at two points so that $W_1l_1=W_2l_2$, the portion of the beam lying between W_1 and W_2 is subjected to a simple bending stress. On any section AB the only stress consists of pull and push, and has for its resultant a couple whose moment $M=W_1l_1=W_2l_2$. This is called the *bending moment* at the section. If the stress be within the elastic limits it will be distributed as in fig. 30, with the neutral axis at the centre of gravity of the section. The greatest intensities of push and of pull, at the top and bottom edge respectively, are (by § 54) $p_1=My_1/I$ and $p_2=My_2/I$, and the intensity at any point at a distance y above or below C is $p=My/I$.

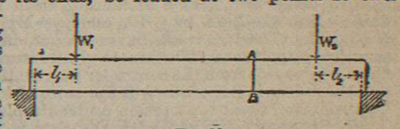

Fig. 29.

57. Let the bending moment now be increased; non-elastic strain will begin as soon as either p_1 or p_2 exceeds the corresponding limit of elasticity, and the distribution of stress will be changed in consequence of the fact that the outer layers of the beam are taking set while the inner layers are still following Hooke's law. As a simple instance we may consider the case of a material strictly elastic up to a certain stress, and then so plastic that a relatively very large amount of strain is produced without further change of stress, a case not very far from being realized by soft wrought-iron and mild steel. The diagram of stress will now take the form sketched in fig. 31. If the elastic limit is (say) less for compression than for tension, the diagram will be as in fig. 32, with the neutral axis shifted towards the tension side. When the beam is relieved from external load it will be left in a state of internal stress, represented, for the case of fig. 31, by the dotted lines in that figure.

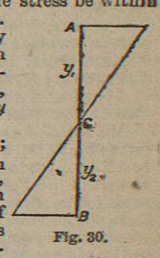

Fig. 30.

58. In consequence of the action which has been illustrated (in a somewhat crude fashion) by figs. 31 and 32, the moment required

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[1] Stromeyer, "The Injurious Effect of a Blue Heat on Steel and Iron," *Min. Proc. Inst. C.E.*, vol. lxxxiv., 1886.