each point the angle between the back and forward lines; he runs his lines as much as possible over level and open ground, avoiding obstacles by working round them. The system is well suited for laying down roads, boundary-lines, and circuitous features of the ground, and is very generally resorted to for filling in the interior details of surveys based on triangulation. It has been largely employed in certain districts of British India, which had to be surveyed in a manner to satisfy fiscal as well as topographical requirements; for, the village being the administrative unit of the district, the boundary of every village had to be laid down, and this necessitated the survey of an enormous number of circuits. Moreover, the traverse system was better adapted for the country than a network of triangulation, as the ground was generally very flat and covered with trees, villages, and other obstacles to distant vision, and was also devoid of hills and other commanding points of view. The principal triangulation had been carried across it, but by chains executed with great difficulty and expense, and therefore at wide intervals apart, with the intention that the intermediate spaces should be provided with points as a basis for the general topography in some other way. A system of traverses was obviously the best that could be adopted under the circumstances, as it not only gave all the village boundaries but was practically easier to execute than a network of minor triangulation.

Procedure of the Indian Survey.—The traverses are executed in minor circuits following the periphery of each village and in major circuits comprising groups of several villages; the former are done with 4″ to 6″ theodolites and a single chain, the latter with 7″ to 10″ theodolites and a pair of chains, which are compared frequently with a standard. The main circuits are connected with every station of the principal triangulation within reach. The meridian of the origin is determined by astronomical observations; the angle at the origin between the meridian and the next station is measured, and then at each of the successive stations the angle between the immediately preceding and following stations; summing these together, the "inclinations" of the lines between the stations to the meridian of the origin are successively determined. The distances between the stations, multiplied by the cosines and sines of the inclinations, give the distance of each station from the one preceding it, resolved in the directions parallel and perpendicular respectively to the meridian of the origin; and the algebraical sums of these quantities give the corresponding rectangular coordinates of the successive stations relatively to the origin and its meridian. The area included in any circuit is expressed by the formula

$$\text{area} = \text{half algebraical sum of products } (x_1+x_2)\,(y_2-y_1) \quad (30),$$

x_1, y_1 being the coordinates of the first, and x_2, y_2 those of the second station, of every line of the traverse in succession round the circuit.

Of geometrical tests there are two, both applicable at the close of a circuit: the first is angular, viz., the sum of all the interior angles of the described polygon should be equal to twice as many right angles as the figure has sides, less four; the second is linear, viz., the algebraical sum of the x coordinates and that of the y coordinates should each be $=0$. The astronomical test is this: at any station of the traverse the azimuth of a referring mark may be determined by astronomical observations; the inclination of the line between the station and the referring mark to the meridian of the origin is given by the traverse; the two should differ by the convergency of the meridians of the station and the origin. In practice the angles of the traverse are usually adjusted to satisfy their special geometrical and astronomical tests in the first instance, and then the coordinates of the stations are calculated and adjusted by corrections applied to the longest, that the angles may be least disturbed, as no further corrections are given them.

Convergency of meridians.

Convergency of Meridians.—The exact value of the convergence, when the distance and azimuth of the second astronomical station from the first are known, is that of $B-(\pi+A)$ of equation (11); but, as the first term is sufficient for a traverse, we have

$$\text{convergency} = x \tan\lambda \,\frac{\operatorname{cosec} 1''}{\nu},$$

substituting x, the coordinate of the second station perpendicular to the meridian of the origin, for $c \sin A$.

Adjustment of a System of Traverses to a Triangulation.—The coordinates of the principal stations of a trigonometrical survey are usually the spherical coordinates of latitude and longitude; those of a traverse survey are always rectangular, plane for a small area but spherical for a large one. It is often necessary, therefore, for purposes of comparison and check at stations common to surveys of both descriptions, to convert either rectangular coordinates into latitudes and longitudes, or *vice versa*, in order that the errors of traverses may be dispersed by proportion over the coordinates of the traverse stations, if desired, or adjusted in the final mapping. The latter is generally all that is necessary, more particularly when the traverses are referred to successive trigonometrical stations as origins, as the operations are being extended, in order to prevent any large accumulation of error. Similar conversions are also frequently necessary in map projections. The method of effecting them will now be indicated.

Transformation of coordinates.

Transformation of Latitude and Longitude Coordinates into Rectangular Spherical Coordinates, and vice versa.—Let A and B be any two points, Aa the meridian of A, Bb the parallel of latitude of B; then Ab, Bb will be their differences in latitude and longitude; from B draw BP perpendicular to Aa; then AP, BP will be the rectangular spherical coordinates of B relatively to A. Put $BP=x$, $AP=y$, the arc $Pb=\eta$, and the arc Bb, the difference of longitude, $=\omega$; also let λ_a, λ_b, and λ_p be the latitudes of A, B, and the point P, ρ_p the radius of curvature of the meridian, and ν_p the normal terminating in the axis minor for the latitude λ_p; and let ρ_0 be the radius of curvature for the latitude $\frac{1}{2}(\lambda_a+\lambda_p)$. Then, when the rectangular coordinates are given, we have, taking A as the origin, the latitude of which is known

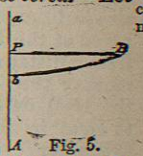

Fig. 5.

$$\left.\begin{array}{l} \lambda_p=\lambda_a+\dfrac{y}{\rho_0}\operatorname{cosec}1''; \quad \eta=\dfrac{x^2}{2\rho_p\nu_p}\tan\lambda_p\operatorname{cosec}1''; \\ \lambda_b-\lambda_a=\dfrac{y}{\rho_0}\operatorname{cosec}1''-\eta; \quad \omega=\dfrac{x}{\nu_p}\sec(\lambda_b+\frac{1}{3}\eta)\operatorname{cosec}1'' \end{array}\right\}(31).$$

And, when the latitude and longitude are given, we have[1]

$$\left.\begin{array}{l} \eta=\left(\dfrac{\omega}{2}\right)^2\dfrac{\nu_b}{\rho_b}\sin 2\lambda_b \sin 1'' \\ y=\rho_0\{\lambda_b-\lambda_a+\eta\}\sin 1'' \\ x=\omega\nu_p\cos(\lambda_b+\frac{1}{3}\eta)\sin 1'' \end{array}\right\}\ldots\ldots\ldots\ldots\ldots(32).$$

Graphic Method of Determining the Coordinates of an Unvisited Point observed from Several Stations.—When a hill peak or other prominent object has been observed from a number of stations whose coordinates are already fixed, the converging rays may be projected graphically, and from an examination of their several intersections the most probable position of the object may be obtained almost as accurately as by calculations by the method of least squares, which are very laborious and out of place for the determination of a secondary point. The following is a description of the application of this method to points on a plane surface in the calculations of the Ordnance Survey. Let $s_1, s_2, \ldots$ be stations whose rectangular coordinates, $x_1, x_2, \ldots$ perpendicular, and $y_1, y_2, \ldots$ parallel, to the meridian of the origin are given; let $\alpha_1, \alpha_2, \ldots$ be the bearings—here the direction-inclinations with the meridian of the origin—of any point P, as observed at the several stations; and let p be an approximate position of P, with coordinates x_p, y_p, as determined by graphical projection on a district map or by rough calculation. Construct a diagram of the rays converging around p, by taking a point to represent p and drawing two lines through it at right angles to each other to indicate the directions of north, south, east, and west. Calculate accurately $(y_p-y_1)\tan\alpha_1$, and compare with (x_p-x_1); the difference will show how far the direction of the ray from s_1 falls to the east or west of p. Or calculate $(x_p-x_1)\cot\alpha_1$, and compare with (y_p-y_1) to find how far the direction falls to the north or south of p. Set off the distance on the corresponding axis of p, and through the point thus fixed draw the direction α_1 with a common protractor. All the other rays around p may be drawn in like manner; they will intersect each other in a number of points, the centre of which may be adopted as the most probable position of P. The coordinates of P will then be readily obtained from those of $p \pm$ the distances on the meridian and perpendicular. In the annexed diagram (fig. 6) P is supposed to have been observed from five stations, giving as many intersecting rays, (1, 1), (2, 2), . . . ; there are ten points of intersection, the mean position of which gives the true position of P, the assumed position being p. The advantages claimed for the method are that, the bearings being independent, an erroneous bearing may be redrawn without disturbing those that are correct; similarly new bearings may be introduced without disturbing previous work, and observations from a large number of stations may be readily utilized, whereas, when calculation is resorted to, observations in excess of the minimum number required are frequently rejected because of the labour of computing them.[1]

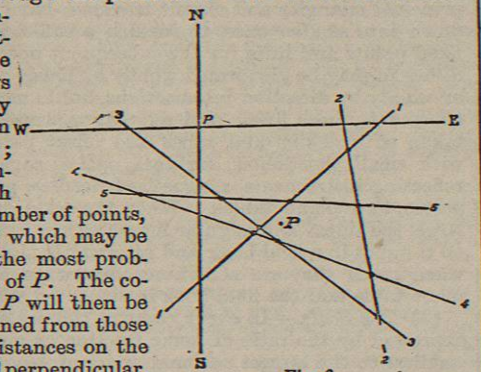

Fig. 6.

III. LEVELLING.

Levelling is the art of determining the relative heights of points on the surface of the ground as referred to a hypothetical surface which cuts the direction of gravity everywhere at right angles. When a line of instrumental levels is commenced at the sea-level, a series of heights is determined corresponding to what would be found by perpendicular measurements upwards from the surface of water communicating freely with the sea in underground channels; thus the line traced indicates a hypothetical prolongation of the surface of the sea inland, which is everywhere conformable to the earth's curvature.

The trigonometrical determination of the relative heights of points at known distances apart, by the measurements of their mutual vertical angles,—as already described in section I.—is a method of levelling. But the method to which the term "levelling" is always applied is that of the direct determination of the differences of height from the readings of the lines at which graduated staves, held vertically over the points, are cut by the horizontal plane which passes through the eye of the observer. Each method has its own advantages. The former is less accurate, but best suited for the requirements of a general geographical survey, to obtain the heights of all the more prominent objects on the surface of the ground, whether accessible or not. The latter may be conducted with extreme precision, and is specially valuable for the determination of the relative levels, however minute, of easily accessible points, however numerous, which succeed each other at short intervals apart; thus it is very generally undertaken *pari passu* with geographical surveys, to furnish lines of level for ready reference as a check on the accuracy of the trigonometrical heights. In levelling with staves the measurements are always taken from the horizontal plane which passes through the eye of the observer; but the line of levels which it is the object of the operations to trace is a curved line, everywhere conforming to the normal curvature of the earth's surface, and deviating more and more from the plane of reference as the distance from the station of observation increases. Thus, either a correction for curvature (see footnote, page 705) must be applied to every staff reading, or the instrument must be set up at equal distances from the staves; the curvature correction, being the same for each staff, will then be eliminated from the difference of the readings, which will thus give the true difference of level of the points on which the staves are set up.

Levelling is an essentially simple operation; but, as it has to be repeated very frequently in executing a long line of levels—say seven times on an average in every mile—it must be conducted with every precaution against errors of various kinds, instrumental and personal, some accidental and tending to cancel each other, others systematic and cumulative. Instrumental errors arise when the visual axis of the telescope is not perpendicular to the axis of rotation, and when the focusing tube does not move truly parallel to the visual axis on a change of focus. The first error is eliminated, and the second avoided, by placing the instrument at equal distances from the staves; and, as this procedure has also the advantage of eliminating the corrections for both curvature and refraction, it should invariably be adopted. Errors of staff readings should be guarded against by having the staves graduated on both faces, but differently figured, so that the observer may not be biassed to repeat an error of the first reading in the second. The staves of the Indian survey have one face painted white with black divisions—feet, tenths, and hundredths—from 0 to 10, the other black with white divisions from 5·55 to 15·55. Deflexion from horizontality may either be measured and allowed for by taking the readings of the ends of the bubble of the spirit-level and applying corresponding corrections to the staff readings, or be eliminated by setting the bubble to the same position on its scale at the reading of the second staff as at that of the first, both being equidistant from the observer.

Certain errors are liable to recur in a constant order and to accumulate to a considerable magnitude, though they may be too minute to attract notice at any single station, as when the work is carried on under a uniformly sinking or rising refraction—from morning to midday or from midday to evening—or when the instrument takes some time to settle down on its bearings after being set up for observation. They may be eliminated (i.) by alternating the order of observation of the staves, taking the back staff first at one station and the forward first at the next; (ii.) by working in a circuit, or returning over the same line back to the origin; (iii.) by dividing a line into sections and reversing the direction of operation in alternate sections. Cumulative error, not eliminable by working in a circuit, may be caused when there is much northing or southing in the direction of the line, for then the sun's light will often fall endwise on the bubble of the level, illuminating the outer edge of the rim at the nearer end and the inner edge at the further end, and so biassing the observer to take scale readings of edges which are not equidistant from the centre of the bubble; this introduces a tendency to raise the south or depress the north

---

[1] In the Indian Survey, tables are employed for these calculations which give the value of 1″ of arc in feet on the meridian, and on each parallel of latitude at intervals of 5′ apart; also a corresponding table of arc-versines (Pb) of spheroidal arcs of parallel (Bb) 1° in length, from which the arc-versines for shorter or longer arcs are obtained proportionally to the squares of the arcs; x is taken as the difference of longitude converted into linear measure.

[1] For fuller details and an application to spherical surfaces, see *Account of the Graphic Method of the Ordnance Survey*, by J. O'Farrell, London, 1886.

ends of lines of level in the northern hemisphere. On long lines, the employment of a second observer, working independently over the same ground as the first, station by station, is very desirable. The great lines are usually carried over the main roads of the country, a number of "bench marks" being fixed for future reference. In the Ordnance Survey of Great Britain lines have been carried across from coast to coast, in such a manner that the level of any common crossing point may be found by several independent lines. Of these points there are 166 in England, Scotland, and Wales; the discrepancies met with at them were adjusted simultaneously by the method of minimum squares.

Sea-Level.—The sea-level is the natural datum plane for levelling operations, more particularly in countries bordering on the ocean. The earliest surveys of coasts were made for the use of navigators, and, as it was considered very important that the charts should everywhere show the minimum depth of water which a vessel would meet with, low water of spring-tides was adopted as the datum. But this does not answer the requirements of a land survey, because the tidal range between extreme high and low water differs greatly at different points on coast-lines. Thus the generally adopted datum plane for land surveys is the mean-sea level, which, if not absolutely uniform all the world over, is much more nearly so than low water. Tidal observations have been taken at nearly fifty points on the coasts of Great Britain, which were connected by levelling operations; the local levels of mean sea were found to differ by larger magnitudes than could fairly be attributed to errors in the lines of level, having a range of 12 to 15 inches above or below the mean of all at points on the open coast, and more in tidal rivers.[1] But the general mean of the coast stations for England and Wales was practically identical with that for Scotland. The observations, however, were seldom of longer duration than a fortnight, which is insufficient for an exact determination of even the short period components of the tides, and ignores the annual and semi-annual components, which occasionally attain considerable magnitudes. The mean-sea levels at Port Said in the Mediterranean and at Suez in the Red Sea have been found to be identical, and a similar identity is said to exist in the levels of the Atlantic and the Pacific Oceans on the opposite coasts of the isthmus of Panama. This is in favour of a uniform level all the world over; but, on the other hand, lines of level carried across the continent of Europe make the mean-sea level of the Mediterranean at Marseilles and Trieste from 2 to 5 feet below that of the North Sea and the Atlantic at Amsterdam and Brest,—a result which it is not easy to explain on mechanical principles. In India various tidal stations on the east and west coasts, at which the mean-sea level has been determined from several years' observations, have been connected by lines of level run along the coasts and across the continent; the differences between the results were in all cases due with greater probability to error generated in levelling over lines of great length than to actual differences of sea-level in different localities.

The sea-level, however, may not coincide everywhere with the geometrical figure which most closely represents the earth's surface, but may be raised or lowered, here and there, under the influence of local and abnormal attractions, presenting an equipotential surface—an ellipsoid or spheroid of revolution slightly deformed by bumps and hollows—which Bruns calls a "geoid." Archdeacon Pratt has shown that, under the combined influence of the positive attraction of the Himalayan Mountains and the negative attraction of the Indian Ocean, the sea-level may be some 560 feet higher at Kurrachee than at Cape Comorin; but, on the other hand, the Indian pendulum operations have shown that there is a deficiency of density under the Himalayas and an increase under the bed of the ocean, which may wholly compensate for the excess of the mountain masses and deficiency of the ocean, and leave the surface undisturbed. If any bumps and hollows exist, they cannot be measured instrumentally; for the instrumental levels will be affected by the local attractions precisely as is the sea-level, and will thus invariably show level surfaces even should there be considerable deviations from the geometrical figure.

IV. Survey of Interior Detail.

(1) *General Principles.*—We have seen that the skeleton framework of a survey may be either a triangulation or a system of traverses; very generally it is a combination of both. The method of filling in the details is necessarily influenced to some extent by the nature of the framework, but it depends mainly on the magnitude of the scale and the requisite degree of minutiæ. In all instances the principal triangles and circuit traverses have to be broken down into smaller ones, to furnish a sufficient number of fixed points and lines for the subsequent operations. The filling in may be performed wholly by linear measurements or wholly by direction intersections, but is most frequently effected by both linear and angular measures, the former taken with chains and tapes and offset poles, the latter with small theodolites, sextants, optical squares, or other reflecting instruments, magnetized needles, prismatic compasses, and plane tables. When the scale of a survey is large, the linear and angular measures are usually recorded on the spot in a field-book and afterwards plotted in office; when small they are sometimes drawn on the spot on a plane table and the field-book is dispensed with.

(2) *The Scale.*—In every country the scale is generally expressed by the ratio of some fraction or multiple of the smallest to the largest national units of length, but sometimes by the fraction which indicates the ratio of the length of a line on the paper to that of the corresponding line on the ground. The latter form is obviously preferable being international and independent of the various units of length adopted by different nations. See table of maps and scales under MAP, vol. xv. p. 522. In the Ordnance Survey of Great Britain and Ireland both forms of expression are adopted, the smaller scales being 1 inch and 6 inches to a mile for provinces and counties, the larger $\frac{1}{2500}$ for parishes and $\frac{1}{500}$ for towns. In the Indian Survey the standard topographical scale is 1 inch to a mile, diminishing to $\frac{1}{2}$ and $\frac{1}{4}$ inch for geographical reconnaissance, and rising by multiples of 2 to higher scales, of which the greatest, for other than city surveys, is 32 inches, for cadastral purposes. In both surveys the double unit of the foot and the Gunter's link ($=\frac{66}{100}$th of a foot) is employed, the former invariably in the triangulation, the latter very generally in the traversing and filling in, because of its convenience in calculations and measurements of area, a square chain of 100 Gunter's links being exactly one-tenth of an acre.

(3) *Ordnance Survey Methods.*—All linear measures are made with the Gunter's chain, all angular with small theodolites only; neither magnetized nor reflecting instruments nor plane tables are ever employed, except in hill sketching, when bearings are taken with the prismatic compass. As a rule the filling in is done by triangle-chaining only; traverses with theodolite and chain are occasionally resorted to, but only when it is necessary to work round woods and hill tracts across which right lines cannot be carried.

(*a*) *Detail Surveying by Triangles.*—This is based on the points of the minor triangulation. The sides are first chained perfectly straight, all the points where the lines of interior detail cross the sides being fixed; the alignment is effected with a small theodolite, and marks are established at the crossing points and at any other points on the sides where they may be of use in the subsequent operations. The surveyor is given a diagram of the triangulation, but no side lengths, as the accuracy of his chaining is tested by comparison with the trigonometrical values. Then straight lines are carried across the intermediate detail between the points established on the sides; they constitute the principal "cutting up or split lines"; their crossings of detail are marked in turn and straight lines are run between them. The process is continued until a sufficient number of lines and marks have been established on the ground to enable all houses, roads, fences, streams, railways, canals, rivers, boundaries, and other detail to be conveniently measured up to and fixed. Perpendicular offsets are limited to eighty and twenty links for the respective scales of 6 inches to a mile and $\frac{1}{2500}$.

---

[1] In tidal estuaries and rivers the mean-water level rises above the mean-sea level as the distance from the open coast-line increases; for instance, in the Hooghly river, passing Calcutta, there is a rise of 10 inches in 42 miles between Sagar (Saugor) Island at the mouth of the river and Diamond Harbour, and a further rise of 20 inches in 43 miles between Diamond Harbour and Kidderpur.

(*b*) *Detail Surveying by Traverses.*—When a considerable area has to be thus treated it is divided into a number of blocks of convenient size, bounded by roads, rivers, or parish boundaries, and a "traverse on the meridian of the origin" is carried round the periphery of each block. Commencing at a trigonometrical station, the theodolite is set to circle reading 0° 0′ with the telescope pointing to the north, and at every "forward" station of the traverse the circle is set to the same reading when the telescope is pointed at the "back" station as was obtained at the back station when the telescope was pointing to the forward one. When the circuit is completed and the theodolite again put up at the origin and set on the last back station with the appropriate circle reading, the circle reading, with the telescope again pointed to the first forward station, will be the same as at first, if no error has been committed. This system establishes a convenient check on the accuracy of the operations and enables the angles to be readily protracted on a system of lines parallel to the meridian of the origin. As a further check the traverse is connected with all contiguous trigonometrical stations by measured angles and distances. Traverses are frequently carried between the points already fixed on the sides of the minor triangles; the initial side is then adopted, instead of the meridian, as the axis of coordinates for the plotting, he telescope being pointed with circle reading 0° 0′ to either of he trigonometrical stations at the extremities of the side.

(*c*) *Plotting and Examination.*—The plotting is done from the eld-books of the surveyors by a separate agency. Its accuracy tested by examination on the ground, when all necessary addenda e made. The examiner—who should be both surveyor, plotter, d draftsman—mounts the plot on his sketching block, and verifies the accuracy of the detail by intersections and productions and occasional direct measurements, and generally endeavours to cause the details under examination to prove the accuracy of each other rather than to obtain direct proof by remeasurement. He fixes conspicuous trees and delineates the woods, footpaths, rocks, precipices, steep slopes, embankments, &c., and supplies the requisite information regarding minor objects—whether pit, shaft, level, spring, well, conduit, weir, quarry, refuse heap, waste, orchard, stack-yard, railway, canal, manufacturing and mineral works, viaducts, bridges, tramways, plantations, &c.—to enable a draftsman to make a perfect representation according to the scale of the map. In examining a coast-line he delineates the foreshore and sketches the strike and dip of the stratified rocks. In tidal rivers he ascertains and marks the highest points to which the ordinary tides flow. The examiner on the 25·344 inch scale ($=\frac{1}{2500}$) is required to give all necessary information regarding the parcels of ground of different character—whether arable, pasture, wood, moor, moss, sandy—defining the limits of each on a separate tracing if necessary. He has also to distinguish between turnpike, parish, and occupation roads, to collect all names, and to furnish notes of military, baronial, and ecclesiastical antiquities to enable them to be appropriately represented in the final maps. The latter are subjected to a double examination,—first in the office, secondly on the ground; they are then handed over to the officer in charge of the levelling to have the levels and contour lines inserted, and finally to the hill sketchers, whose duty it is to make an artistic representation of the features of the ground.

(4) *Indian Survey Methods.*—All filling in is invariably done by plane-tabling on a basis of points previously fixed; the methods differ simply in the extent to which linear measures are introduced to supplement the direction rays of the plane table. When the scale of the survey is small, direct measurements of distance are rarely made and the filling is usually done wholly by direction intersections, which fix all the principal points, and by eye-sketching; but as the scale is increased linear measures with chains and offset poles are introduced to the extent that may be desirable. A sheet of drawing paper is mounted on cloth over the face of the plane table; the points, previously fixed by triangulation or otherwise, are projected on it—the collateral meridians and parallels, or the rectangular coördinates, when these are more convenient for employment than the spherical, having first been drawn; the plane table is then ready for use. Operations are commenced at a fixed point by aligning with the sight rule on another fixed point, which brings the meridian line of the table on that of the station. The magnetic needle may now be placed on the table and a position assigned to it for future reference. Rays are drawn from the station point on the table to all conspicuous objects around with the aid of the sight rule. The table is then taken to other fixed points, and the process of ray-drawing is repeated at each; thus a number of objects, some of which may become available as stations of observation, are fixed. Additional stations may be established by setting up the table on a ray, adjusting it on the back station—that from which the ray was drawn—and then obtaining a cross intersection with the sight rule laid on some other fixed point, also by interpolating between three fixed points situated around the observer. The magnetic needle may not be relied on for correct orientation, but is of service in enabling the table to be set so nearly true at the outset that it has to be very slightly altered afterwards. The error in the setting is indicated by the rays from the surrounding fixed points intersecting in a small triangle instead of a point, and a slight change in azimuth suffices to reduce the triangle to a point, which will indicate the position of the station exactly. Azimuthal error being less apparent on short than on long lines, interpolation is best performed by rays drawn from near points, and checked by rays drawn to distant points, as the latter show most strongly the magnitude of any error of the primary magnetic setting. In this way, and by self-verificatory traverses "on the back ray" between fixed points, plane-table stations are established over the ground at appropriate intervals, depending on the scale of the survey; and from these stations all surrounding objects which the scale permits of being shown are laid down on the table, sometimes by rays only, sometimes by a single ray and a measured distance. The general configuration of the ground is delineated simultaneously.

Checking and examination.

Checking and Examination.—Various methods are followed. For large scale work in plains it is customary to run arbitrary lines across it and make an independent survey of the belt of ground to a distance of a few chains on either side for comparison with the original survey; the smaller scale hill topography is checked by examination from commanding points, and also by traverses run across the finished work on the table.

V. Representation of Ground.

Main lines of ground.

The master lines of ground are the main ridges and water-partings of the hills, the watercourses, and the horizontal contour lines of the coasts; the subordinate lines are those which define the undulations and minor features falling between the low-lying plains and the crests of the hills. These lines must first be laid down on a horizontal projection to fix the dimensions of each feature of the ground, after which the slopes must be indicated with sufficient relief and character to present a true picture of the corrugations of surface. In ancient maps the hills are represented as seen against the sky in profile by a spectator standing on the ground below at some distance off. This system of "natural representation," as it was called, was serviceable in enabling persons looking at the hills from the quarter from which they had been sketched to identify them readily, for which reason such views of distant inland hills are still commonly given on the margins of marine charts of coast-lines for the assistance of navigators. But, when all other objects except the hills are shown in a map by their horizontal projections, hills represented in perspective are false to their surroundings, and misleading to all who approach them from other directions than that of the adopted point of view, for the vertical projection of the profile is practically turned over and confused with the horizontal plane. Hence in course of time hills came to be drawn as if seen from a high bird's-eye point of view, the position of which was shifted until at last the point of sight was supposed to be vertically over them; thus the evils of the perspective system were diminished, whilst something of natural representation was still preserved. About the end of the 18th century the perspective and the bird's-eye systems gave way to the true method of indicating the forms of hills, viz., by their horizontal projections, like all the other details of the ground, and by adding the requisite shading to bring every feature into proper relief.

Hill-Shading.—There are two rival methods of hill-shading,—one by horizontal contours, the other by vertical hachures. A contour being the line of intersection of a hill by a horizontal plane, contour lines indicate the markings which would be made by the successive risings of a flood to different levels above the sea; vertical hachures indicate the directions which the particles of a volume of water, equally disseminated over the top of a hill, would naturally take in running down the sides and slopes. The most perfect representation of ground is obtained when the shade lines, whether horizontal or vertical, are sufficiently close and well graduated in tone and intensity to imitate good mezzotint shading in Indian ink. A good effect may be and is frequently produced by assuming light to fall on the hills obliquely from a specific direction, illuminating them on one side and throwing the reverse slopes into shadow. But this has the disadvantage of giving similar slopes different intensities of shade according to their position with reference to the assumed direction of the light; on the other hand, vertical lighting, which gives the same intensity to the same slope wherever situated, fails in relief and perspicacity. A commission of citizens appointed by the republican Government of France in 1803 to formulate rules on the subject of topography, condemned the representation of hills in demi-perspective as absurd, but approved the system of oblique side-light; it also condemned contours, except for engineering works, and recommended vertical hachures, under the idea that the slope lines of the fall of water represent a material effect of which the eye is witness every moment, and recalls the general cause, if not of the formation, at least of the figure and characteristics, of the mountains.

Scale of shade. *Scale of Shade.*—For military purposes it is very desirable that maps should be so drawn as to enable the angles of inclination of all slopes to be readily ascertained, with a view to determining what portions of the ground are suited for the manœuvres of each of the three arms,—infantry, cavalry, and artillery. Thus military topographers of different nationalities have proposed a variety of scales to regulate the thickness and distance apart of the shade lines, and generally the proportion of black to white, for different angles of slope, that the map may convey to the mind as accurate a knowledge of the slopes of the ground as of the horizontal outlines. All slopes, however, are not of equal practical importance, but only those which are of most common occurrence and most liable to be gone over by men and horses and wheeled vehicles, and their inclination rarely exceeds 25°; consequently it is of most importance to be able to distinguish variations of slope below that angle: it is occasionally desirable to know the sharper slopes up to 45° or 50°, but greater inclinations are comparatively of rare occurrence and unimportant. Now in a true scale of shade the intensity increases with the inclination from 0° to 90°; thus, putting black + white = 1, the proportion of black to white for any inclination i by a scale of cosines will be black $= 1 - \cos i$, white $= \cos i$. But that scale does not sufficiently accentuate the lower inclinations, which are the most important, and have therefore to be dealt with more emphatically; this has led to the introduction of a variety of conventional scales, each with the special characteristics which commended themselves to its author. Major Lehmann of the German army supposed light to be admitted in parallel vertical rays and gave the horizontal plane the fullest light, because the reflected coincides with the vertical ray; at an inclination of 45° the reflected ray is perfectly horizontal, and this slope was therefore least illumined. Disregarding all greater slopes, he placed 45° at the head of his scale and represented it by absolute black; the scale was divided into nine equal parts of 5° each, from 0° to 45°, up to which the illumination varies inversely as the angle of inclination. General van Gorkum of the Netherlands army improved on Lehmann's system: he adopted certain groups of contours arranged according to the slope, making the vertical distances between the contours equal in each group but greater in the higher groups, and between the contours he drew vertical hachures the lengths of which showed by reference to a scale the angles of slope. His lowest group included all angles up to 25°, the vertical distance between the contours being so regulated with reference to the scale of the map as to permit the draftsman to represent the slopes without inconveniently long hachures. For higher angles he doubled and trebled the vertical interval of his contours and the thickness of his hachures. Thus the relative altitudes of any required points might be deduced with comparative facility by noting the thickness and counting the number of the vertical hachures between them. In this respect the system satisfies the requirements of a military map, but the effect is unpleasing and unsuggestive of hill forms. In 1828 a second French commission, having Laplace for its president, was appointed to report on topographical drawing. It reversed the decision of the first commission in favour of oblique side light, as being difficult to execute and inaccurate in giving different intensities to the same angles of slope facing differently; and, after trying various scales of shade, it determined to increase the intensity in proportion to the sines of double the angles of inclination diminished by $\frac{1}{15}$, which gives a more rapid increase of shade for the gentle than the steep slopes. In subsequent instructions of the "depôt de la guerre" the proportion of black to white is fixed at one and a half times the angle of slope. In England various scales of shade have been proposed, by Colonel Scott and Captain Webber of the Royal Engineers, and by the Council of Military Education. Colonel Scott's scale is interesting as having been derived from the average of measurements taken from the best examples of hill sketching in the Ordnance and other surveys, whereas all the others were deduced from a conventional application of geometrical principles. The following table (III.) gives the several scales:—

Table showing the Proportion of Black to White on any Unit of Area, in Horizontal Plan.

Angle of Slope.	Scale of Cosines.		Major Lehmann.		First French.		Second French.		Colonel Scott.		Captain Webber.		Council of Military Education.	
	B.	W.	B.	W.	B.	W.	B.	W.	B.	W.	B.	W.	B.	W.
90°	1·000	·000	..	..	..	..	..	..	..	..	..	..	..	..
45°	·293	·707	1·000	0·000	·600	·400	·675	·325	·708	·292	·803	·197	..	..
35°	·181	·819	·780	·220	·512	·488	·425	·575	..	..	·724	·276	·640	·360
25°	·094	·906	·560	·440	·380	·620	·375	·625	·339	·661	·550	·450	·457	·543
20°	·060	·940	·450	·550	·350	·650	·300	·700	·255	·745	·455	·545	·333	·667
15°	·034	·966	·340	·660	·286	·714	·225	·775	·189	·811	·338	·662	·254	·746
10°	·015	·985	·230	·770	·209	·791	·150	·850	·126	·874	·250	·750	·160	·840
7°	·007	·993	·155	·845	·155	·845	·105	·895	·083	·917	·173	·827	..	..
5°	·004	·996	·120	·880	·110	·890	·075	·925	·055	·945	·108	·892	·082	·918
4°	·002	·998	·077	·923	·095	·905	·060	·940	·049	·951	·065	·935	..	..
3°	·001	·999	·066	·934	·073	·927	·045	·955	·038	·962	·033	·967	·047	·953
2°	·001	·999	·044	·956	·050	·950	·030	·970	·025	·975	·014	·986	·025	·975

Mezzo-tint shading. Of late years the system of shading by lines has been abandoned for the English army, and a method of representing slopes by mezzotint shading over a few governing contour lines, laid down by actual survey, has been introduced instead. The effect aimed at is a transparent shade, dark in proportion to the steepness of the ground represented; its object is to give body and expression to the contours and to explain and develop minor features of the ground which may lie between them. This style of shading, being distinct from all line drawing, may be applied over the most crowded details without causing confusion, such as would be produced by hachure shading. The contours are indicated by continuous red lines of constant thickness, strong enough to be everywhere visible through the shading, which is effected by applying lead with a soft pencil over the parts where it is wanted, and then rubbing it in firmly with a piece of chamois

leather folded into a small pad. No pencil marking is allowed; lightening is done with india-rubber; the shading is finally fixed with a wash of thin gum-water.

It is to be noted that the several scales of shade above given were devised for military maps to be drawn on a scale of not less than 4 inches to the mile and possibly much greater. The harshness and mannerism to which all line-shading by rule is liable are of less importance in maps of small areas represented on large scales than on maps of large areas on small scales. In the former the sacrifice of pictorial effect is more than compensated by the additional information regarding the slopes of the ground; in the latter any attempt to introduce so much information would tend to crowd the map objectionably, and confuse the vertical with the horizontal details. The smaller the scale of a map of hill country the more necessary it is to abandon mechanical conventionalism, and to aim at achieving an artistic representation which will convey an immediate and accurate impression of the general character of the ground.

In India the topography has been mainly executed on scales of or less than 1 inch = 1 mile and rarely exceeding 2 inches, and, as the range of altitude varies considerably in different parts of the country, from plains and undulations little above the sea-level to mountains rising to an altitude of 29,000 feet, scales of shade were long deemed wholly unsuitable for employment. The higher mountains had necessarily to be brought into prominence over the lower by giving them a darker shade than was due simply to their slopes, and similarly the elevated plateaus had to be more lightened and illuminated than the low-lying plains. But in course of time, as the number of hands employed in the operations increased more rapidly than the available supply of artistic draughtsmen, the introduction of a scale of shade became necessary, in order that the multitude of workers might be put more nearly on a par with the few. For men who have been accustomed to associate a certain depth of shade with a certain angle of slope will work together within narrower limits of error and divergence than if left entirely to their own unaided judgment and untutored proclivities. The field sketchers should therefore learn to work on a system which gives every hachure line a definite meaning, so that their sketches may be rightly interpreted and appropriately translated and rendered in the final representation of the ground, when it is the duty of the draughtsman to enhance the tone of the map as much as possible while maintaining its truthfulness.

Ordnance Survey delineation of round. *Ordnance Survey System of Delineating Ground.*—As a rule the features of the ground are sketched in the field on the 6-inch scale, and afterwards reduced and published on the 1-inch scale. The Highlands of Scotland were sketched partly on the 1-inch and partly on the 2-inch scale; in Ireland the 1-inch scale only was used; and this scale is now being adopted for hill sketching in England and Wales. In the parts where the 6-inch scale was used the ground was first contoured instrumentally; a plan of the contours and of all surveyed outlines was supplied to the sketcher, who proceeded to insert the hill features with the aid of a prismatic compass, protractor, plotting scale, and a "hill-sketcher's scale," graduated to show the horizontal intervals between the contours which correspond to various angles of inclination from 0° to 45°. He was required to delineate slopes up to 45° by horizontal hachures, and slopes beyond 45° by vertical hachures. The thickness and number of the strokes, the relation to light and shade, and the character of the touch were left to the skill and experience of the sketcher. The introduction of scales of shade adapted to various inclinations and altitudes was frequently mooted, with a view to securing greater uniformity; but no such scale was adopted, for it was found that, though at first different workmen produced different results, long practice and constant comparison, together with the aid derived from the instrumental contours, effected all desirable uniformity. Thus in good sketches it was found that the maximum breadth of stroke used in the representation of very steep mountain slopes was $\frac{1}{45}$ inch, and the minimum used in low and nearly flat country, $\frac{1}{500}$ inch, also that the average proportions of light to shade were 1 to 3 at the maximum and 25 to 1 at the minimum inclinations. In the field sketches the light is supposed to fall vertically, and all slopes of like altitude and inclination are similarly expressed. The 6-inch sketches are reduced to the 1-inch scale for publication by an artist working with Indian ink and the camel-hair brush on an impression in outline of the 1-inch map. He makes a careful study of the several sketches which he has to combine together, in order to determine which features should be retained and which omitted in the reduction, and he divides the ground into zones of different altitude to guide him in giving a strength of shade proportioned to the altitude rather than to the slope as in the field sketches; and in drawing he increases the contrasts between light and shade and introduces light from a corner of the map to give a stronger relief, and to attract the eye to the highest points and enable it to distinguish readily the higher from the lower ground. His general aim is to produce a more pictorial and less mechanical study of the ground than is supplied by the field sketches. Many exquisite maps have been thus produced and afterwards engraved; see sheets 32, 33, 38, 53, and 64 of Scotland, 38 and 48 of England, 75 of Wales, 93, 94, 191, and 192 of Ireland. These sheets, however, though admirable specimens of engraving, fall short of the original drawings in tone and relief, for in them the hill-shading is necessarily shown by line-etching, and it does not produce such effective contrasts and gradations of light and shade as the original brush work.[1]

Delineation by Instrumental Contouring.—A very precise knowledge of the configuration of surface may be acquired by carrying true contour lines over the ground and projecting them on the map of the survey. But the contours do not give a true representation of the ground, for they seldom represent actual lines on the surface, as do the lines on the map which indicate roads, watercourses, walls, enclosures, &c.; they give, however, a conventional representation which is sufficient *per se* for the engineer and the expert, and they furnish guiding lines for all shading, whether by hachures or mezzotint, which may be subsequently executed to produce an artistic delineation of the features of the ground. In instrumental contouring we have first to decide on the vertical intervals to be maintained between the contours. They depend on the scale of the survey and the nature of the ground. In the Ordnance Survey they are made as small as from 5 to 10 feet, when special plans on large scales are being prepared for engineering requirements; but for the general maps they are 50 feet up to an elevation of 100 feet above the sea-level, and 100 beyond up to 900 feet, which elevation, being the practical limit of cultivation, is the highest generally marked, though in the northern counties of England and in parts of Scotland additional contours have been executed at the altitudes of 1000, 1250, 1500, 1750, 2000, 2500, 3000, &c., feet. The intervals having been determined on, instrumental levelling is commenced at either the top or the bottom of those slopes which best define the general lay of the ground, or at some previously established bench mark of which the height above the sea is known. Points are marked out on the slopes with pickets at the prescribed vertical intervals, and then the contour lines of the horizontal planes passing in succession through each of these points are traced with a levelling instrument and staff and surveyed by traverse, the two processes being performed either simultaneously or consecutively as may be most convenient.

The instruments generally used in the Ordnance Survey are a 5-inch theodolite—employed as a levelling instrument—and a contouring staff, 8 feet long, provided with a sliding vane which may be fixed at any required height; the staff is shifted about until the vane is brought into the horizontal plane of the theodolite, when the bottom of the staff will be on the contour line. A serviceable contouring instrument of very simple construction is the water-level, which consists of a pair of transparent phials partially filled with water; the phials are placed upside down at the ends of a hollow bar fixed on a rotatory vertical axis, and have their mouths connected with piping of any available material,—brass, tin, or gutta-percha. The water in both phials is in free communication, and the water surfaces indicate the horizontal plane naturally, without any mechanical contrivance. The instrument is well suited for short sights not requiring a telescope, and may be readily manipulated by persons ignorant of the use of instruments of a higher class. Eye-reflecting levels, clinometers, orometers, and other light instruments, which may be held in the hand and do not require a fixed support, are frequently employed for interpolating minor between major contours. In military sketching on large scales hypothenusal inclinations and lengths are sometimes measured; the bases and perpendiculars are deduced on the spot from a table

---

[1] With certain exceptions, principally of a military nature, the hill features are now sketched on the 1-inch scale, on photographic reductions of the 6-inch contoured sheets, faintly printed in orange colour as a guide to the sketchers.