Both are held over the circle at the extremity of a radial arm pivoted over its centre. The vernier moves in contact with the surface of the circle, while the microscope views it a short distance off; the former is usually applied to circles whose diameters do not exceed 12 inches, the latter to those of larger diameter. Both kinds of reader are applicable to linear scales as well as to graduated circles, the microscope being usually employed when most precision is desired.

The vernier is so called after its inventor, a Frenchman. Its principle is very simple. The space between any convenient number, n, of graduations on the circle is set off on the vernier and divided into $(n+1)$ equal parts; then some one division of the vernier will always coincide with a graduation of the circle. On counting the divisions from the index onwards it is found that the coinciding division, say the mth, of the vernier is opposite the mth graduation of the circle, counting from the last one passed by the index. This indicates that the distance of the index from the last graduation is $\frac{m}{n+1}$ parts of the space between the graduations; n is invariably taken as an odd number, such that the unit of $(n+1)$ may be some convenient aliquot part of the circle, as a minute for a circle divided into degree spaces.

The micrometer microscope presents the combination of object and eye glasses met with in ordinary microscopes, with the addition of a wire-carrying diaphragm, movable by a screw, for micrometric measurements in the plane of the focus of the object-glass. The tube is conical at the object end and cylindrical at the eye end; the box of the micrometer is mounted between these two parts at right angles with the visual axis. The tube is held at the extremity of the arm of an alidade, in a collar in which it may be moved closer to or away from the surface of the circle, or be turned round so as to place the micrometer tangentially to the circle. The distance between the micrometer and the object-glass is usually about four times that between the object-glass and the face of the circle, and thus a correspondingly magnified image of the spaces between the graduations is obtained in the plane of measurement. The object-glass is held in a small tube which can be screwed in or out of the principal tube, to enable the length of the image to be adjusted to an exact integral number of revolutions of the micrometer. The box of the micrometer and the wire diaphragm are rectangular, the latter sliding to the right or left within the former. Slow motion is communicated to the diaphragm by the micrometer screw, which passes into it through a collar in one side of the box, against which the shoulder of the screw is pressed by an internal spiral spring acting against the sides of the diaphragm and the box. The screw is furnished with a circular head divided into a number of equal parts—usually 60, each equivalent to 1″ for circular arcs, and 100 for linear scales—and is rotated opposite an index arm fixed on the box; complete revolutions are marked by the teeth of a stationary comb, which is fixed above the wire of the diaphragm and viewed with it through the eye-piece.

Spirit level. The spirit-level consists of a glass tube not quite filled with alcohol, a small quantity of air being left, which rises as a bubble to the highest part of the tube. In small and coarse levels the diameter of the tube is largest in the middle and decreases uniformly towards the ends, which are closed by the blow-pipe; in long and delicate levels the tube is cylindrical, but with a longitudinal portion of the interior surface ground to the curvature of a circle of greater or less radius according as the level is designed to be more or less sensitive, and it is sometimes closed by circular glass stoppers cemented into the ends. When the tube is held horizontally, with the curved surface of the interior uppermost, the middle part is occupied by the air bubble. Lines are etched on the outer surface at equal distances from the central point, to enable the tube to be set with the bubble exactly in the middle, or a scale graduated throughout its entire length is provided, to enable any deviation from centricality to be measured and the corresponding dislevelment to be calculated and allowed for subsequently in the reduction of the observations. The glass tube is commonly fixed in a metal tube, with plaster of Paris for protection; but, as it is then liable, under changes of temperature, to torsion and strain, which may sensibly alter its curvature, it is preferable to place it in a metallic cradle and rest it on cork bearings, with due provision against sliding, the whole being covered with a glass cylinder if need be for further protection. The metallic cradle or tube is attached to any instrument on which the level is to be mounted by adjusting screws, for setting it correctly with reference to the axis of rotation with which it is associated. The value of a division of the scale, in seconds of arc, is usually called the "run," and is determined by attaching the level with its scale to a (generally) vertical circle, and taking both the circle and the bubble end readings in different positions of the circle. As the length of the bubble is much affected by changes of temperature, and the curvature of the tube may not be identical at all points, values of the run are commonly obtained under widely differing temperatures.

The telescope consists of a tube, carrying an achromatic object-glass and an eye-piece which holds either a pair of lenses for viewing the inverted image transmitted by the object-glass or a combination of four lenses for inverting the image and causing all objects to be viewed naturally. The former is usually employed for observing celestial objects, the latter for observing terrestrial. The field of view being more or less extensive, a central point is established in the tube, usually by the intersection of a pair of fine wires or spider lines—one vertical, the other horizontal—in the plane of the image, and the telescope is directed by bringing this point on any specific object in the field. As the interval between the object-glass and the image varies with the distance of the object, a tube is provided to slide within the telescope tube and carry the object-glass at one end, while the telescope tube carries the diaphragm and eye-piece at the other end, or *vice versa*. The image and the wires are brought into the same plane by a focusing screw, which acts on the inner through the outer tube. The wires are attached to the surface of an adjustable annular diaphragm, which is held in position by two pairs of antagonizing screws—one pair horizontal, the other vertical—with shoulders working against the exterior of the tube in which the diaphragm is contained, so as to move it to the right or left and up or down, in order to bring the point of intersection of the wires into the visual axis of the telescope. In practice the first adjustment is to set the eye-piece to distinct vision of the wires; the object-glass is then set truly to focus, which is accomplished when no apparent parallax, or movement of the image relatively to the wires, is seen on shifting the position of the eye, for this would indicate that the image is either in front of or behind the plane of the wires. The line joining the point of intersection of the wires with the centre of the object-glass is called the "line of collimation," and the diaphragm should be so fixed that this line may always be perpendicular to the axis on which the telescope revolves.

Surveying compass. The surveying compass gives the magnetic bearing of any object, and is the simplest of all instruments for measuring horizontal angles. It consists of a magnetized needle, with an agate centre, poised on the point of an upright pivot in the centre of the bottom of a circular box and carrying a concentric circular card or silver ring, the circumference of which is graduated into 360°, and is sometimes further subdivided. The aligner is constituted by a pair of sight vanes attached to the box at opposite extremities of a diameter, one vane having a narrow slit for the eye to look through, the other with a wider opening bisected by a vertical wire to be set on the observed object. There is no circle reader, the prolongation of the wire on to the graduations being estimated by the eye; and there is no level, for the circle poises itself horizontally on the supporting pivot.

Prismatic compass. The prismatic compass is similar to the surveying compass, with the addition of a prism in the eye vane through which the wire of the sight vane and the divisions of the circle are viewed apparently together; the division with which the wire coincides when the needle is at rest indicates the magnetic azimuth of any object bisected by the wire. The sight vane carries a mirror turning on a hinge, to enable objects to be seen by reflexion which may be too high to be seen on the wire; the eye vane is furnished with a pair of dark glasses to be employed when the sun is being observed.

Magnetic instruments are useful for rapid reconnaissance and rough survey, and for filling in the minor details of an exact survey, but they are not to be relied on to give bearings with errors less than ten to fifteen minutes. In plotting, however, bearings are preferable to angles, for, by drawing a number of meridional lines parallel to each other on the paper, each bearing may be plotted from an independent meridian without any accumulation of error, such as arises when a number of angles are plotted in succession with the protractor adjusted on short lines.

The plane table is in its usual form simply a rectangular board mounted horizontally on a stand, on which it may be turned round and set in any required position; it is furnished with a flat sight rule, which usually carries a pair of sight vanes and has a bevelled edge, parallel to the line of sight, to serve as a ruler, also with a magnetic needle. Occasionally the construction is more elaborate, and the board is surrounded by a marginal frame with graduations radiating from the centre as the degrees of a circle, so that it may be used as an instrument for measuring horizontal angles, while the sight rule is furnished with a telescope, which takes the place of the vanes and is mounted on an axle to measure vertical angles. The size is made as great as is consistent with the limits of portability in each instance, so that the sheet of paper to be drawn on may be as large as possible. The standard plane table of the Indian Survey measures 30 inches by 24, and is made of planks of well-seasoned wood 1 inch thick, with transverse edge bars below to prevent warping and buckling. It is set up on a stand, usually a braced tripod, to which it is clamped by a powerful hand screw passing through the head of the stand into a brass socket fixed centrally under the table; the screw when relaxed serves as a pivot, round which the table may be turned in azimuth and set in any required position. The table is then firmly clamped so as to maintain a constant position during all the subsequent laying off of bearings. The sight rule is 30 inches long, 2 wide, and one-

third of an inch thick, of ebony, with a brass sight vane at each end, and a fiducial edge parallel to the line of sight; the vanes are about 5 inches high, which gives sufficient elevation and depression for general use. The magnetic needle is about 6 inches long and is held in a rectangular brass box an inch broad, placed on the table whilst it is being set and afterwards removed. Heights may be determined on the spot with the aid of a clinometer, formed of a bar carrying a spirit-level and a pair of sights, one of which has a scale of tangents graduated to radius = the interval between the sights. For the method of employing the table see § 4, p. 709 above.

The theodolite, the most important of all instruments for the purposes of a survey, is a combination of two graduated circles placed at right angles to each other, for the measurement of horizontal and vertical angles, a telescope, which turns on axes mounted centrically to the circles, and an alidade for each circle, which carries two or more readers of the arcs through which the telescope is moved. The whole is supported by a pedestal resting on foot-screws, which are also employed to level the instrument. The size varies from a minimum with circles 3 inches in diameter to a maximum with a 36-inch horizontal and an 18-inch vertical circle, the weight ranging from 4 ℔ to 1000 ℔; the dimensions and magnifying powers of the telescope increase with the diameter of the horizontal circle. The telescope may be connected rigidly with the alidade and move with it while the circle remains stationary, or with the circle and move with it while the alidade remains stationary. The varieties of form as well as of size are numerous: in some the telescope may be completely turned round in altitude as well as azimuth, and pointed to any object celestial or terrestrial; in others the range of movement in altitude is restricted to about 25° above and 25° below the horizon, and a pair of sectors are substituted for the complete vertical circle; in some the telescope and vertical circle are placed between, in others outside of, the pillars which support their common axis; in some the pedestal is a simple tribrach resting on three foot-screws, in others it takes the objectionable form of a ball carrying the vertical axis and a socket holding the ball between two parallel plates, which are antagonized and set firm by two pairs of foot-screws, turning in sockets fixed to the lower plate, while their heads are pressed against the upper plate, to fix it and bring the instrument into level at the same time. There are numerous other specialities of form which have been introduced to meet specific requirements; but these cannot be noticed here.

Transit theodolite. The transit theodolite is an alt-azimuth instrument with the graduated circles of equal diameter, usually 6 to 8 inches. The telescope is mounted between a pair of conical arms which taper outwards and end in cylindrical pivots, constituting what is called the transit axis of the instrument. The pivots rest on Y's or in semicircular collars, on the heads of a pair of pillars, which are made of sufficient height to enable the telescope to revolve between them and be pointed to stars in the zenith. These pillars stand on a circular plate, which serves as the alidade of the horizontal circle and is usually constructed to revolve round a vertical axis fixed in the centre of the plate of the horizontal circle; this axis passes downwards into a socket in the centre of a tribrach, which forms the pedestal of the instrument and rests on three mill-headed foot-screws by which the instrument is levelled. The vertical circle is mounted centrically on one of the cones of the transit axis, near the pivot end; its alidade, usually a rectangular plate carrying a pair of verniers, is fitted centrically over that axis, in contact with the circle but nearer the shoulder of the pivot, and, while the telescope and the circle revolve together, it is held stationary by an adjustable arm the end of which is pinched between a pair of antagonizing screws mounted on the nearest pillar. The alidade of the horizontal circle carries two or three equidistant verniers, because any error in centring an alidade over a circle is eliminated in the mean of the readings whenever two or more verniers, placed at equal distances apart round the circle, are read. A clamp, with a tangent screw for communicating slow motion, is attached to the nearest pillar, to act on the vertical circle and the telescope; another is attached to the plate of the horizontal circle, to act on the alidade of that circle and so also on the telescope for azimuthal motion; and a third to the pedestal, to act on the plate of the horizontal circle. The first two are employed in measuring the vertical and azimuthal angles, the third in setting the zero-diameter of the horizontal circle in any specific direction, with a view to the repetition of the measurements of the azimuthal angles at different parts of the circle. For levelling the instrument, two levels are fixed at right angles to each other on the plate of the alidade of the horizontal circle; a third is attached to the telescope, or, preferably, to the alidade of the vertical circle; a fourth is mounted on the transit axis when levelling for astronomical observations. A magnetic compass or needle is added, and also a plummet for centring the instrument over the station mark.

Theodolites are designed to measure horizontal angles with greater accuracy than vertical, because it is on the former that the most important work of a survey depends, and they are measurable with greatest accuracy; measures of vertical angles are liable to be much impaired by variations in the refractive condition of the lower strata of the atmosphere, more particularly on long lines, so that when heights have to be determined with much accuracy the theodolite must be discarded for a levelling instrument, to be set up repeatedly with staves at short distances. When truly adjusted the theodolite measures the horizontal angle between any two objects, however much they may differ in altitude, as the pole star and any terrestrial object; but, as adjustments are not always made with accuracy nor permanently maintained, it is desirable always to take the observations in pairs, with the face of the vertical circle alternately to the right and left of the observer, for this eliminates collimation error from the horizontal angles and index error in the setting of the spirit-level from the vertical angles.

When a horizontal angle is measured several times for greater accuracy, one of two methods of procedure is adopted. (1) The angle is measured once or oftener in the usual way, the horizontal circle remaining clamped and the telescope and alidade moving over it; then the position of the horizontal circle is shifted[1] as often as may be desired, and after each shifting the angle is again measured as formerly; thus a separate numerical result is obtained for each operation. Or (2), the first object A having been observed and the telescope set on the second object B, the horizontal circle is unclamped and turned round until the telescope is brought back on A, when it is again clamped; then the alidade is unclamped and the telescope again moved over the horizontal circle to be set on B. The operation is repeated as often as may be desired. The vernier readings are only taken for the first telescope pointing to A and the last to B; their difference + 360° for every complete revolution of the circle, divided by the number of repetitions, gives the angle. This method is objectionable when a round of several angles has to be measured, but it enables the value of a single angle—more particularly a small one, as between objects in the same field of the telescope—to be determined accurately with much greater rapidity than the first method.

An auxiliary telescope is sometimes fixed below the plate of the horizontal circle of a theodolite, to be pointed to a referring mark while the upper telescope is being moved about, and thus to serve as a check on the general stability of the instrument and on the permanence of the initial setting of the circle during the measurement of a round of angles. When a theodolite is set up on a lofty scaffolding which is liable to be swayed by the wind, or on a stand which cannot be readily isolated from the observer, horizontal angles may be measured accurately by employing a second observer to keep the auxiliary telescope truly pointed to a referring mark while the observing telescope is being pointed.

The subtense transit theodolite differs from the ordinary transit theodolite merely in having a pair of wire-carrying micrometers mounted in the telescope tube, in order that the small angle subtended by a distant object of known dimensions, or by two objects sufficiently near each other to be seen in the same field of the telescope, may be measured with greater facility and precision than on the graduated circles in the usual way. The micrometers are held in a rectangular box, one on the right hand, the other on the left, with the wires brought as closely as possible into the plane of the fixed wires in the ordinary diaphragm; the box can be turned on the telescope tube through an angle of rather more than 90°, to enable the micrometer wires to be set parallel to either the horizontal or the vertical wire of the diaphragm, or to be placed at any desired angle of inclination. The subtense object usually employed in survey work is a pole of known length; if held perpendicularly to the line of sight of the telescope, its direct distance may be determined from the angle measured by the micrometers with a sufficiently small percentage of error to make this method preferable to chaining over rough ground. The instrument has been advantageously employed in carrying traverses of considerable length over ground which was impracticable for direct linear measurements. The micrometers are also serviceable in astronomical observations for time and longitude, for they give additional wires on which to observe the passage of a star, at distances from the fixed wire which may be varied with the speed of the star; and for determining the longitude they permit numerous measures of the distance between the edge of the moon and a star to be taken, immediately before and after occultation.

Eckhold's omnimeter is a theodolite furnished with a microscope of considerable magnitude facing a graduated linear scale; the tube of the microscope is rigidly attached to the telescope tube, either at right angles or parallel to it, so that the two always move together. The scale is fixed either parallel or perpendicular to the alidade plate of the horizontal circle; thus, when the telescope is moved through vertical arcs within the range of the scale, the tangents of the arcs are measured by the microscope on the scale. The latest and best form of the instrument is shown in fig. 8, which represents a transit theodolite converted into an omnimeter by the application of a microscope *AB* to the telescope at right

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[1] This is often done arbitrarily, but systematic shifts which bring equidistant graduations of the circle under the verniers during all the telescope pointings to any one object are always preferable (see sect. I., § 9, p. 698 above).

angles to it, and of a scale *C* to the plate of the alidade of the horizontal circle in a plane parallel to that of the vertical circle. The microscope is furnished with a diagonal eye-piece, through which the observer looks down on the scale. The scale is divided into 100 equal parts, and is movable in its bed-plate through the length of one of these divisions by one rotation of a micrometer screw, with a large head, *D*, the circumference of which is divided into 100 equal parts, each divisible into fifths by a vernier. The microscope has a fixed wire in a diaphragm at its eye end, *A*, and, when the telescope is set on an object and the wire is seen between a pair of divisions on the scale, the scale is moved by the micrometer screw until the nearest division is brought under the wire; the scale reading corresponding to the horizontal position of the telescope being known, the difference between it and the reading when the telescope is pointing above or below the horizontal plane is the tangent of the arc of elevation or depression, to radius = the perpendicular from the axis of rotation of the telescope to the scale. Thus both the distance and the height of any point over which a staff of known length has been set up vertically may be readily determined with fair accuracy. Let *O* (fig. 9) be the position of the transit axis of the telescope, *OA* the direction of the telescope when horizontal, and *Oa* the corresponding direction of the microscope at right angles to the scale *amn*; let *M* be a distant point over which the staff *MN* has been set up vertically, and let *m* and *n* be the graduations under the microscope when the telescope is pointing to the bottom and top of the staff; then, since *MN* and *Oa* are known, the horizontal distance *OA* and the height *AM* are determined from the proportions

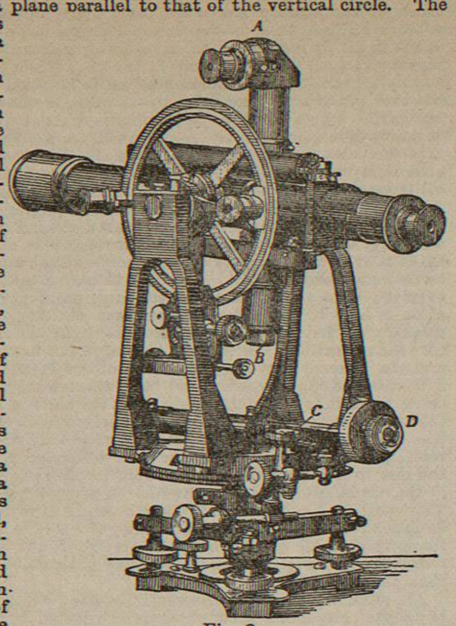

Fig. 8.

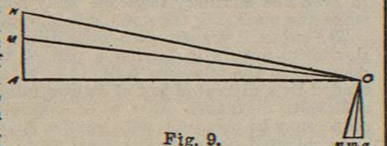

Fig. 9.

$$\left.\begin{array}{l}OA:Oa\\ AM:am\end{array}\right\}::MN:mn.$$

It is essential that the focusing tube of the microscope should always move parallel to the visual axis when different divisions of the scale are being brought into focus, otherwise errors materially exceeding the quantities appreciable by the micrometer may be caused. The linear results thus obtained are satisfactory when the subtense staff is set up at a moderate distance; the instrument has often been used with advantage in localities where measuring chains could not be conveniently employed. As an angular instrument it is identical with the ordinary transit theodolite, as will be seen from the figure, which may be referred to as illustrating the description of that instrument; the foot-screws are represented as resting on a plate such as is usually fixed on the head of a folding tripod stand, their lower extremities, as well as the grooves in which they are placed on the plate, being concealed from view by a capping upper plate, which is clamped over their shoulders to prevent the instrument from falling off the stand.

In any theodolite with a telescope of the ordinary form the height of the pillars must necessarily be somewhat greater than half the length of the telescope if stars in the zenith are to be observed or if the telescope is to be completely rotated on its transit axis; the higher the pillars the higher the centre of gravity, the less perfect the stability of the instrument when set up for observation, and the greater its weight and cumbersomeness for transport. In Germany and Russia theodolites and transit instruments are sometimes employed in which the eye end of the telescope tube is removed—a counterpoise to the object end being substituted in its place; and a prism is inserted at the intersection of the visual axis with the transit axis, so that the rays of light from the object-glass may be reflected through one of the tubes of the transit axis to an eye-piece in the pivot of this tube. In this case the pillars need only be high enough for the counterpoise to pass freely over the plate of the horizontal circle; but the observer has always to place himself at right angles to the direction of the object he is observing.

The levelling instrument consists of a telescope which carries a long spirit-level parallel to itself and is mounted on a horizontal plate, which is fixed rigidly either on the head of a vertical axis revolving within a socket in the centre of the pedestal or on that of a hollow cone revolving round a vertical axis which projects upwards from the pedestal. There are various forms of the instrument; in the Y-level the telescope rests on a pair of Y's, in which it can be both rotated and turned end for end; in the dumpy level the telescope is rigidly attached to its supports, and its tube is made shorter and of greater diameter, to carry an object-glass of shorter focal length and larger aperture. A magnetic compass is attached to the instrument to enable the bearings of the levelling staves to be taken whenever desired. Levelling staves are of a variety of patterns and are graduated in various ways, best on both faces and dissimilarly, for a check on accidental errors of reading, as indicated in sect. III.

Reflecting levels.

Reflecting levels are portable instruments which may be held by the hand for rough and rapid survey work. They are of two forms: in one an image of the eye of the observer, in the other an image of the bubble of a spirit-level, is seen by reflexion on a level with the observed object. The first consists of a square of common looking-glass, which is set in a frame suspended from a ring on the line of prolongation of one of the diagonals in such a manner as to swing freely but not turn round on its axis of suspension; the frame is weighted by a metal plate behind, to which it is so adjusted that, when suspended, the plane of the surface of the mirror will be vertical. A small portion of the glass at one end of the horizontal diagonal is either cut away or unsilvered. When the image of the observer's eye is seen on the diagonal, all objects bisected by the diagonal, whether viewed through the opening in the mirror or by reflexion, are on the level of the eye. The second consists of a tube open at the object end and closed at the eye end by a disk which is perforated with a sight hole; a mirror filling up half the section is fixed in the tube, facing the eye end at an angle of 45° with the axis; and an all-round transparent spirit-level is mounted over an opening above the mirror, and its bubble is seen by reflexion in the axis of the tube. Abney's level is of the latter construction, but with the spirit-level attached to the alidade of a graduated arc fixed to one side of the (rectangular) tube; thus vertical angles as well as levels may be determined with it.

Abney level.

Optical square.

The optical square is a reflecting instrument indicating a right angle, and is of great use in laying off perpendiculars for the measurement of offsets from a line of survey. It consists of two glass plates, one wholly the other partially silvered, which are fixed permanently in a shallow circular box at an angle of 45°, so that any two objects seen together through a sight hole in the box—one directly through the transparent portion, the other by reflexion in the mirror of the partially silvered glass plate—subtend an angle of 90° at the point where the observer is standing.

Plotting instruments.

Plotting and Plot-measuring Instruments.—These comprise linear scales, common compasses, and angular protractors for laying off distances and angles measured on the ground, proportional compasses and pantagraphs for reproducing a finished plot on some other scale, and opisometers and planimeters for measuring plotted lines and areas.

Scales.

Scales are divided, either decimally or fractionally, into equal parts, each of which is a portion of a fixed unit of length, as a foot or an inch; some are subdivided more or less minutely throughout their entire length between a pair of parallel lines; others are subdivided at their extremities only. Diagonal scales are formed by eleven equidistant parallel lines, the outer ones of which are divided primarily and subdivided into tenths at their extremities. The primary divisions are joined by cross lines perpendicular to the eleven parallel lines; the end subdivisions are joined diagonally, the first on the lower line with the second on the upper, and so on, each diagonal cutting every horizontal line in a point a tenth of a subdivision beyond the cutting point on the parallel line below, as measured from any one of the perpendicular lines; and each of these tenths is further divisible into tenths by measuring from the perpendicular at intervals of tenths between the parallel lines; thus great precision of measurement is obtained.

Marquois scale and triangle.

The Marquois scale and triangle consist of a scale divided throughout into equal parts more or less minutely and a right-angled triangle of which the hypothenuse is three times the shortest side. An arrow is drawn perpendicular to the hypothenuse to serve as a pointer to the divisions of the scale. The third side has a bevelled edge for ruling. When the triangle is placed with its hypothenuse against the scale and is moved along it, all lines drawn along the bevelled edge are parallel to each other, their distances apart being one-third of the distances travelled by the arrow along the scale.

Compasses usually take the form of a pair of legs movable about a joint, so that their extremities, which are of steel, finely pointed, may be set at any required distance apart; the legs may be knee-jointed, and one is usually adapted to hold either a pencil, a ruling pen, or a steel pointer, as may be desired. A beam compass is employed when long lengths are laid off; it consists of a light tubular metal bar, or a rectangular deal rod, fitted with a pair of boxes, which slide along it and carry either pen, pencil, or pointer, and may be set and clamped at any desired distance apart.

Proportional compasses consist of two parts so exactly similar that when held in contact throughout they appear as one; each is pointed at both ends, flat and grooved through one-half its length, and tapering to a point in the other half. The two are coupled together by a pair of similar sliders, one for each groove, turning on a common axle which carries a disk at one end and a clamping screw at the other; by shifting the position of the sliders in the grooves the distances between the points at the opposite ends can be brought into any desired proportion. The settings for different proportions are effected by bringing a line on the slider opposite the lines of a fractional scale engraved on one side of the groove.

Protractors are of two forms circular (or semicircular) and rectangular; the circumferences of the former are divided into 360° or 180°; the latter are divided on three sides of their periphery by lines drawn from the centre of the fourth side to the degree points on the circumference of a semicircle of which that side is the diameter. The protractor being set with its centre on a given point and its zero line on a given line passing through the point, any angle with this line at the point can be readily laid off. Protractors for plotting traverses are commonly annular, that they may be centred over the station of origin with the zero diameter on the initial meridian; their bearings at any other station may be laid off without moving the protractor by drawing lines parallel to the same bearings at the origin. Rectangular protractors sometimes have parallel lines engraved on their faces at equal distances, for setting over paper ruled with parallel lines at unequal distances, and their backs engraved with scales of rhumbs, sines, secants, and tangents and common scales of equal parts.

Station pointer.

The station pointer enables the position of any station at which angles between three fixed points have been measured to be plotted on paper. It consists of three arms: the centre arm carries a graduated circle fixed over an axis at one end; the other two are movable round this axis, and each carries a vernier for reading the circle. Each arm has a straight edge bevelled as a ruler, and the lines on the prolongations of these edges meet in the centre of the axis, where there is a small opening through which a point may be pricked on the paper. The arms having been set to the observed angles, the instrument is moved about until each edge is over one of the fixed points on the paper, when its centre will be exactly over the position of the station if none of the angles are very acute. The instrument is much used in nautical surveying, for laying down the position of a vessel at sea by angles measured to fixed objects on shore.

Triangular compass.

The triangular compass is serviceable in reproducing plans to full scale; it is formed by jointing a third leg to the centre pin of the joint of an ordinary pair of compasses, so as to be movable in any direction.

Pantagraph.

The pantagraph is employed in reproducing a map on a different—generally a smaller—scale. It consists of two long arms, *AB* and *AC*, jointed together at *A*, and two short arms, *FD* and *FE*, jointed together at *F* and with the long arms at *D* and *E*; *FD* is made exactly equal to *AE* and *FE* to *AD*, so that *ADFE* is a true parallelogram whatever the angle at *A*. The instrument is supported parallel to the paper on ivory castors, on which it moves freely. A tube is usually fixed vertically at *c*, near the extremity of the long arm *AC*, and similar tubes are mounted on plates which slide along the short arms *BD* and *FD*; they are intended to hold either the axle pin on a weighted fulcrum round which the instrument turns, or a steel pointer, or a pencil, interchangeably. When the centres of the tubes are exactly in a straight line, as on the dotted line *bfc*, the small triangle *bfD* will always be similar to the large triangle *bcA*; and then, if the fulcrum is placed under *b*, the pencil at *f*, and the pointer at *c*, when the instrument is moved round the fulcrum as a pivot, the pencil and the pointer will move parallel to each other through distances which will be respectively in the proportion of *bf* to *bc*; thus the pencil at *f* draws a reduced copy of the map under the pointer at *c*; if the pencil and the pointer were interchanged an enlarged copy would be drawn; if the fulcrum and pencil were interchanged, and the sliders set for *f* to bisect *bc*, the map would be copied exactly. Lines are engraved on the arms *BD* and *FD*, to indicate the positions to which the sliders must be set for the ratios $\frac{1}{2}$, $\frac{1}{3}$, . . ., which are commonly required.

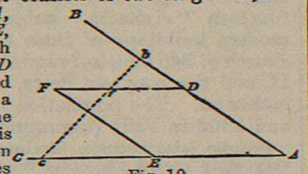

Fig. 10.

The square pantagraph of Adrian Gavard consists of two graduated arms which are pivoted on a plain bar and connected by a graduated bar sliding between them throughout their entire length, to be set at any required distance from the plain bar; a sliding plate carrying a vertical tube, to hold either the axle of the fulcrum, the pencil, or the pointer, is mounted on one of the arms and on a prolongation of the plain bar beyond the other arm, and also on the graduated connecting bar; and an additional arm is provided by means of which reductions below or enlargements above the scales given on the instrument can be readily effected.

The eidograph is designed to supersede the pantagraph, which is somewhat unsteady, having several supports and joints. It is composed of three graduated bars, one of which is held over a fulcrum and carries the others, which are lighter, one at each extremity. The three bars are movable from end to end in box-sockets, each having an index and a vernier in contact with the graduated scale. The box-socket of the principal bar turns round the vertical axle of the fulcrum; that of each side bar is attached to a vertical axle, which also carries a grooved wheel of large diameter and turns in a collar at either end of the principal bar. The two wheels are of exactly the same diameter and are connected by a steel band fitting tightly into the grooves, so that they always turn together through identical arcs; thus the side bars over which they are respectively mounted, when once set parallel, turn with them and always remain parallel. A pointer is held at the end of one of the side bars and a pencil at the diagonally opposite end of the other. The bars may be readily set by their graduated scales to positions in which the distances of the pencil and the pointer from the fulcrum will always be in the ratio of the given and the required map scales.

The opisometer is intended to measure the lengths of roads, rivers, and other lines on a map. It consists simply of a milled wheel mounted in a forked handle on a steel screw with a very fine thread. The wheel, being turned up to one end of the screw, is put down on the map with the handle held vertically over the point at which the measurement is to commence, and is run over the road or line until the point is reached at which the measurement is to stop; it is then lifted off the paper, placed on the scale of the map, and run backwards to the initial end of the screw, over a length of the scale which corresponds to the length run over on the map.

Polar planimeter.

The polar planimeter was invented by Professor Amsler of Schaffhausen for the measurement of areas on maps and plans. It consists essentially of two arms jointed together and a roller, carried at right angles to one of the arms and moving in touch with the paper, which by its revolutions records the area of a figure whose perimeter is traced by a point on that arm, while the instrument is turned bodily on a point on the other arm as a fixed centre. There are two forms of the instrument: in one the position of the roller is fixed and the arms are jointed on a common pinion; in the other the roller and a pinion, to which the holding arm is attached, are both carried by a slider, which is movable along the tracing arm and can be set at any required distance from the tracing point. The first form gives areas in a single unit of measure only; the second in various units. The annexed figure represents the first form, showing the joint *A*, the tracing point *P*, the fixed point *O*, and the roller with its graduated dial and vernier, for indicating the lengths of line rolled over while the tracer moves round the perimeter of the area under measurement.

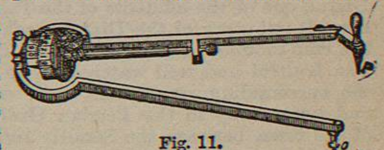

Fig. 11.

The following explanation of the theory of the instrument is due to Professor Greenhill. Let *OA*, *AP* be the two arms jointed at *A*, with the fixed point at *O* and the tracer at *P*, and suppose the wheel to be fixed at *R* on the prolongation of the arm *PA*. Let $OA=a$, $AP=b$, $AR=c$, and the radius of the roller $=r$; and let the direction of a positive rotation of the roller, as marked by the graduations, be that of rotation on a right-handed screw on the axle of *R* which would give motion in the direction *AR*. Drop the perpendicular

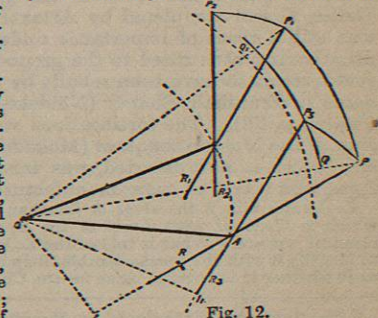

Fig. 12.

OI from O on AR, and first suppose the joint A to be clamped. Then, if I is in AR produced, a rotation of the instrument about O with angular velocity ω will give to R the component velocities $OI.\omega$ in the direction IR and $IR.\omega$ in the direction perpendicular to IR, and will therefore compel the roller to turn with the angular velocity $\frac{RI}{r}\omega$; but, if I is on the other side of R, the angular velocity of the roller will be $-\frac{RI}{r}\omega$. Therefore, keeping A clamped, the roller will turn through an angle $\frac{RI}{r}\theta$ or $-\frac{RI}{r}\theta$, according as I is or is not on the same side of R as A, when the instrument is rotated through an angle θ about O; but, when I coincides with R, the roller will not turn, and then P describes a circle, called the "zero circle," represented by the middle dotted circular line, of radius

$$\sqrt{(OR^2+RP^2)}=\sqrt{\{a^2-c^2+(b+c)^2\}}=\sqrt{(a^2+b^2+2bc)}.$$

Next unclamp the joint A and clamp the arm OA; then the roller will turn through an angle $-\frac{c}{r}\phi$, while AP turns through an angle ϕ. Now suppose P to travel round the finite circuit $PP_1P_2P_3$ by a combination of the preceding motions in the following order. (1) Clamp the joint, and move P to P_1 and A to A_1 on arcs of circles of centre O; then the roller will turn through an angle $\frac{RI}{r}\theta$, θ being $=AOA_1=POP_1$. (2) Unclamp the joint and clamp the arm, and move the pointer from P_1 to P_2 on the arc of a circle of centre A_1; then the roller will turn through an angle $=-\frac{c}{r}\phi$, ϕ being $=P_1A_1P_2$. (3) Unclamp the arm and clamp the joint, and move the pointer from P_2 backwards to P_3 and A_1 to A, on arcs of circles of centre O, through an angle θ; then the roller will turn through an angle $-\frac{RI_1}{}\theta$, if OI_1 is the perpendicular from O on P_3A. (4) Unclamp the joint and clamp the arm, and move the pointer from P_3 to P on the arc of a circle of centre A, and consequently through an angle ϕ; the roller will turn through an angle $\frac{c}{r}\phi$, which cancels the angle due to motion (2). Thus in completing the finite circuit $PP_1P_2P_3$ the roller will have turned through an angle $(RI-RI_1)\frac{\theta}{r}=(AI-AI_1)\frac{\theta}{r}$.

$$\begin{aligned}\text{But the area } PP_1P_2P_3 &= \text{area } PP_1Q_1Q,\\ &= \text{sector } OPP_1-\text{sector } OQQ_1=\tfrac{1}{2}(OP^2-OP_3^2)\theta,\\ &= \tfrac{1}{2}\{OA^2+AP^2+2AI.AP-(OA^2+AP^2\\ &\qquad +2AI_1.AP)\}\,\theta\\ &= (AI-AI_1)\,b\theta,\\ &= br \text{ times the angle turned through by the roller.}\end{aligned}$$

The area $PP_1P_2P_3$ is therefore b times the travel of the circumference of the roller.

Any irregular area, supposed to be built up of infinitesimal elements found in the same manner as $PP_1P_2P_3$, will be accurately measured by the roller when the point P completes a circuit of the perimeter, the arm AP being free to turn on the joint at A and the arm OA on a fixed point O. If, however, O is inside the area, the area of the zero circle must be added to the area deduced from the readings of the roller. When the roller is fixed permanently, this area is constant, and is usually engraved on the arm in units of the adopted length b; when the roller is held on a slider which also carries the pinion of the arm OA, the length b may be so adjusted that the areas described will be expressed in any desired unit of measure.

Literature and Authorities consulted.—Accounts of the Operations of the Great Trigonometrical Survey of India; *Manual of Survey for India*; Col. A. R. Clarke, *Geodesy*; *Methods and Processes of the Ordnance Survey*; Col. Waterhouse, *On the Application of Photography to Maps and Plans*; and *Professional Papers of the Royal Engineers.* (J. T. W.)

SUSA, the Biblical SHUSHAN, capital of Susiana or Elam and from the time of Darius I. the chief residence of the Achæmenian kings, was a very ancient city, which had been the centre of the old monarchy of Elam and undergone many vicissitudes before it fell into the hands of the Persians (see ELAM). The site of the town, which has been fixed by the explorations of Loftus and Churchill, lies in the plain, but within sight of the mountains, between the courses of the Kerkha (Choaspes) and the Dizful, one of the affluents of the Pasitigris. The Shápúr, a small tributary of the Dizful, washes the eastern base of the ruin-mounds of Sús or Shúsh. Thus the whole district was fruitful and well watered, fit to support a great city; the surrounding rivers with their canals gave protection and a waterway to the Persian Gulf; while the position of the town between the Semitic and Iranian lands of the empire was convenient for administrative purposes. It is not therefore surprising that Susa became a vast and populous capital; Greek writers assign to it a circuit of 15 or 20 miles,—a statement which is fairly well borne out by the remains. These include three main mounds, of which one is identified with the strong citadel[1] and a second shows the relics of the great palace built by Darius I. and completed by Artaxerxes Mnemon. Susa was still a place of importance under the Sasanians, and after having been razed to the ground in consequence of a revolt seems to have been rebuilt by Shápúr II. under the name of Éránshahr-Shápúr (Nöldeke, *Gesch. d. Perser aus Tabari*, p. 58). The fortifications were destroyed at the time of the Moslem conquest (Moḳaddasi, p. 307); but the site, which is now deserted, was inhabited in the Middle Ages, and a seat of sugar-manufacture.

In Daniel viii. 2 the river of Shushan is called Ulai, a name which is identical with Avrai of the Bundehesh and Eulæus of classical writers. What is told of the Eulæus makes it impossible to identify it with the inconsiderable Shápúr; but authorities differ as to whether it is another name for the Choaspes or rather denotes the Dizful or the Pasitigris. Susa in the days of its greatness must have stretched nearly from river to river. There is a sanctuary of the tomb of Daniel on the banks of the Shápúr, and Arabic geographers relate that this tomb was a frequented shrine before the Moslem conquest and that the Arabs turned the stream over the grave.

SUSA, a city of Italy, in the province of Turin, 33½ miles west of Turin by the railway which passes by the Mont Cenis tunnel into France, is situated on the Dora Riparia (tributary of the Po) at 1625 feet above the sea, and is so protected from the northern winds by the Rocciamelone that it enjoys a milder winter climate than Turin itself. The city walls, 20 to 30 feet broad at the base, were about 50 feet in height, but in 1789 their ruinous condition caused them to be reduced by about half their elevation. Numerous remains of Roman buildings and works of art still show the importance of the ancient town; and the triumphal arch erected by Cottius in honour of Augustus still stands on the old Roman road between Italy and Gaul,—a noble structure, 45 feet high, 39 broad, and 23 deep. The inscription, now illegible, mentioned fourteen "civitates" subject to Cottius. Among the modern buildings of Susa the first place belongs to the church of San Giusto, founded in 1029 by Olderico Manfredi II. and the countess Berta, and in 1772 raised to be the cathedral. The population of the city was 3254 in 1871 and 3305 in 1881 (commune, 4418).

Segusio (also Secusio, Siosium, Seutium, Seucia, &c.) was at a very early period the chief town of this Alpine region, and the Cottian Alps themselves preserve the name of the Segusian chief Cottius, who with the title of præfectus became a tributary and ally of Rome in the reign of Augustus, and left his state strong enough to maintain its independence till the reign of Nero. As a Roman municipium and military post Segusio continued to flourish. After the time of Charlemagne a marquisate of Susa was established; and the town became in the 11th century the capital of the famous countess Adelaide, who was mistress of the whole of Piedmont. On his retreat from Legnano, Barbarossa set fire to Susa; but the town became more than ever important when Emanuel Philibert fortified it at great expense in the 16th century.

SUSA (*Súsa*), a city of Tunis, on the coast of the gulf of Hamáma, 33 miles south of Ḥamáma. It occupies the side of a hill sloping seawards, and is still, as far as the town proper is concerned, surrounded with heavy white-

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[1] The Greeks called the citadel the Μεμνόνιον (Strabo, xv. 3, 2), and supposed it to have been founded by the Ethiopian Memnon. It was strong enough to withstand Molon in his war with Antiochus the Great (Polyb., v. 48).