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Sunday Feb 16, 96

THE

NORMAL ELEMENTARY GEOMETRY:

EMBRACING A BRIEF TREATISE ON

Mensuration and Trigonometry.

DESIGNED FOR

ACADEMIES, SEMINARIES, HIGH SCHOOLS, NORMAL SCHOOLS, AND ADVANCED CLASSES IN COMMON SCHOOLS.

REVISED EDITION,

BY

EDWARD BROOKS, A.M., PH.D.,

LATE PRINCIPAL OF STATE NORMAL SCHOOL PENNSYLVANIA, AND AUTHOR OF THE NORMAL PRIMARY ARITHMETIC, NORMAL MENTAL ARITHMETIC, NORMAL WRITTEN ARITHMETIC, NORMAL UNION ARITHMETIC, PHILOSOPHY OF ARITHMETIC, METHODS OF TEACHING, MENTAL SCIENCE AND CULTURE, ETC.

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PREFACE.

PROGRESS in education is symbolized in the multiplication and improvement of text-books. They are showered upon us like the flowers of spring-time, until a new text-book is no longer a novelty. To-day the author sends forth this little volume on what it is hoped may be a mission of usefulness. It comes modestly claiming a welcome from the public as an addition to our educational literature, and in support of such claim the following statement of its object and peculiarities is presented.

Our text-books upon Geometry, though well adapted to our higher institutions, are, for a large class of schools, both too voluminous and difficult. In many of our Academies, Seminaries, High and Normal Schools, the time allotted to Geometry is too brief to allow the pupil to complete more than four or five books of the ordinary text-book: in consequence of which, all of that most important and practical part treating of the measurement of the surface and volume of prisms, pyramids, cylinders, cones, and spheres, must be omitted. To supply this defect and enable the pupil to acquire a fuller knowledge of the subject, the present volume has been written.

GENERAL FEATURES.—In its adaptation to the class of pupils designated, this work is characterized by four general features. First, an abbreviation of the ordinary text-books; second, a simplification, so far as possible, of the methods of demonstration usually employed; third, examples to impart the power of making a practical application of the principles of the science; fourth, undemonstrated theorems, to cultivate the power of original thought and investigation. These general characteristics will be briefly noticed.

ABBREVIATION.—In the abbreviation of the subject, the object has been to present the most valuable part of Geometry in about one-half of the space usually devoted to it. This object has been accomplished in two ways:—first, by an omission of all that is not essential to the final results; and secondly, by such a modification of the remainder as to preserve the chain of logic intact. The difficulty of this will be appreciated by those who remember that many propositions, apparently of little importance, are essential to the proof of others which follow them.

SIMPLIFICATION.—Much care has been taken to simplify the subject as far as possible. The author has endeavored to give the very simplest methods of treating special subjects, such as parallels, areas, volumes, the circle, etc.; and also the clearest and most concise methods of demonstrating individual theorems. The *method of infinites*, as applied to incommensurable quantities, the circle, and the sphere, has contributed largely to this simplification and abbreviation. This method is regarded by some as less satisfactory than the method of *reductio ad absurdum*, or the *method of limits;* but it will be remembered that it is supported by the authority of our most eminent mathematicians, and, being so much more simple and concise, is believed to be preferable in a brief work like this.

APPLICATIONS.—A radical defect of most of our text-books upon Geometry is that they present the subject so abstractly, that when the pupil has completed his course he is often unable to make any practical application of what he has learned. This defect has been supplied by the presentation of a collection of *practical examples* at the close of each book. With these, the pupil can see the application, the practical value of what he is doing, and will not only be able to make use of his knowledge, but will be incited to study the subject with more interest and earnestness.

THEOREMS FOR ORIGINAL THOUGHT.—Another general defect of

our text-books upon Geometry is the lack of matter for original thought, for the training of the inventive powers of the student. The pupil is required to learn the demonstrations of the text-book, but he has no undemonstrated theorems to test his own geometrical powers, and to train him in reasoning independently of the text-book. In view of this general defect, a collection of *theorems for original thought* has been given at the close of each book.

Geometry, in both the respects mentioned, has been treated quite differently from arithmetic and algebra. In these latter works, we have generally a large class of problems both for the application of the principles and the exercise of original thought. It is proper to remark, too, that several authors have realized this defect of Geometry, and have occasionally given some practical problems, and, in one or two instances, a collection of undemonstrated theorems. In the present work, such problems and theorems are an essential and prominent part of the plan.

SPECIAL FEATURES.—The attention of teachers is also respectfully invited to the following special features of the work:

1. The *Systematic Arrangement* of the subject-matter, it is thought, will be an acceptable feature of the work.

2. The *Analysis* at the beginning of each book is supposed to be valuable in giving the pupil a general idea of the object of the book, and thus often indicating the course of reasoning in its development.

3. The *Doctrine of Parallels*, resulting from the modern definition of an angle and parallel lines, now adopted by several authors, is here presented in its most simple and concise form.

4. The *Subject of Areas* in Book III., and *Volumes* in Book VI., are presented with great conciseness and simplicity by the use of the doctrine of infinites and indivisibles.

5. The *treatment of the circumference ana area of a circle*, in their relation to π, is much more simple and logical than any thing the author has met. It is confidently believed that it will give pupils a clearer view of the subject than they usually acquire in the study of other text-books.

MENSURATION.—A formal treatise upon Mensuration is also appended, although the most of it is really presented in the practical exercises. A few rules are given without demonstration; such omissions may be supplied by the teacher.

TRIGONOMETRY.—The little treatise on Trigonometry presents the elements of the subject briefly, and contains about as much as the advanced pupils in our ordinary academies, seminaries, etc. should be required to learn. A short treatise on Analytical Trigonometry is appended for those who have time to study this very interesting and useful subject. The Trigonometry and Geometry will be bound together, and also separately, to accommodate schools of different grades.

A general acknowledgment of indebtedness is due to those who have previously written upon the subject of Geometry, especially to American and French authors, many of whose works have been examined with great interest and profit.

Thanking my friends for the generous appreciation bestowed upon my previous labors, I send forth this little volume, hoping that it may be as kindly welcomed, and that, in its mission of usefulness, it may aid in awakening a deeper interest in the beautiful science of *form*,—a science over which the ancient sages mused with such deep enthusiasm, and to which the achievements of modern art and invention are so largely indebted.

EDWARD BROOKS.

STATE NORMAL SCHOOL, Jan. 10, 1865.

CONTENTS.

GEOMETRY.

TRIGONOMETRY.

HISTORY
OF
GEOMETRY AND TRIGONOMETRY.

Geometry is generally supposed to have had its origin in Egypt, where the annual overflowing of the Nile obliterated the landmarks and rendered it necessary to have recourse to mathematical measurement to re-establish them. This origin is indicated by the term itself, Geometry being from two Greek words, γη, *earth*, and μετρον, *measure*, signifying, literally, the measurement of the earth. But, whatever may have been the origin of the term, the natural tendency of the human mind to compare things in respect of their forms and magnitudes is so universal, that a geometry more or less perfect must have existed since the first dawn of civilization.

Geometry, originating in Egypt, is supposed to have been introduced into Greece by Thales, who lived about the year 650 B.C. Pythagoras, who lived about 570 B.C., was one of the earliest Greek geometers. He is supposed to have discovered the following principles:—1. Only three plane figures can fill up the space about a point; 2. The sum of the angles of a triangle equals two right angles; 3. The celebrated proposition of the square on the hypothenuse. Some say that in honor of this last discovery he sacrificed one hundred oxen. Plutarch says but one ox. Cicero doubts even that, as it was in opposition to his doctrines to offer bloody sacrifices, and suggests that they may have been images made of flour or clay.

The next geometer of eminence was Anaxagoras, who composed a treatise on the *quadrature of the circle*. Plato, the "poetical philosopher," delighted in the science, and cultivated it with great success, as is proved by his simple and elegant solution of the *duplication of the cube*. About fifty years after the time of Plato, Euclid collected the propositions which had been discovered by his predecessors, and formed of them his famous "*Elements*,"—a work of such eminent excellence that by many it is regarded, even at the present day, as the best text-book upon the subject of Elementary Geometry. It consists of fifteen books, thirteen of which are known to have been written by Euclid; but the fourteenth and fifteenth are supposed to have been added by Hypsicles of Alexandria.

Apollonius of Perga, about 250 years B.C., composed a treatise on *Conic*

Sections, in eight books. He is said to have given them their names, *parabola*, etc. About the same time flourished Archimedes, who distinguished himself in Geometry by the discovery of the beautiful relation between the sphere and cylinder. See Theorem XI. Book VII. He also distinguished himself by his work on *conoids* and *spheroids*, by his discovery of the exact *quadrature of the parabola*, and his very ingenious approximation to that of the circle.

Other geometers of eminence followed, among whom the most illustrious, perhaps, were Pappus and Diophantus; but the Greek Geometry, though it was afterward enriched by many new theorems, may be said to have reached its limits in the hands of Archimedes and Apollonius, and a long interval of seventeen centuries elapsed before this limit was passed. In 1637, Descartes published his Geometry, which contained the first systematic application of algebra to the solution of geometrical propositions. Soon after this followed the discovery of the infinitesimal calculus of Leibnitz and Newton; and from that time to the present Geometry has shared in the general progress of all mathematical sciences.

TRIGONOMETRY.—Trigonometry, it is generally believed, originated with the Greek astronomers of Alexandria. The solutions of the most useful cases of spherical triangles have been known from the time of Hipparchus, and the fundamental formulæ appear in the Analemma of Ptolemy.

The Greeks used the *chords* of the *double arcs*, instead of the *sines*. The *sines*, or *semi-chords*, were introduced by the Arabians, probably by the astronomer Albategnius. To the Arabs, who preserved and cultivated the sciences during the dark ages, this science is indebted also for several other improvements. Regiomontanus introduced the *tangents*, which did much to simplify the calculations.

The term *sine* seems to be derived from the Latin *sinus*, a bosom; the arc is supposed to represent a *bow*, and thus gets its name; the string, half of which represents the sine of half the arc, would come against the heart or bosom; hence the name *sine*. The terms *tangent* and *secant* are naturally derived from the old geometrical definitions. The *cosine* and *co-secant* of an arc mean the sine and secant of the complement, the *co* being merely an abbreviation of *complement*. They were first introduced by Gunter.

There are two methods of treating Trigonometry, known as the analytical and synthetic methods. The synthetic method regards the trigonometrical functions as lines, or geometrical magnitudes, and develops the science according to the laws of geometrical reasoning. The analytical method regards these functions as ratios or numbers, and develops the science by means of analytical formulas.

The modern or analytical method is superseding the ancient or geometrical method. This method is said to have been first introduced by Dr. Peacock. Professor De Morgan, however, one of the first English authorities, tells us that "Rheticus, who gave the first complete trigonometrical table, and invented the secant and co-secant to complete it, used the method of ratios."

LOGARITHMS.—Logarithms were invented by Lord Napier, Baron of Merchiston, in Scotland. His work upon them was first published in 1614; though it is probable that he had commenced the investigation of them as early as 1594. The invention is regarded as one of the most useful ever made. It gave the author so high a reputation that Kepler dedicated a work to him in 1617, and succeeding mathematicians have paid him the highest compliments.

Napier's system of logarithms was afterward improved by Henry Briggs, a contemporary of the inventor, and Professor of Geometry in Gresham College. Assuming 10 for the basis, he constructed a system of logarithms corresponding to our system of numeration, which is much more convenient for the ordinary purposes of calculation. The two systems are distinguished as the *Napierian* and *Briggean*, or the *Hyperbolic* and *Common* logarithms. The former are called *Hyperbolic* because they represent the area of a rectangular hyperbola between its asymptotes; the latter are called *Common* because they are those in common use.

Briggs calculated the logarithms to 14 places, with the index of all numbers between 1 and 20,000, and between 90,000 and 100,000, and published them in 1624. Adrian Vlacq, a native of Holland, computed the logarithms of numbers between 20,000 and 90,000, and thus completed what Briggs had begun: he reduced the tables, however, to 10 decimal places. Vlacq's treatise was published in 1628, and contained the logarithms of all numbers up to 100,000, and also the logarithms of the sines, tangents, and secants of every minute of the quadrant. In 1623, he published a work containing the logarithmic sines, cosines, tangents, and cotangents for every ten seconds of the quadrant, calculated from the natural sines, etc. of the *Opus Palatinum* of Rheticus.

In the same year the *Trigonometrica Britannica* was published at Gouda, which contained the logarithmic sines and tangents for the 100th part of every degree of the quadrant, together with a table of natural sines, tangents, and secants. These had been computed by Briggs. Since then, many different tables have been published. The most complete are those of Vlacq; but these are very scarce. *Hutton's Logarithms* and *Babbage's Logarithms of Numbers* are among the most accurate and convenient. For more information upon the subject, see *Brande's Encyclopedia*, from which most of this history is collated.

SUGGESTIONS TO TEACHERS.

The author desires to present the following suggestions to those who may use this work:

1. Young pupils should have a preliminary drill upon concrete Geometry before taking up the text-book. Let them be required to cut out triangles, squares, etc. from paper, give their names, compare them, and draw them upon the board. In this manner a general idea of the subject, the figures treated of, and even the method of reasoning, may be obtained, and the transition from this to the abstract will be simple and easy.

2. In the recitation, the pupil should be required to construct his diagram upon the blackboard without the aid of the text-book, and then enunciate and demonstrate the theorem, care being taken that the language and reasoning be accurate. At the close of the demonstration, those of the class who have noticed errors, upon being called upon by the teacher, should rise and point them out; after which the teacher may make any criticisms or explanations he may think proper.

3. With quite young pupils, and those whose time for the study is limited, the *theorems for original thought* may be omitted; with others, however, these exercises will be found to be of great value. They can be given in connection with the demonstrations of the book, or lessons may be assigned upon them after completing the book to which they belong, or they may be omitted until review. The latter method will be generally preferred.

The *Practical Exercises* should be solved by all classes. The easier problems may be assigned in connection with the theorems which they illustrate; the others may be deferred until the book upon which they depend is completed. The most difficult problems may be omitted until the whole Geometry is completed.

ELEMENTARY GEOMETRY.

INTRODUCTION.

LESSON I.

SUBJECT-MATTER OF GEOMETRY.

Every object that we can see occupies some portion of space, and has extent and form. If we consider some object, as this book, for instance, we will perceive that it has length, breadth, and thickness. These are called the dimensions of the book.

If, now, we remove the book from before us, we can still imagine the space which it filled to be in the form of a book. This space, of course, will not be a material thing like the book, but it will have form and extent the same as the book had. Such definite portions of space, their forms and extent, are the things considered in Geometry.

These limited portions of space are called *Volumes*. A volume has length, breadth, and thickness, and these are called its dimensions. We should be careful to distinguish the geometrical volume, which is a portion of space, from the solid body, which occupies space. The one is material, the other is immaterial; the one is *real body*, the other is *ideal body* or *pure form*. It is ideal body or pure form that is treated of in Geometry.

2