SUGGESTIONS TO TEACHERS.

THE author desires to present the following suggestions to those who may use this work:

1. Young pupils should have a preliminary drill upon concrete Geometry before taking up the text-book. Let them be required to cut out triangles, squares, etc. from paper, give their names, compare them, and draw them upon the board. In this manner a general idea of the subject, the figures treated of, and even the method of reasoning, may be obtained, and the transition from this to the abstract will be simple and easy.

2. In the recitation, the pupil should be required to construct his diagram upon the blackboard without the aid of the text-book, and then enunciate and demonstrate the theorem, care being taken that the language and reasoning be accurate. At the close of the demonstration, those of the class who have noticed errors, upon being called upon by the teacher, should rise and point them out; after which the teacher may make any criticisms or explanations he may think proper.

3. With quite young pupils, and those whose time for the study is limited, the *theorems for original thought* may be omitted; with others, however, these exercises will be found to be of great value. They can be given in connection with the demonstrations of the book, or lessons may be assigned upon them after completing the book to which they belong, or they may be omitted until review. The latter method will be generally preferred.

The *Practical Exercises* should be solved by all classes. The easier problems may be assigned in connection with the theorems which they illustrate; the others may be deferred until the book upon which they depend is completed. The most difficult problems may be omitted until the whole Geometry is completed.

ELEMENTARY GEOMETRY.

INTRODUCTION.

LESSON I.

SUBJECT-MATTER OF GEOMETRY.

EVERY object that we can see occupies some portion of space, and has extent and form. If we consider some object, as this book, for instance, we will perceive that it has length, breadth, and thickness. These are called the dimensions of the book.

If, now, we remove the book from before us, we can still imagine the space which it filled to be in the form of a book. This space, of course, will not be a material thing like the book, but it will have form and extent the same as the book had. Such definite portions of space, their forms and extent, are the things considered in Geometry.

These limited portions of space are called *Volumes*. A volume has length, breadth, and thickness, and these are called its dimensions. We should be careful to distinguish the geometrical volume, which is a portion of space, from the solid body, which occupies space. The one is material, the other is immaterial; the one is *real body*, the other is *ideal body* or *pure form*. It is ideal body or pure form that is treated of in Geometry.

Let us now consider this ideal body or volume a little more closely. First, we will notice that it is distinctly separated from the surrounding space. That which so separates it is called a *Surface;* and, since that which bounds the volume forms no part of the volume, it will be seen that a surface has no thickness, and possesses, therefore, but two dimensions,—length and breadth.

If we consider one of these bounding surfaces, we will see that it also is limited or bounded. That which limits a surface is called a *Line;* and since that which limits a surface forms no part of the surface, it is seen that a line has only one dimension,—length.

Again, if we examine one of these lines, we will see that its ends are limited. This limit is called a *Point;* and, since the limit forms no part of the line, a point has neither length, breadth, nor thickness, but position only.

Now, although we have considered a point as the limit of a line, a line as the limit of a surface, and a surface as the limit of a volume, yet each of these may be regarded in a purely abstract manner, distinct from each other. Thus, we may consider points without regard to lines, lines without reference to surfaces, and surfaces without reference to volumes.

We have now attained a conception of the ideas of Geometry by passing from a body to an abstract volume, from this volume to a surface, from a surface to a line, and from a line to a point. This is the method of analysis, and is, without doubt, the method in which these ideas were primarily attained. They may, however, also be attained by synthesis, in the following manner.

Fix upon the idea of a point in space. Now, suppose

this point to move, and we have a line; suppose the line to move in a particular manner, and we have a surface; suppose the surface to move, and we have a volume.

These lines, surfaces, and volumes, of which we have attained the idea, are the fundamental quantities of Geometry. A quantity, you will remember, is any thing that can be measured. When one line crosses another, their divergence may be measured: hence we have a fourth kind of geometrical quantity, called *angles.* We are now prepared to define Geometry.

Geometry is that science which treats of the properties and relations of geometrical magnitudes. Its subject-matter are lines, surfaces, volumes, and angles.

LESSON II.

REASONING OF GEOMETRY.

The subject-matter of Geometry, we have seen, are lines, surfaces, volumes, and angles. These general conceptions give rise to many special forms; these special forms are described, and such descriptions constitute the *Definitions* of the science.

When we consider these special forms of quantity, as well as quantity in general, we perceive some truths concerning them that are self-evident,—that must be true, since they cannot be conceived as untrue. These self-evident truths are called *Axioms.*

The science of Geometry begins with these primary ideas of space and the self-evident truths arising out of them, and from these, as a basis, rises to the higher truths by a process of reasoning. The axioms and definitions are, therefore, said to be the basis of the science of Geometry. The definitions present the subjects upon which we reason; the axioms give the laws which guide us in the reasoning process. From these we trace our way, step by step, to the loftiest and most beautiful truths of the science, by the simple process of comparison. This process of comparison is called *reasoning;* and to this we now call attention.

Reasoning.—All Reasoning is *comparison.* A comparison requires a standard or basis, and this standard is the *simple,* the *axiomatic,* the *known.* To these we bring the *complex,* the *theoretic,* the *unknown,* and learn to understand them by comparing the *complex* with the *simple,* the *theoretic* with the *axiomatic,* the *unknown* with the *known.*

There are two distinct methods of geometrical reasoning, which may be distinguished as the *analytic* and *synthetic* methods. The analytic method is adapted to the discovery of truth; the synthetic method, to the proving of a truth when it has already been discovered.

Synthetic Method.—The synthetic method, which is generally employed in proving a truth which is already known, is called *demonstration.* There are two distinct methods of demonstration, called the *Direct* and the *Indirect Method.*

The simplest form of the *Direct Method* is that in which figures are directly compared by applying one to another. This is called the method by *superposition.* The more general form of the direct method is that in which truths are proven by a reference to the definitions and axioms, or some principle previously proven.

The *Indirect Method,* known as the *reductio ad absurdum,* consists in supposing the proposition to be proven not to be true, and then showing that such an hypothesis leads to a contradiction of some known truth. This is frequently used to prove the converse of a proposition, when there is no good direct method; it is also used in incommensurable quantities.

There are two errors of demonstration into which young pupils are liable to fall. The first is called *Reasoning in a Circle;* the second is *Begging the Question.* We reason in a circle when, in demonstrating a truth, we employ a second truth which cannot be proven without the aid of the first. We are said to beg the question when, in order to establish a proposition, we employ the proposition itself.

Analytic Method.—The analytic method begins with the thing required, and by tracing the relations of the various parts we arrive at some known truth. It is a kind of going back from the result sought by a chain of relations to what has been previously established. In a demonstration, we pass through every step from the simplest self-evident truth to the highest deductions of the science; in the process of analysis, we pass over every step from the latter truths down to the simplest.

Analysis is the method of discovery; synthesis, of demonstration. The one has for its object to find unknown truths; the other, to prove known ones. Frequently both methods are employed simultaneously, when the object is to discover new relations, or the solution of new problems; but when we wish to prove to others the truths we have discovered, the synthetical method is usually preferred.

LESSON III.

GEOMETRICAL LANGUAGE.

LANGUAGE is the instrument of thought and the medium of expression. All thinking is by means of language; and the more concise and perfect the language, the more profound and searching is our thought. The language of mathematics differs somewhat from that of ordinary usage, in being more concise and more definite in its use.

Much of the language of mathematics is symbolical; that is, a symbol is used in place of the written word. There are three classes of symbols in Geometry: *symbols of quantity*, *symbols of operation*, and *symbols of relation.*

The SYMBOLS OF QUANTITY are usually pictured representations of the quantities considered. Sometimes, however, the letters of the alphabet are used to indicate them.

The SYMBOLS OF OPERATION are as follow:—

The *Sign of Addition*, $+$, called *plus;* thus, $A + B$, denotes that B is to be added to A.

The *Sign of Subtraction*, $-$, called *minus;* thus, $A - B$, denotes that B is to be subtracted from A.

The *Sign of Multiplication*, $\times$; thus, $A \times B$, denotes that A is to be multiplied by B.

The *Sign of Division*, $\div$; thus, $A \div B$, denotes that A is to be divided by B.

The *Exponential Sign;* thus, A^4, denotes that A is used four times as a factor, or is raised to the fourth power.

The *Radical Sign*, $\sqrt{\ }$; thus, $\sqrt{A}$, $\sqrt[3]{B}$, denotes that the square root of A and the cube root of B are to be extracted.

The *Parenthesis* and *Vinculum* denote that the quantity

is to be operated upon as a whole; thus, $(A + B) \times C$, or $\overline{A + B} \times C$, denotes that the sum of A and B is to be multiplied by C.

The SYMBOLS OF RELATION are as follow:—

The *Sign of Equality*, $=$; thus, $A = B + C$, denotes that A is equal to the sum of B and C.

The expression of the equality of two quantities is an equation; thus, $A = B + C$, is an equation. The part on the left of the sign of equality is the *first member;* that on the right is the *second member.*

The *Sign of Inequality*, $>$ or $<$; thus, $A > B$, denotes that A is greater than B. The greater quantity is at the opening of the sign.

The *Sign of Ratio*, :; thus, $A : B$, denotes the ratio of A to B.

The *Sign of Equal Ratios*, : :; thus, $A : B :: C : D$, denotes that the ratio of A to B *equals* the ratio of C to D.

We present also a few combinations of these symbols, called *formulas*, which will be found valuable in some of the demonstrations.

1. $A \times B + C \times B = (A + C) \times B.$
2. $\frac{1}{2} A \times B - \frac{1}{2} B \times C = \frac{1}{2}(A - C) \times B.$
3. $(A + B)^2 = A^2 + 2A \times B + B^2.$
4. $(A - B)^2 = A^2 - 2A \times B + B^2.$
5. $(A + B) \times (A - B) = A^2 - B^2.$

DEFINITION OF TERMS.

An AXIOM is a self-evident truth.

A THEOREM is a truth to be demonstrated.

A PROBLEM is a question to be solved.

A POSTULATE is a problem whose solution is self-evident.

A COROLLARY is an obvious consequence of, or a theorem suggested by, one or more propositions.

A SCHOLIUM is a remark upon one or more propositions.

Theorems, Axioms, Problems, and Postulates, are all called *Propositions*.

An HYPOTHESIS is a supposition made in the statement of a proposition, or in its demonstration.

NOTE.—In making references, A. stands for Axiom; B. for Book; C. for Corollary; D. for Definition; I. for Introduction; Th. for Theorem; P. for Problem; S. for Scholium. In referring to another Book, the number of the book is given; in referring to the same Book, the number of the Book is not given.

ELEMENTARY GEOMETRY.

BOOK I.

DEFINITIONS.

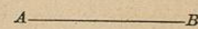
1. GEOMETRY is the science which treats of the properties and relations of geometrical magnitudes.

2. A GEOMETRICAL MAGNITUDE is some definite element of space. It is a line, a surface, a volume, or an angle.

3. A POINT is that which has position, but no magnitude.

4. A LINE is that which has length, but no breadth or thickness. Lines are *straight* or *curved*.

5. A STRAIGHT LINE is one which has the same direction at every point: as, *AB*.



6. A CURVED LINE is one which changes its direction at every point: as, *CD*.

The word *line* used alone, means a *straight line;* the word *curve*, alone, means a *curved line*.

7. A SURFACE is that which has length and breadth, without thickness. Surfaces are *plane* or *curved*.

8. A PLANE is a surface such that if any two of its points be joined by a straight line, every part of that line will lie in the surface.