A Corollary is an obvious consequence of, or a theorem suggested by, one or more propositions.

A Scholium is a remark upon one or more propositions.

Theorems, Axioms, Problems, and Postulates, are all called *Propositions*.

An Hypothesis is a supposition made in the statement of a proposition, or in its demonstration.

Note.—In making references, A. stands for Axiom; B. for Book; C. for Corollary; D. for Definition; I. for Introduction; Th. for Theorem; P. for Problem; S. for Scholium. In referring to another Book, the number of the book is given; in referring to the same Book, the number of the Book is not given.

ELEMENTARY GEOMETRY.

BOOK I.

DEFINITIONS.

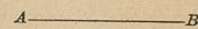
1. Geometry is the science which treats of the properties and relations of geometrical magnitudes.

2. A Geometrical Magnitude is some definite element of space. It is a line, a surface, a volume, or an angle.

3. A Point is that which has position, but no magnitude.

4. A Line is that which has length, but no breadth or thickness. Lines are *straight* or *curved*.

5. A Straight Line is one which has the same direction at every point: as, *AB*.



6. A Curved Line is one which changes its direction at every point: as, *CD*.

The word *line* used alone, means a *straight line;* the word *curve*, alone, means a *curved line.*

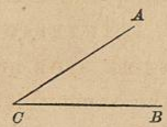
7. A Surface is that which has length and breadth, without thickness. Surfaces are *plane* or *curved.*

8. A Plane is a surface such that if any two of its points be joined by a straight line, every part of that line will lie in the surface.

9. A VOLUME is that which has length, breadth, and thickness.

PLANE ANGLES.

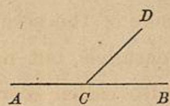
10. An ANGLE is the difference of direction, or the divergence, of two lines proceeding from a common point.



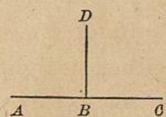
The point from which the lines proceed is called the *vertex* of the angle; the lines themselves are the *sides* of the angle.

An angle is named by the letter at the vertex, or by the three letters with the letter at the vertex in the middle. Thus, we say the angle *C*, or the angle *ACB*.

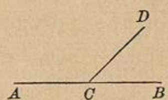
11. ADJACENT ANGLES are angles which have a common vertex with a common side between them; thus, *ACD* and *BCD* are adjacent angles.



12. A RIGHT ANGLE is an angle formed by one straight line meeting another, making the adjacent angles equal. The first line is then said to be *perpendicular* to the other.



13. An OBTUSE ANGLE is one which is greater than a right angle; as, *ACD*.

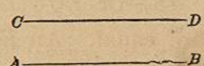


An ACUTE ANGLE is one which is less than a right angle; as, *DCB*.

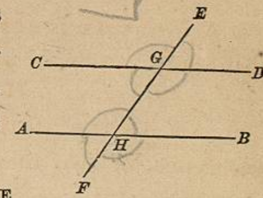
Obtuse and acute angles are called *oblique* angles, in distinction from right angles.

PARALLEL LINES.

14. PARALLEL LINES are those which have the same direction; as, *AB* and *CD*.



When a straight line intersects two parallel straight lines, the angles formed take particular names. Suppose the line *EF* to intersect the parallels *AB* and *CD*; then—



1. INTERIOR ANGLES ON THE SAME SIDE are those which lie within the parallels, on the same side of the *secant*, or intersecting line; thus, *CGH* and *AHG;* also, *HGD* and *GHB;*

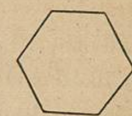
2. ALTERNATE INTERIOR ANGLES lie within the parallels, on different sides of the secant line, but not adjacent; as, *CGH* and *GHB;*

3. EXTERIOR-INTERIOR ANGLES lie on the same side of the secant line, one without and the other within the parallels, but not adjacent; as *EGD* and *GHB*. They are also called *corresponding* angles.

PLANE FIGURES.

15. A PLANE FIGURE is a plane bounded by lines either straight or curved.

16. A POLYGON is a plane figure bounded by straight lines. These lines are called *sides* of the polygon; taken together, they form the *perimeter* of the polygon.



17. A POLYGON of three sides is called a *triangle;* of four sides, a *quadrilateral;* of five sides, a *pentagon;* of six sides, a *hexagon;* of seven sides, a *heptagon;* of eight sides, an *octagon;* of nine sides, a *nonagon;* of ten sides, a *decagon*, etc.

18. An EQUILATERAL POLYGON is one whose sides are equal. An *Equiangular Polygon* is one whose angles are equal.

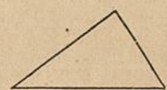
Two polygons are *mutually equilateral* when their sides are respectively equal. Two polygons are *mutually equi-angular* when their angles are respectively equal.

19. A Diagonal of a polygon is a line joining the vertices of two angles, not consecutive.

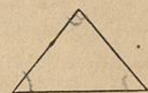
TRIANGLES.

20. A Triangle is a polygon of three sides and three angles. Triangles are classified by their sides and their angles.

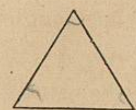
21. A Scalene Triangle is one in which the three sides are unequal.



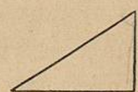
22. An Isosceles Triangle is one which has two of its sides equal.



23. An Equilateral Triangle is one which has its three sides equal.



24. A Right-Angled Triangle is one which has one right angle. The side opposite the right angle is called the *hypothenuse.*



25. An Acute-Angled Triangle is one in which all the angles are acute.

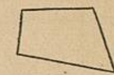
26. An Obtuse-Angled Triangle is one which has one obtuse angle.

Triangles are the simplest of all polygons, since three sides are the least number that can bound a plane figure. The properties of polygons are determined by analyzing them into triangles.

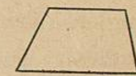
QUADRILATERALS.

27. A Quadrilateral is a polygon of four sides and four angles. There are three classes:—

1. The Trapezium is a quadrilateral having no two sides parallel.



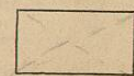
2. The Trapezoid is a quadrilateral having two of its opposite sides parallel.



3. The Parallelogram is a quadrilateral having its opposite sides parallel.

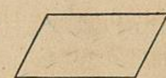
28. Parallelograms are divided, from their angles, into two classes,—right-angled and oblique-angled parallelograms.

1. A Rectangle is a parallelogram whose angles are right angles.



A Square is an equilateral rectangle.

2. A Rhomboid is a parallelogram whose angles are oblique.

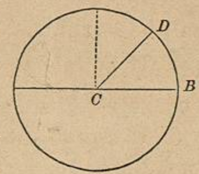


A Rhombus is an equilateral rhomboid.

THE CIRCLE.

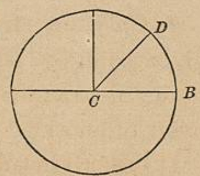
29. A Circle is a plane figure bounded by a curve line, every point of which is equally distant from a point within, called the *centre.*

The CIRCUMFERENCE is the bounding line; any part of the circumference is called an *arc*. A line through the centre having its ends in the circumference is a *diameter;* a line from the centre to the circumference is the *radius*.



30. The arcs of circles are used to measure angles. An angle having its vertex at the centre of a circle is measured by the arc included between its sides; thus, the arc *BD* measures the angle *DCB*.

To measure angles, the circumference is divided into 360 equal parts, called *degrees*. Each degree is divided into 60 equal parts, called *minutes;* each minute into 60 equal parts, called *seconds*. Degrees, minutes, and seconds are marked thus, ° ′ ″; 16° 24′ 32″ are read, 16 degrees, 24 minutes, and 32 seconds.



A right angle, it will be seen, is measured by 90°; half a right angle, by 45°; two right angles, by 180°; four right angles, by 360°.

AXIOMS.

31. An AXIOM is a self-evident truth. There are two classes of axioms in Geometry. First, those which pertain to quantity in general; second, those which arise out of the special forms of geometrical quantity.

FIRST CLASS.

1. Things which are equal to the same thing are equal to each other.

2. If equals be added to equals, the sums will be equal.

3. If equals be subtracted from equals, the remainders will be equal.

4. If equals be added to or subtracted from unequals, the results will be unequal.

5. If equals be multiplied by equals, the products will be equal.

6. If equals be divided by equals, the quotients will be equal.

7. The whole is greater than any of its parts.

8. The whole equals the sum of all its parts.

SECOND CLASS.

9. Only one straight line can be drawn connecting two given points.

10. A straight line is the shortest distance from one point to another.

11. All right angles are equal to each other.

12. Parallel straight lines cannot meet each other when produced.

13. Through a given point only one straight line can be drawn parallel to a given line.

Corollary. From axiom 10, it is evident that any side of a triangle is less than the sum of the other two sides.

POSTULATES.

32. The following postulates are self-evident problems resulting from the preceding definitions:—

1. A straight line can be drawn from one point to another.

2. A straight line may be prolonged to any length.

3. A line or an angle may be bisected.

4. An angle may be described equal to a given angle.

5. A line may be drawn through a given point parallel to a given line.

6. A perpendicular may be drawn to a given line from a point without the line or in the line.

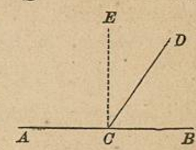
Analysis of Book I.—Book I. treats mainly of angles, parallel lines, triangles, and parallelograms. It treats of the angles formed by one line meeting or cutting another, of the angles formed by one line cutting two parallel lines, of the equality and inequality of triangles, of the sum of the angles of a triangle, of the relation of the angles and sides of a parallelogram, and of the exterior and interior angles of a polygon. It is thus seen that the idea of angles is a prominent, if not the principal one of the book.

OF ANGLES.

THEOREM I.

When one straight line meets another straight line, the sum of the two adjacent angles equals two right angles.

Let the straight line DC meet the straight line AB at the point C; then will $ACD + DCB =$ two right angles.



For, at the point C, erect CE perpendicular to AB; then (D. 12,) the angles ACE and ECB are both right angles. Now,

$ACD = a \text{ right angle} + ECD$; and

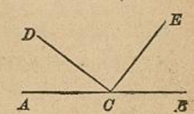
$DCB = a \text{ right angle} - ECD$; hence, adding,

we have, $ACD + DCB = \text{two right angles}$.

Therefore, when a straight line meets another straight line, the sum of the two adjacent angles equals two right angles.

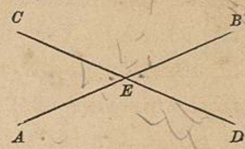
Cor. 1. If one of the angles ACD or DCB is a right angle, the other is also a right angle.

Cor. 2. The sum of all the angles formed on the same side of a straight line by drawing lines to any point of that line, is equal to two right angles. For, their sum is equal to the sum of ACD and DCB, which is equal to two right angles, according to the proposition.



THEOREM II.

When two straight lines intersect each other, the opposite or vertical angles are equal.



Let the two straight lines AB and CD intersect each other at the point E; then will AEC be equal to BED.

For, since CE meets AB, the angle $AEC + CEB =$ two right angles (Th. I.); and since BE meets CD, the angle $CEB + BED =$ two right angles; but things which are equal to the same thing are equal to each other (A. 1); hence,

$$AEC + CEB = CEB + BED.$$

Taking from each sum the common angle CEB, there remains (A. 3),

$$AEC = BED.$$

In a similar manner it may be shown that the angle AED equals CEB. Therefore, etc.

Cor. 1. The sum of the four angles formed by the intersection of two lines is equal to four right angles.

Cor. 2. The sum of all the angles that can be formed about a point is equal to four right angles.

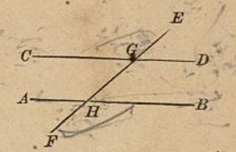
PARALLEL LINES.

THEOREM III.

If a line intersect two parallel lines;

1. *The exterior-interior angles will be equal.*
2. *The alternate interior angles will be equal.*
3. *The sum of the interior angles on the same side will be equal to two right angles.*

Let the line EF intersect the two parallels AB and CD; then,



First. The angle EGD is equal to GHB. For, since HB and GD are parallel, they have the same direction; hence, they must diverge equally from the line EF; therefore, the difference of direction or divergence of GE and GD must be equal to the divergence of HE and HB, or the angle EGD equal to GHB. In the same way it may be shown that $FHB = HGD$.

Second. The two alternate angles CGH and EHB will be equal. For, CGH equals EGD (Th. II.); but EGD equals EHB; therefore, $CGH = GHB$ (A. 1); and in the same manner it may be shown that AHG equals HGD.

Third. The sum of the two interior angles GHB and HGD equals two right angles. For, $EGD + DGH =$ two right angles (Th. I.); but $EGD = EHB$; hence, $GHB + HGD =$ two right angles. In the same way it may be shown that $AHG + CGH$ equals two right angles. Therefore, etc.

Cor. If a line is perpendicular to one of two parallels, it is perpendicular to the other also. For, if EGD were a right angle, its equal EHB would be a right angle also.

THEOREM IV.

Conversely.—*If a straight line meets two other straight lines, these two lines will be parallel;*

1. *When the exterior-interior angles are equal.*
2. *When the alternate interior angles are equal.*
3. *When the sum of the two interior angles on the same side is equal to two right angles.*

Let the straight line EF meet the two straight lines AB and CD; then,