BOOK II.

RATIO AND PROPORTION.

1. All reasoning is by comparison. In comparing two quantities, we see that they bear a certain relation to each other.

2. Ratio is the measure of the relation of two similar quantities. It is found by dividing the first by the second; thus, the ratio of 8 to 4 is $\frac{8}{4}$, or 2, the ratio of A to B is $\frac{A}{B}$.

3. The two quantities compared are called the *Terms* of the ratio. The first is called the *Antecedent*, the second the *Consequent*, and the two constitute a *Couplet*.

4. A ratio is indicated by placing a colon between the quantities, or by writing the consequent under the antecedent, as in division; thus, the ratio of A to B is written, $A : B$, or $\frac{A}{B}$.

5. A Proportion is an expression of equality between equal ratios; thus, the ratio of 8 to 4 equals the ratio of 6 to 3, and a formal comparison of these, as $8 : 4 = 6 : 3$, is a proportion.

6. The equality of ratios is usually indicated by a double colon; thus, $8 : 4 :: 6 : 3$. This is read, the ratio of 8 to 4 equals the ratio of 6 to 3, or, 8 is to 4 as 6 is to 3.

7. There are four terms in a proportion; the first and fourth are called the *extremes;* the second and third, the *means.* The first and second together are the *first couplet;* the third and fourth, the *second couplet.*

8. Quantities are in proportion by *Alternation*, when antecedent is compared with antecedent, and consequent with consequent; thus, if $A : B :: C : D$, by alternation we have $A : C :: B : D$.

9. Quantities are in proportion by *Inversion*, when the antecedents are made consequents and the consequents antecedents; thus, if $A : B :: C : D$, by inversion we have $B : A :: D : C$.

10. Quantities are in proportion by *Composition*, when the sum of antecedent and consequent is compared with either antecedent or consequent; thus, if $A : B :: C : D$, by composition we have, $A : A + B :: C : C + D$.

11. Quantities are in proportion by *Division*, when the difference of antecedent and consequent is compared with either antecedent or consequent; thus, if $A : B :: C : D$, we have, $A : A - B :: C : C - D$.

A Continued Proportion is a series of equal ratios; as, $A : B :: C : D :: E : F ::$, etc.

Analysis.—The object of the theorems of this book is to derive the principles of proportion. These principles are employed in the books which follow. The method consists in regarding a proportion as an *equation*, which it really is,—an *equality of ratios*. Thus, the pupil should be taught to regard the proportion $A : B :: C : D$ as equivalent to $A \div B = C \div D$, and as soon as this idea is clearly fixed in the mind the subject becomes simple and easy. The first proportion is the basis of demonstration for the others, and may be used as a test of the truth of all others.

ac + ad = [illegible] + db

¡Que cachaza!

THEOREM I.

If four quantities are in proportion, the product of the means will equal the product of the extremes.

Take the proportion

$A : B :: C : D$; then we wish to prove

that $A \times D = B \times C$.

For, from the proportion we have

$\frac{A}{B} = \frac{C}{D}$; multiplying by $B \times D$,

we have, $A \times D = B \times C$.

Therefore, *if four quantities are*, etc.

Nov 8

¡Que estupidez!

Por la Virgen del Pilar, que gente tan burra

THEOREM II.

If the product of two quantities equals the product of two other quantities, the quantities forming one product may be made the means, and the other two the extremes of a proportion.

Suppose we have

$A \times D = B \times C$; dividing by $B \times D$,

we have, $\frac{A}{B} = \frac{C}{D}$; placing this in another form,

we have, $A : B :: C : D$.

Therefore, etc.

THEOREM III.

A mean proportional between two quantities equals the square root of their product.

Let B be a mean proportional between A and C; then we have,

$A : B :: B : C$;

whence (Th. I.), $B^2 = A \times C$,

or, $B = \sqrt{A \times C}$.

Therefore, etc.

THEOREM IV.

If four quantities are in proportion, they will be in proportion by alternation.

Suppose $A : B :: C : D$; from this (Th. I.)

we have, $A \times D = B \times C$; dividing by $D \times C$,

we have, $\frac{A}{C} = \frac{B}{D}$; whence,

$A : C :: B : D$.

Therefore, etc.

REMARK.—The proposition is evidently true, since we have the same products when we take the product of the means and extremes as before the change. This principle may be applied to several other propositions.

THEOREM V.

If four quantities are in proportion, they will be in proportion by inversion.

Suppose $A : B :: C : D$; from this

we have, $\frac{A}{B} = \frac{C}{D}$; taking the reciprocal,

we have, $\frac{B}{A} = \frac{D}{C}$; whence,

$B : A :: D : C$.

Therefore, etc.

THEOREM VI.

If four quantities are in proportion, they will be in proportion by composition.

Suppose $A : B :: C : D$; then

we have, $\frac{A}{B} = \frac{C}{D}$. Adding *one* to each

we have, $\frac{A}{B} + 1 = \frac{C}{D} + 1$; reducing to a common denominator, we have, $\frac{A+B}{B}=\frac{C+D}{D}$; whence,

$$A+B:B::C+D:D.$$

Therefore, etc.

THEOREM VII.

If four quantities are in proportion, they will be in proportion by division.

Suppose $A:B::C:D$; then

we have, $\frac{A}{B}=\frac{C}{D}$; subtracting 1,

we have, $\frac{A}{B}-1=\frac{C}{D}-1$; reducing,

we have, $\frac{A-B}{B}=\frac{C-D}{D}$; whence,

$$A-B:B::C-D:D.$$

Therefore, etc.

THEOREM VIII.

If two proportions have a couplet in each the same, the other couplets will form a proportion.

Suppose $A:B::C:D$; and

$A:B::E:F$; then,

$\frac{A}{B}=\frac{C}{D}$ and $\frac{A}{B}=\frac{E}{F}$; hence (A. 1),

$\frac{C}{D}=\frac{E}{F}$; whence $C:D::E:F$.

Cor. If two proportions have a couplet in each proportional, the other couplets will form a proportion.

THEOREM IX.

Equimultiples of two quantities are proportional to the quantities themselves.

Let A and B be any two quantities; then

$\frac{A}{B}=\frac{A}{B}$; multiply both terms of the first by m,

3:4::6:8
6:8::12:16

we have, $\frac{mA}{mB}=\frac{A}{B}$; whence,

$$mA:mB::A:B.$$

Therefore, etc.

THEOREM X.

If four quantities are in proportion, any equimultiples of the first couplet will be proportional to any equimultiples of the second couplet.

Suppose $A:B::C:D$; then

$\frac{A}{B}=\frac{C}{D}$; hence, also,

we have, $\frac{mA}{mB}=\frac{nC}{nD}$; whence

we have, $mA:mB::nC:nD$.

Therefore, etc.

5:8::15:24

$\frac{5}{8}=\frac{15}{24}$

$\frac{20}{32}=\frac{30}{48}$

20:32::30:48

THEOREM XI.

The products of the corresponding terms of two or more proportions are proportional.

96
96

Suppose $A:B::C:D$, and

$M:N::P:Q$; then

we have, $A\times D=B\times C$,

$M\times Q=N\times P$; taking their product,

we have,

$$A\times M\times D\times Q=B\times N\times C\times P;\text{ whence (Th. II.)}$$

we have, $A\times M:B\times N::C\times P:D\times Q$.

THEOREM XII.

In any continued proportion, any antecedent will be to its consequent as the sum of the antecedents is to the sum of the consequents.

Let $A:B::C:D::E:F$, etc.

Then, since $A:B::C:D$, and

$A : B :: E : F$; we have

$A \times D = B \times C$, and

$A \times F = B \times E$; adding

to these, $A \times B = A \times B$, we have,

$$A \times B + A \times D + A \times F = A \times B + B \times C + B \times E,$$

or, $A \times (B + D + F) = B(A + C + E)$;

whence, $A : B :: A + C + E : B + D + F$.

PRACTICAL EXERCISES.

1. If the first three terms of a proportion are 12, 14, and 18, what is the fourth term? *Ans.* 21.

2. Given the proportion $3 : 12 :: 5 : 20$; what proportion have we by composition?

3. Find a mean proportional to 12 and 27; to m and n. *Ans.* 18; $\sqrt{m \times n}$.

4. If the ratio of A to B is $\frac{1}{3}$, what is the ratio of $3\,A$ to $2\,B$? *Ans.* $\frac{2}{3}$.

5. If the ratio of $3\,A$ to $2\,B$ is $\frac{3}{4}$, what is the ratio of A to B? *Ans.* $\frac{1}{2}$.

6. What proportion is deducible from the equation $M \times N = A^2 - B^2$. *Ans.* $M : A + B :: A - B : N$.

7. What proportion is deducible from the equation $(C + D) \times A = (A + B) \times C$? *Ans.* $A : B :: C : D$

THEOREMS FOR ORIGINAL THOUGHT.

1. If $a : b :: c : d$, prove that $am : bn :: cm : dn$.
2. If $a : b :: c : d$, prove that $\frac{a}{m} : \frac{b}{n} :: \frac{c}{m} : \frac{d}{n}$.
3. If $a : b :: c : d$, prove that $a : a + b :: c : c + d$.
4. If $a : b :: c : d$, prove that $a + b : a - b :: c + d : c - d$.
5. If $a : b :: c : d$ and $m : c :: n : d$, prove that $a : b :: m : n$.

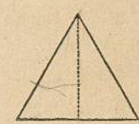
BOOK III.

AREAS AND RELATIONS OF POLYGONS.

1. THIS book treats of the area of polygons and their relation to each other.

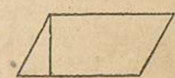
2. The AREA of a polygon is its quantity of surface: it is expressed by the number of times which the polygon contains some other area assumed as a *unit of measure.*

3. The ALTITUDE OF A TRIANGLE is the perpendicular distance from the vertex of either angle to the opposite side, or the opposite side produced.



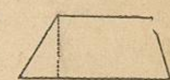
The vertex of the angle from which the altitude is drawn is called the *vertex of the Triangle;* the opposite side is called the *base* of the triangle.

4. The ALTITUDE OF A PARALLELOGRAM is the perpendicular distance between two opposite sides.



These opposite sides are called *bases,* one is the *upper base,* the other the *lower base.*

5. The ALTITUDE OF A TRAPEZOID is the perpendicular distance between its parallel sides.



These sides are called bases; one is called the *upper base,* the other the *lower base.*

6. SIMILAR POLYGONS are those which are mutually equiangular, and in which the corresponding sides are proportional.