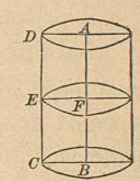
BOOK VII.

THE CYLINDER, THE CONE, AND THE SPHERE.

1. A CYLINDER is a volume which may be generated by the revolution of a rectangle about one of its sides as an axis.

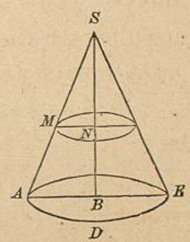
Thus, if the rectangle $ABCD$ be revolved around the side AB as an axis, it will generate the cylinder in the margin. The line AB is called the *axis;* the surface described by CD is called the *convex surface;* the circle BC is the *lower base;* the circle AD is the *upper base.*



It is evident that the circle described by the line EF perpendicular to the axis is equal to either base; hence, if a cylinder be cut by a plane parallel to the base, the section will be a circle equal to the base.

2. A CONE is a volume which may be generated by the revolution of a right-angled triangle about one of its sides adjacent to the right angle.

Thus, if the right-angled triangle SBA be revolved around SB as an axis, it will generate the cone $ADE—S$. The side SB is the *axis* of the cone; the circle described by AB is the base; the hypothenuse SA is the *slant height;* the surface generated by SA is the *convex surface.*

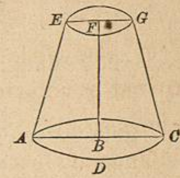


It is evident that the circle described by any line MN perpendicular to the

axis is a circle; hence, the section of a cone by a plane parallel to the base is a circle.

3. A FRUSTUM OF A CONE is the part which remains after cutting off the top with a plane parallel to the base.

Thus, $ADC—G$ is the frustum of a cone; FB is its *altitude; EA* is its *slant height.* The frustum of a cone may be generated by the revolution of the trapezoid $ABFE$.

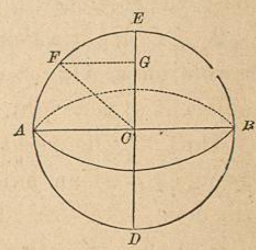


4. SIMILAR CYLINDERS or CONES are those whose axes are proportional to the radii or the diameters of their bases.

5. A prism may be *inscribed in a cylinder* by inscribing similar polygons with their sides parallel in each base, and uniting the vertices of the angles with straight lines. The cylinder is then said to circumscribe the prism.

6. A pyramid may be *inscribed in a cone;* and a frustum of a pyramid may be inscribed in the frustum of a cone.

7. A SPHERE is a volume bounded by a curved surface, every point of which is equally distant from a point within, called the *centre.*



The distance from the centre to the circumference is called the *radius.* The *diameter* is a line passing through the centre and limited at both extremities by the surface.

8. A SPHERICAL SECTOR is a volume generated by the revolution of a sector of a circle about the diameter. Thus, the revolution of ACF will generate a spherical sector.

9. A ZONE is a portion of the surface of a sphere in-

cluded between two parallel planes. The bounding lines of the zone are called its *bases;* the distance between the planes is its *altitude.*

10. A SPHERICAL SEGMENT is a portion of the sphere included between two parallel planes.

11. If a semi-circumference be divided into equal arcs, the chords of these arcs form half of the perimeter of an inscribed polygon. The half perimeter is called a *regular semi-perimeter.*

The figure bounded by the diameter of the semi-circle and the regular semi-perimeter is called a *regular semi-polygon.* The diameter is called the *axis* of the semi-polygon.

12. The CYLINDER, the CONE and the SPHERE are the THREE ROUND BODIES of Geometry.

ANALYSIS.—This book treats of the *cylinder*, the *cone*, and the *sphere*. Its object is to find the *convex surface* and *volume* of each of these bodies, and also their *relation* to each other.

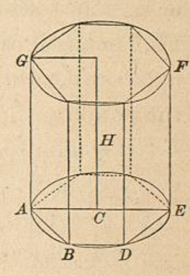
The method of treatment consists in regarding these volumes as polyedrons of an infinite number of sides. Thus, the cylinder is regarded as a right prism of an infinite number of sides, the cone as a right pyramid, and the sphere as a polyedron having its centre at the centre of the sphere.

CYLINDER, CONE, AND FRUSTUM.

THEOREM I.

The convex surface of a cylinder is equal to the circumference of its base multiplied by its altitude.

Let $ABDE$ be the base of a cylinder whose altitude is H; then will its convex surface be equal to circumference $CA \times H$.



For, inscribe in the cylinder a prism whose base is a regular polygon. Now, the convex surface of this prism will be equal to the perimeter of its base multiplied by its altitude (B. VI. Th. I.); and this is true whatever the number of sides; hence, it is true when the number of sides is infinite. But when the number of sides is infinite, the convex surface of the prism becomes the convex surface of the cylinder, the perimeter of the base of the prism becomes the circumference of the base of the cylinder, and the altitudes being the same, therefore, the convex surface of the cylinder equals the circumference of its base multiplied by its altitude.

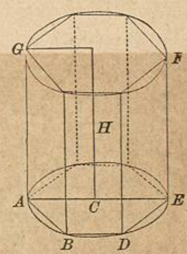
Cor. 1. Since the circumference of the base is $2\pi R$, the expression for the convex surface of a cylinder is $2\pi R \times H$.

Cor. 2. The convex surfaces of cylinders which have equal altitudes are to each other as the circumferences of their bases.

THEOREM II.

The volume of a cylinder is equal to the area of its base multiplied by the altitude.

Let $ABD—F$ be a cylinder, whose altitude is H; then will its volume be equal to the area of its base multiplied by its altitude.



For, inscribe in the cylinder a prism whose base is a regular polygon. Now, the volume of this prism is equal to its base multiplied by its altitude (B. VI. Th. IX.), and this is true whatever the number of sides, and therefore true when the number of sides is infinite. But when the number of sides is infinite, the prism coincides with the cylinder in every respect; hence, the volume of the cylinder is equal to its base multiplied by its altitude. Therefore, etc.

Cor. 1. Since the area of the base is πR^2, the expression for the volume of a cylinder is $\pi R^2 \times H$.

Cor. 2. Cylinders are to each other as the products of their bases and altitudes. Cylinders having equal bases are to each other as their altitudes; cylinders having equal altitudes are to each other as their bases.

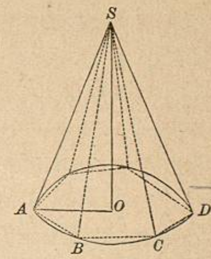
Cor. 3. Similar cylinders are to each other as the cubes of their altitudes, or of the radii of the bases. Let the pupil prove it.

THEOREM III.

The convex surface of a cone is equal to the circumference of its base multiplied by one-half of the slant height.

Let $S—ABCD$ be a cone whose base is ABD and slant height SA; then will its convex surface be equal to the

circumference of its base multiplied by one-half of its slant height.



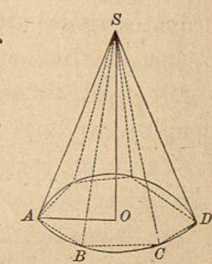
For, inscribe in the cone a right pyramid. The convex surface of this pyramid is equal to the perimeter of its base multiplied by one-half of the slant height (B. VI. Th. XII.); and this is true whatever the number of sides of the base; hence, it is true when the number of sides is infinite. But when the number of sides is infinite, the pyramid coincides with the cone in every respect; hence, the convex surface of the cone is equal to the circumference of its base multiplied by one-half of the slant height.

Cor. 1. If S represents the slant height, the expression for the convex surface of a cone is $2\pi R \times \frac{1}{2}S$, or $\pi R \times S$.

THEOREM IV.

The volume of a cone is equal to the base multiplied by one-third of the altitude.

Let $S—ABCD$ be a cone whose base is $ABCD$ and altitude SO; then will its volume be equal to its base multiplied by one-third of its altitude.



For, inscribe in the cone a right pyramid. The volume of this pyramid is equal to the base $ABCD$ multiplied by one-third of its altitude SO (B. VI. Th. XVII.); and this is true whatever the number of sides of the base; hence, it is true when the number of sides is infinite. But when the number of sides of the base is infinite, the pyramid becomes the cone; hence, the volume of a cone is equal to its base multiplied by one-third of its altitude. . Therefore, etc.

Cor. 1. The expression for the volume of a cone is $\pi R^2 \times \frac{1}{3} H$, or, $\frac{1}{3} \pi R^2 \times H$.

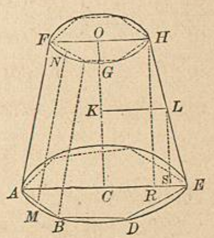
Cor. 2. A cone is one-third of a cylinder having an equal base and altitude.

Cor. 3. Cones are to each other as the products of their bases and altitudes; cones having equal bases are to each other as their altitudes; cones having equal altitudes are to each other as their bases.

THEOREM V.

The convex surface of a frustum of a cone is equal to one-half of the sum of the circumferences of the upper and lower bases multiplied by the slant height.

Let $ABDE$—H be a frustum of a cone, OC its altitude, FA its slant height; then will its convex surface be equal to one-half the sum of the circumferences of its two bases multiplied by its slant height.



For, inscribe within the frustum of a cone the frustum of a right pyramid. The convex surface of this frustum is equal to one-half the sum of the perimeters of its bases multiplied by the slant height (B. VI. Th. XIX.); and this is true whatever the number of lateral faces; hence, it is true when the number of faces is infinite. But when the number of faces is infinite, the frustum of a pyramid becomes the frustum of a cone, the perimeters of its bases become the circumferences of the bases of the

frustum of the cone, and the slant height of the frustum of a pyramid becomes the slant height of the frustum of a cone; hence, the convex surface of the frustum of a cone equals one-half the sum of the circumferences of its bases multiplied by the slant height.

Cor. The expression for the convex surface of a frustum of a cone is $\frac{1}{2}(2\pi R + 2\pi R') \times S$, where R and R' represent the radii of the bases, and S the slant height.

Scholium. Through L, the middle point of HE, draw LK parallel to EC, and HR and LS perpendicular to EC; now $RS = SE$ (Bk. III. Th. IX.); $OH = CR$, and $KL = CR + RS$ (Bk. I. Th. XV. C. 2). But $CR + RS = SE + OH$; hence,

$$KL = \tfrac{1}{2}(CR + RS + SE + OH) \text{ or } \tfrac{1}{2}(CE + OH).$$

Multiplying this by 2π, we have,

$$2\pi KL = \tfrac{1}{2}(2\pi CE + 2\pi OH);$$

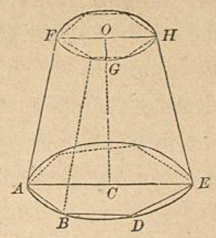
that is, *circ. KL* equals $\frac{1}{2}$ of the sum of the circumferences of the two bases; hence, *the convex surface of the frustum of a cone, generated by the revolution of the line HE, is equal to the circumference of a circle generated by its middle point into the length of the line.*

THEOREM VI.

The volume of the frustum of a cone is equal to the sum of the volume of three cones, having for a common altitude the altitude of the frustum, and for bases the two bases of the frustum and a mean proportional between them.

Let $ABDE$—H be a frustum of a cone, OC its altitude; then will its volume be equal to the sum of the volumes of three cones whose common altitude is OC, and whose bases are the two bases and a mean proportional between them.

For, inscribe in the frustum the frustum of a right pyramid. The volume of this frustum is equal to the sum of the volumes of three pyramids having the common altitude of the frustum, and whose bases are the two bases of the frustum and a mean proportional between them (B. VI. Th. XX.); and this is true whatever the number of lateral faces, and, hence, true when the number of faces is infinite. But when the number of lateral faces is infinite, the frustum of the pyramid becomes the frustum of a cone, and the three pyramids become cones; hence, the volume of the frustum of a cone equals the sum of the volumes of three cones, whose common altitude is the altitude of the frustum, and whose bases are the two bases of the frustum and a mean proportional between them.



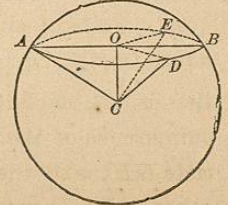
Cor. The expression for the volume of a frustum of a cone is $(\pi R^2 + \pi r^2 + \pi R \times r) \times \frac{1}{3} H$.

THE SPHERE.

THEOREM VII.

Every section of a sphere made by a plane is a circle.

Let C be the centre of a sphere whose radius is CA, and ADB any section made by a plane; then will this section be a circle.



For, draw CO perpendicular to the section ADB, and draw the lines OD and OE to different points of the

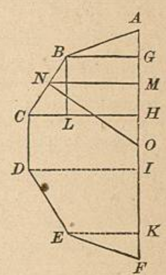
curve ADB; draw also the radii CD and CE. Then, since the radii CD and CE are equal, the lines OD and OE must be equal (B. V. Th. III. C. 1); hence, the section ADB is a circle. Therefore, etc.

Cor. If the plane pass through the centre of the sphere, the radius of the section will be equal to the radius of the sphere. The section is then called a *great circle*. All other sections are called *small circles*.

THEOREM VIII.

If a regular semi-polygon be revolved about its axis, the surface generated by the semi-perimeter will be equal to the circumference of the inscribed circle multiplied by the axis.

Let $ABCDEF$ be a regular semi-polygon, AF the axis, ON the radius of the inscribed circle; then will the surface generated by the revolution of the semi-polygon be equal to *circ.* $ON \times AF$.



For, from the extremities of any side, as BC, draw BG and CH perpendicular to AF; from N, the middle point of BC, draw NM perpendicular to AF; draw also BL perpendicular to CH. Now, the surface described by BC is equal to *circ.* $MN \times BC$ (Th. V. S.). But, since the triangles BCL and NOM are similar, we have,

$$BC : BL \text{ or } GH :: ON : NM :: circ.\ ON : circ.\ NM;$$

hence, $$circ.\ NM \times BC = circ.\ ON \times GH;$$

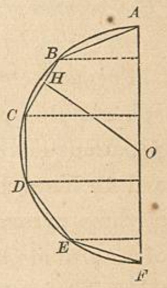
that is, the surface generated by BC is equal to the circumference of the inscribed circle multiplied by the altitude GH; and the same may be shown for each of the other sides; hence, the surface described by the entire semi-perimeter is equal to the circumference of the inscribed circle multiplied by the sum of AG, GH, HI, etc., or the axis AF. Therefore, etc.

Cor. The surface described by any portion of the perimeter, as BCD, is equal to *circ.* $ON \times GI$.

THEOREM IX.

The surface of a sphere is equal to the circumference of a great circle multiplied by the diameter.

Let $ABCDEF$ be a semicircle, O its centre, and AF its diameter; then will the surface of the sphere generated by the revolution of the semi-circumference about the diameter be equal to *circ.* $OA \times AF$.



For, inscribe in the semi-circumference a regular semi-polygon. The surface described by the revolution of the polygon is equal to *circ.* $OH \times AF$ (Th. VIII.); and this is true whatever the number of sides; hence, it is true when the number of sides is infinite, in which case the volume becomes a sphere with the radius OA; hence, the surface of a sphere is equal to *circ.* $OA \times AF$. Therefore, etc.

Cor. 1. *The surface of a sphere is equal to four of its great circles.* For, *sur.* $=$ *circ.* $OA \times 2\,OA$; but *circ.* $OA = 2\pi OA$; hence, *sur.* $= 2\pi OA \times 2\,OA$; which gives *sur.* $= 4\pi OA^2$; but πOA^2 is the area of a great circle; hence, $4\pi OA^2$ is the area of *four* great circles.

Cor. 2. The expression for the surface of a sphere is $4\pi R^2$, or πD^2, in which R is the radius and D the diameter.

Cor. 3. The surfaces of spheres are to each other as the

squares of their radii or diameters. For, *sur.* $S = 4\pi R^2$, and *sur.* $s = 4\pi r^2$; hence,

$$S : s :: 4\pi R^2 : 4\pi r^2, \text{ or } R^2 : r^2.$$

Cor. 4. The surface of a zone is equal to the circumference of a great circle multiplied by its altitude.

Cor. 5. Zones on the same sphere, or on equal spheres, are to each other as their altitudes. A zone is to the surface of a sphere as the altitude of the zone is to the diameter of the sphere.

THEOREM X.

The volume of a sphere is equal to its surface multiplied by one-third of its radius.

For, conceive a regular polyedron to be inscribed in a sphere; this polyedron may be conceived as consisting of pyramids having their vertices at the centre of the sphere, and for bases the faces of the polyedron. The volume of each of these pyramids is equal to its base multiplied by one-third of its altitude, and, their altitudes being equal, the volume of the polyedron will be equal to the sum of all their bases, which is the surface of the polyedron, multiplied by one-third of the common altitude. Now, the sphere may be regarded as a polyedron consisting of an infinite number of pyramids, having their vertices at the centre of the sphere and their bases at its surface, their altitudes being equal to the radius of the sphere; hence, the volume of a sphere is equal to its surface multiplied by one-third of the radius.

Cor. 1. If we represent the volume of a sphere by *vol.* S, and the surface by *sur.* S, we will have,

vol. $S =$ *sur.* $S \times \frac{1}{3}R$; and since *sur.* $S = 4\pi R^2$,

we have, *vol.* $S = 4\pi R^2 \times \frac{1}{3}R$; which, reduced, gives, (1) $vol.\ S = \frac{4}{3}\pi R^3$.

But, $R = \frac{1}{2}D$, or $R^3 = \frac{1}{8}D^3$,

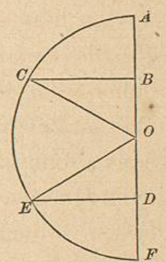
Hence, (2) $vol.\ S = \frac{1}{6}\pi D^3$.

Cor. 2. Spheres are to each other as the cubes of their radii, or diameters.

Cor. 3. The volume of a spherical sector or pyramid is equal to its base multiplied by one-third of the radius.

For the sector or pyramid may be conceived as consisting of an infinite number of pyramids having their vertices at the centre of the sphere, and the volume of the sum of these will be the sum of their bases multiplied by one-third of the radius.

Cor. 4. The volume of a spherical segment of one base and less than a hemisphere, as that generated by *ACB* revolving about *AF*, is equal to the volume of the spherical sector *AOC* minus the volume of the cone formed by *OCB*.



The volume of a spherical segment of one base and greater than a hemisphere, as *AED*, is equal to the volume of the spherical sector *AOE* plus the volume of the cone formed by *EDO*.

The volume of a spherical segment of two bases, as that generated by *BCED*, is equal to the volume of the sector, formed by *COE*, plus the volume of the cones formed by *OCB* and *OED*. If the points *C* and *E* fall on the same side of the centre, the last cone must be subtracted. The measure is as follows:

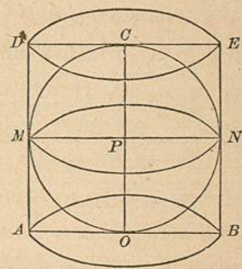
$$Segment\ BCED = zone\ CE \times \tfrac{1}{3}OC + \pi\overline{BC}^2 \times \tfrac{1}{3}OB + \pi\overline{DE}^2 \times \tfrac{1}{3}OD.$$

THEOREM XI.

The surface of a sphere is to the entire surface of the circumscribed cylinder, including its bases, as 2 *to* 3; *and their volumes are to each other in the same ratio.*

Let *AE* be a cylinder circumscribed about a sphere whose centre is *P*; then,

First. The surface of the sphere is to the entire surface of the cylinder as 2 is to 3.



For, the surface of the cylinder equals *circumference* $AO \times OC$ (Th. I.); that is, the circumference of a great circle of the sphere multiplied by the diameter of the sphere; but this is equal to the surface of a sphere (Th. IX.); hence, the surface of the cylinder equals the surface of the sphere; but the surface of the sphere equals four great circles; hence, the convex surface of the cylinder equals four great circles, and adding the two bases, we have the entire surface of the cylinder equal to six great circles; hence, the surface of the sphere is to the surface of the cylinder as 4 great circles is to 6 great circles, or as 4 to 6, or 2 to 3.

Second. The volume of the sphere is to the volume of the cylinder as 2 is to 3.

For, the volume of the sphere is $\frac{4}{3}\pi R^3$ (Th. X. C. I.), and the volume of the cylinder is $\pi R^2 \times CO$ (Th. II.), or $\pi R^2 \times 2R = \frac{6}{3}\pi R^3$; hence,

$$vol.\ S. : vol.\ cyl. :: \tfrac{4}{3}\pi R^3 : \tfrac{6}{3}\pi R^3, \text{ or},$$

$$:: 4 : 6, \text{ or } 2 : 3.$$

Therefore, etc.

PRACTICAL EXAMPLES.

1. Required the convex surface and contents of a cylinder whose altitude is 16 inches, and diameter of the base 8 inches.
Ans. 402.12; 804.25.

2. Required the convex surface and volume of a cone whose altitude is 24 inches, and radius of the base 10 inches.
Ans. 816.816; 2513.28.

3. Required the convex surface of the frustum of a cone whose altitude is 36 inches, the radius of the upper base 6 inches, and lower base 21 inches. *Ans.* 3308.1048.

4. Required the volume of a frustum of a cone whose altitude is 9 feet, diameter of lower base 4 feet, and of upper base 2 feet.
Ans. 65.9736.

5. Required the surface and contents of a sphere whose diameter is 16 inches. *Ans.* 804.2496; 2144.6656.

6. The surface of a sphere is 1809.5616 square inches; required its diameter and its volume. *Ans.* D. = 24 inches.

7. The volume of a sphere is 113.0976 cubic inches; required its diameter and its surface. *Ans.* D. = 6 inches.

8. Given the volume of a sphere 268.0832 cubic inches; required the altitude of the circumscribing cylinder. *Ans.* 8 inches.

9. What is the surface of a zone of a single base whose altitude is 10 feet, the diameter of the sphere being 100 feet?
Ans. 3141.6 sq. ft.

10. Required the volume of a spherical segment of one base whose altitude is 2 feet, the diameter of the sphere being 8 feet.
Ans. 41.888 cubic feet.

11. Required the volume of a spherical segment whose greater diameter is 24 inches, less diameter 20 inches, and distance of bases 4 inches. *Ans.* 1566.6112 cubic inches.

THEOREMS FOR ORIGINAL THOUGHT.

1. Prove that two great circles of a sphere bisect each other.
2. Prove that every great circle divides the sphere into two equal parts.

3. Prove that the centres of a small circle and the sphere are in a line perpendicular to the small circle.
4. Prove that the radius of a small circle is less than the radius of the sphere.
5. Prove that circles whose planes are equidistant from the centre are equal.
6. Prove that the intersection of two spheres is a circle.
7. Prove that the arc of a great circle may be made to pass through any two points on the surface of a sphere.
8. Prove that if a cone and sphere be inscribed in a cylinder, that these bodies are to each other as 1, 2, and 3.

MISCELLANEOUS PROBLEMS.—PLANE FIGURES.

1. How many bricks 8 inches long and 4 inches wide will it take to pave a yard 20 feet by 16 feet? *Ans.* 1440.

2. How much will it cost to plaster a room whose length is 24 ft., width 18 ft., and height 12 ft., at 16 cts. a square yard? *Ans.* $25.60.

3. What is the difference in area between a rectangle 60 feet by 40 feet, and a square which has the same perimeter? *Ans.* 100 sq. ft.

4. What is the diagonal of a square whose area is equal to the area of a rectangle 16 inches by 25 inches? *Ans.* 28.28 inches.

5. The diagonal of a square is $\sqrt{50}$ inches; required the side of the square. *Ans.* 5 inches.

6. Required the diagonal of a room whose length is 48 feet, width 20 feet, and height 39 feet. *Ans.* 65 feet.

7. A vessel sailed north 20 miles, then west 30 miles, then north 60 miles, then west 70 miles; how far was it then from the point at which it started? *Ans.* 128.06 miles.

8. The gable ends of a house are 48 ft. wide, and the ridge-pole is 10 ft. above the eaves; required the length of the rafters. *Ans.* 26 ft.

9. Required the area of an isosceles triangle whose base is 20 feet, and each of its equal sides 15 feet. *Ans.* 111.803 square feet.

10. A flag-staff was broken, and fell, the broken part resting upon the upright, so that the end struck 48 feet from the foot; the upright part measured 36 feet; how long was the staff? *Ans.* 96 feet.

11. I wish to enclose a square rod in the form of an equilateral triangle; what must be the length of each side? *Ans.* 25.076.

12. Given the area of a circle 19.635 square inches; required the diameter and circumference. *Ans.* D. = 5 inches.

13. The equal sides of an equilateral triangle are each 16 feet; what is the side of the inscribed square? *Ans.* 7.425.

14. I have a plank 12 feet long which contains 15 square feet; what is the width of each end, if they are as 2 to 3? *Ans.* 12 in.; 18 in.

15. If the minute-hand of a clock is 6 inches long, over how much space does it pass in 40 minutes? *Ans.* 75.398 square inches.

16. What is the circumference of a circle whose diameter equals the diagonal of a square which contains 25 sq. rds.? *Ans.* 22.211112.

17. What is the diameter of a wheel which makes 200 revolutions in a minute, when the cars are going 30 miles an hour? *Ans.* $4\frac{1}{5}$ + feet.

18. A horse is fastened in a meadow, by a halter 20 feet long, to the top of a post 6 feet high; what is the area of the circle over which he can graze? *Ans.* 127.06 square yards.

19. Required the area of a circle in which the number expressing its area equals the number expressing its circumference.

Ans. 12.5664.

20. The area of a circular park is 4 acres; how long will it take to drive round it at the rate of 6 miles an hour? *Ans.* 2 min. 48 sec.

21. A circular garden containing 2 acres is bordered by a gravel walk of uniform width, which takes up $\frac{1}{4}$ of its area; required the width of the walk. *Ans.* 22.308 feet.

22. If the hour-hand of a clock is 4 inches long, and the minute-hand 6 inches, what is the difference of the surfaces over which they travel in an hour? *Ans.* 108.91 square inches.

MISCELLANEOUS PROBLEMS.—VOLUMES.

1. Required the surface of a brick 8 inches long, 4 inches wide, and 2 inches thick. *Ans.* 112 square inches.

2. Required the entire surface of a right pyramid whose base is a square 4 in. long, and the slant height 12 inches. *Ans.* 112 sq. in.

3. Required the entire surface of a cylinder whose altitude is 16 in., the radius of the base being 6 inches. *Ans.* 829.3824 sq. in.

4. Required the entire surface of a cone whose height is 16 feet, the radius of the base being 12 feet. *Ans.* 1206.3744 sq. ft.

5. Required the surface and contents of a sphere inscribed in a cube whose edge is 20 inches; and also the space between them.

6. The surface of a sphere is 6.305 square feet; required its diameter and volume. *Ans.* Vol. 1.48868 cubic feet.

*7. The volume of a sphere is 1.2411 cubic feet; required the diameter and surface. *Ans.* D. 16 inches.

8. The convex surface of a cylinder whose altitude is 14 feet is 116.666 square feet; required the diameter of its base. *Ans.* 2.65 ft.

9. What is the volume of a cylinder whose height is 20 feet, and the circumference of the base is 20 feet also? *Ans.* 636.64 feet.

10. The volume of a cylinder is 15.708 cubic feet; what is the altitude, if the diameter of the base is 2 feet? *Ans.* 5 feet.

11. The convex surface of a cone is 141.372 square feet, and the diameter of the base 4.5 feet; required the slant height and altitude.

Ans. 20 feet.

12. If a segment of 6 feet slant height be cut off of a cone whose slant height is 30 feet, the circumference of the base being 10 feet, what is the surface of the frustum? *Ans.* 144 square feet.

13. The convex surface of a frustum is 376.992 square feet, the slant height 20 feet, and the diameter of the less end 4 feet; what is the diameter of the greater end? *Ans.* 8 feet.

14. The volume of a cone is 8.83575 cubic feet, the altitude 15 feet; what is the diameter of the base? *Ans.* 18 inches.

15. The volume of a frustum of a cone is 65.9736 cubic feet, the diameter of one end is 4 feet, and of the other 2 feet; required the altitude. *Ans.* 9 feet.

16. Required the entire surface of the frustum of a cone whose height is 12 feet, the radius of the lower base being 9 feet and the upper base 4 feet. *Ans.* 266 π.

17. Required the entire surface of the frustum of a pyramid whose bases are squares, the lower 9 feet, the upper 4 feet, on a side, the altitude being 12 feet. *Ans.* 415.68 sq. ft.

18. How far must a person ascend above the earth that he may see one-third of the surface? *Ans.* 2 times the radius.