5. What is the area of a bi-rectangular triangle whose vertical angle is 108°? *Ans.* $\frac{3}{5}\pi R^2$.

6. Find the area of a spherical triangle whose angles are 60°, 90°, and 120°, the diameter of the sphere being 8. *Ans.* 8π.

7. Find the area of a lune, its angle being 45°. *Ans.* $\frac{1}{2}\pi R^2$.

8. Find the area of a lune, the angle being 54° and radius of the sphere being 5. *Ans.* 15π.

9. Find the volume of a spherical wedge, the angle of the lune being 72°. *Ans.* $\frac{4}{15}\pi R^3$.

10. Find the volume of a spherical wedge, the angle of the lune being 36° and the diameter of the sphere 10. *Ans.* $16\frac{2}{3}\pi$.

11. Find the angles of an equilateral spherical triangle whose area is equal to the surface of a great circle. *Ans.* 120°.

12. What must be the angles of an equilateral spherical triangle that its area may be equal to an equilateral spherical hexagon, each of whose angles is 130°? *Ans.* 80°.

THEOREMS FOR ORIGINAL THOUGHT.

1. Prove that in an isosceles spherical triangle the angles opposite the equal sides are equal.

2. Prove that a spherical triangle having two equal angles is isosceles.

3. In any spherical triangle the greater side is opposite the greater angle, and conversely.

4. If from any point of a hemisphere two arcs of great circles are drawn perpendicular to its circumference, the shorter of the two arcs is the shortest arc that can be drawn from the given point to the circumference.

5. Two oblique arcs drawn from the same point to points of the circumference at equal distances from the foot of the perpendicular are equal.

6. Of two oblique arcs, that which meets the circumference at the greatest distance from the foot of the perpendicular is the longer.

7. Prove that the area of a spherical triangle, each of whose angles is $\frac{4}{3}$ of a right angle, is equal to the surface of a great circle.

MENSURATION.

MENSURATION OF LENGTHS AND SURFACES.

1. MENSURATION is the science which treats of the measurement of geometrical magnitudes.

2. The AREA of a figure is its quantity of surface; it is expressed by the number of times which it contains the unit of measure.

3. This *Unit of Measure* is a square whose side is some known length; as, an inch, a foot, etc.

4. The unit of surface has generally the same name as the linear unit; thus, if the linear unit is *one foot*, the surface unit is one *square foot*, etc.

5. Some superficial units have no corresponding linear unit of the same name; as, the *rood* and *acre*.

6. To refresh the memory, we give a few of the more important measures of surfaces.

1 rood = 40 perches, or square rods.
1 acre = 4 roods.
1 square mile = 640 acres.

Also,

1 chain = 100 links = 4 rods.
10 chains = 1 furlong.
1 square chain = 100 × 100 = 10,000 square links.
1 acre = 10 square chains = 100,000 square links.

THE TRIANGLE.

7. The AREA is found by the following rules:

RULE 1.—*Multiply the base by one-half of the altitude;* or,

RULE 2.—*Take half the sum of the sides, subtract from it each side separately, multiply the half sum and these remainders together, and take the square root of the product.*

1. What is the area of a triangular field whose base is 40 rods and altitude 16 rods? *Ans.* 2 acres.

2. Required the area of a triangle whose sides are 20, 30, and 40 chains respectively. *Ans.* 29 A. 8 P.

3. A man has a triangular garden whose sides are 150, 200, and 250 feet respectively; required the area.
Ans. 1666.66 yards.

THE QUADRILATERAL.

8. PARALLELOGRAM.—The AREA is found as follows:

RULE.—*Multiply the base by the altitude.*

1. What is the area of a parallelogram 9 feet long and 7 feet wide? *Ans.* 63 square feet.

2. How many acres in a square field whose side is $70\frac{1}{2}$ chains? *Ans.* 497 A. 4 P.

3. A man has a lot in the form of a rhombus, whose length is 333 feet and altitude 33.35 feet; required its area. *Ans.* 1233.95 square yards.

9. TRAPEZOID.—The AREA is found as follows:

RULE.—*Multiply one-half of the sum of the parallel sides by the altitude.*

1. Required the area of a trapezoid, one side being 192 inches and the other 96 inches, and altitude 12 feet.
Ans. 144 square feet.

2. What is the area of a plank 24 feet long, 18 inches wide at one end and 12 inches at the other?
Ans. 30 square feet.

3. A farmer has a field in the form of a trapezoid, whose parallel sides are 95 and 75 rods respectively, and the perpendicular distance between them 65 rods; how much land in the field? *Ans.* 34 A. 2 R. 5 P.

10. TRAPEZIUM.—The AREA is found as follows:

RULE.—*Divide the trapezium into two triangles by a diagonal, find the area of each triangle, and take their sum.*

1. What is the area of a trapezium whose diagonal is 290 inches, and the altitudes of the triangles, the diagonal being the base, are 60 and 80 inches respectively?
Ans. 140 square feet, 140 square inches.

2. Required the area of a trapezium the lengths of whose sides are respectively 40, 60, 50, and 70 chains, and the diagonal 80 chains. *Ans.* 289 A. 1 R. 24 P.

POLYGONS OF ANY NUMBER OF SIDES.

11. REGULAR POLYGONS.—The AREA is found as follows:

RULE.—*Multiply half the perimeter by the perpendicular let fall from the centre on one of the sides.*

1. What is the area of a regular hexagon whose side is 14.6 feet and perpendicular 12.64 feet?
Ans. 61.5147 square yards.

2. Required the area of an octagon whose sides are 9.941 feet and its perpendicular 12 feet.
Ans. 477.168 square feet.

12. The following table shows the areas of ten regular polygons when the side is 1:—

Triangle	0.4330127	Octagon	4.8284271
Square	1.0000000	Nonagon	6.1818242
Pentagon	1.7204774	Decagon	7.6942088
Hexagon	2.5980762	Undecagon	9.3656404
Heptagon	3.6339124	Dodecagon	11.1961524

Now, since the areas of similar polygons are to each other as the squares of their homologous sides, to find the area of a regular polygon we have the following

RULE.—*Square the side of the polygon, and multiply by the tabular area set opposite the polygon.*

3. What is the area of a regular hexagon whose side is 5 inches long? *Ans.* 64.9519 square inches.

4. Required the area of an octagon whose sides are each 3 feet 4 inches. *Ans.* 53.649 square feet.

13. IRREGULAR POLYGON.—The AREA is found as follows:

RULE.—*Draw diagonals dividing the polygon into triangles, find the area of these triangles, and take the sum.*

1. In the irregular pentagon $ABCDE$, the diagonal AC is 24 inches, the diagonal AD is 18 inches, the altitude of the triangle ABC is 8 inches, of ACD is 10 inches, and of AED 6 inches; required the area.

Ans. 240 square feet.

2. In the irregular hexagon $ABCDEF$, the side AB is 268, BC 249, CD 310, DE 290, EF 199, and AF 246 links, and the diagonals AC 459, CE 524, and AE 326 links; required the area. *Ans.* 1 A. 2 R. 22 P. 13 yd. 47 ft.

THE CIRCLE.

14. The CIRCUMFERENCE is found by the following

RULE.—*Multiply the diameter by* 3.1416.

NOTE.—Hence, the diameter equals the circumference divided by 3.1416, or multiplied by .31831.

1. What is the circumference of a circle whose diameter is 50 inches? *Ans.* 157.08 inches.

2. A man has a circular fish-pond 32 rods in diameter; what is the distance around it? *Ans.* 100.5312 rods.

3. Required the diameter of a water-wheel whose circumference is 78.54 feet. *Ans.* 25 feet.

4. A man has a garden in the form of a circle, the diameter being 45 rods; what is the distance around it?

Ans. 141.372 rods.

15. The LENGTH OF AN ARC, when its degrees and radius are given, is found as follows:

RULE.—*Multiply the number of degrees by the decimal* .01745, *and the product by the radius.*

1. The degrees in an arc are 45, and the radius 10; what is the length of the arc? *Ans.* 7.852.

2. What is the length of an arc of 32° 38′ 42″, the radius being 25 inches? *Ans.* 14.2414 inches.

16. When the *chord* and *chord of the half* arc are given.

RULE.—*From* 8 *times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by* 3.

1. The chord of an arc is 96 inches, and the chord of half the arc is 60 inches; what is the length of the arc?

Ans. 128 inches.

2. The chord of an arc is 16 inches, and the diameter of the circle is 20 inches; what is the length of the arc? *Ans.* 18.5178 inches.

17. The AREA OF A CIRCLE is found as follows:

RULE I.—*Multiply the circumference by one-fourth of the diameter, or the square of the radius by* 3.1416.

RULE II.—*Multiply the square of the diameter by* .7854, *or the square of the circumference by* .07958.

(Let the pupil prove the last rule from the previous principles.)

1. What is the area of a circle whose diameter is 50 inches and circumference 157.08 inches?

Ans. $1963\frac{1}{2}$ square inches.

2. Required the area of a circle whose diameter is 18 inches. *Ans.* 254.4696 square inches.

3. What is the area of a circular garden whose circumference is 90 rods? *Ans.* 644.598 square rods.

18. The AREA OF A SECTOR is found as follows:

RULE.—I. *Multiply the arc by one-half the radius;* or,

II. *The sector is to the circle as the number of degrees in the sector is to* 360°.

1. What is the area of a circular sector whose arc contains 18°, the diameter of the circle being 6 feet?

Ans. 1.4137 square feet.

2. Required the area of a sector, the chord of half the arc being 30 inches, and the radius 50 inches.

Ans. 1523.45 square inches.

19. The AREA OF A SEGMENT is found as follows:

RULE.—*Find the area of the sector having the same arc, and also the area of the triangle formed by the chord of the segment and the radii of the sector.*

If the segment is greater than a semicircle, add the two areas; if less, subtract them.

1. Required the area of a segment whose height is 2 inches, and chord 20 inches. *Ans.* 26.864 square inches.

2. What is the area of a segment whose height is 18 inches, the diameter of the circle being 50 inches?

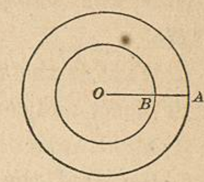
Ans. 632 sq. in.

3. Required the area of a segment whose arc is 180°, and radius of circle 12 feet. *Ans.* 226.1952

20. The AREA OF A CIRCULAR RING is found as follows:

RULE.—*Find the difference of the squares of the radii, and multiply it by* 3.1416.

DEMONSTRATION.—Let the figure represent two circles having a common centre O; then the difference between them will be a circular ring. The area of circle OA is πOA^2, and of OB is πOB^2; the difference is $\pi OA^2 - \pi OB^2 = \pi(OA^2 - OB^2)$, which proves the rule.



1. What is the area of the circular ring when the diameters are 20 and 30?

Ans. 392.70.

2. A circular park 400 feet in diameter has a carriage-way around it 24 feet wide; required the area of the carriage-way. *Ans.* 3149.9776 square yards.

21. The SIDE OF AN INSCRIBED SQUARE is found thus:

RULE.—*Multiply the diameter by* .7071, *or multiply the circumference by* .2251.

1. What is the side of a square that can be cut out of a circular board whose diameter is 14 inches?

Ans. 9.899 inches.

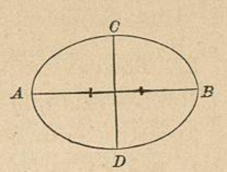
2. How large a square can be cut out of a circular board whose circumference is 400 inches?

Ans. 90.04 inches.

THE ELLIPSE.

22. An ELLIPSE is a plane figure bounded by a curve, the sum of the distances from every point of which to two fixed points is equal to the line drawn through those points and terminated by the curve.

The two points are called *foci;* the line through the foci is the *transverse axis;* a line perpendicular to this through the centre is the *conjugate axis.*



23. The AREA is found by the following

RULE.—*Multiply half of the two axes together, and multiply that product by 3.1416.*

1. What is the area of an ellipse whose transverse axis is 20 inches and conjugate axis 16 inches?

Ans. 251.328.

2. Required the area of an elliptical mirror whose length is 6 feet and breadth 5 feet.

Ans. 23.562 square feet.

MENSURATION OF VOLUMES.

24. MENSURATION OF VOLUMES is the process of determining their surface and contents.

25. The CONTENTS of a volume is the number of times it contains a given *unit* of measure.

26. The UNIT OF MEASURE of a volume is a small cube whose dimensions are known.

MEASURES OF VOLUMES.

1 cubic foot	= 1728	cubic inches.
1 " yard	= 27	" feet.
1 " rod	= $4492\frac{1}{8}$	" feet.
1 wine gallon	= 231	" inches.
1 ale gallon	= 282	" inches.
1 bushel	= 2150.42	" inches.
1 cord	= 128	" feet.

THE PRISM.

27. The CONVEX SURFACE OF A RIGHT PRISM is found thus:

RULE.—*Multiply the perimeter of the base by the altitude.*

To find the entire surface, we add the bases.

1. What is the convex surface of a triangular prism, the three sides of whose base are respectively 6, 7, and 8 inches, and the height 50 inches?

Ans. 1050 square inches.

2. What is the entire surface of a cube, the length of each side being 16 inches? *Ans.* $10\frac{2}{3}$ square feet.

3. What is the entire surface of the triangular prism given in the first problem? *Ans.* 1090.66 square inches.

28. The CONTENTS OF A PRISM are found thus:

RULE.—*Multiply the area of the base by the altitude of the prism.*

1. Required the contents of a cube whose sides are 30 inches. *Ans.* 15.625 cubic feet.

2. Required the contents of a square prism whose altitude is 27 feet, and the side of the base 4 feet?

Ans. 432 cubic feet.

3. Required the contents of a triangular prism whose altitude is 24 feet, the sides of the base being 3, 4, and 5 feet respectively. *Ans.* 144 cubic feet.

THE PYRAMID.

29. The CONVEX SURFACE OF A RIGHT PYRAMID is found thus:

RULE.—*Multiply the perimeter of the base by one-half the slant height.*

1. What is the convex surface of a triangular pyramid whose sides are 3, 4, and 5 feet, and slant height 20 feet?

Ans. 120 square feet.

2. Required the convex surface of a pentangular pyramid whose sides are each 5 feet, and slant height 60 feet.

Ans. 750 square feet.

30. The CONTENTS OF A PYRAMID are found thus:

RULE.—*Multiply the base by one-third of the altitude.*

1. Required the contents of a pyramid whose base is a hexagon, each side being 5 feet, and whose altitude is 20 feet. *Ans.* 433.013.

2. The pyramid of Cheops is 480 feet high, and the base is a square 763.4 feet on a side; required its solid contents.

Ans. $93244729\frac{3}{5}$ cubic feet.

THE CYLINDER.

31. The CONVEX SURFACE and CONTENTS are found thus:

RULE 1.—*The surface equals the circumference of the base multiplied by the altitude.*

RULE 2.—*The contents equal the area of the base multiplied by the altitude.*

1. What is the convex surface of a cylinder 12 feet long and 6 feet in diameter? *Ans.* 226.1952 square feet.

2. Required the convex surface of a cylinder whose length is 20 feet and the diameter of the base 8 feet.

Ans. 502.656 square feet.

3. A man has a log 12 feet long and about $6\frac{2}{3}$ feet in diameter; required its contents. *Ans.* 418.88 cubic feet.

4. The Winchester bushel is a cylinder containing 2150.42 cubic inches, its height being 8 inches; what is its diameter? *Ans.* $18\frac{1}{2}$ inches.

THE CONE.

32. The CONVEX SURFACE and CONTENTS are found thus:

RULE 1.—*The surface equals the circumference of the base into one-half of the slant height.*

RULE 2.—*The contents equal the area of the base into one third of the altitude.*

1. Find the convex surface and contents of a cone, the diameter of the base being 6 ft. and altitude 4 ft.

Ans. Sur. = 47.124.

2. Find the surface and contents of a cone whose slant height is 26 in. and radius of the base 10 in.

Ans. Vol. = 2513.28.

THE FRUSTUM OF A PYRAMID AND CONE.

33. The CONVEX SURFACE is found by the following

RULE.—*Find the sum of the perimeters or circumferences of the two bases, and multiply it by one-half of the slant height.*

1. Required the convex surface of the frustum of a square pyramid whose slant height is 24 ft., the side of the lower base 12 ft., and of the upper base 8 ft. *Ans.* 960 sq. ft.

2. Required the surface of a frustum of a cone whose slant height is 20 ft., the diameter of the lower base being 12 ft., and of the upper base 8 ft. *Ans.* 628.32 sq. ft.

34. The CONTENTS OF A FRUSTUM are found as follows:

RULE.—*Find the sum of the two bases and the square root of their product, and multiply this sum by one-third of the altitude of the frustum.*

NOTE.—In a frustum of a cone the following formula gives a shorter rule:—$V = \frac{\pi}{3}(R^2 + r^2 + R.r) \times h$.

1. What is the amount of timber in a log which measures 40 feet in length, the radius of one base being 6 feet and of the other 3 feet? *Ans.* 2638.944 cubic feet.

2. Required the contents of the frustum of a regular hexagonal pyramid, the side of the greater end being 3 feet, that of the less 2 feet, the height being 24 feet.

Ans. 394.9075 cubic feet.

3. A cask, consisting of two equal conic frustums joined at their larger ends, has its bung diameter 30 inches, and its head diameter 20 inches; how many gallons of wine will it hold if $3\frac{1}{2}$ feet long? *Ans.* 90.44 gallons.

THE SPHERE.

35. The SURFACE OF A SPHERE is found as follows:

RULE.—*Multiply the diameter by the circumference; or, Square the radius, and multiply it by* 4 *and* 3.1416.

1. Required the surface of a sphere whose diameter is 17 inches. *Ans.* 6.305 square feet.

2. How many square miles on the surface of the earth, the diameter being about 7912 miles?

Ans. 196,663,355 square miles.

36. The SURFACE OF A ZONE is found as follows:

RULE.—*Multiply the height of a zone by the circumference of a great circle of the sphere.*

1. The diameter of a sphere is 25 feet, and the height of the zone 6 feet; what is the surface of the zone?

Ans. 471.24 square feet.

2. Required the surface of the torrid zone, the diameter of the earth being 7912 miles.

Ans. 78,419,272 square miles.

NOTE.—This is to be solved after the pupil has completed Trigonometry.

37. The CONTENTS OF A SPHERE are found as follows:

RULE.—*Multiply the surface by one-third of the radius; or, Multiply the cube of the diameter by* $\frac{1}{6}$ *of* 3.1416.

1. Required the contents of a sphere whose diameter is 17 inches. *Ans.* 2572.4468 cubic inches.

2. Required the contents of the planet Mars, the diameter being about 4500 miles. *Ans.* 47713050000.

38. The contents of a SPHERICAL SEGMENT OF ONE BASE are found thus:

RULE.—*Add the square of the height to three times the square of the radius of the base; multiply this sum by the height, and the product by* .5236; or, see Th. X. B. VII.

NOTE.—For the volume of a segment of two bases, see B. VII. Th. X. C. 4.

1. Required the contents of the segment of a sphere whose height is 4 inches, and radius of the base 8 inches.

Ans. 435.635 cubic inches.

2. Find the volume of either temperate zone, the diameter of the earth being 7912 miles.

Ans. 54,919,403,678 cubic miles.

NOTE.—The pupil will solve this after completing Trigonometry.

CYLINDRICAL RINGS.

39. A CYLINDRICAL RING is formed by bending a cylinder until the two ends meet. We find its surface by the following

RULE.—*To the thickness of the ring add the inner diameter; multiply this sum by the thickness of the ring, and the product by* 9.8696.

NOTE.—For contents, multiply the *sum* by the *square of* $\frac{1}{2}$ *the thickness*, instead of the thickness, the other part of the rule being the same as for surface.

1. The thickness of a cylindrical ring is 4 inches, and the inner diameter 18 inches; what is the convex surface?

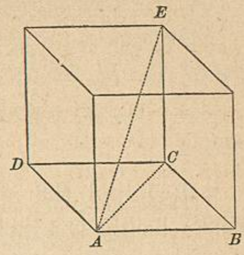
Ans. 868.52 square inches.

2. The thickness of a cylindrical ring is 2 inches, and the inner diameter 1 foot; required its contents.

Ans. 138.1744 cubic inches.

40. The SIDE OF AN INSCRIBED CUBE is found thus:

RULE.—*Multiply the diameter by .57736, or the radius by 1.15472.*



DEMONSTRATION.—Let the cube in the margin represent an inscribed cube; then will AE be the diameter of the sphere. Let the diameter be denoted by D, and the radius by R. Now, $\overline{AE}^2$ or $D^2 = AC^2 + \overline{CE}^2$; but $AC^2 = AB^2 + \overline{BC}^2$; hence, $D^2 = AB^2 + BC^2 + CE^2$, or, since the sides are equal, $D^2 = 3\,AB^2$, or, $D = AB \times \sqrt{3} = AB \times 1.73205$, and, consequently, $AB = D \times \frac{1}{1.73205} = D \times .57736$, or $AB = R \times 1.15472$.

1. Required the side of a cube that can be cut out of a sphere whose diameter is 16 inches. *Ans.* 9.23776.

2. Required the volume of a cube inscribed in a sphere whose circumference is 18.849552 inches.

Ans. 41.571219 cubic inches.

41. The VOLUME OF AN IRREGULAR BODY is found thus:

RULE.—*Immerse the body in a vessel of known dimensions, containing water; note the rise in the water, and calculate accordingly.*

1. A stone immersed in a cylindrical vessel 10 inches in diameter, raised the water 5 inches; required the volume of the stone. *Ans.* 392.70. cubic inches.

2. A man put a stone into a vessel 14 cubic feet in capacity, and it then required $2\frac{1}{2}$ quarts of water to fill the vessel; required the volume of the stone.

Ans. 13.9164 cubic feet.

MISCELLANEOUS PROBLEMS—PLANE FIGURES.

1. How many yards of paper that is 30 inches wide will it require to cover the wall of a room $15\frac{1}{2}$ feet long, $11\frac{1}{4}$ feet wide, and $7\frac{3}{4}$ feet high? *Ans.* 55.2833 yards.

2. A ladder 130 feet long, with its foot in the street, will reach on one side to a window 78 feet high, and on the other to a window 50 feet high; what was the width of the street? *Ans.* 224 feet.

3. The diameter of a circle is 4 feet; required the area of the inscribed equilateral triangle. *Ans.* $3\sqrt{3}$ square feet.

4. From a plank 16 inches broad, 6 square ft. are to be sawed off; at what distance from the end must the line be struck? *Ans.* $4\frac{1}{2}$ feet.

5. The ball on the top of a church is 6 feet in diameter; what did the gilding of it cost, at 8 cents per square inch? *Ans.* $1302.884.

6. The area of an equilateral triangle, whose base falls on the diameter and its vertex in the middle of the arc of a semicircle, is 100 square feet; what is the diameter of the semicircle? *Ans.* 26.32148.

7. The cost of paving a semicircular plot of ground, at 20 cents a square foot, amounted to $20; required its diameter. *Ans.* 15.9576.

8. A gentleman has a garden 80 feet long and 60 feet wide; what must be the width of a walk extending around the garden, which shall occupy one-half of the ground? *Ans.* 10 feet.

9. Required the perimeter of a regular dodecagon which shall contain the same area as a circle whose circumference is 1000 feet.

Ans. 1011.67 feet.

10. If a horse tied to a post in the centre of a field by a rope 1 chain 78 links can graze upon an acre, what length of rope would allow it to graze upon $5\frac{4}{9}$ acres? *Ans.* 4 chains $15\frac{1}{3}$ links.

11. A has a circular garden which is 20 rods, and B has a circular garden whose area is $6\frac{1}{4}$ times as great; what is the diameter of B's garden? *Ans.* 50 rods.

12. A has a circular garden, and B a square one; the distance around each is 64 rods; which contains the most land, and how much?

Ans. 69.948 square rods.

13. Atherton has a circular garden and Fell has a square one, and

they contain 4 acres; how much farther around is one than the other? *Ans.* 11.512 rods.

14. Mr. Thompson has a square yard containing $\frac{1}{10}$ of an acre; he makes a gravel walk around it which occupies $\frac{15}{64}$ of the whole yard; what is the width of the walk? *Ans.* 4 feet $1\frac{1}{2}$ inches.

15. A general, attempting to draw up his division in the form of a square, found he lacked 100 men to complete the square; he then received a reinforcement of five companies of 100 men each, and found he could increase the side of the square by 3 men and have 1 man remaining; how many men had he at first? *Ans.* 4125 men.

VOLUMES.

1. The volume of a sphere is 606.132 cubic feet; required its diameter. *Ans.* 10.5 feet.

2. The edge of a cube is 6 feet; what is the volume of a sphere that may be inscribed within it? *Ans.* 113.0976 cubic feet.

3. I have a cistern in the form of the frustum of a cone, its top diameter being 12 feet, its bottom diameter 9 feet, and its depth 5 feet; how many barrels of water will it contain? *Ans.* 103.515 barrels.

4. Bunker Hill Monument is 220 feet high, 30 feet square at the base, and 15 feet at the vertex; what is its volume? *Ans.* 115500 cubic ft.

5. Mr. Wilson has a pond which covers 100 acres, the average depth being 10 feet; how many cubic feet of water does it contain?

Ans. 43560000 cubic feet.

6. A man has a log of wood 20 ft. long, the larger end being 3 ft. in diameter, and the smaller 2 ft.; required the contents of the largest square stick, 20 ft. long, that can be sawed out of it. *Ans.* $63\frac{1}{3}$ cubic feet.

7. A bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep; what should be the dimensions of a measure of similar form to contain 64 bushels? *Ans.* Diameter, 74 inches; depth, 32 inches.

8. Mr. Benson can dig a shaft 5 feet each way in one day; how long will it take him to dig a shaft 20 feet each way? *Ans.* 64 days.

9. A man has a square garden 100 feet long, and wishes to make a gravel walk half-way around it; what will be the width of the walk if it takes up one-half of the garden? *Ans.* 29.289 feet.

10. A wishes to enclose his garden, which is 100 feet long and 80 feet wide, with a ditch 4 feet wide; how deep must it be dug that the soil taken out may raise the surface 1 foot? *Ans.* 5.319 feet.

11. A cubic foot of brass is to be drawn into a wire $\frac{1}{30}$ of an inch in diameter; required the length of the wire, supposing there is no loss of metal in the process. *Ans.* 31.252 miles.

12. Mr. Bonnycastle mentions a globe whose volume and surface are represented by the same number; what was the diameter of this globe?

Ans. 6.

13. Mr. Haswell requires the weight of an iron shell 4 inches in diameter, the thickness of the metal being 1 inch, estimating a cubic inch of iron at $\frac{1}{4}$ of a pound. *Ans.* 7.3304 pounds.

14. Bunker Hill Monument is 220 feet high, the lower base being 30 feet square, the upper 15 feet square; through its centre runs a cylindrical opening 15 feet in diameter at the bottom and 11 feet at the top; how many cubic feet of material in the monument?

Ans. 86068.444 cubic feet.

15. A gentleman has a bowling-green 300 feet long and 200 feet broad, which he wishes to raise 1 foot higher by means of the earth that is to be taken from a ditch that is to go around it; to what depth must the ditch be dug, supposing its breadth to be 8 feet?

Ans. 7 feet 3.21 inches.

16. A man having a garden 100 feet long and 80 feet broad, wishes to make a gravel walk half-way around it; what will be the width of the walk if it takes up one-half of the garden? *Ans.* 25.9688 feet.

17. Three persons having bought a sugar loaf, want to divide it equally among them by sections parallel to the base; what is the altitude of each person's share, supposing the loaf is a cone 20 inches high?

Ans. 13.867 upper part; 3.604 middle; 2.528 lower.

Suggestion.—Solve it by the principle of similar cones being to each other as the cubes of their altitudes.

Note.—Several of these problems are from the old writers on Mensuration. For more methods and exercises, see Bonnycastle's and Haswell's works on Mensuration.