1.3342 (Krause). The lens is completely enveloped by a structureless elastic capsule thicker in front than behind, which resembles in its reactions the sarcolemma of muscle or the membrana propria of glands.

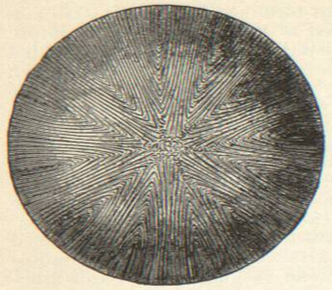

FIG. 2016.—Anterior Surface of Adult Lens. (Arnold.)

The development of the lens is the key to its structure. It will be remembered that it is formed from an invagination of the external epithelium that becomes isolated as the lenticular vesicle and that the cells of the anterior wall retain their epithelial characters, becoming more and more columnar toward the margin or equator of the lens, while those on the retinal aspect become lengthened into flattened fibres that vary in size and length in the central and the peripheral portions (Fig. 2019). The superficial fibres are from 0.010 to 0.012 mm. wide, 0.005 mm. thick, and 8 mm. long, while those of the deeper portion are 0.0075 wide, 0.0025 thick, and 4 mm. long. In the middle and interior portions of the lens the edges of the fibres are finely serrated (Figs. 2020, 2021), but these serrations do not fit into each other and there are thus left between the fibres fine spaces that serve for the passage of lymph. As the lens has no blood-vessels such a provision seems necessary for its nutrition. Between the epithelium and the lens fibres is the original cavity of the lenticular vesicle reduced to a narrow slit filled with semifluid material. This rapidly liquefies after death and becomes the so-called *liquor Morgagni*.

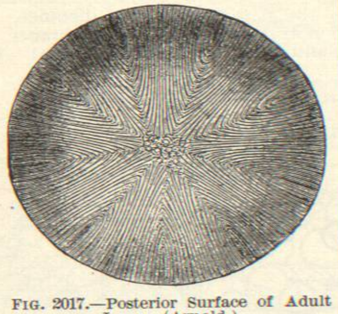

FIG. 2017.—Posterior Surface of Adult Lens. (Arnold.)

When treated by appropriate reagents the fibres of the peripheral third of the lens may be separated into concentric layers like those of an onion (Fig. 2022). In each layer the fibres have approximately the same length and terminate along nearly regular radiating lines called the *lens stars* (Fig. 2023). In the young lens these have behind an arrangement like a Y, in front that of the same letter reversed (⅄). They are more complicated in the adult.

The *zonula*, formerly thought to consist of two folds of a tenuous membrane enclosing a triangular canal (canal of Petit) that encircles the lens, is now known to be a system of delicate fibrillæ that extend from the ciliary region and the hyaloid membrane to be implanted upon the capsule of the lens. The origin and exact nature of these fibrillæ is still under discussion. It seems probable that they originate like the vitreous from mesenchymal connective tissue. They form an annular structure about 6 mm. wide that completely encircles the lens, serves to hold it in position and to keep the capsule tense.

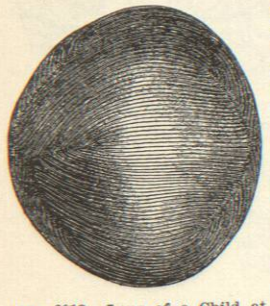

FIG. 2018.—Lens of a Child at Birth Seen from the Side. (Arnold.)

FIG. 2019.—Meridional Section Through Lens of Calf. (Arnold.)

The anterior chamber has the character of a lymph space developed in the connective tissue that primitively separates the lens from the external epithelium. This cavity is filled by the *aqueous humor*, a sparkling, transparent fluid containing 98.71 per cent. of water and having a considerable similarity to lymph. The quantity of this is from 231 to 323 c.mm. and remains nearly constant during life, being sufficient to maintain an intraocular tension equal to that of a column of mercury 26 mm. in height. Its specific gravity is 1.0053 and its index of refraction 1.3420, that of distilled water being 1.3342 (Krause). Apparently it is constantly renewed by a secretion from the vessels of the choroid, particularly those of the ciliary processes, and removed from the chamber by infiltration through the spaces of Fontana into the scleral sinus and perhaps by the lymph crypts of the iris. Normally it contains no morphological elements.

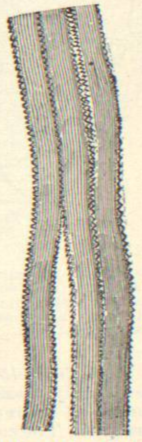

FIG. 2020.—Isolated Fibres from Lens. (Arnold.)

Behind the lens the cavity of the eyeball is filled with the *vitreous humor* or *body*, a transparent, jelly-like material that has the characters of a primitive form of very watery connective tissue, showing widely scattered cells embedded in a matrix of gelatinous material in which appropriate reagents develop a rich fibrillary network. Leucocytes are also found in it, especially where it is in contact with the retina. It has a very delicate envelope, the *hyaloid membrane*, adherent to the retina at the optic disc and the ciliary region where arteries enter during fœtal life. This membrane does not pass down into the intervals between the ciliary bodies, but spans them over, forming thus small meridional spaces, the *recesses of the posterior chamber*, which are filled with aqueous humor infiltrated through the zonula from the anterior chamber. In front the vitreous has a depression, the *hyaloid* or *patellar fossa*, into which the lens is received. The hyaloid membrane appears to be lost upon the zonula in front and not to be continued over the hyaloid fossa, the consistence of the vitreous at that place being maintained by a condensation of its substance. From the optic disc forward there runs in fœtal life a small vessel, the *hyaloid artery*, for the nourishment of the lens. This is represented in the adult by a central lymph space, the *hyaloid canal* (Fig. 1952). Just in front of the disc this canal enlarges to a space known as the *area Martegiani*.

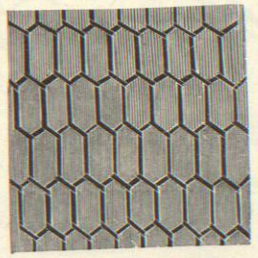

FIG. 2021.—Section of Frozen Lens Showing Hexagonal Shape of Fibres. (Arnold.)

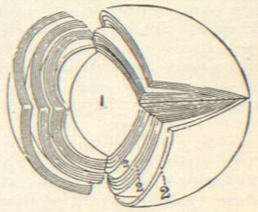

FIG. 2022.—Lamination of the Lens as Shown After Treatment with Dilute Alcohol. (Arnold.) 1, Former central portion called the nucleus lentis; 2, 2, layers of the so-called cortical substance.

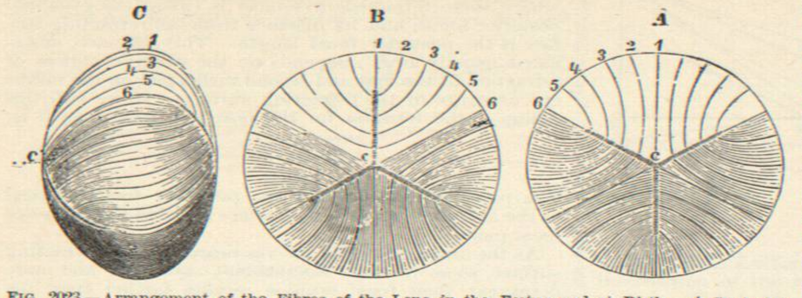

FIG. 2023.—Arrangement of the Fibres of the Lens in the Fœtus and at Birth. *A*, Posterior surface; *B*, anterior surface; *C*, side; *c*, centre of the lens stars, being the anterior and posterior poles of the lens; 1 to 6, course of fibres taken at equal distances.

The refractive index of the vitreous body is nearly that of distilled water, its weight is from 6.7 to 8.3 gm. and its specific gravity is 1.005. Its reaction is alkaline, and the proportion of water found in it is very high, being 98.40 to 98.64 per cent.

Frank Baker.

EYE, DIOPTRICS OF.—REFRACTION.—1. The term refraction refers in physics to the deviation of a ray of light from its straight path on passing from one transparent medium into another of different nature. In physiological optics, however, this term is used with a special significance, denoting the relation of the focal length of the eye to the position of the retina. The eye has a normal, or emmetropic (from ἔμμετρος, according to measure, and ὤψ, eye), refraction, if images of distant objects are sharply defined on the retina; the refraction is myopic, or near-sighted, if distant objects form images in front of the retina; while hypermetropia is that refractive state in which images of distant objects can be sharply defined only behind the actual place of the retina. Any refractive state other than emmetropia is referred to in general as ametropia (from ἄμετρος, disproportionate, and ὤψ, eye).

The eye of all vertebrates is an optic instrument, the principle of which is illustrated by the photographer's camera obscura. By means of a convex lens, the rays of light coming from the different points of external objects are so reunited as to form inverted, but geometrically correct, images of those objects on a screen. The screen is the retina, while the convex lens is constituted by all the transparent media of the eyeball.

A correct knowledge of the optic properties of the eye is not possible without some familiarity with the laws of physical dioptrics. Hence we must begin with a résumé of the laws of the refraction of light. In order to keep this article within the allotted space, we will not attempt to follow out the mathematical deduction of all the various formulæ of which we must make use. For the complete mathematical proof of all the statements the reader must consult some of the works mentioned in the bibliography, especially those of Helmholtz and Donders.

The paths of the rays of light entering the eye, and the influence of the different media upon them, can be deduced from the following optic principles:

2. *Law of Refraction.*—From every point of a luminous or illuminated object there proceed rays of light in all directions. Every ray pursues a straight course as long as it passes through a uniform medium. When a ray passes from one transparent medium into another of different optic properties, it is refracted or deflected from its straight path, except when its original direction is vertical to the surface of the second medium. The extent of deflection depends on a specific property of each medium, viz., its refractive power. The relation of the refractive power of any one medium to that of another is termed the refractive index, and is usually designated by the letter n. Air is taken as the standard of comparison, and its index of refraction is called 1. Compared with air the refractive index of water is 1.334; of crown glass, 1.533; of flint glass, 1.664. A medium having the greater refractive index is said—in a somewhat loose manner—to have a greater optic density than another rarer medium.

The extent to which a ray of light is deflected from its straight path, by refraction, depends on the angle at which it strikes the surface, as well as on the refractive indices of the media.

Let A, in Fig. 2024, be air, with the refractive index $= n$, and B be glass, with the index $= n'$, the two being separated by the surface $s\,s'$. A ray of light, having the direction $a\,c$ in the air, will be bent in the direction $c\,d$ on entering the glass. If we erect the normal $b\,c\,e$ vertical to the surface at the point c, the angle $a\,c\,b$ is the angle of incidence of the ray $a\,c$; the angle $e\,c\,d$ is the angle of refraction. The latter is here smaller than the former, for, in the denser medium, the refracted ray is bent toward the perpendicular. The relation of one angle to the other depends on the relation of the refractive indices of the two media, so that the sine of either angle is to the sine of the other inversely as the index of the corresponding medium is to that of the other, or

$$\frac{\text{sine } a\,c\,b}{\text{sine } e\,c\,d} = \frac{n'}{n} \qquad (1)$$

Since the sine of an angle zero is likewise zero in value, it follows that when the incident ray is itself perpendicular to the refracting surface, it continues to be perpendicular after its entrance into the second medium, *i.e.*, it is not deflected at all from its course.

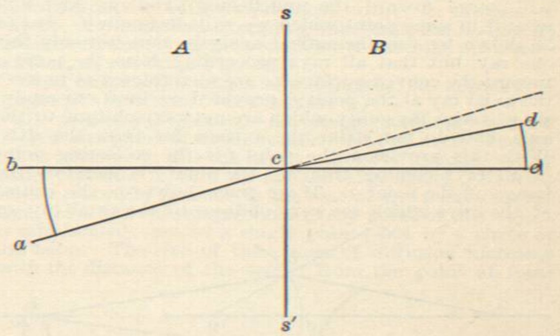

FIG. 2024.

The path which a ray describes in passing through one or more refracting surfaces is the same, whether that ray travels forward or backward from any given point in its course.

3. If the denser medium is in the form of a plate bounded by parallel surfaces and surrounded by the same rarer medium on both sides, a ray, after passing through the plate, follows a direction parallel to its original course, but is displaced laterally.

For in Fig. 2025, where A represents the air, and B the plate of glass, the ray $a\,c$ on entering the glass is bent toward the perpendicular $d\,c$ to an extent exactly counterbalanced by its deflection from the perpendicular

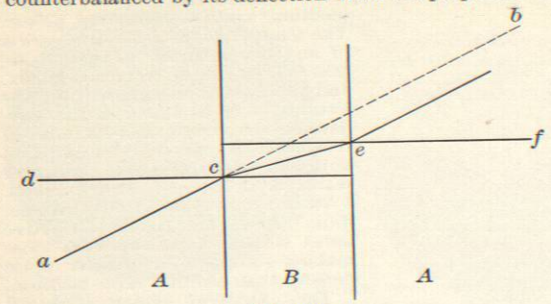

FIG. 2025.

$e\,f$ on leaving the glass. Hence the ray $e\,b$ after passing through the glass is parallel to the prolonged direction of the original ray $a\,c$.

4. *Refraction by a Single Refracting Surface.*—When the denser medium is bounded by a convex spherical surface, all rays (except one) coming from any point in the rarer medium are deflected in such a manner as to be less divergent after their refraction than they were before. The one ray not deflected is the one striking the surface vertically, hence coinciding in the radius or axis of that point of the surface. It is hence termed the *axial ray*.

In Fig. 2026 let a be a point in the air from which rays proceed toward the convex surface $b\,c$ separating the air from the glass. Let o be the centre of curvature of the refracting surface $b\,c$. Let the ray $a\,c$ be vertical to the surface, *i.e.*, coinciding with the direction of the radius $o\,c$; it will hence not be deflected from its course. The ray $a\,b$ is, however, refracted. If $o\,b$ is the radius of the surface, and is hence vertical to the point b, the angle of refraction $f\,b\,o$ can be found by the formula:

$$\text{sine}\,f\,b\,o : \text{sine}\,a\,b\,d = n : n'$$

It is evident from Fig. 2026, that if the point a be not too near to the refracting surface, the refracted ray $b\,f$ will verge toward the undeflected axial ray, and will meet it in some point which we will designate f. It can be shown by a mathematical analysis, that not only this one ray, but that all rays proceeding from the point a toward the convex surface $b\,c$ are so refracted as to meet the axial ray at the point f, provided we limit our analysis to those rays only which are not very oblique to the axis, and do not strike the surface far from the axis. With this provision, the point f is the collecting point of all rays coming from a; the point f is therefore the image of the point a. If we trace, however, the course of the rays which are very oblique to the axial ray, or

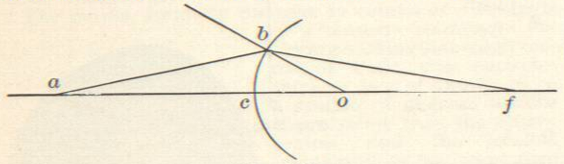

FIG. 2026.

which strike the surface far from its axis, we shall find that they meet the axial ray at various points different from f. Such a want of exact reunion of all the rays, when too large an extent of the refracting surface is exposed, constitutes the fault of optic instruments known as *spherical aberration*. The refraction of such very oblique rays is represented, in a somewhat exaggerated manner, in Fig. 2027.

5. *Focal Length.*—When the luminous point is situated in the rarer medium at a distance infinitely great compared with the dimensions of the refracting surface, the different rays emanating from that point strike the refracting surface with so little divergence as to be practically parallel to each other. These parallel rays are rendered convergent by their refraction. The point where these refracted rays meet is called the principal posterior focus, and its distance from the refracting surface is the posterior focal length. This distance, designated usually as F'', depends on the relative indices of refraction of the first and second media and on the radius of curvature of the refracting surface. If we call this radius r, the formula for the posterior focal length is:

$$F'' = \frac{n'\,r}{n' - n} \tag{2}$$

The plane drawn through the posterior focus vertical to the axis of the refracting surface is called the posterior focal plane.

As the luminous point moves nearer to the refracting surface, so as to make the incident rays more and more divergent, their focal reunion recedes farther from the surface, until a certain point in front of the convex surface is reached, the rays proceeding from which are so divergent that they can no longer be rendered convergent by their refraction, but only parallel. Their point of focal reunion may then be considered as infinitely far behind the surface. The point in front of the surface from which these rays proceed is the principal anterior focus, and its distance from the surface, which we will call F',

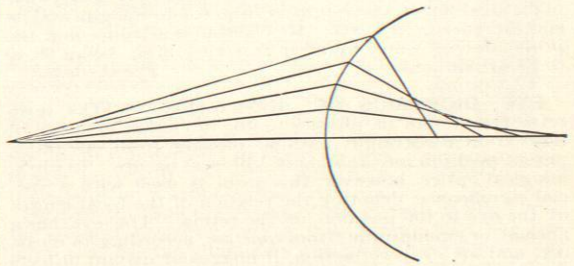

FIG. 2027.

is the anterior focal length. A vertical plane laid through it is the anterior focal plane. F' can be found by the formula:

$$F' = \frac{n\,r}{n' - n} \tag{2a}$$

A comparison of formulæ (2) and (2a) shows that the anterior and posterior focal lengths are not alike under the circumstances, but that

$$F' : F'' = n : n' \tag{2b}$$

Remembering that the path of the rays is the same, whether they travel forward or backward, we may also define the anterior focus as the point in which rays unite, which proceed parallel to each other in the denser medium toward the refracting surface.

6. The distance of the focal reunion from the refracting surface can be calculated for any set of rays, coming from any point, if we know the distance of that point from the surface, and also the anterior and posterior focal lengths of the refracting surface. If f' be the known distance of the luminous point, the distance of the corresponding focal reunion f'' will be:

$$f'' = \frac{F''\,f'}{f' - F'} \tag{3}$$

Conversely, the distance of a luminous point f' can be found, if we know the distance of its corresponding focal reunion or image f'', according to the formula:

$$f' = \frac{F'\,f''}{f'' - F''} \tag{3a}$$

A very convenient formula for finding the place of the focal reunion of rays coming from a point at a known distance can be obtained by the transformation of (3) and (3a). If we designate the distance from the luminous point to the anterior focus, viz.: $f' - F'$ as l' (counting l' negative if f' is nearer to the surface than F'), and the

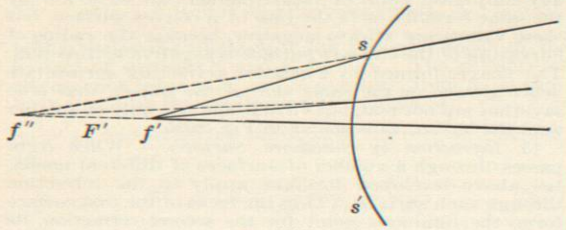

FIG. 2028.

distance from the posterior focus to the point of focal reunion of the rays in question, viz.: $f'' - F''$ as l'', then

$$l'\,l'' = F'\,F'' \tag{3b}$$

In all these instances the relation of luminous point and corresponding image can be reversed without change of position; in other words, if the point x is the image of the point a, then for rays proceeding in the reverse direction, a is also the image of the point x. Any two points having such a relation of luminous point and corresponding image are called *conjugate points*.

7. *Virtual Image.*—When the rays, coming from a point nearer than the anterior focus, strike the refracting surface, their divergence is too great to be entirely overcome by the refraction. They cannot therefore be united to form an actual image; but if their direction, after the refraction, be prolonged backward, their prolongations meet to form a *virtual* image. The focal reunion of such rays is therefore negative.

Thus, in Fig. 2028, if F' be the principal anterior focus of the refracting surface $s\,s'$, and f' the luminous point, the rays proceeding from f' will have a divergence after their refraction, as if they came from the point f'', which is therefore the image of f'. Since f'' is on the same side of the surface as f', its distance is counted negative. It can be determined by formula (3a) or (3b).

8. *Formation of Images.*—Since an object is made up of an infinitely great number of points, and since of every such point in front of the refracting surface an image is formed somewhere behind the surface, therefore an image must also be formed of the entire object. From every point of the object there proceeds one ray, which is not bent from its course by refraction, viz.: the ray

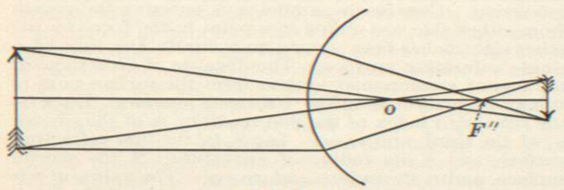

FIG. 2029.

which strikes the surface perpendicularly, and hence coincides in direction with the radius of the point of the surface. Such rays are termed *rays of direction*. Since all other rays proceeding from any one point of the object meet the ray of direction at the corresponding distance of focal reunion, the image of every point of the object is situated somewhere in the path of the ray of direction coming from that point. Since every ray of direction coincides with one of the radii of the refracting surface, the rays of direction must, therefore, like the radii, intersect at the centre of curvature of the surface. Since the focal length of a refracting surface is always greater than its radius, according to formulæ (2) to (3a), therefore the actual images of objects farther off than the

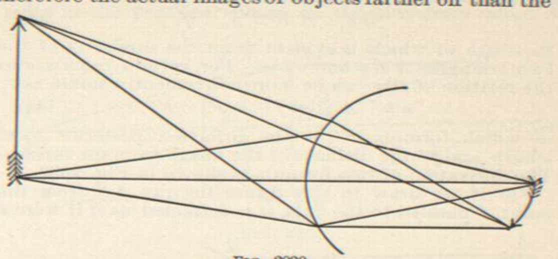

FIG. 2030.

anterior focus are formed somewhere beyond the point of intersection of the rays of direction, and are hence inverted. This is illustrated by Fig. 2029.

The point where the rays of direction cross, viz.: the centre of curvature of the surface, is also called the *optic centre*.

9. When the object lies in a plane vertical to the axis of the refracting surface, all points of the object near the axis are sensibly at the same distance from the refracting surface, as measured by the length of the rays of direction. Their images are therefore likewise situated at equal distances behind the surface, and hence lie in a plane vertical to the axis. But this is true only as long as the angle included between the ray of direction coming from that point of the object, and the axis of the refract-

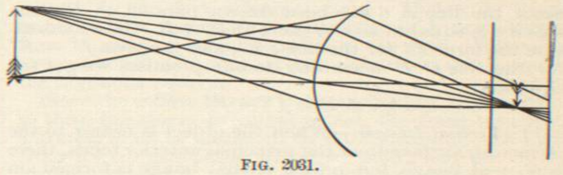

FIG. 2031.

ing surface is so small as to permit the substitution of its chord for its arc without practical error. In the case of objects of dimensions exceeding this limit, the images lie in a curved line, with the concavity toward the refracting surface, as shown in Fig. 2030.

Such extensive images are blurred, on account of spherical aberration. (See Section 4.)

10. *Circles of Diffusion.*—An image is sharply defined only in the plane of focal reunion of the rays. In any plane anterior to this the rays are not yet united; in any plane posterior to this the rays diverge again. Hence a screen placed in front or in the rear of the focal reunion receives a blurred image, since every point of the object is represented—not by a single point—but by a circle of diffusion. The size of the circles of diffusion increases with the distance of the screen from the point of focal

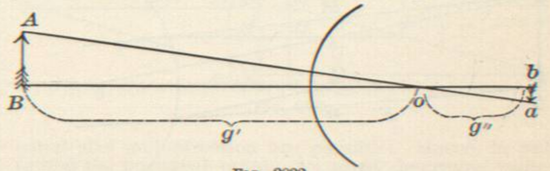

FIG. 2032.

reunion in either direction, and with the extent of the refracting surface through which rays pass. This is evident from Fig. 2031.

11. *Size of Images.*—The size of the image is to the size of the object as the distance of the image from the optic centre is to the distance of the object from the same point. If we designate, in Fig. 2032, the distance of the

object $A\ B$ from o, the optic centre, as g', and the distance of the image $a\ b$ from o as g'', we can express the relation of size by the formula:

$$a\ b : A\ B = g'' : g' \qquad (4)$$

the truth of which is evident from the similarity of the two triangles $A\ B\ o$ and $a\ b\ o$. For certain calculations the relation of size can be more conveniently stated as

$$a\ b : A\ B = f'' - F'' : F'' \qquad (4a)$$

in which formula F'' is the principal posterior focal length, and f'' the distance of the image from the surface. The derivation of this formula is shown in Fig. 2033.

For if we draw in this figure the ray $A\ d$ from the point A parallel to the axis, it is refracted as if it were a

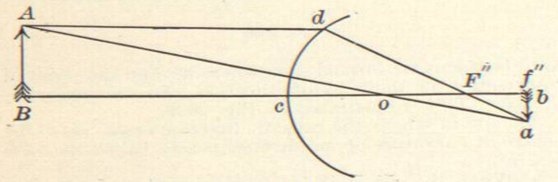

FIG. 2033.

ray coming from some point in the axis at an infinite distance, that is to say, it is deflected to F'' the posterior focus, and proceeds beyond F'' until it meets the other rays coming from A in the point of focal reunion a. Thereby are formed the two similar triangles $d\ c\ F''$ and $a\ b\ F''$. Hence,

$$a\ b : d\ c = f''\ F'' : c\ F''$$

Since the line $A\ d$ has been drawn parallel to $B\ c$, the side $d\ c$ is equal in size to the object $A\ B$. If we substitute the term F'' for the line $c\ F''$, and the term $f'' - F''$ for the line $f''\ F''$ according to our premises we get the equation:

$$a\ b : A\ B = f'' - F'' : F''$$

12. *Virtual Images.*—When the object is nearer to the refracting surface than the principal anterior focus, there is no real image, but only a virtual image is formed on the same side of the surface as the object is itself. In this case f'' is hence negative, otherwise the same formulæ apply as above. Since the rays of direction do not cross, this virtual image is erect, and since it is always farther from the optic centre than the object, it is larger than the object. The amplification of the image diminishes as the object approaches the refracting surface, for when the object has just passed through the anterior focus the image is at an infinitely great distance, and is therefore most enlarged, while when the object touches the refracting surface, object and image coincide in position and size. The formation of virtual images is shown in Fig. 2034.

13. All the facts above stated with reference to images formed in the denser medium of objects situated in front

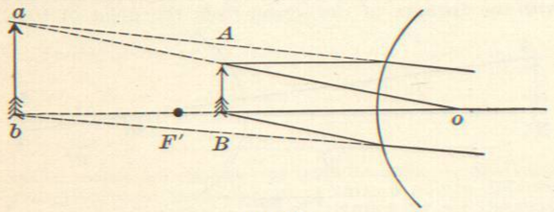

FIG. 2034.

of the refracting surface, are equally true conversely of images formed in the rarer medium of objects situated behind the refracting surface.

14. *Refraction by Concave Surfaces.*—When the refracting surface is concave on the side of the rarer medium, rays coming from any one point are not reunited in an actual focus. But if the direction which they assume after their refraction be prolonged backward, the prolonged rays will meet in a point representing the focus. This focal length is therefore always negative, that is to say, the focus is always on the same side of the surface as the object itself. The position of the principal foci or any conjugate point of focal reunion can be found by the same formulæ as in the case of a convex surface, but these values are always negative, because the radius of curvature of the concave surface is negative in direction. The images formed by a concave refracting surface are hence always on the same side as the object, that is to say, they are not real, but virtual, and can only be smaller than the object, as is shown in Fig. 2035.

15. *Refraction by Successive Surfaces.* — When light passes through a number of surfaces of different media, the above-developed formulæ apply to the refraction through each surface. Thus the focus of the first surface forms the luminous point for the second refraction, its distance being counted positive when it is in front of the second surface, and negative when it happens to be behind that surface. Similarly the image formed by the second surface is the object for the third surface, and so on. But such calculations become very cumbersome if we try to follow actually the course of the rays through their successive refractions. The matter is simplified by treating the series of surfaces as one system having certain fixed points, the position of which determines the path of all rays. These are called the cardinal points. Their position remains constant in any given system, and can be calculated, if the refractive indices of the media, the radii of curvature of all the surfaces, and the distances of all the surfaces from each other are known, provided the centres of curvature of all the surfaces lie in a straight line, the axis of the system. If the latter

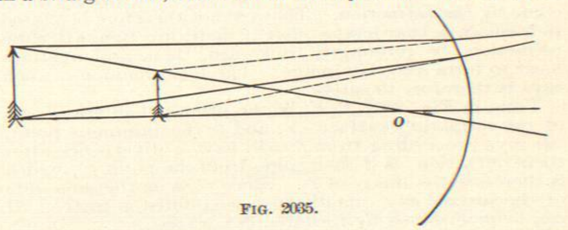

FIG. 2035.

condition is fulfilled, the system is said to be centred. The cardinal points are the two foci, the two principal points and the two nodal points.

16. *The Foci.*—The foci, anterior or first, and posterior or second, are the two points—one on either side of the system—at such distances that rays coming from the focus are rendered parallel to each other by their last refraction. Conversely parallel rays, entering the system from either side, are united in a point in the focus on the other side, as has been shown to occur in the case of a single refracting surface. The distance of either one of the foci of a compound system from the surface next to it may be computed in the following manner: Let n be the refractive index of the first medium, n_1 of the second, n_2 of the third, and so on. Let S_1 be the first refracting surface, and r_1 its radius of curvature; S_2 the second surface, and r_2 its radius, and so on. The value of r is positive if the surface is convex on the side of the luminous point; negative if it is concave. Let d_1 be the distance from the first surface to the second, d_2 from the second to the third, and so on. Determine the foci of each surface by itself, without reference to the other surfaces. According to formulæ (2) and (2a), we get for the first surface,

$$F_1' = \frac{nr_1}{n_1 - n} \text{ and } F_1'' = \frac{n_1\ r_1}{n_1 - n}$$

and for the second surface,

$$F_2' = \frac{n_1\ r_2}{n_2 - n_1} \text{ and } F_2'' = \frac{n_2\ r_2}{n_2 - n_1}$$

The posterior focus for the first and second surfaces taken together will now be found by considering F_1'' as the luminous point for S_2; and, on applying the formula (3), viz., $f'' = \frac{F''\ f'}{f' - F'}$ we get the equation:

$$F({}_1 + {}_2)'' = \frac{F_2''\ (d' - F_1'')}{(d' - F_1'') - F_2'} \qquad (5)$$

which states the distance of the posterior focus of the system of surfaces S_1 and S_2 from S_2 The distance of the anterior focus of S_1 and S_2 together, from S_1, is similarly found by the application of formula (3a), viz., $f' = \frac{F'\ f''}{f' - F''}$ and following the light in the reverse direction, hence

$$F({}_1 + {}_2)' = \frac{F_1'\ (d_1 - F_2')}{(d_1 - F_2') - F_1''} \qquad (5a)$$

By treating the surfaces S_1 and S_2 as one system, we can then proceed to determine the foci for $S({}_1 + {}_2)$ and S_3, and so on.

The foci of an optic system having been determined, the focal length is now measured by the distance of each

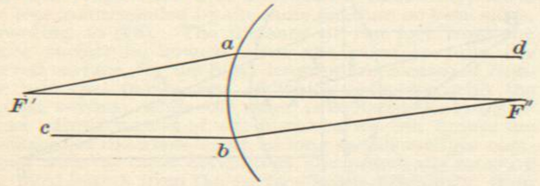

FIG. 2036.

focus from its corresponding principal point. If we call the anterior or first principal point H', and the posterior or second principal point H'', then $F'\ H'$ is the anterior focal length, and $H''\ F''$ is the posterior focal length.

17. *The Principal Points.*—The principal points are the two points where the axis of the system is cut by the two principal planes. The significance of the principal planes can be best explained on comparing the refraction by a single surface with the course of rays through a compound system. In the case of the single surface $a\ b$, Fig. 2036, the ray $F'\ a$, coming from the anterior focus, is, by its refraction, made parallel to the axial ray $F'\ F''$. Incident and refracted rays intersect here in the plane of the refracting surface. The same is true of the ray $F''\ b$ coming from the posterior focus, and its continuation, $b\ c$, after refraction. In Fig. 2037, however, where there are two refracting surfaces, $a\ b$ and $a'\ b'$, the ray, $F'\ a$, coming from the anterior focus, is twice bent from its course, viz., at each surface, so as to finally assume the direction $a'\ d$, parallel to the axial ray. In this case the incident ray, $F''\ a$, and the refracted ray, $a'\ d$, do not intersect, being separated by the space between a and a'; but if we prolong them through this space, their prolongations intersect at the point h'. A plane, $h'\ H'$, laid through this point, vertical to the axis, is the first principal plane. Similarly, if we prolong the ray $F''\ b'$, which proceeds from the posterior focus to the surface $a'\ b'$, it will cut the backward prolongation of the refracted ray $b\ c$ at the point h'', which point determines the position of the second principal plane. It is evident, from Fig. 2037, that the ray which before entering the system is directed toward the point h' in the first principal plane, has a direction after its last refraction as if it came from a point in the second principal plane at the same distance from the axis as the point h'. Likewise the ray which proceeds in the opposite direction from F'' toward the point h'' has a direction, after its last refraction, as if it came from a point in the first principal plane at the same distance from the axis as the point h''. This mutual relation of the two principal planes is the same for all rays, not merely those coming from F' or F''; and it is true also for any distance at which the points h' and h'' may be from the axis within the limitations of Section 4. In other words, any ray which is directed toward a given point in the principal plane on the side from which it

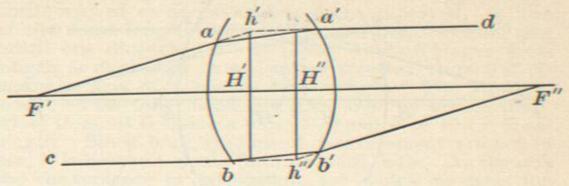

FIG. 2037.

comes, apparently emerges from the other principal plane at the same distance from the axis. We can thus determine by construction the direction of any ray after its refraction through a compound system, and thereby find the places of the image formed by such rays, if we know the position of the foci and the two principal planes. In Fig. 2038, let $a\ a'$ be the first, and $X\ X'$ be the last refracting surface of a compound system; let H' be the first, and H'' be the second principal point, F' the anterior, and F'' the posterior focus. Draw the axis $F'\ F''$. From the point B there proceeds the ray $B\ a$, parallel to the axis, and verging toward the point m' in the first principal plane. Hence, after its last refraction, that ray has a direction as if it came from the point m'' in the second principal plane, going through the posterior focus F''. Another ray convenient to follow is the one coming from B and passing through F'. This verges toward the point n' in the first principal plane. Hence, by its refraction, it is turned in the direction $n''\ b$, as if coming from n'' and proceeding parallel to the axis. Where the two refracted rays meet, at the point b, is the image of the luminous point B.

Since the course of rays is determined by their relation to these imaginary principal planes, the focal lengths of a system must be measured by the distance of each focus from its corresponding principal point, and not by the distance of the foci from the refracting surfaces. Indeed, for all subsequent purposes, we can practically ignore the position of the refracting surfaces of any system, after we have once determined the position of the foci and of the principal planes. For now all the problems relative to the position of object and image can be solved by the same formulæ as in the case of a single refracting surface, by measuring the focal lengths from the principal planes.

18. The position of the two principal points is determined by the position of the two planes in which the images of a certain other plane are of equal size and direction. In every refracting system there exist only two such planes, and these are the principal planes; and there exists, moreover, only one plane of which two images of

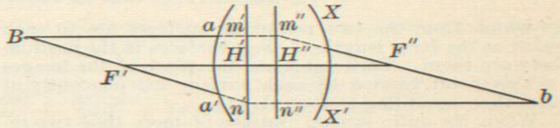

FIG. 2038.

equal size and direction are possible. Hence, in order to find the principal points, we must determine where a line must be in order to form two images of the same size and direction, and then learn the place of these two images.

This proposition can be demonstrated by means of Fig. 2039, in which we have a refracting system bounded by the surfaces $s'\ S'$ and $s''\ S''$. F' is the first and F'' the second focus. Draw the ray $F'\ c$, which is deflected toward s'' by the surface $s'\ S'$, and is made parallel to the axis by the surface $s'' S''$. By the intersection of the prolonged incident and refracted rays the point h' determines the position of the first principal plane. Similarly a reverse ray going from F'' to e is bent twice, so as to fol-

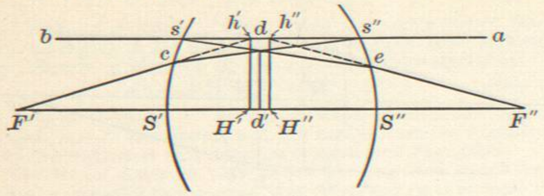

FIG. 2039.

low ultimately the direction $s' b$ parallel to the axis. The prolongations of the ray before entering and after leaving the system give us the point h'', and thereby the position of the second principal plane. If we draw a line $d d'$ vertical to the axis from the point d, where the rays coming from each side intersect after their first refraction, the virtual image of $d d'$ produced by the surface $s' S'$ coincides with the first principal plane $h' H'$, while the image formed by the surface $s'' S''$ coincides with the second principal plane $h'' H''$, as is evident from the refraction of the rays according to the construction; and these two images are alike in size. The distance of d' from each surface is determined by the equation (4a).

$$a\, b\ (image) : A\, B\ (object) = f'' - F'' : F''$$

or more conveniently, on account of the erect position of the virtual image,

$$a\, b : A\, B = F'' - f'' : F''$$

Each surface is here considered independently of the other. Hence it can be deduced that the distances of d' from each surface must be to each other as the focal lengths of the surfaces, in order to have the images formed by the two surfaces of equal size. This is shown in Fig. 2040, where $P' S''$ is the focal length of the surface $s'' S''$ in the medium between the two surfaces and $P'' S'$ of the surface $s' S'$.

A consideration of the two triangles $P' S'' s''$ and $P'' S' s'$ and their segmentation by the line $d\, d'$ proves that

$$d' S' : d' S'' = P'' S' : P' S'' \qquad (6)$$

Hence, in order to find the principal planes of a system, determine the position of a point in the axis, the distances

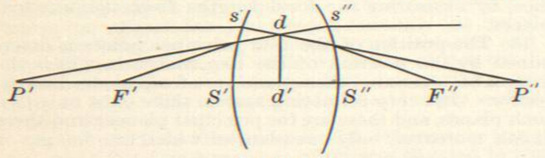

FIG. 2040.

of which from the two refracting surfaces are to each other as the focal lengths of these surfaces in the medium between them. Then calculate the place of the images of this point formed by each surface independently of the other, according to formula (3).

When the optic system consists of more than two refracting surfaces, determine the principal points for two adjoining surfaces, and then divide the distance between the one principal point next to the third surface and the third surface into two parts, which are to each other as the corresponding focal lengths in the intervening medium. The images of the dividing point formed independently by the third surface, and by the system of the other two surfaces, are then the principal points of the system of three surfaces.

19. *The Nodal Points.*—The nodal points of a compound system replace the optic centre of a single refracting surface. For while all rays of direction pass undeflected through the optic centre of a single surface, a second surface will deflect all rays with the exception only of the axial ray. Hence a single optic centre cannot exist in a compound system. But there exist two points in the axis, viz., the nodal points, of such properties that a ray directed toward the first before entering the system pursues a course after its final refraction as if it had passed through the second nodal point parallel to its original direction.

That a pair of points of such properties must exist in any compound system is evident from Fig. 2041, where F' is the first and F'' the second focus, and $m' H' n'$ the first and $m'' H'' n''$ the second principal plane. A ray $B\, m'$, coming from the luminous points B, forms with the refracted ray $m'' b$ any angle of less than 180° as seen from below, while the ray $B\, n'$ forms with its refracted prolongation $n'' b$ an angle greater than 180° as seen from below. Somewhere between m' and n' there must be a level where the incident ray includes with its continuation beyond the second principal plane an angle of 180° exactly, in other words, where the two are parallel. Let this be at the level l'. If we prolong the incident ray $B\, l'$, the points where this prolonged ray and the refracted

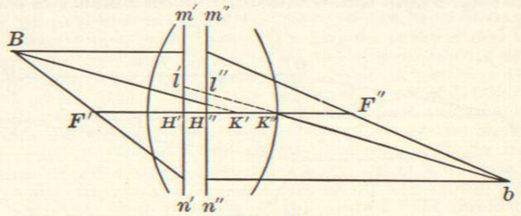

FIG. 2041.

ray $l'' b$ cut the axis, viz., K' and K'', answer the requirements of the nodal points. The position of the nodal points relative to the principal points is made evident in Fig. 2042.

In this figure we will draw the line $B\, F'$ on F' vertical to the axis. From the point B there proceeds a ray of direction $B\, K'$ to the first nodal point K'. According to our premises the ray $K'' b$ must be parallel to $B\, K'$ and in the direction $K'' b$ lies the image of B, at an infinite distance from the optic system. Since the point B lies in the anterior focal plane, every ray proceeding from it is rendered parallel by its refraction to the ray of direction coming from B. Hence the ray $B\, h'$ parallel to the axis is continued after passing through the second principal plane, as $h'' F''$ parallel to $K'' b$, and hence also to $B\, K$. From the similarity of the triangle $B\, F' K'$ and $h'' H'' F''$ it is evident that the distance

$$F' K' = H'' F'' \qquad (7)$$

and by reversing the figure and constructing the course of rays coming from the posterior focal plane, we can similarly learn that the distance

$$F'' K'' = H' F' \qquad (7a)$$

and that hence

$$H' H'' = K' K'' \qquad (7b)$$

which latter corollary is also apparent from Fig. 2041. The position of the nodal points can hence be at once

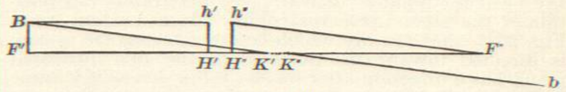

FIG. 2042.

learned by formulæ (7) and (7a), after knowing the position of the principal points and of the foci. And thus we have all data necessary to follow the course of rays through any compound optic system.

20. *Lenses.*—It will be best to refer briefly to the optic properties of glass lenses before we proceed to the eye

itself. According to the curvature of their surfaces and the distance between them, lenses either reunite into one point all the rays coming from one point, or disperse them. The former, or collecting lenses, have a positive focal length. They form inverted actual images of distant objects, and erect, virtual, and enlarged images of objects nearer than their focus. The latter, or diverging lenses, have a negative focal length, and can form only erect, virtual images, smaller than their object. When both surfaces of a lens are convex, or one is convex and the other plane, the lens belongs to the former variety; when one surface is concave and the other plane, or when both are concave, we have a diverging lens with negative focus. When one surface is convex and the other concave, the positive or negative value of the resulting focal length depends not only on the curvature of the two surfaces, but also on the distance between them.

The refractive index of the glass used for spectacles is so near 1.5 that we can calculate on that basis without important error, at least for spectacles. Taking this index we find that in a plano-concave, or plano-convex lens, the focal length is twice the length of the radius of the curved surface, measuring from the curved surface, according to formula (2) or (2a). The first and second focal lengths are of course equal to each other, when the lens is surrounded by the same medium on both sides, according to (2b). The distance of the foci from the plane surface is, however, not the same as from the curved surface, for the focal lengths are measured from the principal points, one of which coincides with the curved surface, while the other is between the two surfaces. But for most of our purposes we can ignore the thickness of the glass lenses, as this is slight compared with the other dimensions, and practically measure the focal length from the surface, or, indifferently, from the centre of the lens. If, hence, we express the focal length of a plano-convex lens as

$$F' = F'' = 2\, r$$

we get for a biconvex lens the formula

$$F = \frac{r' + r_2}{2}$$

If the radii of curvature of the two surfaces are alike, we find the focal length equal to the radius, provided the index of refraction is practically 1.5. This applies similarly to biconcave lenses, F being, however, negative. In lenses with one convex and one concave surface, the measurement of the focal length is not quite so simple, because the thickness of such a lens cannot be ignored without error, and the principal points can in such a combination be outside of the substance of the lens.

The so-called strength of lenses is measured differently, according to the unit which we adopt. Formerly a lens of the focal length of one inch—either negative or positive—was taken as the standard and called 1. Any lens of longer focal distance could be named only in fractions, $\frac{1}{2}$ being one-half of that strength; that is to say, having a focal distance of two inches, and $\frac{1}{12}$ having twelve inches focal length, while a stronger lens, greater than 1, had a corresponding shorter focal distance. Since the adoption of the metric system in ophthalmology, the opposite way of enumeration has been employed. A lens of the focal length of 1 metre is now taken as unit, and called one dioptric, or 1 D. Any lens of longer focal length is expressed in a decimal fraction, thus, 0.25 D means a lens of 4 metres focal length. The stronger lenses, on the other hand, are measured by several dioptrics; thus, 10 D, being a lens of $\frac{1}{10}$ metre, or 10 c.c. focal length. Since both systems of measurement are yet in use, it is best to become familiar with both. As regards the convenience in calculation, for which purpose the dioptric system was introduced, there is really not much difference between them. In order to convert the number of a lens from one system into another, it is to be remembered that the metre is equal to 39.37 English inches, or 36.94 French inches. On account of inaccuracies in grinding glasses, the whole numbers, 40 (English), and 36 (French), are close enough for practical purposes. Hence, to get the dioptric equivalent of an English number, divide that number into 40, and to translate a certain number of dioptrics into the French inch system, divide the number into 36.

21. In order to examine the refraction in the eye we must determine the refractive indices of the different media, the curvature of the surfaces, and the distances between the separate surfaces.

22. *Refractive Indices of the Media of the Eye.*—The refractive indices have been measured by various observers in dead eyes by means of different physical methods. On account of less perfect methods former results must be taken with some caution. Even the more recent determinations made with Abbe's refractometer have yielded not inconsiderable discrepancies among different observers, which may perhaps be attributed to individual variations. The table of refractive indices, given below, is copied from Zehender and Matthieson.[1] It is the most complete of all recent determinations.

The indices of cornea, aqueous humor, and vitreous body are so nearly alike that an average index of the three can be used as a basis for calculation without appreciable error. Helmholtz assumes 1.3365 as the most nearly correct average in his latest publication. While the index of the capsule of the lens is considerably above this figure, this membrane is so thin and its surfaces are so nearly parallel to each other that its influence on the rays can be neglected without error. The index of the lens itself increases from the external layer to the nucleus, so that the lens really consists of a large number of layers of increasing optic density. By reason of this stratified arrangement the total refractive power of the lens is greater than it would be were the entire lens of the refractive index of its nucleus. For each layer, as we proceed toward the nucleus, increases not only in refractive

TABLE OF REFRACTIVE INDICES (ZEHENDER AND MATTHIESON).

Subject.	Cornea.	Aqueous humor.	Anterior capsule of lens.	Lens.			Posterior capsule of lens.	Vitreous body.	Distilled water.
				Cortical layer.	Middle layer.	Nucleus.			
Male, 50 years, I	......	......	......	1.3953	1.4087	1.4121	1.3455		
II	1.3770	......	......	1.3853	1.4067	......	1.3658	1.3348	
Female, 45 years, I	......	......	......	......	1.4044	1.4112			
II	......	......	......	......	1.4044	1.4094			
Female, 26 years, I	......	......	1.3600	1.3867	1.4056	1.4154			
Male, I	1.3772	......	......	1.3902	1.4062	1.4077	......	1.3342	
Unknown, I	......	......	......	......	1.4076	1.4091			
II	......	......	......	......	......	1.4096			
Female, 45 years, I	......	......	......	1.3930	1.4018	1.4101			
II	......	......	......	1.3811	1.4073	1.4107			
Child of two days, I	1.3721	1.3338	1.3821	......	......	......	1.3503	1.3340	
II	......	......	1.3780	......	......	......	1.3572		
Average	1.3754	1.3338	1.3734	1.3886	1.4059	1.4106	1.3547	1.3343	1.3326