BROOKS

THE NORMAL ELEMENTARY ALGEBRA

H. Villareal

THE

NORMAL ELEMENTARY ALGEBRA:

CONTAINING THE

FIRST PRINCIPLES OF THE SCIENCE,

DEVELOPED WITH CONCISENESS AND SIMPLICITY,

FOR

COMMON SCHOOLS, ACADEMIES, SEMINARIES AND NORMAL SCHOOLS.

REVISED EDITION.

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"*Mathematical studies cultivate clearness of thought, acuteness of analysis and accuracy of expression.*"

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PREFACE.

Education aims at mental culture and practical skill; and for the attainment of both of these objects the mathematical sciences have in all ages been highly valued. They train the mind to logical methods of thought, give vigor and intensity to its operations, and lead to the important habit of resting only in certainty of results; while as instruments of investigation they stand pre-eminent.

Among the three fundamental branches of mathematics, Algebra occupies a prominent position in view of both of these objects. As a method of calculation it is the most powerful of them all, and for giving mental acuteness and the habit of analytic thought it is unequaled. With the advance of education this science is growing in popularity, and is being introduced into our best public schools, as well as academies and seminaries. Many teachers are beginning to see that a knowledge of elementary algebra is worth more than a knowledge of higher arithmetic, and are omitting the arithmetic, when necessary, for the algebra. This has increased the demand for good text-books upon the subject; and to assist in meeting this demand the present work has been prepared. Some of its general and special features will be briefly stated.

General Features.—This work is not a mere collection of problems and solutions, but the evolution of a carefully-matured plan, the embodiment of an ideal formed by long and thoughtful experience in the school-room. Attention is called to its extent, its matter and its method.

Extent.—The work embraces just about as many topics as it is thought the ordinary pupil in elementary algebra should be required to study. These topics have been presented, not superficially, but with comparative thoroughness, so that the knowledge given may be of actual use in calculation, and afford a basis for the study of a higher work if desired. While not presenting quite as much as some teachers might prefer, the author has been careful not to make the work too elementary. Superficial scholarship is one of the growing evils of our country, and teachers and text-books are responsible for it. It should never be forgotten that *it is better to know much of a few things than to know a little of many things.* While endeavoring to avoid superficiality, the author has been careful to so simplify the subject as to render it suitable to those beginning the study. The object has been to hit the golden mean, and thus adapt it to the wants of the majority of pupils and teachers.

Matter.—In the development of the various topics, care has been taken to combine in due proportion theory and practice. The French works on algebra are very complete in the discussion of principles, but are deficient in matter for the attainment of skill in their application. The English works abound with practical examples, but are usually less complete in their theoretical discussions. I have endeavored to combine both of these features, by giving an ample collection of problems, as well as a thorough discussion of principles. The aim has been to make the work both philosophical and practical.

The problems were prepared with especial reference to the principles and methods which they are designed to illustrate. They are often so related that one problem prepares the way for the succeeding one, thus making a problem which alone might be quite difficult, comparatively easy. The miscellaneous examples at the close of the book embrace a choice selection from American and English works, especially from the excellent collection given by Todhunter in his Elementary Algebra.

Method of Treatment.—In the treatment of the various topics I have aimed to simplify the subject without impairing its logical completeness and thoroughness. In this respect the skill of an author is particularly shown, and in this consist the principal merits of a text-book. The difficulty of a subject is not so much in itself as in the manner in which it is presented; special pains have therefore been taken to pass so gradually from the simple to the complex as to make easy what otherwise might be complicated and difficult.

I have also been careful to give conciseness, clearness and simplicity to its methods of explanation. There is a simple and direct method of stating a solution or demonstration that is much more readily comprehended by the learner than a talking about it in a popular sort of style. The scientific method is usually the simplest method. Text-books on algebra have been especially defective in this respect. The methods of mental arithmetic have created a revolution in the forms of explanation in the science of numbers, giving beauty and simplicity to that which was before awkward and complicated. Geometry, coming to us as the product of the Greek mind, is characterized by the simplicity and elegance of its demonstrations; while algebra, mainly the product of modern thought, has been less clear, logical and finished in its methods of development. I have endeavored to make an improvement in this respect, by combining the simple and natural analytical methods of mental arithmetic with the elegance of form and logical exactness of the geometrical methods.

Special Features.—The principal merits of the work are supposed to consist in its methods of treatment, its solutions, discussions and explanations—in brief, in the general spirit that pervades it, giving simplicity and unity to the work as a whole. There are, however, several special features to which the attention of teachers is respectfully invited:

1. The *definition of algebra* seems simpler and more complete than any which the author has met.

2. The *classification of algebraic symbols* under three distinct heads is different from anything heretofore presented.

3. The *method of explaining* addition and subtraction by means of an *auxiliary* quantity is a new feature which merits notice.

4. The new topic, called *Composition*, as a synthetic process correlative to the analytic process of *Factoring*, will no doubt arrest attention.

5. The variety of the cases in factoring, the demonstration of the divisibility of $a^n - b^n$ by $a - b$, and the explanation of the greatest common divisor and least common multiple, solicit notice.

6. The treatment of involution and evolution possesses some points of novelty, and especial attention is called to the second method of cube root.

7. Particular attention is invited to the discussion of the Courier Problem, and to the development of the properties of the incomplete quadratic equation.

8. The variety and appropriateness of the problems, the frequent generalization from a particular to a general problem, and the variation of old problems, are designed to add interest to the study and give discipline to the student.

9. Unusual care has been given to the typography of the work, in order to make it attractive and interesting to the pupil. The introduction of the small symbols for addition, subtraction, multiplication and division, as used in the best English works, is regarded as a great improvement.

Encouraged by the approval of many of the best teachers of the country who have used my former works, and yielding to their wishes as well as my own inclination, I have found time and strength, amid the cares and duties of a large institution, to prepare the present work; and I now send it forth, trusting that it may afford discipline and knowledge to many youthful minds of the present generation, and convey a kindly remembrance of my own labors to teachers and pupils of the future.

State Normal School,
Millersville, Pa., May, 1871.

EDWARD BROOKS.

SUGGESTIONS TO TEACHERS.

THE following suggestions are respectfully made for the benefit of young or inexperienced teachers:

1. It will be well to make frequent use of the inductive method of teaching, as suggested in the *Introduction*, leading pupils from the ideas and methods of arithmetic to those of algebra.

2. Drill thoroughly upon the fundamental operations, especially upon the use of the minus sign, the use of exponents in multiplication and division, and the methods of factoring. Slowness here is speed afterward.

3. Where there are two solutions, the teacher may select the one he prefers, and drill the pupils thoroughly upon it before they attempt the other. The first of two given solutions is usually regarded as the simplest, though not always the best.

4. Frequently require pupils to change a particular problem into a general one, as on page 107, and also to make special problems out of general ones. Have pupils to make problems by changing the conditions of given problems, or by using a required condition and requiring some given condition, as in problems on page 111. Pupils should be encouraged to form new problems, and to originate new methods of explanation and solution. We should always aim to make *thinkers* of our pupils rather than *mathematical machines*.

5. *A Shorter Course.*—While this work is the author's ideal of the extent of an elementary algebra, yet it may be used by teachers who desire a shorter course. For such the following omissions may be made without impairing the unity of the subject:

Omit the second methods of Greatest Common Divisor and Least Common Multiple, the Supplement to Simple Equations, Imaginary Quantities, Principles of Quadratics, Quadratics of Two Unknown Quantities, and the Miscellaneous Examples at the close of the work. A still shorter course may be attained by the further omission of the latter half of the examples under each subject.

6. In conclusion, the author suggests to teachers of public schools to give their pupils a course in elementary algebra before completing higher arithmetic. His own experience is, that pupils cannot thoroughly understand arithmetic until they have studied algebra.

THE

NORMAL ELEMENTARY ALGEBRA.

INTRODUCTION.

THE object of the exercises in the *Introduction* is to lead pupils from the ideas and operations of Arithmetic to those of Algebra.

LESSON I.

1. Henry's number of apples, increased by three times his number, equals 24; how many apples has he?

SOLUTION. By arithmetic this problem is solved in the following manner:

Henry's number, plus *three times his number*, equals 24;
hence, 4 times *Henry's number* equals 24,
and once *Henry's number* equals one-fourth of 24, or 6.

ABBREVIATED SOLUTION. If we represent the expression *Henry's number* by some character, as the letter x, the solution will be much shorter; thus:

x plus 3 times x equals 24;
hence, 4 times x equals 24,
and once x equals 6.

ALGEBRAIC SOLUTION. If we use $3x$ and $4x$ to denote "3 *times* x" and "4 *times* x," the sign = for the word "equals," and the sign + for the word "plus," the solution will be purely algebraic; thus:

$$x+3x=24;$$
$$4x=24,$$
$$x=6. \quad \textit{Ans.}$$

Symbols.—It will be seen that this last solution is the same as the first, except that we use *characters* instead of *words*.

These characters are called *symbols*. The method of solving problems by means of symbols is called *Algebra*.

Addition is denoted by the symbol +, read *plus*; *subtraction* is denoted by the symbol −, read *minus*.

The expressions $2x$, $3x$, $4x$, etc., mean 2 *times* x, 3 *times* x 4 *times* x, etc. *One-half* of x, *one-third* of x, *two-thirds* of x, etc., are denoted by $\frac{1}{2}x$, $\frac{1}{3}x$, $\frac{2}{3}x$, etc., or $\frac{x}{2}$, $\frac{x}{3}$, $\frac{2x}{3}$, etc.

The symbol = denotes *equality*, and is read *equals*, or *is equal to*. The expression $x+3x=24$ is called an *equation*.

The pupil will now express the following in algebraic symbols:

2. Three times John's number of apples equals 27.
3. One-half of Mary's number of peaches equals 12.
4. A's number of books, plus three times his number, equals 16.
5. B's number of dollars, minus half his number, equals 18.
6. Two-thirds of C's fortune, minus one-half of his fortune, equals $50.

Note.—The pupil may be led to see, from the solution given, which is the *unknown* quantity and which the *known* quantity, and how each is represented.

Also, that the symbols of Algebra are of three classes: symbols of *quantity*, symbols of *operation*, and symbols of *relation*.

LESSON II.

1. John has a certain number of peaches, and James has three times as many, and they both have 40; how many has each?

Solution. Let x equal John's number; then, since James has 3 times as many as John, $3x$ will equal James' number, and since they together have 40, $x+3x$ will equal 40, or, adding, $4x$ will equal 40. If $4x$ equals 40, x will equal one-fourth of 40, which is 10, John's number; and $3x$ will equal 3 times 10, or 30, James' number.

OPERATION.

Let x = John's number.
$3x$ = James' number.
$x+3x=40$
$4x=40$
$x=10$, John's number.
$3x=30$, James' number

2. Mary's age is twice Sarah's, and the sum of their ages is 36 years; what is the age of each?
Ans. Sarah, 12 years; Mary, 24 years.

3. A man bought a coat and a vest for $40, and the coat cost 4 times as much as the vest; required the cost of each.
Ans. Coat, $32; vest, $8.

4. In a mixture of 360 bushels of grain there is 5 times as much wheat as corn; how many bushels of each?
Ans. Wheat, 300 bushels; corn, 60 bushels.

5. Divide the number 144 into two parts, such that the larger part will be 5 times the smaller part. *Ans.* 120; 24.

6. The sum of two numbers is 120, and the larger number is 4 times the smaller number; required the two numbers.
Ans. 24; 96.

7. A man bought a span of horses and a carriage for $1000, paying three times as much for the horses as the carriage; required the cost of each. *Ans.* Horses, $750; carriage, $250.

8. The salary of a clerk for a year was $1500, and he spent five times as much of it as he saved; how much did he save?
Ans. $250.

LESSON III.

1. The difference of two numbers is 24, and the larger equals 4 times the smaller; required the numbers.

Solution. Let x equal the smaller number; then $4x$ will equal the larger number. And since the difference of the two numbers is 24, $4x-x$ will equal 24, or, subtracting, $3x$ will equal 24. If $3x$ equals 24, x will equal one-third of 24, which is 8, the smaller number; and $4x$ will equal 4 times 8, or 32, the larger number.

OPERATION.

Let x = the smaller.
$4x$ = the larger.
$4x-x=24$
$3x=24$
$x=8$, the smaller
$4x=32$, the larger.

2. The difference of two numbers is 28; and 5 times the smaller equals the greater; what are the numbers?
Ans. Smaller, 7; greater, 35.

3. A has 28 cents more than B, and 3 times B's number equals A's; how many has each? *Ans.* A, 42; B, 14.

4. Mary gathered 21 flowers more than her sister; how many did each gather if Mary gathered 4 times as many as her sister? *Ans.* Mary, 28; sister, 7.

5. Seven times a number, diminished by 3 times the number, equals 48; what is the number? *Ans.* 12.

6. Marie has 40 cherries more than Jane, and 5 times Jane's number equals Marie's number; how many has each? *Ans.* Marie, 50; Jane, 10.

7. A bought a house and lot, paying 5 times as much for the house as the lot; what did he pay for each if the house cost $2560 more than the lot? *Ans.* House, $3200; lot, $640.

8. A and B enter into a copartnership, in which A's interest is 6 times as great as B's: A's gain was $650 more than B's gain; what was the gain of each? *Ans.* A's, $780; B's, $130.

LESSON IV.

1. Julia and Anna had 24 oranges, and Julia had one-half as many as Anna; how many had each?

SOLUTION. Let x equal Anna's number; then will $\frac{x}{2}$ equal Julia's number; and since they together have 24, $x+\frac{x}{2}$ equals 24. Adding, x plus $\frac{1}{2}$ of x, or $\frac{3}{2}$ of x, equals 24. If $\frac{3}{2}$ of x equals 24, $\frac{1}{2}$ of x equals $\frac{1}{3}$ of 24, which is 8, Julia's number; and $\frac{2}{2}$ of x, or x, equals 2 times 8, or 16, Anna's number.

OPERATION.

Let x = Anna's number.

$\frac{x}{2}$ = Julia's number.

$$x+\frac{x}{2}=24$$

$$\frac{3x}{2}=24$$

$\frac{x}{2}=8$, Julia's number.

$x=16$, Anna's number.

2. A's money, increased by one-half of his money, equals $60; what is his money? *Ans.* $40.

3. What number is that to which if its one-third be added, the sum will be 36? *Ans.* 27.

4. What number is that to which if its two-thirds be added, the sum will be 45? *Ans.* 27.

5. What number is that which being diminished by its three-eighths, the remainder will be 30? *Ans.* 48.

6. Mary's age, diminished by its three-fifths, equals 6 years; how old is Mary? *Ans.* 15 years.

7. If one-half of my age be increased by one-third of my age, the sum will be 40 years; what is my age? *Ans.* 48 years.

8. Four times the distance from Philadelphia to Lancaster, diminished by $2\frac{1}{2}$ times the distance, equals 102 miles; required the distance. *Ans.* 68 miles.

9. Benton lost four-fifths of his money, and then found three-fourths as much as he lost, and then had $120; how much money had he at first? *Ans.* $150.

10. Bessie gave three-fourths of her money to the poor, and then found two-thirds as much as she gave away, and then had $30; how much had she at first? *Ans.* $40.

LESSON V.

1. A man bought a hat, vest and coat for $35; the vest cost twice as much as the hat, and the coat cost four times as much as the hat; required the cost of each.

SOLUTION. Let x equal the cost of the hat; then will $2x$ equal the cost of the vest, and $4x$ equal the cost of the coat, and their sum, $x+2x+4x$, will equal the cost of all, or $35. Adding, we have $7x$ equals $35. If $7x$ equals $35, x equals one-seventh of $35, or $5, the cost of the hat; $2x$ equals 2 times $5, or $10, the cost of the vest; and $4x$ equals 4 times $5, or $20, the cost of the coat.

OPERATION.

Let x = cost of the hat.

$2x$ = cost of the vest.

$4x$ = cost of the coat.

$$x+2x+4x=35$$

$$7x=35$$

$x=5$, hat;

$2x=10$, vest;

$4x=20$, coat.

2. Divide the number 105 into three such parts that the first shall be twice the second and the second twice the third. *Ans.* 60; 30; 15.

3. Three men, A, B and C, earned $216; A earned twice as much as B, and C earned as much as both A and B; how much did each earn? *Ans.* A, $72; B, $36; C, $108.

4. The sum of three numbers is 63; the second is one-half of the first, and the third one-fourth of the first; what are the numbers? *Ans.* 1st, 36; 2d, 18; 3d, 9.

5. A man, with his wife and son, earned $22 in a week; the man earned twice as much as his wife, and three times as much as his son; what did each earn?

Ans. Man, $12; wife, $6; son, $4.

6. A man bought a horse, a cow and a sheep for $315; the cow cost 5 times as much as the sheep, and the horse cost three times as much as the cow; required the cost of each.

Ans. Horse, $225; cow, $75; sheep, $15.

7. A tax of $450 is assessed upon three persons according to the relative value of their property; A is worth two-thirds as much as B, and B is worth three-fourths as much as C; what is each man's tax? *Ans.* A's, $100; B's, $150; C's, $200.

LESSON VI.

1. A being asked how much money he had, replied that three times his money increased by $8 equals $80; how much had he?

Solution. Let x equal A's money; then, by the condition of the problem we shall have $3x+8=80$. Now, if $3x$ *increased* by 8 equals 80, $3x$ will equal 80 *diminished* by 8, which is 72; if $3x$ equals 72, x will equal one-third of 72, which is 24. Hence A had 24 dollars.

OPERATION.

Let x = A's money.

$$3x+8=80$$
$$3x=72$$
$$x=24$$

2. If three times a number increased by 12 equals 57, what is that number? *Ans.* 15.

3. If three-fourths of the distance from New York to Troy be diminished by 21 miles, the result will be 90 miles; what is the distance? *Ans.* 148 miles.

4. If A's age be increased by its two-thirds and 7 years more, it will equal 32 years; what is his age? *Ans.* 15 years.

5. If $2\frac{3}{4}$ times the money a boy spent on the Fourth of July be diminished by 40 cents, the result will be $5.65; how much did he spend? *Ans.* $2.20.

6. One-half of my fortune, plus one-third of it and $380 more, equals $2580; what is my fortune? *Ans.* $2640.

7. One-third of the trees in an orchard bear apples, one-fourth bear peaches, and the remainder, which is 100, bear plums; required the number of trees in the orchard. *Ans.* 240.

8. If 4 times what Mr. Jones spent during a summer vacation be diminished by three-fifths of the sum spent and $680 the result will be $5100; what did he spend? *Ans.* $1700.

LESSON VII.

1. Anna has 8 oranges more than William, and they together have 36; how many has each?

Solution. Let x equal William's number; then, since Anna has 8 oranges more than William, $x+8$ will equal Anna's number; and since they both have 36, x plus $x+8$ will equal 36. Adding, we have $2x+8=36$. If $2x$ *increased* by 8 equals 36, $2x$ will equal 36 *diminished* by 8, or 28. If $2x$ equals 28, x equals one-half of 28, or 14, William's number; and $x+8$ equals $14+8$, or 22, Anna's number.

OPERATION.

Let x = William's number.

$x+8$ = Anna's number.

$$x+x+8=36$$
$$2x+8=36$$
$$2x=28$$
$$x=14, \text{ William's number.}$$
$$x+8=22, \text{ Anna's number.}$$

2. A and B together have $35, and A's money, plus $9 equals B's; how much has each? *Ans.* A, $13; B, $22.

3. The sum of two numbers is 100, and the smaller number equals the larger diminished by 16; what are the numbers?

Ans. 42; 58.

4. A watch and chain cost $220, and the chain cost $20 less than five-sevenths of the cost of the watch; required the cost of each. *Ans.* Watch, $140; chain, $80.

5. A house and lot cost $5800; required the cost of each if the lot cost $300 more than three-eighths as much as the house. *Ans.* House, $4000; lot, $1800.

6. Blanche, Lidie and Kate went a-shopping and spent $70. Lidie spent $4 more than Kate, and Blanche spent $6 more than Kate; how much did each spend?

Ans. Blanche, $26; Lidie, $24; Kate, $20.

7. A, B and C contributed $125 to a Sabbath-school; A gave $10 less than twice as much as C, and B gave $10 more than twice as much as C; what did each contribute?

Ans. A, $40; B, $60; C, $25.

8. A lady bought a hat, cloak and shawl for \$78; what did she pay for each, supposing that the cloak cost twice as much as the hat, plus \$4, and the shawl twice as much as the cloak, lacking \$4? *Ans.* Hat, \$10; cloak, \$24; shawl, \$44.

LESSON VIII.

1. If the height of a tree be increased by its two-thirds and 10 feet more, the sum will be twice the height; what is the height of the tree?

SOLUTION. Let x equal the height of the tree; then, by the condition of the problem, we have the equation $x+\frac{2}{3}x+10=2x$. Adding, we have $\frac{5}{3}x+10=2x$. If $\frac{5}{3}x+10$ equals $2x$, then 10 will equal $2x$ minus $\frac{5}{3}x$, or $\frac{x}{3}$; or $\frac{x}{3}$ will equal 10; hence x equals three times 10, or 30.

OPERATION.

Let x = the height.

$$x+\frac{2x}{3}+10=2x$$
$$\frac{5}{3}x+10=2x$$
$$10=\frac{x}{3}$$
$$\frac{x}{3}=10$$
$$x=30, \text{ height.}$$

2. If twice the length of a pole be increased by two-thirds of its length and 8 feet more, the sum will equal three times its length; what is its length? *Ans.* 24 feet.

3. Three-fourths of Nelson's age, increased by 6 years, equals four-fifths of his age, increased by 5 years; how old is he? *Ans.* 20 years.

4. Five times the money Jennie paid for her bracelets, diminished by \$8, equals three times the money she paid, increased by \$8; what did she pay? *Ans.* \$8.

5. Emma bought a fan, shawl and cloak; the fan cost \$4; the shawl cost \$4 more than two-thirds of the cost of the cloak, and the cloak cost \$4 more than the fan and shawl; required the cost of the shawl and the cloak. *Ans.* Shawl, \$28; cloak, \$36.

6. A's money, plus \$12, equals B's, and B's, plus \$6, equals C's, and the sum of their moneys equals four and one-half times A's money; how much money has each? *Ans.* A, \$20; B, \$32; C, \$38.

7. Three times what it cost Harry to attend college a year increased by \$50 equals twice the sum obtained by increasing the amount by \$150; what did it cost him? *Ans.* \$250.

8. A man driving his geese to market was met by another, who said, "Good-morning, master, with your hundred geese." He replied, "I have not a *hundred*, but if I had as many more, and half as many more, and two geese and a half, I would have a hundred." How many geese had he? *Ans.* 39.

LESSON IX.

In Problem 1, Lesson I., we supposed Henry's number, plus three times his number, to be equal to *twenty-four*. Suppose now, instead of representing the number *twenty-four* by the figures 2 and 4, we use one of the first letters of the alphabet, as a, to represent it. The problem will then become—

1. Henry's number of apples, plus three times his number, equals a; how many apples has he?

SOLUTION. Let x equal Henry's number; then, by the condition of the problem, we will have $x+3x=a$, or $4x=a$; hence x equals a divided by 4, which we express thus, $\frac{a}{4}$.

OPERATION.

Let x = Henry's number.

$$x+3x=a$$
$$4x=a$$
$$x=\frac{a}{4}$$

Known Numbers.—Now, it is evident that a may represent any other number, as 12, 16, etc. Hence, we see we may represent *known* numbers by *letters* as well as by *figures*.

A number represented by figures expresses a *definite* number of units, and may therefore be called a *definite number*. A number represented by a letter does not express a definite number of units, and may therefore be called an *indefinite number*.

Substitution.—Since a represents any number, let us suppose, as at first, its value to be 24; if we then use 24 for a in the result $x=\frac{a}{4}$, we will have $x=\frac{24}{4}$, or 6, which is the same result as we obtained by using 24 in the solution in Lesson I.

This using some particular value of a general quantity in an expression containing the quantity is called *Substitution.*

PROBLEMS

2. Mary's age is twice Sarah's, and the sum of their ages is a years; how old is each? *Ans.* Sarah, $\frac{a}{3}$; Mary, $\frac{2a}{3}$.

3. Find the age of each when $a=36$, by substituting the value of a in the result. *Ans.* 12 years; 24 years.

4. Divide the number m into two parts, such that the larger part will be 5 times the smaller. *Ans.* $\frac{m}{6}$; $\frac{5m}{6}$.

5. Find the value of each part when $m=144$, by substituting the value of m in the results. *Ans.* 24; 120.

6. The difference of two numbers is a, and 5 times the smaller equals the larger; what are the numbers? *Ans.* $\frac{a}{4}$; $\frac{5a}{4}$.

7. Find the value of each part when $a=24$, by substituting the value of a in the results. *Ans.* 6; 30.

8. What number is that to which if its one-third be added the sum will be b? *Ans.* $\frac{3b}{4}$.

9. Divide the number c into three such parts that the first shall be twice the second, and the second twice the third.

Ans. 1st, $\frac{4c}{7}$; 2d, $\frac{2c}{7}$; 3d, $\frac{c}{7}$.

10. If three times a number increased by n equals a, what is the number? *Ans.* $\frac{a-n}{3}$.

11. The sum of two numbers is a, and the smaller equals the larger diminished by c; what are the numbers?

Ans. $\frac{a+c}{2}$; $\frac{a-c}{2}$.

12. One-half the length of a pole is in the mud, one-third in the water, and h feet in the air; what is the length of the pole? *Ans.* $6h$.

NOTE.—Let the pupil give special values to the general quantities in each of the above problems, and find the results by Substitution.

ELEMENTARY ALGEBRA.

SECTION I.

DEFINITIONS AND EXPLANATIONS.

1. Mathematics is the science of quantity. It treats of the properties and relations of quantity.

2. Quantity is anything that can be measured. It is of two kinds, *Number* and *Extension.*

3. Arithmetic is the science of *Number; Geometry* is the science of *Extension.*

4. Algebra is a method of investigating quantity by means of general characters called *symbols.*

5. Algebraic Symbols are the characters used to represent quantities, their relations and the operations performed upon them.

6. The **Symbols of Algebra** are of three kinds, namely—

1. Symbols of Quantity; 2. Symbols of Operation;
3. Symbols of Relation.

NOTES.—1. With beginners we regard Algebra as restricted to *numbers,* or as a kind of *general Arithmetic.* They may afterward be led to see how general symbols introduce ideas not found in Arithmetic; and eventually, that Algebra is a general method of investigation that may be applied to all kinds of quantity.

2. Some writers divide Algebra into *Arithmetical* Algebra and *Symbolical* Algebra. Newton called it *Universal Arithmetic,* and many writers speak of it as *General Arithmetic.* D'Alembert divides Arithmetic into *Numérique,* Special Arithmetic, and *Algebra,* General Arithmetic.