MISCELLANEOUS EXAMPLES.

1. Add $(a+2b)x^n$ and $(2a-b)x^n$. *Ans.* $(3a+b)x^n$.

2. Subtract $3(m-an)$ from $3a(m-n)$. *Ans.* $3m(a-1)$.

3. Multiply a^{n-2} by $3a^{m+2}$. *Ans.* $3a^{m+n}$.

4. Multiply $a^{\frac{2}{3}}-b^{\frac{2}{3}}$ by $a^{\frac{2}{3}}+b^{\frac{2}{3}}$. *Ans.* $a^{\frac{4}{3}}-b^{\frac{4}{3}}$.

5. Divide a^nb^{m-n} by $a^{n-m}b^{-n}$. *Ans.* a^mb^n.

6. Divide $a^{\frac{3}{4}}-z^{\frac{3}{4}}$ by $a^{\frac{1}{4}}-z^{\frac{1}{4}}$. *Ans.* $a^{\frac{1}{2}}+a^{\frac{1}{4}}z^{\frac{1}{4}}+z^{\frac{1}{2}}$.

7. Divide a^n+b^n by $a^{\frac{n}{3}}+b^{\frac{n}{3}}$. *Ans.* $a^{\frac{2n}{3}}-a^{\frac{n}{3}}b^{\frac{n}{3}}+b^{\frac{2n}{3}}$.

8. Multiply $a^2+2ax-x^2$ by $a^2+2ax+x^2$.
Ans. $a^4+4a^3x+4a^2x^2-x^4$.

9. Divide $2a^4+27ab^3-81b^4$ by $a+3b$.
Ans. $2a^3-6a^2b+18ab^2-27b^3$.

10. Divide a^6-2a^3+1 by a^2-2a+1.
Ans. $a^4+2a^3+3a^2+2a+1$.

11. Value of $a-\{2b-(3c+2b-a)\}$. *Ans.* $3c$.

12. Value of $16-\{5-2x-[1-(3-x)]\}$. *Ans.* $9+3x$.

13. Value of $15x-\{4-[3-5x-(3x-7)]\}$. *Ans.* $7x+6$.

14. Value of $2x-[3y-\{4x-(5y-6x+7y)\}]$.
Ans. $12x-15y$.

15. Value of $a-[5b-\{a-(5c-\overline{2c-b}-4b)+2a-(a-\overline{2b+c})\}]$. *Ans.* $3a-2c$.

16. Prove $(a-b)^3+b^3-a^3=3ab(b-a)$.

17. Prove $(a^2+ab+b^2)^2-(a^2-ab+b^2)^2=4ab(a^2+b^2)$.

18. Prove $(a+b+c)^3-(a^3+b^3+c^3)=3(a+b)(b+c)(a+c)$.

19. Expand $(a^n-2)(a^n+2)(a^n+3)(a^n-3)$.
Ans $a^{4n}-13a^{2n}+36$.

20. Factor a^4+b^4; a^5-b^5; a^5+b^5; $a^{4n}-b^{4n}$.

21. Factor a^6-b^6; a^6+b^6; a^8-b^8; a^8+b^8; $a^{8n}-b^{8n}$.

22. Factor $a^2+9ab+20b^2$; $a^nc^n-b^nc^n+a^nd^n-b^nd^n$

23. Find the greatest common divisor of $a^2+8a+15$ and $a^2+9a+20$. *Ans.* $a+5$.

24. Find the greatest common divisor of $5(x^2-x+1)$, $4(x^6-1)$ and $2(x^3+1)$. *Ans.* x^2-x+1.

25. Find the least common multiple of x^3+1, x^3-1 and x^2-x+1. *Ans.* x^6-1.

26. Find the least common multiple of a^2-1, a^2+1, a^4+1 and a^8-1. *Ans.* a^8-1.

27. Value of $\dfrac{x^2+3x+2}{x^2+6x+5}$; $\dfrac{x^2+10x+21}{x^2-2x-15}$. *Ans.* $\dfrac{x+2}{x+5}$; $\dfrac{x+7}{x-5}$.

28. Value of $\dfrac{x^2+(a+b)x+ab}{x^2+(a+c)x+ac}$; $\dfrac{x^4+a^2x^2+a^4}{x^6-a^6}$.
Ans. $\dfrac{x+b}{x+c}$; $\dfrac{1}{x^2-a^2}$.

29. Value of $\dfrac{a-b}{b}+\dfrac{2a}{a-b}-\dfrac{a^3+a^2b}{a^2b-b^3}$. *Ans.* $\dfrac{b}{a-b}$.

30. Value of $\dfrac{a^2}{(a-b)(a-c)}+\dfrac{b^2}{(b-a)(b-c)}+\dfrac{c^2}{(c-a)(c-b)}$.
Ans. 1.

31. Value of $\left(b^n+\dfrac{a^{2n}}{b^n}\right)\left(a^n-\dfrac{b^{2n}}{a^n}\right)$. *Ans.* $\dfrac{a^{4n}-b^{4n}}{a^nb^n}$.

32. Value of $\dfrac{x^2+xy}{x^2+y^2}\times\left(\dfrac{x}{x-y}-\dfrac{y}{x+y}\right)$. *Ans.* $\dfrac{x}{x-y}$.

33. Value of $\left(\dfrac{x}{a}-\dfrac{a}{x}+\dfrac{y}{b}-\dfrac{b}{y}\right)\times\left(\dfrac{x}{a}-\dfrac{a}{x}-\dfrac{y}{b}+\dfrac{b}{y}\right)$.
Ans. $\dfrac{x^2}{a^2}+\dfrac{a^2}{x^2}-\dfrac{y^2}{b^2}-\dfrac{b^2}{y^2}$.

34. Value of $\dfrac{x^2+(a+c)x+ac}{x^2+(b+c)x+bc}\div\dfrac{x^2-a^2}{x^2-b^2}$. *Ans.* $\dfrac{x-b}{x-a}$.

35. Value of $\left(1+\dfrac{x}{y}\right)\left(1-\dfrac{x}{y}\right)\div\dfrac{y}{x^2+y^2}$. *Ans.* $\dfrac{y^4-x^4}{y^3}$

36. Value of $\left(a^3-\dfrac{1}{a^3}\right)\div\left(a-\dfrac{1}{a}\right)$. *Ans.* $\dfrac{a^4+a^2+1}{a^2}$.

37. Value of $\left(\frac{a^2}{x^2}+1+\frac{x^2}{a^2}\right) \div \left(\frac{a}{x}-1+\frac{x}{a}\right)$. *Ans.* $\frac{a^2+ax+x^2}{ax}$.

38. Value of $\dfrac{x-1+\dfrac{6}{x-6}}{x-2+\dfrac{3}{x-6}}$. *Ans.* $\frac{x-4}{x-5}$.

39. Value of $1+\dfrac{x}{1+x+\dfrac{2x^2}{1-x}}$. *Ans.* $\frac{1+x}{1+x^2}$.

40. Value of $\dfrac{1}{1+\dfrac{a}{1+a+\dfrac{2a^2}{1-a}}}$. *Ans.* $\frac{1+a^2}{1+a}$.

41. Value of $\frac{3x-9}{x^2-7x+12}$, when $x=3$. *Ans.* -3.

42. Value of $\frac{a^2-x^2}{(a-x)^2}$, when $x=a$. *Ans.* ∞.

43. Value of $\frac{ax^2+ac^2-2acx}{bx^2-2bcx+bc^2}$, when $x=c$. *Ans.* $\frac{a}{b}$.

44. Given $\frac{x+1}{3}-\frac{3x-1}{5}=x-2$, to find x. *Ans.* $x=2$.

45. Given $\frac{5x-7}{2}-\frac{2x+7}{3}=3x-14$, to find x. *Ans.* $x=7$.

46. Given $\frac{3x-4}{2}-\frac{6x-5}{8}=\frac{3x-1}{16}$, to find x. *Ans.* $x=2\frac{1}{3}$.

47. Given $5x-[8x-3\{16-6x-(4-5x)\}]=6$. *Ans.* $x=5$.

48. Given $\frac{x+3}{2}-\frac{x-2}{3}=\frac{3x-5}{12}+\frac{1}{4}$, to find x. *Ans.* $x=28$.

49. Given $\frac{3+x}{3-x}-\frac{2+x}{2-x}-\frac{1+x}{1-x}=1$, to find x. *Ans.* $x=1\frac{1}{2}$.

50. Given $\frac{x^2-x+1}{x-1}+\frac{x^2+x+1}{x+1}=2x$, to find x. *Ans.* $x=0$.

51. Given $\frac{a(a-x)}{b}-\frac{b(b+x)}{a}=x$, to find x. *Ans.* $x=a-b$.

52. Given $\frac{a(x-a)}{b}+\frac{b(x-b)}{a}=x$, to find x. *Ans.* $x=a+b$.

53. Given $\frac{1}{x-a}-\frac{1}{x-b}=\frac{a-b}{x^2-ab}$, to find x. *Ans.* $x=\frac{2ab}{a+b}$.

54. Given $\left\{\begin{array}{l} 2x+\frac{y-2}{5}=21 \\ 4y+\frac{x-4}{6}=29 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=10, \\ y=7. \end{array}\right.$

55. Given $\left\{\begin{array}{l} \frac{x+y}{3}+\frac{y-x}{2}=9 \\ \frac{x}{2}+\frac{x+y}{9}=5 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=6, \\ y=12. \end{array}\right.$

56. Given $\left\{\begin{array}{l} 3x+9y=2.4 \\ .21x-.06y=.03 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=.2, \\ y=.2. \end{array}\right.$

57. Given $\left\{\begin{array}{l} .3x+.125y=x-6 \\ 3x-.5y=28-.25y \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=10, \\ y=8. \end{array}\right.$

58. Given $\left\{\begin{array}{l} x+y=a+b \\ bx+ay=2ab \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=a, \\ y=b. \end{array}\right.$

59. Given $\left\{\begin{array}{l} \frac{x}{a}+\frac{y}{b}=c \\ \frac{x}{b}-\frac{y}{a}=0 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=\frac{ab^2c}{a^2+b^2}, \\ y=\frac{a^2bc}{a^2+b^2}. \end{array}\right.$

60. Given $\left\{\begin{array}{l} a(x+y)+b(x-y)=1 \\ a(x-y)+b(x+y)=1 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=\frac{1}{a+b}, \\ y=0. \end{array}\right.$

61. Given $\left\{\begin{array}{l} 4x-5y+z=6 \\ 7x-11y+2z=9 \\ x+y+3z=12 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=2, \\ y=1, \\ z=3. \end{array}\right.$

62. Given $\left\{\begin{array}{l} y+z=a \\ x+z=b \\ x+y=c \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=\frac{1}{2}(b+c-a) \\ y=\frac{1}{2}(a-b+c), \\ z=\frac{1}{2}(a+b-c). \end{array}\right.$

63. Given $\left\{\begin{array}{l} y+z-x=a \\ x+z-y=b \\ x+y-z=c \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=\frac{1}{2}(b+c), \\ y=\frac{1}{2}(a+c), \\ z=\frac{1}{2}(a+b). \end{array}\right.$

64. Given $\left.\begin{matrix}\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1\\ \frac{x}{a}+\frac{y}{c}+\frac{z}{b}=1\\ \frac{x}{b}+\frac{y}{a}+\frac{z}{c}=1\end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix}x\\y\\z\end{matrix}\right\}=\frac{abc}{ab+bc+ac}$.

65. Given $\left.\begin{matrix}\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=3\\ \frac{a}{x}+\frac{b}{y}-\frac{c}{z}=1\\ \frac{2a}{x}-\frac{b}{y}-\frac{c}{z}=0\end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix}x=a,\\y=b,\\z=c.\end{matrix}\right.$

66. Value $\sqrt{\left(\frac{a^{m+n}}{a^{m-n}}\right)}$; 5th root of $\frac{a}{2}\sqrt[4]{\frac{a}{2}}$. *Ans.* a^n; $\frac{1}{2}\sqrt[4]{8a}$.

67. Value $(ax+by)^3+(ax-by)^3$. *Ans.* $2a^3x^3+6axb^2y^2$.

68. Value $(a+b+c+d)^2-(a-b+c-d)^2$. *Ans.* $4(a+c)(b+d)$.

69. Square root of $a^6-12a^5+60a^4-160a^3+240a^2-192a+64$. *Ans.* $a^3-6a^2+12a-8$.

70. Fourth root of $16a^4-96a^3y+216a^2y^2-216ay^3+81y^4$. *Ans.* $2a-3y$.

71. Cube root of $x^6+3x^5+6x^4+7x^3+6x^2+3x+1$. *Ans.* x^2+x+1.

72. Sixth root of $1+12a+60a^2+160a^3+240a^4+192a^5+64a^6$. *Ans.* $1+2a$.

73. Simplify $\{-(x^3)^{\frac{1}{2}}\}^{-\frac{1}{3}}\times\{-(-x)^{-3}\}^{\frac{1}{2}}$. *Ans.* $-x^{-2}$.

74. If $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}$, and $y=\frac{\sqrt{3}-1}{\sqrt{3}+1}$, find the value of the expression x^2+xy+y^2. *Ans.* 15.

75. Given $\sqrt{(x-a)}=\frac{a}{\sqrt{(x+a)}}$, to find x. *Ans.* $x=\pm a\sqrt{2}$.

76. Given $\frac{x-4}{2x+1}=\frac{2x-1}{x+4}$, to find x. *Ans.* $x=\pm\sqrt{(-5)}$.

77. Given $\frac{\sqrt{x}-2}{\sqrt{x}+10}=\frac{\sqrt{x}-1}{\sqrt{x}+23}$, to find x. *Ans.* $x=9$.

78. Given $4x-\frac{12-x}{x-3}=22$, to find x. *Ans.* $x=6$, or $2\frac{1}{4}$.

79. Given $\frac{2x+11}{x}=5-\frac{x-5}{3}$, to find x. *Ans.* $x=11$, or 3.

80 Given $\frac{x-2}{x-3}-\frac{x-4}{x-1}=\frac{14}{15}$, to find x. *Ans.* $x=6$, or $2\frac{2}{7}$.

81. Given $\frac{x-3}{x-2}-\frac{x-1}{x-4}=-\frac{6}{5}$, to find x. *Ans.* $x=7$, or $2\frac{1}{3}$.

82. Given $\frac{x-1}{x-4}-\frac{x-3}{x-2}=\frac{11}{12}$, to find x. *Ans.* $x=8$, or $2\frac{4}{11}$.

83. Given $\frac{x+1}{x+2}+\frac{x-1}{x-2}=\frac{2x-1}{x-1}$. *Ans.* $x=4$, or 0.

84. Given $\frac{x-2}{x+2}+\frac{x+2}{x-2}=\frac{2(x+3)}{x-3}$. *Ans.* $x=1\frac{1}{3}$, or 0.

85. Given $\frac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}=b$. *Ans.* $x=\pm\frac{2a\sqrt{b}}{1+b}$.

86. Given $\left\{\begin{matrix}x+y=4(x-y)\\xy=15\end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix}x=\pm5,\\y=\pm3.\end{matrix}\right.$

87. Given $\left\{\begin{matrix}x^4+y^4=97\\4x^2=9y^2\end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix}x=\pm3,\\y=\pm2.\end{matrix}\right.$

88. Given $\left\{\begin{matrix}x+y=3(x-y)\\x^3-y^3=56\end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix}x=4,\\y=2.\end{matrix}\right.$

89. Given $\frac{x+a}{x-a}-\frac{x-a}{x+a}=\frac{b+x}{b-x}-\frac{b-x}{b+x}$. *Ans.* $x=\pm\sqrt{(ab)}$.

90 Given $\left\{\begin{matrix}x-y=1\\x^2-xy+y^2=21\end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix}x=5,\text{ or }-4,\\y=4,\text{ or }-5.\end{matrix}\right.$

91. Given $\left\{\begin{matrix}3x+2y=5xy\\15x-4y=4xy\end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix}x=\frac{2}{3},\text{ or }0,\\y=\frac{3}{2},\text{ or }0.\end{matrix}\right.$

92. Given $\left\{\begin{matrix} x+y=xy \\ ax=by \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=\frac{a+b}{a}, \text{ or } 0, \\ y=\frac{a+b}{b}, \text{ or } 0. \end{matrix}\right.$

93. Given $\left\{\begin{matrix} \frac{x}{a}+\frac{y}{b}=2 \\ x^2+y^2=ax+by \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=a, \text{ or } \frac{2ab^2}{a^2+b^2}, \\ y=b, \text{ or } \frac{2a^2b}{a^2+b^2}. \end{matrix}\right.$

94. Given $\left\{\begin{matrix} x^2+xy=28 \\ xy-y^2=3 \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=\pm 4, \text{ or } \pm\frac{7}{2}\sqrt{2}, \\ y=\pm 3, \text{ or } \pm\frac{1}{2}\sqrt{2}. \end{matrix}\right.$

95. Given $\left\{\begin{matrix} x+y+\sqrt{x+y}=12 \\ x^2+y^2=41 \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=5, \text{ or } 4, \\ y=4, \text{ or } 5. \end{matrix}\right.$

96. Given $\left\{\begin{matrix} x^2+y^2+2x+2y=23 \\ xy=6 \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=3, \text{ or } 2, \\ y=2, \text{ or } 3. \end{matrix}\right.$

97. Given $\left\{\begin{matrix} x^2-y^2 : x-y :: 6 : 1 \\ xy=8 \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=4, \\ y=2. \end{matrix}\right.$

98. Given $\left\{\begin{matrix} x^3-y^3 : (x-y)^3 :: 61 : 1 \\ xy=20 \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=5, \\ y=4. \end{matrix}\right.$

99. Given $\left\{\begin{matrix} x^3+y^3 : x^3-y^3 :: 210 : 114 \\ xy^3=24 \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=3, \\ y=2. \end{matrix}\right.$

100. Given $\left\{\begin{matrix} x^2+y^2=34 \\ x^2-y^2+\sqrt{(x^2-y^2)}=20 \end{matrix}\right\}$. *Ans.* $\left\{\begin{matrix} x=\pm 5, \\ y=\pm 3. \end{matrix}\right.$

MISCELLANEOUS PROBLEMS.

1. A child was born in November, and on the tenth day of December he is as many days old as the month was on the day of his birth; when was he born? *Ans.* Nov. 20th.

2. After A has received \$10 from B he has as much money as B and \$6 more; and between them they have \$40; how much money had each at first? *Ans.* A, \$13; B, \$27.

3. A father has 6 sons, each of whom is 4 years older than his next younger brother, and the eldest is 3 times as old as the youngest; find their respective ages.

Ans. 10 yrs., 14 yrs., 18 yrs., 22 yrs., 26 yrs., 30 yrs.

4. Four men club to buy a set of ten-pins, but by clubbing with 2 more the expense of each is diminished \$1.75; what did the set cost? *Ans.* \$21.

5. A pudding consists of 2 parts of flour, 3 parts of raisins and 4 parts of suet; flour costs 3d. a pound, raisins 6d., and suet 8d.; find the cost of the several ingredients of the pudding when the whole cost is 2s. 4d. *Ans.* 3d., 9d., 1s. 4d.

6. A market-woman being asked what she paid for eggs, replied, "Six dozen eggs cost as many pence as you can buy eggs for eightpence." What was the price a dozen?

Ans. 4d.

7. The tens' digit of a number is less by 2 than the units digit, and if the digits are inverted the new number is to the former as 7 is to 4; find the number. *Ans.* 24.

8. In paying two bills, one of which exceeded the other by $\frac{1}{3}$ of the less, the change out of a five-dollar note was $\frac{1}{2}$ the difference of the bills; find the amount of each bill.

Ans. \$2; \$2$\frac{2}{3}$.

9. Two persons, A and B, own together 175 shares in a railway company: they agree to divide, and A takes 85 shares, while B takes 90 shares and pays \$250 to A; find the value of a share. *Ans.* \$100.

10. Find the fraction such that if you quadruple the numerator and add 3 to the denominator, the fraction is doubled; but if you add 2 to the numerator and quadruple the denominator, the fraction is halved. *Ans.* $\frac{2}{3}$.

11. How many sheep must a person buy at \$35 each that, after paying \$1.50 a score for folding them at night, he may gain \$394 by selling them at \$40 each? *Ans.* 80.

12. A colonel, on attempting to draw up his regiment in the form of a solid square, finds that he has 31 men over, and that he would require 24 men more in his regiment in order to increase the side of the square by 1 man; how many men were there in the regiment? *Ans.* 760.

13. In a certain weight of gunpowder the saltpetre com-

posed 6 pounds more than $\frac{1}{2}$ of the weight, the sulphur 5 pounds less than $\frac{1}{3}$, and the charcoal 3 pounds less than $\frac{1}{4}$; how many pounds were there of each of the three ingredients?

Ans. 18; 3; 3.

14. A cistern could be filled in 12 minutes by 2 pipes which run into it; and it could be filled in 20 minutes by 1 alone; in what time could it be filled by the other alone?

Ans. 30 min.

15. Divide the number 88 into 4 parts, such that the first increased by 2, the second diminished by 3, the third multiplied by 4, and the fourth divided by 5, may all be equal.

Ans. 10, 15, 3, 60.

16. A and B began to play together with equal sums of money: A first won \$100, but afterward lost $\frac{1}{2}$ of all he then had, and then his money was $\frac{1}{2}$ as much as that of B; what money had each at first? *Ans.* \$300.

17. A and B shoot by turns at a target: A puts 7 bullets out of 12 into the bull's-eye, and B puts 9 out of 12; between them they put in 32 bullets; how many shots did each fire?

Ans. 24.

18. A person buys a piece of land at \$150 an acre, and by selling it in lots finds the value increased threefold, so that he clears \$750, and retains 25 acres for himself; how many acres were there? *Ans.* 40.

19. A and B play at a game, agreeing that the loser shall always pay to the winner \$1 less than $\frac{1}{2}$ the money the loser has: they commence with equal sums of money, and after B has lost the first game and won the second he has \$2 more than A; how much had each at the commencement?

Ans. \$6.

20. It is between 11 and 12 o'clock, and it is observed that the number of minute-spaces between the hands is $\frac{2}{3}$ of what it was ten minutes previously; find the time.

Ans. 20 min. of 12 o'clock.

21. A clock has two hands turning on the same centre: the swifter makes a revolution every 12 hours, and the slower every 16 hours; in what time will the swifter gain one complete revolution on the slower? *Ans.* 48 hrs.

22. An officer can form his men into a hollow square 4 deep, and also into a hollow square 8 deep: the front in the latter formation contains 16 men fewer than in the former formation; find the number of men. *Ans.* 640.

23. A certain number of 2 digits is equal to 4 times the sum of its digits; and if 18 be added to the number the digits are reversed; find the number. *Ans.* 24.

24. Prove that if 2 numbers differ by a, the difference of their squares equals $2a$ times their arithmetical mean.

25. If two integers differ by 2, show that the difference of their squares equals 4 times the integer between them.

26. The two digits which form a number change places on the addition of 9, and the sum of the original and the resulting number is 33; find the digits. *Ans.* 1 and 2.

27. Which is the greater, and how much—the square of a number, or the product of the number a unit less than it by the number a unit greater than it?

Ans. The square is 1 greater.

28. If a certain rectangular floor had been 2 feet broader and 3 feet longer, it would have been 64 square feet larger; but if it had been 3 feet broader and 2 feet longer, it would have been 68 square feet larger; find the length and breadth.

Ans. Length, 14 ft.; breadth, 10 ft.

29. When a certain number of 2 digits is doubled and increased by 36, the result is the same as if the number had been reversed and doubled, and then diminished by 36; also the number itself exceeds 4 times the sum of its digits by 3; find the number. *Ans.* 59.

30. A sets out from M to N, going $3\frac{1}{2}$ miles an hour; forty minutes afterward B sets out from N to M, going $4\frac{1}{2}$ miles an hour, and he goes half a mile beyond the middle point before he meets A; find the distance between M and N. *Ans.* 29 mi.

31. A person walked a certain distance at the rate of $3\frac{1}{2}$

miles an hour, and then ran part of the way back at the rate of 7 miles an hour, walking the remaining distance in 5 minutes: he was gone 25 minutes; how far did he run?

Ans. $\frac{7}{12}$ mi.

32. Two trains, 92 feet long and 84 feet long respectively, are moving with uniform velocities on parallel rails: when they move in opposite directions they are observed to pass each other in $1\frac{1}{2}$ seconds, but when they move in the same direction the faster train is observed to pass the other in 6 seconds; find the rate at which each train moves.

Ans. 30 mi. and 50 mi. per hour.

33. If the sum of 2 fractions is unity, show that the first, together with the square of the second, is equal to the second, together with the square of the first.

34. A man has a rectangular field containing 4 acres whose length is to its breadth as 8 : 5; required the length and breadth of the field. *Ans.* 32 rods; 20 rods.

35. The sum of two numbers is 17, and the less divided by the greater is to the greater divided by the less as 64 : 81; what are the numbers? *Ans.* 8 and 9.

36. There are two quantities whose product is a and quotient b; required the quantities. *Ans.* $\pm\sqrt{ab}$ and $\pm\sqrt{\frac{a}{b}}$.

37. A father gave to each of his children on New Year's day as many books as he had children: for each book he gave 12 times as many cents as there were children, and the cost of the whole was \$15; how many children had he? *Ans.* 5.

38. A man bought a field whose length was to its breadth as 3 to 2; the price per acre was equal to the number of rods in the length of the field, and $4\frac{1}{2}$ times the distance around the field equaled the number of dollars that it cost; what was the length and breadth of the field? *Ans.* 60 rods; 40 rods.

39. A and B carried 100 eggs to market, and each received the same sum: if A had carried as many as B, he would have received 36 cents for them, and if B had carried only as many as A, he would have received only 16 cents for them, how many eggs had each? *Ans.* A, 40; B, 60.

40. Several gentlemen made an excursion, each taking \$484: each had as many servants as there were gentlemen, and the number of dollars which each had was 4 times the number of all the servants; how many gentlemen were there?

Ans. 11.

41. There is a rectangular field whose breadth is $\frac{4}{5}$ of the length. After laying out $\frac{1}{5}$ of the whole ground for a garden, it was found that there were left 400 square rods for mowing; required the length and breadth of the field.

Ans. 25 rods; 20 rods.

42. There are two square grass-plats, a side of one of which is 10 yards longer than a side of the other, and their areas are as 25 to 9; what are the lengths of the sides?

Ans. 25 yds.; 15 yds.

43. A farmer bought a number of sheep for \$80: if he had bought 4 more for the same money, he would have paid \$1 less for each; how many did he buy? *Ans.* 16 sheep.

44. A man divided 110 bushels of coal among a number of poor persons: if each had received 1 bushel more, he would have received as many bushels as there were persons; find the number of persons. *Ans.* 11.

45. A person bought a certain number of yards of cloth for \$36, which he sold again at \$4 per yard, and gained as much on the whole as 4 yards cost; find the number of yards.

Ans. 12.

46. A cistern can be supplied with water by two pipes, one of which would fill it 6 hours sooner than the other, and they both would fill it in 4 hours; how long will it take each pipe alone to fill it? *Ans.* 6 hrs.: 12 hrs.

47. The side of a square is 110 inches long; find the dimensions of a rectangle which shall have its perimeter 4 inches longer than that of the square, and its area 4 square inches less than that of the square.

Ans. Length, 126 in.; breadth, 96 in.

48. The third term of an arithmetical progression is 4 times the first term, and the sixth term is 17; required the first six terms of the series. *Ans.* 2, 5, 8, etc.

49. A square tract of land contains $\frac{1}{8}$ as many acres as there are rods in the fence enclosing it; required the length of the fence. *Ans.* 320 rods.

50. An English woman sells eggs at such a price that 10 more in half a crown's worth lowers the price threepence per score; required the price per score. *Ans.* 15d.

51. An English woman sells eggs at such a price that 10 fewer in half a crown's worth raises the price threepence per score; required the price per score. *Ans.* 12d

52. Find the side of a cube which shall contain 4 times as many solid units as there are linear units in the distance between its two opposite corners. *Ans.* $2\sqrt{3}$.

53. A grass-plat, 18 yards long and 12 wide, is surrounded by a border of flowers of uniform width; the areas of the grass-plat and border are equal; what is the width of the border?

Ans. 3 yds.

54. A and B set out to meet each other from two places 320 miles apart: A traveled 8 miles a day more than B, and the number of days in which they met was equal to half the number of miles B went in a day; how far did each travel before they met? *Ans.* A, 192 mi.; B, 128 mi.

55. Two clerks, A and B, sent ventures in a ship bound to India: A gained \$120, and at this rate he would have gained as many dollars on a hundred as B sent out. B gained \$36, which was but one-fourth as much per cent. as A gained; how much money was sent out by each?

Ans. A, \$100; B, \$120.

56. In a collection containing 27 coins, each silver coin is worth as many cents as there are copper coins; and each copper coin is worth as many cents as there are silver coins; and the whole is worth \$1; how many coins are there of each sort?

Ans. 2 of silver; 25 of copper.

57. The third term of an arithmetical progression is 18, and the seventh term is 30; find the sum of 17 terms.

Ans. 612.

58. A man by selling a horse for 264 dollars gains as much per cent. as the horse cost him; required the cost of the horse.

Ans. \$120.

59 Two numbers are in the ratio of 4 to 5, but if one is increased and the other diminished by 10, the ratio of the resulting numbers is inverted; required the numbers.

Ans. 40 and 50.

60. Show that the difference between the square of a number consisting of 2 digits, and the square of the number formed by changing the places of the digits, is divisible by 99.

61. A laborer, having built 105 rods of fence, found that, had he built 2 rods less a day, he would have been 6 days longer in completing the job; how many rods did he build per day?

Ans. 7.

62. The common difference in an arithmetical progression is equal to 2, and the number of terms is equal to the second term; what is the first term if the sum is 35? *Ans.* 3.

63. If the sum of two fractions is unity, show that the first, together with the square of the second, is equal to the second together with the square of the first.

64. A man having a garden 12 rods long and 10 rods wide, wishes to make a gravel-walk half-way around it; what will be the width of the walk if it takes up $\frac{1}{10}$ of the garden?

Ans. 9.234 ft.

65. If 8 gold coins and 9 silver ones are worth as much as 6 gold coins and 19 silver ones, find the ratio of the value of a silver coin to that of a gold coin. *Ans.* 1 : 5.

66. A rectangular picture is surrounded by a narrow frame which measures altogether 10 linear feet, and costs, at 12 cents per foot, 20 times as many cents as there are square feet in the picture; required the length and breadth of the picture.

Ans. Length, 3 ft.; breadth, 2 ft.

67. A person walked to the top of a mountain at the rate of $2\frac{1}{3}$ miles an hour, and down the same way at the rate of $3\frac{1}{2}$ miles an hour, and was gone 5 hours; how far did he walk altogether?

Ans. 14 mi.

68. A and B hired a pasture, into which A put 4 horses, and B as many as cost him 18 shillings a week. Afterward, B put in 2 additional horses, and found that he must pay 20 shillings a week; at what rate was the pasture hired?

Ans. 30s. a week.

69. Two partners, A and B, gained \$700 by trade: A's money was in trade 3 months, and his gain was \$300 less than his stock; and B's money, which was \$250 more than A's, was in trade 5 months; required A's stock. *Ans.* \$500.

70. A sets out for a certain place, and travels 1 mile the first day, 2 the second, 3 the third, and so on; five days afterward B sets out from the same place, and travels 12 miles a day; how long will A travel before he is overtaken by B?

Ans. 8 or 15 days.

71. A certain number of students go on an excursion: if there were five more, and each should pay \$1 more, the expense would be $\$61\frac{1}{2}$ more; but if there were 3 less, and each should pay $\$1\frac{1}{2}$ less, the expense would be \$42 less; required the number of students and the fare of each.

Ans. Number, 14; fare, $\$8\frac{1}{2}$.

72. A traveler set out from a certain place, and went 1 mile the first day, 3 the second, 5 the third, and so on; after he had been gone three days a second traveler set out, and went 12 miles the first day, 13 the second, and so on; in how many days will the second overtake the first? *Ans.* In 2 or 9 days.

73. A set out from C toward D, and traveled 7 miles a day: after he had gone 32 miles, B set out from D toward C, and went every day $\frac{1}{19}$ of the whole journey; and after he had traveled as many days as he went miles in 1 day, he met A; required the distance between C and D.

Ans. 76 mi., or 152 mi.

74. Two trains start at the same time from two towns, and each proceeds at a uniform rate toward the other town: when they meet it is found that one train has run 108 miles more than the other, and that if they continue to run at the same rate they will finish the journey in 9 and 16 hours respectively; required the distance between the towns and the rates of the trains.

Ans. Distance, 756 mi.; rate, 1st, 36 mi.; 2d, 27 mi. an hour.

75. A criminal having escaped from prison, traveled 10 hours before his escape was known; he was then pursued so as to be gained upon 3 miles an hour; after his pursuers had traveled 8 hours, they met an express going at the same rate as themselves, who had met the criminal 2 hours and 24 minutes before; in what time from the commencement of the pursuit will they overtake him? *Ans.* 20 hrs.