**Rule.**—I. *Find the greatest common divisor of the two quantities; divide one quantity by it, and multiply the other quantity by the quotient.*

II. *When there are more than two quantities, find the least common multiple of two of them, then of this multiple and the third quantity,* etc.

**EXAMPLES.**

2. Find the least common multiple of $x^2-x-12$ and $x^2-4x-21$. *Ans.* $x^3-8x^2-5x+84$.

3. Find the least common multiple of $x^2+5x+6$ and $x^2+6x+8$. *Ans.* $x^3+9x^2+26x+24$.

4. Find the least common multiple of $a^2+4ab+3b^2$ and $a^2-b^2$. *Ans.* $a^3+3a^2b-ab^2-3b^3$.

5. Find the least common multiple of $a^2+3ab+2b^2$ and $a^2-ab-6b^2$. *Ans.* $a^3-7ab^2-6b^3$.

6. Find the least common multiple of $x^2-ax+3x-3a$ and $x^2-3x-ax+3a$. *Ans.* $x^3-ax^2-9x+9a$.

7. Find the least common multiple of $x^2-x-2$, $x^2+3x+2$ and $x^2+5x+4$. *Ans.* $x^4+5x^3-20x-16$.

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REVIEW QUESTIONS.

Define Composition. A Composite Quantity. State the relation of Composition to Factoring. State the four theorems of Composition.

Define Factoring. Factors. A Prime Quantity. Quantities prime to each other. State each case. Give the rule for each case. State the three theorems of Factoring.

Define Common Divisor. Greatest Common Divisor. State the cases. The principles. The rules. Define a Multiple. A Common Multiple. The Least Common Multiple. State the cases. The principles. The rules.

# SECTION IV.

# FRACTIONS.

**122.** A **Fraction** is a number of the equal parts of a unit.

**123.** A **Fraction** in Algebra is expressed by two quantities, one above the other, with a straight line between them; as, $\frac{a}{c}$ or $\frac{a-x}{a-z}$.

**124.** The **Denominator** of a fraction denotes the number of equal parts into which the unit is divided. It is written below the line.

**125.** The **Numerator** of a fraction denotes the number of equal parts taken. It is written above the line.

**126.** An **Entire Quantity** is one that has no fractional part; as, $2a^2$, $a+b$, etc.

**127.** A **Mixed Quantity** is one that has both an entire and a fractional part; as, $a+\frac{b}{c}$, $ax-\frac{2ac}{m+n}$.

**128.** An **Algebraic Fraction** is usually regarded as the expression of one quantity divided by another; thus, $\frac{a}{c}$ means $a$ divided by $c$, etc.

## PRINCIPLES OF FRACTIONS.

**129.** The **Principles of Fractions** are general laws showing the relation of the value of the fraction to its numerator and denominator.

PRIN. 1. *Multiplying the numerator or dividing the denominator of a fraction by any quantity multiplies the value of the fraction by that quantity.*

If we multiply the numerator of a fraction by $n$, there will be $n$ times as many parts taken, each of them the same size as before; hence, the value will be $n$ times as great.

If we divide the denominator by $n$, the unit will be divided into 1-$n$th as many parts; hence, each part will be $n$ times as great; and the same number of parts being taken, the value of the fraction will be $n$ times as great. Therefore, etc.

PRIN. 2. *Dividing the numerator or multiplying the denominator of a fraction by any quantity divides the value of the fraction by that quantity.*

If we divide the numerator of a fraction by $n$, there will be only 1-$n$th as many parts taken, each of the same size as before; hence, the value of the fraction will be 1-$n$th as great.

If we multiply the denominator of a fraction by $n$, the unit will be divided into $n$ times as many parts; hence, each part will be 1-$n$th as great as before; and the same number of parts being taken, the value of the fraction will be 1-$n$th as great. Therefore, etc.

PRIN. 3. *Multiplying or dividing both numerator and denominator of a fraction by the same quantity does not change the value of the fraction.*

For, since multiplying the numerator multiplies the fraction, and multiplying the denominator divides the fraction, multiplying both numerator and denominator by the same number both multiplies and divides the value of the fraction by that number, and hence does not change the value. In the same way it may be shown that dividing both terms does not change the value of the fraction. Therefore, etc.

PRIN. 4. *The value of a fraction is equal to the quotient of its numerator divided by its denominator.*

## SIGNS OF THE FRACTION.

**130.** The **Sign** of a fraction is the sign written before the dividing-line, showing whether the fraction is to be added or subtracted.

NOTE.—This sign is sometimes called the *apparent sign* of the fraction and the sign of its value, the *real* sign.



PRIN. 1. *Changing the sign of the numerator or denominator changes the sign of the fraction.*

For, this is the same as multiplying or dividing the fraction by $-1$, which will change the sign of the quantity (Prin. 1, Art. 129). Thus, $+\frac{a-x}{c+z}$ equals $-\frac{-a+x}{c+z}$, or $-\frac{a-x}{-c-z}$, etc.

PRIN. 2. *Changing the sign of both numerator and denominator does not change the sign of the fraction.*

For, this is the same as multiplying both numerator and denominator by $-1$, which will not change the value of the fraction. (Prin. 3, Art. 129.)

## REDUCTION.

**131. Reduction of Fractions** is the process of changing their form without changing their value.

### CASE I.

**132. To reduce a fraction to its lowest terms.**

A **Fraction** is in its lowest terms when the numerator and denominator are prime to each other.

1. Reduce $\frac{8a^3b^2c}{12a^2b^3c^2}$ to its lowest terms.

SOLUTION. Dividing both terms of the fraction by the common factors, 4, $a^2$, $b^2$ and $c$, we have it equal to $\frac{2a}{3c}$ (Prin. 3, Art. 129); and this is the lowest term of the fraction, since the terms are prime to each other.

OPERATION.

$$\frac{8a^3b^2c}{12a^2b^2c^2}=\frac{2a}{3c}$$

**Rule.**—*Divide both numerator and denominator by their common factors;*

Or, *Divide both terms by their greatest common divisor.*

EXAMPLES.

2. Reduce $\frac{15a^4x^2}{25a^7x}$ to its lowest terms. *Ans.* $\frac{3x}{5a^3}$.

3. Reduce $\frac{54x^3z^6}{36x^5z^2}$ to its lowest terms. *Ans.* $\frac{3z^4}{2x^2}$.

4. Reduce $\frac{24a^5x^6z^3}{56a^4x^9z^7}$ to its lowest terms. *Ans.* $\frac{3a}{7x^3z^4}$.

5. Reduce $\frac{a^2-b^2}{a^4-b^4}$ to its lowest terms. *Ans.* $\frac{1}{a^2+b^2}$.

6 Reduce $\frac{2a+2b}{a^2-b^2}$ to its lowest terms. *Ans.* $\frac{2}{a-b}$.

7. Reduce $\frac{a^2-1}{2(ab-b)}$ to its lowest terms. *Ans.* $\frac{a+1}{2b}$.

8. Reduce $\frac{2a^{m+4}}{3a^{m+2}}$ to its lowest terms. *Ans.* $\frac{2a^2}{3}$.

9. Reduce $\frac{4a^2c^{n+1}}{6a^3c^{n-1}}$ to its lowest terms. *Ans.* $\frac{2c^2}{3a}$.

10. Reduce $\frac{x^2-9}{2x^2+10x+12}$ to its lowest terms. *Ans.* $\frac{x-3}{2x+4}$.

11. Reduce $\frac{a^3-ab^2}{a^2+2ab+b^2}$ to its lowest terms. *Ans.* $\frac{a(a-b)}{a+b}$.

12. Reduce $\frac{x^2-4a^2}{x^2+2ax-8a^2}$ to its lowest terms. *Ans.* $\frac{x+2a}{x+4a}$.

13. Reduce $\frac{x^{2n}-9b^{2n}}{x^{2n}-6b^nx^n+9b^{2n}}$ to its lowest terms.
*Ans.* $\frac{x^n+3b^n}{x^n-3b^n}$.

## CASE II.

**133. To reduce a fraction to an entire or mixed quantity.**

1. Reduce $\frac{ac+b}{c}$ to a mixed quantity.

OPERATION.

$$\frac{ac+b}{c}=a+\frac{b}{c}$$

SOLUTION. The value of a fraction is equal to the quotient of the numerator divided by the denominator. Dividing $ac+b$ by $c$, we have $a$ for the entire part, and $\frac{b}{c}$ for the fractional part. Therefore, etc.

**Rule.**—I. *Divide the numerator by the denominator for the entire part, continuing the division as far as necessary.*

II. *Write the denominator under the remainder, and annex the result to the entire part with the proper sign.*



### EXAMPLES.

2. Reduce $\frac{ac^2+b}{c}$ to a mixed quantity. *Ans.* $ac+\frac{b}{c}$.

3. Reduce $\frac{2ax+x^2}{a+x}$ to a mixed quantity. *Ans.* $2x-\frac{x^2}{a+x}$.

4. Reduce $\frac{a^2-4c^2}{a-c}$ to a mixed quantity. *Ans.* $a+c-\frac{3c^2}{a-c}$.

5. Reduce $\frac{3a^3-3x^3}{a-x}$ to an entire quantity.
*Ans.* $3(a^2+ax+x^2)$.

6. Reduce $\frac{x^3-z^3}{(x-z)^2}$ to a mixed quantity. *Ans.* $x+2z+\frac{3z^2}{x-z}$.

7. Reduce $\frac{a^3-b^3}{a+b}$ to a mixed quantity.
*Ans.* $a^2-ab+b^2-\frac{2b^3}{a+b}$.

8. Reduce $\frac{x^4-z^4}{(x^2-z^2)^2}$ to a mixed quantity. *Ans.* $1+\frac{2z^2}{x^2-z^2}$.

9. Reduce $\frac{x^3+z^3}{(x+z)^3}$ to a mixed quantity. *Ans.* $1-\frac{3xz}{(x+z)^2}$.

## CASE III.

**134. To reduce a mixed quantity to a fraction.**

1. Reduce $a+\frac{c}{x}$ to a fraction.

OPERATION.

$$a+\frac{c}{x}=$$
$$\frac{ax}{x}+\frac{c}{x}=\frac{ax+c}{x}$$

SOLUTION. It is evident that $1=\frac{x}{x}$; hence $a$ equals $a$ times $\frac{x}{x}$, or $\frac{ax}{x}$, which, added to $\frac{c}{x}$, equals $\frac{ax}{x}+\frac{c}{x}$, or $\frac{ax+c}{x}$.

**Rule.**—*Multiply the entire part by the denominator of the fraction; add the numerator when the sign of the fraction is plus, and subtract it when the sign is minus, and write the denominator under the result.*

NOTE.—When the sign of the fraction is minus, remember to change the signs of all the terms.

### EXAMPLES.

2. Reduce $3a+\frac{a-3}{2}$ to a fraction. *Ans.* $\frac{7a-3}{2}$

3. Reduce $2z+\frac{3-2z}{3}$ to a fraction. *Ans.* $\frac{4z+3}{3}$

4. Reduce $x+\frac{ax}{a+x}$ to a fraction. *Ans.* $\frac{2ax+x^2}{a+x}$

5. Reduce $a+c-\frac{3c^2}{a-c}$ to a fraction. *Ans.* $\frac{a^2-4c^2}{a-c}$

6. Reduce $4x-\frac{3-5x}{4}$ to a fraction. *Ans.* $\frac{21x-3}{4}$.

7. Reduce $a-\frac{2ac-c^2}{a}$ to a fraction. *Ans.* $\frac{(a-c)^2}{a}$.

8. Reduce $4a+x+\frac{3ax+x^2}{a-x}$ to a fraction. *Ans.* $\frac{4a^2}{a-x}$.

9. Reduce $2x-5-\frac{2x^2+4}{x-3}$ to a fraction. *Ans.* $\frac{11(1-x)}{x-3}$.

10. Reduce $a+x-\frac{c^2-x^2}{a-x}$ to a fraction. *Ans.* $\frac{a^2-c^2}{a-x}$.

## CASE IV.

**135. To reduce fractions by changing factors from one term to another.**

PRINCIPLE.—*Any factor may be transferred from one term of a fraction to the other if the sign of the exponent be changed.*

This principle is readily proved by applying the principles of Article 94.

1. Change the terms in the fraction $\frac{a^2}{c^3}$.

SOLUTION. First, $\frac{a^2}{c^3}=a^2\times\frac{1}{c^3}$; but $a^2=\frac{1}{a^{-2}}$ (Prin. 3, Art. 94); hence, $a^2\times\frac{1}{c^3}=\frac{1}{a^{-2}}\times\frac{1}{c^3}=\frac{1}{a^{-2}c^3}$.

OPERATION.

1st. $\frac{a^2}{c^3}=a^2\times\frac{1}{c^3}=\frac{1}{a^{-2}}\times\frac{1}{c^3}=\frac{1}{a^{-2}c^3}$

2d. $\frac{a^2}{c^3}=a^2\times\frac{1}{c^3}=a^2\times c^{-3}=a^2c^{-3}$



Second, $\frac{a^2}{c^3}=a^2\times\frac{1}{c^3}$; but $\frac{1}{c^3}=c^{-3}$; hence, $a^2\times\frac{1}{c^3}=a^2\times c^{-3}$, or $a^2c^{-3}$.

**Rule.**—*In changing a factor from one term of a fraction to the other, change the sign of its exponent.*

### EXAMPLES.

2. Change the terms in $\frac{a^5}{c^2x^3}$. *Ans.* $a^5c^{-2}x^{-3}$.

3. Change the terms in $\frac{x^3y^4}{a^{-3}z^5}$. *Ans.* $a^3x^3y^4z^{-5}$.

4. Change $\frac{3c^3}{ab^2}$ to an entire quantity. *Ans.* $3a^{-1}b^{-2}c^3$.

5. Change $\frac{a-x}{a+x}$ to an entire quantity. *Ans.* $(a-x)(a+x)^{-1}$.

6. Change $\frac{3(a+c)}{5c^{-2}}$ to an entire quantity. *Ans.* $3\times5^{-1}(a+c)c^2$.

7. Change $\frac{4(c-z^3)}{6c^{-2}}$ to a simpler form. *Ans.* $\frac{2}{3}(c^3-c^2z^3)$.

8. Change $\frac{x^2-2xy+y^2}{x^2-y^2}$ to an entire quantity. *Ans.* $(x-y)(x+y)^{-1}$.

9. Change $\frac{a(b-c)}{(b+c)^{-1}}$ to an entire quantity. *Ans.* $a(b^2-c^2)$.

10. Change $\frac{4a(c-z)^{-1}}{c+z}$ to positive exponents. *Ans.* $\frac{4a}{c^2-z^2}$.

11. Change $\frac{(a-b)^2(x-y)^{-1}}{(a-b)^{-1}(x-y)}$ to positive exponents. *Ans.* $\frac{(a-b)^3}{(x-y)^2}$.

## CASE V.

**136. To reduce fractions to a common denominator.**

**137. Fractions** have a *common denominator* when they have the same expression for a denominator.

**138.** The **Least Common Denominator** of several fractions is the least denominator to which they may all be reduced.

PRIN. 1. *A common denominator of several fractions is a common multiple of their denominators.*

For, when a fraction is reduced to higher terms, its denominator is multiplied by some number, and the common denominator to which several fractions are reduced must therefore be a multiple of each given denominator.

PRIN. 2. *The least common denominator of several fractions is the least common multiple of their denominators.*

For, since a *common* denominator is a *common* multiple of the denominators, it is evident that a *least common denominator* is a *least common* multiple of the denominators.

1. Reduce $\frac{a}{bc}$ and $\frac{c}{b^2d}$ to their least common denominator.

SOLUTION. The least common multiple of $bc$ and $b^2d$ is $b^2cd$, which is the least common denominator. Dividing $b^2cd$ by $bc$, the denominator of the first fraction, we find we must multiply both terms of $\frac{a}{bc}$ by $bd$ to reduce it to the common denominator. Multiplying both terms of $\frac{a}{bc}$ by $bd$, we have $\frac{abd}{b^2cd}$. Dividing $b^2cd$ by $b^2d$, we have $c$. Multiplying both terms of $\frac{c}{b^2d}$ by $c$, we have $\frac{c^2}{b^2cd}$.

OPERATION.

$$\frac{a}{bc},\ \frac{c}{b^2d};\quad \text{L.C.M.} = b^2cd$$

$$b^2cd \div bc = bd;\quad b^2cd \div b^2d = c$$

$$\frac{a}{bc} = \frac{a \times bd}{bc \times bd} = \frac{abd}{b^2cd}$$

$$\frac{c}{b^2d} = \frac{c \times c}{b^2d \times c} = \frac{c^2}{b^2cd}$$

**Rule.**—*Find the least common multiple of the denominators; divide this by the denominator of each fraction, and multiply the numerator by the quotient;*

Or, *Multiply both terms of each fraction by the denominators of the other fractions.*

NOTE.—Fractions may be reduced to a common denominator by multiplying both terms of one or more fractions by such quantities as will make the denominators equal.



EXAMPLES.

Reduce to a common denominator—

2. $\frac{a}{mn}$, $\frac{b}{m^2}$ and $\frac{c}{mn^2}$. *Ans.* $\frac{amn}{m^2n^2}$, $\frac{bn^2}{m^2n^2}$, $\frac{cm}{m^2n^2}$.

3. $\frac{2a}{3x}$, $\frac{3b}{4z}$ and $\frac{c}{ax^2}$. *Ans.* $\frac{8a^2xz}{12ax^2z}$, $\frac{9abx^2}{12ax^2z}$, $\frac{12cz}{12ax^2z}$.

4. $\frac{c}{4a}$, $\frac{5}{6c}$ and $\frac{2ac}{3n^2}$. *Ans.* $\frac{3c^2n^2}{12acn^2}$, $\frac{10an^2}{12acn^2}$, $\frac{8a^2c^2}{12acn^2}$.

5. $\frac{3}{2a^2c}$, $\frac{4b^2}{3ac^2}$ and $\frac{2ac}{4c^2z}$. *Ans.* $\frac{18cz}{12a^2c^2z}$, $\frac{16ab^2z}{12a^2c^2z}$, $\frac{6a^3c}{12a^2c^2z}$.

6. $\frac{a-b}{a^2c}$, $\frac{a+b}{3ac^2}$ and $5\frac{1}{2}$. *Ans.* $\frac{6c(a-b)}{6a^2c^2}$, $\frac{2a(a+b)}{6a^2c^2}$, $\frac{33a^2c^2}{6a^2c^2}$.

7. $\frac{ab}{a-b}$, $\frac{bc}{a+b}$ and $\frac{cd}{a^2-b^2}$. *Ans.* $\frac{ab(a+b)}{a^2-b^2}$, $\frac{bc(a-b)}{a^2-b^2}$, $\frac{cd}{a^2-b^2}$.

8. $\frac{2ax}{x-1}$, $\frac{3ax}{x+1}$ and $\frac{4ax}{x^2-1}$.
*Ans.* $\frac{2ax^2+2ax}{x^2-1}$, $\frac{3ax^2-3ax}{x^2-1}$, $\frac{4ax}{x^2-1}$.

9. $a$, $\frac{a}{c}$ and $\frac{a-b}{a-c}$. *Ans.* $\frac{a^2c-ac^2}{c(a-c)}$, $\frac{a^2-ac}{c(a-c)}$, $\frac{c(a-b)}{c(a-c)}$.

10. $\frac{a-c}{(a+c)^2}$ and $\frac{a+c}{(a-c)^2}$. *Ans.* $\frac{(a-c)^3}{(a^2-c^2)^2}$, $\frac{(a+c)^3}{(a^2-c^2)^2}$.

11. $\frac{a+c}{a-c}$, $\frac{a-c}{a+c}$ and $\frac{a^2+c^2}{a^2-c^2}$. *Ans.* $\frac{(a+c)^2}{a^2-c^2}$, $\frac{(a-c)^2}{a^2-c^2}$, $\frac{a^2+c^2}{a^2-c^2}$.

12. $\frac{a}{a^2+1}$, $\frac{a^2}{a^2-1}$ and $\frac{a^4}{a^4-1}$.
*Ans.* $\frac{a(a^2-1)}{a^4-1}$, $\frac{a^2(a^2+1)}{a^4-1}$, $\frac{a^4}{a^4-1}$.

13. $\frac{a}{(a+b)(b-c)}$ and $\frac{b}{(a+b)(c-b)}$.
*Ans.* $\frac{a}{(a+b)(b-c)}$, $\frac{-b}{(a+b)(b-c)}$.

## ADDITION.

**139. Addition of Fractions** is the process of finding the simplest expression for the *sum* of two or more fractions.

PRINCIPLE.—*To be added, fractions must have a common denominator.*

For, they then express similar fractional units, and only similar units can be united into one sum.

1. Find the sum of $\frac{a}{n}$ and $\frac{b}{n}$.

SOLUTION. $a$ divided by $n$, plus $b$ divided by $n$, equals $(a+b)$ divided by $n$.

OPERATION.

$$\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}$$

2. Find the sum of $\frac{a}{n}$ and $\frac{b}{m}$.

SOLUTION. Since the denominators are not alike, we must first reduce the fractions to a common denominator. The common denominator is $mn$. $\frac{a}{n}$ equals $\frac{am}{mn}$, and $\frac{b}{m}$ equals $\frac{bn}{mn}$; $am$ divided by $mn$, plus $bn$ divided by $mn$, equals $(am+bn)$ divided by $mn$.

OPERATION.

$$\frac{a}{n}+\frac{b}{m}=\frac{am}{mn}+\frac{bn}{mn}=\frac{am+bn}{mn}$$

**Rule.**—I. *Reduce the fractions, when necessary, to their least common denominator.*

II. *Add the numerators, and write the common denominator under their sum.*

NOTES.—1. Reduce each fraction to its lowest terms before adding, and also the result after addition.

2. Mixed quantities may be added by adding the integral parts and uniting the sum with the sum of the fractions.

### EXAMPLES.

Find the sum—

3. Of $\frac{a}{b}+\frac{c}{d}$. *Ans.* $\frac{ad+bc}{bd}$.

4. Of $\frac{a}{b}+\frac{b}{d}+\frac{d}{c}$. *Ans.* $\frac{adc+b^2c+bd^2}{bdc}$.



5. Of $\frac{2a}{n}+\frac{3c}{mn}+\frac{4b}{m^2n}$. *Ans.* $\frac{2am^2+3cm+4b}{m^2n}$.

6. Of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$. *Ans.* $\frac{yz+xz+xy}{xyz}$.

7. Of $\frac{2a}{3}+a-\frac{2x^2}{3ac}$. *Ans.* $\frac{5a^2c-2x^2}{3ac}$.

8. Of $\frac{a+b}{2}+\frac{a-b}{2}$. *Ans.* $a$.

9. Of $\frac{1}{m+n}+\frac{1}{m-n}$. *Ans.* $\frac{2m}{m^2-n^2}$.

10. Of $\frac{a}{a+b}+\frac{b}{a-b}$. *Ans.* $\frac{a^2+b^2}{a^2-b^2}$.

11. Of $\frac{a}{2a-2b}+\frac{b}{2b-2a}$. *Ans.* $\frac{1}{2}$.

12. Of $\frac{a}{2c}$, $\frac{a-c}{ac}$ and $\frac{c-a}{ac}$. *Ans.* $\frac{a}{2c}$.

13. Of $\frac{a^n}{2x}$, $\frac{x-a^{2n}}{a^nx}$ and $\frac{a^{2n}x-2z^2}{2a^nx^2}$. *Ans.* $\frac{x^2-z^2}{a^nx^2}$.

14. Of $\frac{1+a}{1-a}$ and $\frac{1-a}{1+a}$. *Ans.* $\frac{2(1+a^2)}{1-a^2}$.

15. Of $\frac{x-y}{xy}$, $\frac{y-z}{yz}$ and $\frac{z-x}{xz}$. *Ans.* 0.

16. Of $3a^2+\frac{x-3}{3}$ and $4a^2+\frac{2a-z}{2a}$. *Ans.* $7a^2+\frac{2ax-3z}{6a}$.

17. Of $\frac{2}{z+1}$ and $\frac{1+z^2}{z^2+z}$. *Ans.* $\frac{1+z}{z}$.

18. Of $\frac{1+x^2}{1-x^2}$ and $\frac{1-x^2}{1+x^2}$. *Ans.* $\frac{2(1+x^4)}{1-x^4}$.

19. Of $\frac{a}{(a-b)(b-c)}$ and $\frac{b}{(a-b)(c-b)}$. *Ans.* $\frac{1}{b-c}$.

20. Of $\frac{a+b}{(b-c)(c-a)}+\frac{b+c}{(c-a)(a-b)}+\frac{c+a}{(a-b)(b-c)}$. *Ans.* 0.

## SUBTRACTION.

**140. Subtraction of Fractions** is the process of finding the simplest expression for the *difference* of two fractions.

PRINCIPLE.—*To be subtracted, fractions must have a common denominator.*

For, they then express similar fractional units, and only similar units can be subtracted.

1. Subtract $\frac{b}{n}$ from $\frac{a}{n}$.

SOLUTION. $a$ divided by $n$, minus $b$ divided by $n$, equals $(a-b)$ divided by $n$.

OPERATION.

$$\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}$$

2. Subtract $\frac{b}{n}$ from $\frac{a}{m}$.

SOLUTION. Since the denominators are not alike, we must first reduce the fractions to a common denominator. The common denominator is $mn$. $\frac{a}{m}=\frac{an}{mn}$ and $\frac{b}{n}=\frac{bm}{mn}$; $an$ divided by $mn$, minus $bm$ divided by $mn$, equals $(an-bm)$ divided by $mn$.

OPERATION.

$$\frac{a}{m}-\frac{b}{n}=\frac{an}{mn}-\frac{bm}{mn}=\frac{an-bm}{mn}$$

**Rule.**—I. *Reduce the fractions, when necessary, to their least common denominator.*

II. *Subtract the numerator of the subtrahend from the numerator of the minuend, and write the common denominator under the result.*

NOTES.—1. Reduce each fraction to its lowest terms before subtracting, and also the result after subtraction.

2. Mixed quantities may be subtracted by subtracting the integral parts, and uniting the difference with the difference of the fractions.

EXAMPLES.

3. From $\frac{a}{b}$ take $\frac{c}{d}$. *Ans.* $\frac{ad-bc}{bd}$.

4. From $\frac{2a}{n}$ take $\frac{2c}{mn}$. *Ans.* $\frac{2(am-c)}{mn}$.

5. From $\frac{3ax}{2c^2}$ take $\frac{4x}{3ac}$. *Ans.* $\frac{9a^2x-8cx}{6ac^2}$.



6. From $\frac{a+b}{2}$ take $\frac{a-b}{2}$. *Ans.* $b$.

7. From $\frac{a+b}{a}$ take $\frac{a-b}{a}$. *Ans.* $\frac{2b}{a}$.

8. From $5a^2+\frac{3x}{ac}$ take $3a^2+\frac{2a}{ax}$. *Ans.* $2a^2+\frac{3x^2-2ac}{acx}$

9. From $4c-\frac{a}{a-3}$ take $2c-\frac{a+3}{a}$. *Ans.* $2c-\frac{9}{a^2-3a}$

10. From $\frac{a+b}{a}$ take $\frac{b-a}{b}$. *Ans.* $\frac{a^2+b^2}{ab}$

11. From $\frac{a}{a-b}$ take $\frac{b}{a+b}$. *Ans.* $\frac{a^2+b^2}{a^2-b^2}$

12. From $\frac{1}{a-b}$ take $\frac{b}{a^2-b^2}$. *Ans.* $\frac{a}{a^2-b^2}$.

13. From $\frac{1}{1-a}$ take $\frac{1}{1+a}$. *Ans.* $\frac{2a}{1-a^2}$.

14. From $\frac{x+1}{x-1}$ take $\frac{x-1}{x+1}$. *Ans.* $\frac{4x}{x^2-1}$.

15. From $\frac{1+z^2}{1-z^2}$ take $\frac{1-z^2}{1+z^2}$. *Ans.* $\frac{4z^2}{1-z^4}$.

16. Find value of $\frac{a}{a-x}+\frac{3a}{a+x}-\frac{2ax}{a^2-x^2}$. *Ans.* $\frac{4a}{a+x}$.

17. Find value of $\frac{1}{a+b}+\frac{b}{a^2-b^2}-\frac{a}{a^2+b^2}$. *Ans.* $\frac{2ab^2}{a^4-b^4}$

18. Find value of $\frac{3}{1-2x}-\frac{7}{1+2x}+\frac{4-20x}{1-4x^2}$. *Ans.* 0

19. Find value of $\frac{a}{(a+b)(b-c)}-\frac{b}{(a+b)(c-b)}$. *Ans.* $\frac{1}{b-c}$

20. Find value of $\left(\frac{1}{m}+\frac{1}{n}\right)(a+b)-\left(\frac{a+b}{m}-\frac{a-b}{n}\right)$. *Ans.* $\frac{2a}{n}$

## MULTIPLICATION.

**141. Multiplication of Fractions** is the process of finding a *product* when one or both factors are fractions.

### CASE I.

**142. To multiply a fraction by an entire quantity.**

1. Multiply $\frac{a}{b}$ by $c$.

SOLUTION. Since multiplying the numerator of a fraction multiplies the value of the fraction (Prin. 1, Art. 129), $c$ times $\frac{a}{b}$ equals $\frac{ac}{b}$.

OPERATION.
$$\frac{a}{b}\times c=\frac{ac}{b}$$

2. Multiply $\frac{a}{b^2}$ by $b$.

SOLUTION. Since dividing the denominator of a fraction multiplies the fraction (Prin. 1, Art. 129), $b$ times $\frac{a}{b^2}$ equals $\frac{a}{b}$.

OPERATION
$$\frac{a}{b^2}\times b=\frac{a}{b}$$

**Rule.**—*Multiply the numerator or divide the denominator of the fraction by the multiplier.*

NOTES.—1. The second method is preferred when the denominator is divisible by the multiplier.

2. It is often convenient to indicate the multiplication, and cancel equal factors in numerators and denominators.

#### EXAMPLES.

3. Multiply $\frac{a}{c}$ by $n$. *Ans.* $\frac{an}{c}$

4. Multiply $\frac{a^2b}{c^2d}$ by $cd$. *Ans.* $\frac{a^2b}{c}$.

5. Multiply $\frac{5ax^2}{12cz^3}$ by $4z^2$. *Ans.* $\frac{5ax^2}{3cz}$



6. Multiply $\frac{5an}{c^3(a-x)}$ by $3ac^2$. *Ans.* $\frac{15a^2n}{c(a-x)}$.

7. Multiply $\frac{mx}{(m-x)^2}$ by $2(m-x)$. *Ans.* $\frac{2mx}{m-x}$.

8. Multiply $\frac{3a^2z}{(a-x)^2}$ by $a^2-x^2$. *Ans.* $\frac{3a^2z(a+x)}{a-x}$.

9. Multiply $\frac{x+y}{x^2-2xy+y^2}$ by $x^2-y^2$. *Ans.* $\frac{(x+y)^2}{x-y}$.

10. Multiply $\frac{5a^2x}{a-1}$ by $a^2-1$. *Ans.* $5a^3x+5a^2x$.

11. Multiply $\frac{3a^2z^3}{x^3-x}$ by $2a(x-1)$. *Ans.* $\frac{6a^3z^3}{x(x+1)}$.

### CASE II.

**143. To multiply an entire or fractional quantity by a fraction.**

1. Multiply $a$ by $\frac{b}{c}$.

SOLUTION. $a$ multiplied by $b$ is $ab$; hence, $a$ multiplied by $b$ divided by $c$ is $ab$ divided by $c$, or $\frac{ab}{c}$.

OPERATION.
$$a\times\frac{b}{c}=\frac{ab}{c}$$

2. Multiply $\frac{a}{b}$ by $\frac{c}{d}$.

SOLUTION. $\frac{a}{b}$ multiplied by $c$ is $\frac{ac}{b}$; hence, $\frac{a}{b}$ multiplied by $c$ divided by $d$ must be $\frac{ac}{b}$ divided by $d$, which is $\frac{ac}{bd}$. (Prin. 2, Art. 129.)

OPERATION.
$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

SOLUTION 2D. From Art. 135, Prin., $\frac{a}{b}$ equals $ab^{-1}$ and $\frac{c}{d}$ equals $cd^{-1}$, and $ab^{-1}$ multiplied by $cd^{-1}$ equals $acb^{-1}d^{-1}$, which, by Art. 135, Prin., is equal to $\frac{ac}{bd}$.

OPERATION.
$$\frac{a}{b}\times\frac{c}{d}=ab^{-1}\times cd^{-1}=$$
$$acb^{-1}d^{-1}=\frac{ac}{bd}$$

**Rule.**—*Multiply the numerators together for the numerator, and the denominators together for the denominator, canceling common factors.*

NOTES.—1. When there are common factors in the numerators and denominators, indicate the multiplication and then cancel the common factors.

2. If either factor is a mixed quantity, reduce it to a fraction before multiplying.

**EXAMPLES.**

3. Multiply $\frac{a}{m}$ by $\frac{c}{n}$. *Ans.* $\frac{ac}{mn}$.

4. Multiply $\frac{2ax}{3c}$ by $\frac{3x^2}{2a}$. *Ans.* $\frac{x^3}{c}$.

5. Multiply $\frac{3a^2c}{2n^2}$ by $\frac{4cn}{5a^3}$. *Ans.* $\frac{6c^2}{5an}$.

6. Multiply $\frac{ab^{2n}}{x^5}$ by $\frac{ax^3}{mb^n}$. *Ans.* $\frac{a^2b^n}{mx^2}$.

7. Multiply $\frac{a-x}{a^2}$ by $\frac{a^3x^2}{3b}$. *Ans.* $\frac{ax^2(a-x)}{3b}$.

8. Multiply $\frac{a+c}{c^2}$ by $\frac{a+x}{a+c}$. *Ans.* $\frac{a+x}{c^2}$.

9. Multiply $\frac{a+x}{4ax}$ by $\frac{a-x}{a+x}$. *Ans.* $\frac{a-x}{4ax}$.

10. Multiply $\frac{1-a^2}{6a^3}$ by $\frac{4ab^3}{1-a}$. *Ans.* $\frac{2b^3(1+a)}{3a^2}$.

11. Multiply $\frac{(a-b)^2}{a+b}$ by $\frac{(a+b)^2}{a-b}$. *Ans.* $a^2-b^2$.

12. Multiply $a-\frac{a}{c}$ by $\frac{2bc}{3a}$. *Ans.* $\frac{2b(c-1)}{3}$.

13. Multiply $a+\frac{a}{x}$ by $a-\frac{a}{x}$. *Ans.* $a^2-\frac{a^2}{x^2}$.

14. Multiply $\frac{n^2-z^2}{3m^2}$ by $\frac{6n^2}{n+z}$. *Ans.* $\frac{2n^2(n-z)}{m^2}$.



15. Multiply $\frac{a^2+ab}{(1+b)^2}$ by $\frac{c+bc}{a+b}$. *Ans.* $\frac{ac}{1+b}$.

16. Multiply $\frac{ac+bc}{(a-b)^2}$ by $\frac{a^2-ab}{c^2}$. *Ans.* $\frac{a(a+b)}{c(a-b)}$.

17. Multiply $\frac{a}{b}+\frac{c}{d}$ by $\frac{b}{a}+\frac{d}{c}$. *Ans.* $2+\frac{bc}{ad}+\frac{ad}{bc}$.

18. Multiply $\frac{n}{m+n}$, $\frac{m^2-n^2}{m^2}$ and $\frac{m}{m-n}$ together. *Ans.* $\frac{n}{m}$.

19. Multiply $\frac{1-x^2}{1-c}$ by $\frac{1-c^2}{x+x^2}$. *Ans.* $\frac{(1-x)(1+c)}{x}$.

20. Multiply $\frac{a(a-b)}{a^2+2ab+b^2}$ by $\frac{a(a+b)}{a^2-2ab+b^2}$. *Ans.* $\frac{a^2}{a^2-b^2}$.

21. Multiply $\frac{a^4-b^4}{a^2-2ab+b^2}$ by $\frac{a-b}{a^2+ab}$. *Ans.* $\frac{a^2+b^2}{a}$.

22. Required the value of $\left(\frac{a^2}{b-c}-\frac{b^2}{b-c}\right)\times\left(\frac{b}{a^2-b^2}-\frac{c}{a^2-b^2}\right)$.
*Ans.* 1.

23. Required the value of $\left(\frac{a+b}{b-c}\right)(a-b)\times\left(\frac{b-c}{a+b}\right)\left(\frac{1}{a-b}\right)$.
*Ans.* 1.

## DIVISION.

**144. Division of Fractions** is the process of dividing when one or both terms are fractional.

### CASE I.

**145. To divide a fraction by an entire quantity.**

1. Divide $\frac{ab}{c}$ by $b$.

SOLUTION. Since dividing the numerator of a fraction divides the fraction (Prin. 2, Art. 129), to divide $\frac{ab}{c}$ by $b$ we divide the numerator by $b$, and have $\frac{a}{c}$.

OPERATION.

$\frac{ab}{c}\div b=\frac{a}{c}$

2. Divide $\frac{a}{c}$ by $b$.

SOLUTION. Since multiplying the denominator of a fraction divides the fraction, to divide $\frac{a}{c}$ by $b$ we multiply the denominator by $b$, and have $\frac{a}{bc}$.

OPERATION.

$$\frac{a}{c} \div b = \frac{a}{bc}$$

**Rule.**—*Divide the numerator or multiply the denominator of the fraction, by the divisor.*

NOTE.—It is often convenient to indicate the division and then cancel common factors.

EXAMPLES.

3. Divide $\frac{6ax^3}{bc}$ by $2ax$. *Ans.* $\frac{3x^2}{bc}$.

4. Divide $\frac{12b^2c^4}{ad}$ by $4bc^3$. *Ans.* $\frac{3bc}{ad}$.

5. Divide $\frac{abcd}{mn}$ by $2x^2$. *Ans.* $\frac{abcd}{2mnx^2}$.

6. Divide $\frac{15x^2z^5}{a^3b^2}$ by $5z^4c$. *Ans.* $\frac{3x^2z}{a^3b^2c}$.

7. Divide $\frac{a^2(x-z)}{3c}$ by $2a^4c$. *Ans.* $\frac{x-z}{6a^2c^2}$.

8. Divide $\frac{x^2-1}{ab^2}$ by $a(x+1)$. *Ans.* $\frac{x-1}{a^2b^2}$.

9. Divide $\frac{a^3-ab^2}{a-c}$ by $c^2(a-b)$. *Ans.* $\frac{a(a+b)}{c^2(a-c)}$.

10. Divide $\frac{a^3x-ax^3}{c^n}$ by $ac^n+c^nx$. *Ans.* $\frac{ax(a-x)}{c^{2n}}$.

CASE II.

**146. To divide an entire or a fractional quantity by a fraction.**

1. Divide $a$ by $\frac{b}{c}$.

SOLUTION. $a$ divided by $b$ equals $\frac{a}{b}$; but since the divisor is $b$ divided by $c$, the quotient must be $c$ times as great, or $\frac{a}{b} \times c = \frac{ac}{b}$.

OPERATION.

$$a \div \frac{b}{c} = \frac{ac}{b}$$



2. Divide $\frac{a}{b}$ by $\frac{c}{d}$.

SOLUTION. $\frac{a}{b}$ divided by $c$ equals $\frac{a}{b \times c}$ (Prin. 2, Art. 129); but since the divisor is $c$ divided by $d$, the quotient must be $d$ times as great, or $\frac{a}{b \times c} \times d$, which equals $\frac{a \times d}{b \times c}$, or $\frac{ad}{bc}$.

OPERATION.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c} = \frac{ad}{bc}$$

SOLUTION 2D. From Art. 135, Prin., $\frac{a}{b} = ab^{-1}$, and also $\frac{c}{d} = cd^{-1}$; $ab^{-1}$ divided by $cd^{-1}$ equals $\frac{ab^{-1}}{cd^{-1}}$, which equals $\frac{ad}{bc}$.

OPERATION.

$$\frac{a}{b} \div \frac{c}{d} = ab^{-1} \div cd^{-1} = \frac{ab^{-1}}{cd^{-1}} = \frac{ad}{bc}$$

By inspection, we see that the same result can be obtained by inverting the terms of the divisor and multiplying; hence the following

**Rule.**—*Invert the terms of the divisor and proceed as in multiplication.*

NOTES.—1. When there are common factors in numerators and denominators, indicate the operation and then cancel common factors.

2. If either term is a mixed quantity, reduce it to a fraction before dividing.

EXAMPLES.

3. Divide $\frac{a^2}{c}$ by $\frac{b^2}{d}$. *Ans.* $\frac{a^2d}{b^2c}$.

4. Divide $\frac{ax}{cy}$ by $\frac{a^2}{xy^3}$. *Ans.* $\frac{x^2y^2}{ac}$.

5. Divide $\frac{4c^3x}{5ac^2}$ by $\frac{3cx^2}{5a^2b}$. *Ans.* $\frac{4ab}{3x}$.

6. Divide $(a-x)$ by $\frac{(a-x)c}{2a^2}$. *Ans.* $\frac{2a^2}{c}$.

7. Divide $\frac{(a-x)^2}{3c}$ by $\frac{a-x}{4a^2}$. *Ans.* $\frac{4a^2(a-x)}{3c}$.

8. Divide $3a+\frac{c}{d}$ by $\frac{a}{x}$. *Ans.* $\frac{3adx+cx}{ad}$.

9. Divide $a+\frac{b}{c}$ by $a+\frac{d}{c}$. *Ans.* $\frac{ac+b}{ac+d}$.

10. Divide $\frac{a+1}{2a}$ by $\frac{a-1}{4a^2}$. *Ans.* $\frac{2a(a+1)}{a-1}$

11. Divide $\frac{a^2-x^2}{a-1}$ by $\frac{a+x}{a(a-1)}$. *Ans.* $a(a-x)$.

12. Divide $\frac{ax^2-bx^3}{3a}$ by $\frac{5cx^2}{6ab}$. *Ans.* $\frac{2b(a-bx)}{5c}$

13. Divide $\frac{4a^3n}{a^2-b^2}$ by $\frac{an^3}{a+b}$. *Ans.* $\frac{4a^2}{n^2(a-b)}$.

14. Divide $1+\frac{1}{x}$ by $1-\frac{1}{x^2}$. *Ans.* $\frac{x}{x-1}$

15. Divide $1+\frac{a^n}{x^n}$ by $1+\frac{x^n}{a^n}$. *Ans.* $\frac{a^n}{x^n}$

16. Divide $\frac{(x-1)^2}{a^2-1}$ by $\frac{x^2+x-2}{a-1}$. *Ans.* $\frac{x-1}{(a+1)(x+2)}$.

17. Divide $\frac{a^2-b^2}{a^2+ax}$ by $\frac{(a-b)^2}{a+x}$. *Ans.* $\frac{a+b}{a(a-b)}$.

18. Divide $\frac{x^2+3x+2}{x+3}$ by $\frac{x^2+x}{x+3}$. *Ans.* $1+\frac{2}{x}$.

19. Divide $\frac{x^2-5x+6}{x+4}$ by $\frac{x-2}{x^2+x-12}$. *Ans.* $(x-3)^2$.

20. Divide $1-\frac{a^{2n}}{x^{2n}}$ by $1+\frac{a^n}{x^n}$. *Ans.* $1-\frac{a^n}{x^n}$.

## COMPLEX FRACTIONS.

**147.** A **Complex Fraction** is one in which the numerator or denominator, or both, contain a fraction.

1. Reduce $\frac{\frac{a}{b}}{\frac{c}{d}}$ to a simple fraction.

OPERATION

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\div\frac{c}{d}=$$

$$\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

SOLUTION. This complex fraction may be regarded as an expression of $\frac{a}{b}$ divided by $\frac{c}{d}$, which equals $\frac{a}{b}\times\frac{d}{c}$ or $\frac{ad}{bc}$



OPERATION.

$$\frac{\frac{a}{b}\times bd}{\frac{c}{d}\times bd}=\frac{ad}{bc}$$

SOLUTION 2D Since multiplying both terms of a fraction by the same quantity does not change its value (Prin. 3, Art. 129), if we multiply both terms of the complex fraction by the least common multiple of their denominators, $b$ and $d$, we will have the complex fraction equal to $\frac{ad}{bc}$.

**Rule.**—*Divide the numerator by the denominator, as in division;*

Or, *Multiply both terms of the complex fraction by the least common multiple of their denominators.*

### EXAMPLES.

2. Reduce $\frac{\frac{a}{c}}{\frac{x}{z}}$ to a simple fraction. *Ans.* $\frac{az}{cx}$.

3. Reduce $\frac{\frac{2a^2}{c^3}}{\frac{4ax}{bc}}$ to a simple fraction. *Ans.* $\frac{ab}{2c^2x}$.

4. Reduce $\frac{\frac{a}{2}-\frac{b}{a}}{1+\frac{1}{a}}$ to a simple fraction. *Ans.* $\frac{a^2-2b}{2a+2}$

5. Reduce $\frac{1+\frac{1}{c}}{a+\frac{1}{a}}$ to a simple fraction. *Ans.* $\frac{a(c+1)}{c(a^2+1)}$.

6. Reduce $\frac{\frac{c}{c-1}-1}{1-\frac{c}{c+1}}$ to a simple fraction. *Ans.* $\frac{c+1}{c-1}$.

7. Reduce $\frac{\frac{a+b}{x+y}}{\frac{a^2-b^2}{x^2-y^2}}$ to a simple fraction. *Ans.* $\frac{x-y}{a-b}$

8. Reduce $1-\dfrac{1}{1+\dfrac{1}{a}}$ to a simple fraction. *Ans.* $\dfrac{1}{a+1}$

9. Reduce $\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{n}}}$ to a simple fraction. *Ans.* $n+1$.

10. Reduce $\dfrac{a-1+\dfrac{6}{a-6}}{a-2+\dfrac{3}{a-6}}$ to a simple fraction. *Ans.* $\dfrac{a-4}{a-5}$.

## VANISHING FRACTIONS.

**148.** A **Vanishing Fraction** is one which reduces to the form $\frac{0}{0}$ when certain suppositions are made.

Thus, $\dfrac{x^2-1}{x-1}$, when $x=1$, becomes $\dfrac{1-1}{1-1}$, or $\dfrac{0}{0}$. So, also, $\dfrac{a^2-x^2}{a-x}$, when $a=x$, becomes equal to $\dfrac{0}{0}$.

PRINCIPLE.—*Vanishing fractions contain a common factor in the numerator and denominator, which reduces to zero when a special supposition is made.*

Thus, $\dfrac{x^2-1}{x-1}$ by factoring becomes $\dfrac{(x-1)(x+1)}{x-1}$, in which the factor, $x-1$, is common to both terms, and is equal to 0 when $x=1$.

1. Find the value of $\dfrac{a^2-x^2}{a-x}$ when $a=x$.

SOLUTION. If we substitute $a$ for $x$, we will have $\frac{0}{0}$; but factoring the numerator and dividing by the denominator, we have the fraction equal to $a+x$. Substituting the value of $x$, we have $a+a$, or $2a$. Hence, the value of the given fraction when $a=x$ is $2a$.

OPERATION.

$$\frac{a^2-x^2}{a-x}=\frac{(a+x)(a-x)}{a-x}=$$
$$a+x=2a$$



**Rule.**—*Cancel the common factor which reduces to zero, and then make the supposition which reduced the fraction to* $\frac{0}{0}$.

EXAMPLES.

2. Find the value of $\dfrac{x^2-1}{x-1}$ when $x=1$. *Ans.* 2.

3. Find the value of $\dfrac{x^3-1}{x-1}$ when $x=1$. *Ans.* 3.

4. Find the value of $\dfrac{x^3-a^3}{x-a}$ when $x=a$. *Ans.* $3a^2$.

5. Find the value of $\dfrac{x^3-a^3}{x^2-a^2}$ when $x=a$. *Ans.* $\dfrac{3a}{2}$.

6. Find the value of $\dfrac{x^4-a^4}{x-a}$ when $x=a$. *Ans.* $4a^3$.

7. Find the value of $\dfrac{(x-a)^2}{x^3-a^3}$ when $x=a$. *Ans.* 0.

8. Find the value of $\dfrac{x-x^6}{1-x}$ when $x=1$. *Ans.* 5.

9. Find the value of $\dfrac{x^2+2x-15}{x^2+4x-21}$ when $x=3$. *Ans.* $\dfrac{4}{5}$.

10. Find the value of $\dfrac{1-x^m}{1-x}$ when $x=1$. *Ans.* $m$.

11. Find the value of $\dfrac{x^n-a^n}{x-a}$ when $x=a$. *Ans.* $na^{n-1}$.

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## REVIEW QUESTIONS.

Define a Fraction. The Terms. Numerator. Denominator. A Mixed Quantity. State the principles. What is the sign of a fraction? State the principles of the signs.

Define Reduction of Fractions. State the cases. The rule for each case. Define Addition. Subtraction. Multiplication. Divison. How many cases in each? Give the rule for each case. How is a quantity changed from one term of a fraction to another?

Define a Complex Fraction. Give the rule for the reduction of complex fractions to simple fractions. Define a Vanishing Fraction. When does a fraction become a vanishing fraction? How do we find the value of a vanishing fraction?