SECTION V.

SIMPLE EQUATIONS.

149. An **Equation** is an expression of equality between two equal quantities. Thus, $2x+4=20$ is an equation.

150. The **First Member** of an equation is the quantity on the left of the sign of equality.

151. The **Second Member** of an equation is the quantity on the right of the sign of equality.

152. A **Numerical Equation** is one in which the known quantities are expressed by figures; as, $3x-4=17$.

153. A **Literal Equation** is one in which some or all of the known quantities are expressed by letters; as, $2x-a=b$, or $x-2=b$.

154. An **Identical Equation** is one in which the two members are the same, or will become the same by performing the operations indicated; as, $3x+1=3x+1$; or, $3x+a=2x+2a+x-a$.

155. The **Degree** of an equation containing but one unknown quantity is determined by the highest power of the unknown quantity.

156. A **Simple Equation,** or an *equation of the first degree,* is one in which the first power is the highest power of the unknown quantity; as, $x+a=b$.

157. A **Quadratic Equation,** or an *equation of the second degree,* is one in which the second power is the highest power of the unknown quantity; as, $x^2+ax=b$, or $x^2=a$.

158. A **Cubic Equation,** or an *equation of the third degree,* is one in which the third power is the highest power of the unknown quantity; as, $x^3+4x^2+3x=8$.

NOTE.—The symbol = was introduced by *Robert Recorde,* who gave as his reason for it that "noe 2 thynges can be moare equalle" than two parallel lines.

TRANSFORMATION OF EQUATIONS.

159. The **Transformation** of an equation is the process of changing the terms without affecting the equality of the members.

160. Equations may be transformed by means of the following axiomatic principles:

PRINCIPLES.

1. *The same or equal quantities may be added to both members of an equation.*
2. *The same or equal quantities may be subtracted from both members of an equation.*
3. *Both members of an equation may be multiplied by the same or equal quantities.*
4. *Both members of an equation may be divided by the same or equal quantities.*
5. *Both members of an equation may be raised to the same power.*
6. *Both members of an equation may have the same root extracted.*

161. In the transformation of equations there are two principal cases—

1. Clearing of fractions; 2. Transposition of the terms.

CASE I.

162. To clear an equation of fractions.

1 Clear $\frac{3x}{4}-\frac{2x}{3}=\frac{5}{6}$ of fractions.

SOLUTION. The least common multiple of the denominators is 12; multiplying both members of the equation by 12, we have $9x-8x=10$; hence, multiplying both members of the equation by the least common multiple of the denominators clears it of fractions and does not change the equality of the members. (Prin. 3.)

OPERATION.

$$\frac{3x}{4}-\frac{2x}{3}=\frac{5}{6}$$

$$9x-8x=10$$

Rule.—*Multiply both members of the equation by the least common multiple of the denominators, reducing fractional terms to integers.*

NOTES.—1. An equation may be cleared of fractions by multiplying each term by all the denominators.

2. If a fraction has the minus sign before it, the signs of all the terms of the numerator must be changed when the denominator is removed.

EXAMPLES.

Clear the following equations of fractions:

2. $\frac{x}{2}+\frac{x}{3}=\frac{5}{3}$. *Ans.* $3x+2x=10$.

3. $\frac{2x}{3}+\frac{3x}{4}=\frac{5}{6}$. *Ans.* $8x+9x=10$.

4. $\frac{3x}{4}-\frac{5x}{6}=\frac{7}{8}$. *Ans.* $18x-20x=21$.

5. $\frac{x}{2}+\frac{x}{6}=4-\frac{x}{3}$. *Ans.* $3x+x=24-2x$.

6. $\frac{x}{3}-a+6=\frac{a+x}{6}$. *Ans.* $2x-6a+36=a+x$.

7. $\frac{x}{a-b}-b=\frac{c}{2}$. *Ans.* $2x-2ab+2b^2=ac-bc$.

8. $\frac{ax}{3}-\frac{bx}{4}=\frac{cx-dx}{6}$. *Ans.* $4ax-3bx=2cx-2dx$.

9. $\frac{2x}{a+b}-\frac{3x}{a-b}=4$. *Ans.* $2ax-2bx-3ax-3bx=4a^2-4b^2$.

10. $\frac{3x-4}{a}=2-3a^{-1}$. *Ans.* $3x-4=2a-3$.

11. $\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{3a}{x^2-1}$. *Ans.* $(x+1)^2-(x-1)^2=3a$.

12. $\frac{2-\frac{x}{2}}{\frac{x}{3}}=\frac{1-\frac{1}{2}}{\frac{2}{3}}$. *Ans.* $8-2x=2x-x$.

13. $\frac{x+3}{x-3}-\frac{x-3}{x+3}=6\frac{6}{7}$. *Ans.* $7(x+3)^2-7(x-3)^2=48(x^2-9)$.

CASE II.

163. To transpose the terms of an equation.

164. Transposition is the process of changing a term from one member of an equation to the other without affecting the equality of the members.

1. In $x+b=a$, transpose b to the second member.

SOLUTION. Subtracting b from both members, which does not affect the equality of the members (Prin. 2, Art. 160), we have $x=a-b$.

OPERATION.

$$\begin{array}{r} x+b=a \\ b=b \\ \hline x=a-b \end{array}$$

2. In $ax-a=bx+c$, transpose a and bx.

SOLUTION. Adding a to both members and subtracting bx from both members, which, according to Prin. 1 and 2, will not affect the equality of the members, we have $ax-bx=a+c$.

OPERATION.

$$\begin{array}{r} ax-a=bx+c \\ a=a \\ \hline ax=bx+a+c \\ bx=bx \\ \hline ax-bx=a+c \end{array}$$

In both of these examples we see that in changing a quantity from one member to the other, the *sign* of the quantity is changed; hence the following rule.

Rule.—*A term may be transposed from one member of an equation to the other, if, at the same time, the sign be changed.*

EXAMPLES.

In the following examples transpose the known terms to the second member and the unknown terms to the first member:

3. $2x+c=a$. *Ans.* $2x=a-c$.

4. $3x-2=x+4$. *Ans.* $3x-x=4+2$.

5. $5x+b=3x+a$. *Ans.* $5x-3x=a-b$.

6. $3a-2b=6x-ax$. *Ans.* $ax-6x=2b-3a$.

7. $a^2-a^2x-an=3x^2$. *Ans.* $-3x^2-a^2x=an-a^2$.

8. $ax^2+5ac-2a=cx$. *Ans.* $ax^2-cx=2a-5ac$.

9. $2x^2-\frac{a-b}{2}=5a+\frac{6x^2-x}{3}$. *Ans.* $2x^2-\frac{6x^2-x}{3}=5a+\frac{a-b}{2}$.

SOLUTION OF SIMPLE EQUATIONS,

CONTAINING ONE UNKNOWN QUANTITY.

165. The **Solution** of an equation is the process of finding the value of the unknown quantity.

166. The **Root** of an equation is the value of the unknown quantity.

167. To **Verify** the root of an equation, we substitute its value for the unknown quantity and reduce the members to identity. The equation is then said to be *satisfied.*

NOTE.—The solution of an equation is often called the *reduction of the equation. To reduce an equation* is therefore to solve it.

CASE I.

NUMERICAL EQUATIONS.

1. Find the value of x in the equation $3x-4=12-x$.

SOLUTION. Transposing the unknown terms to the first member and the known terms to the second member, we have $3x+x=12+4$; uniting the terms, we have $4x=16$; dividing by the coefficient of x, we have $x=4$.

VERIFICATION. Substituting for x its value in the given equation, we have $3\times4-4=12-4$; reducing, we have $8=8$; and since this is an identical equation, the root is *verified.*

OPERATION.

$$3x-4=12-x$$
$$3x+x=12+4$$
$$4x=16$$
$$x=4$$

VERIFICATION.

$$3\times4-4=12-4$$
$$8=8$$

2. Find the value of x in the equation $\frac{x}{2}-\frac{5}{6}=\frac{1}{2}+\frac{x}{6}$.

SOLUTION. Clearing the equation of fractions by multiplying by 6, we have $3x-5=3+x$; transposing the terms, we have $3x-x=3+5$; uniting the terms, we have $2x=8$; dividing by the coefficient of x, we have $x=4$.

VERIFICATION.—Substituting the value of x in the equation, we have $\frac{4}{2}-\frac{5}{6}=\frac{1}{2}+\frac{4}{6}$; reducing, we have $\frac{7}{6}=\frac{7}{6}$; and since this is an identical equation, the root is *verified.*

OPERATION

$$\frac{x}{2}-\frac{5}{6}=\frac{1}{2}+\frac{x}{6}$$
$$3x-5=3+x$$
$$3x-x=3+5$$
$$2x=8$$
$$x=4$$

VERIFICATION

$$\frac{4}{2}-\frac{5}{6}=\frac{1}{2}+\frac{4}{6}$$
$$\frac{7}{6}=\frac{7}{6}$$

Rule.—I. *Clear the equation of fractions, if necessary.*

II. *Transpose the unknown terms to the first member of the equation and the known terms to the second member.*

III. *Reduce each member to its simplest form, and divide both members by the coefficient of the unknown quantity.*

To verify the result: *Substitute the value of the unknown quantity in the equation, and if the members are identical the result is correct.*

NOTES.—1. It is sometimes advantageous to transpose and make some reductions before clearing of fractions.

2. When the coefficient of the unknown quantity is negative we may multiply both members by -1, or divide by the negative coefficient.

EXAMPLES.

3. Given $4x+6=2x+10$, to find x. *Ans.* $x=2$.
4. Given $5x+4-2x=10+x$, to find x. *Ans.* $x=3$.
5. Given $18-3x=5x+2$, to find x. *Ans.* $x=2$.
6. Given $7x+6=5x+14$, to find x. *Ans.* $x=4$.
7. Given $4x-17=4x+13-6x$, to find x. *Ans.* $x=5$.
8. Given $2x-12=5x-30$, to find x. *Ans.* $x=6$.
9. Given $6x+16=9x-5$, to find x. *Ans.* $x=7$.
10. Given $\frac{x}{2}+\frac{x}{3}=5\frac{1}{3}+4\frac{2}{3}$, to find x. *Ans.* $x=12$.
11. Given $\frac{x}{3}-\frac{x}{4}+\frac{x}{5}=2$, to find x. *Ans.* $x=7\frac{1}{17}$.
12. Given $\frac{x}{2}-\frac{x}{6}+3=\frac{x}{5}+4\frac{3}{5}$, to find x. *Ans.* $x=12$.
13. Given $\frac{3x}{5}+\frac{5x}{6}=\frac{4x}{5}+1\frac{4}{15}$, to find x. *Ans.* $x=2$.
14. Given $\frac{2x}{7}-\frac{3x}{4}=\frac{7x}{6}-11\frac{5}{12}$, to find x. *Ans.* $x=7$.
15. Given $4(x+1)=3(x+2)$, to find x. *Ans.* $x=2$.
16. Given $\frac{2x}{3}+\frac{x-1}{6}=\frac{3x+1}{2}-10$, to find x. *Ans.* $x=14$.

17. Given $\frac{x+3}{2}+\frac{x}{3}=4-\frac{x-5}{4}$, to find x. *Ans.* $x=3\frac{6}{13}$.

18. Given $x+\frac{2x-4}{3}=12-\frac{3x-5}{2}$, to find x. *Ans.* $x=5$.

19 Given $\frac{x-5}{3}+\frac{x}{2}=12-\frac{x-10}{3}$, to find x. *Ans.* $x=14\frac{4}{7}$

20. Given $\frac{x+3}{2}-\frac{x-2}{3}=\frac{3x-5}{12}+\frac{1}{4}$, to find x. *Ans.* $x=28$.

CASE II.

LITERAL EQUATIONS.

1. Given $ax+bx=ac+bc$, to find x.

SOLUTION. Factoring both members of equation (1), we obtain equation (2); dividing by $(a+b)$, the coefficient of x, we obtain $x=c$.

OPERATION.

$$ax+bx=ac+bc \quad (1)$$
$$(a+b)x=(a+b)c \quad (2)$$
$$x=c \quad (3)$$

EXAMPLES.

2. Given $ax+bx=an+bn$, to find x. *Ans.* $x=n$.

3. Given $ax+d=c-bx$, to find x. *Ans.* $x=\frac{c-d}{a+b}$.

4. Given $nx-c=nc-x$, to find x. *Ans.* $x=c$.

5. Given $mx-n=nx+m$, to find x. *Ans.* $x=\frac{m+n}{m-n}$.

6. Given $nx+m=mx+n$, to find x. *Ans.* $x=1$.

7. Given $ax+b=\frac{x}{a}+\frac{1}{b}$, to find x. *Ans.* $x=\frac{a(1-b^2)}{b(a^2-1)}$.

8. Given $\frac{x}{a}+\frac{x}{b-a}=\frac{a}{b+a}$, to find x. *Ans.* $x=\frac{a^2(b-a)}{b(b+a)}$

9. Given $\frac{1-x}{1+x}=1-\frac{1}{c}$, to find x. *Ans.* $x=\frac{1}{2c-1}$.

10. Given $x+a=\frac{x^2}{a+x}$, to find x. *Ans.* $x=-\frac{a}{2}$.

11 Given $(a+x)(b+x)-a(b+c)=\frac{a^2c}{b}+x^2$, to find x.
Ans. $x=\frac{ac}{b}$.

12. Given $\frac{a+b}{a-x}+\frac{b+c}{c-x}=\frac{a-b}{a-x}$, to find x.
Ans. $x=\frac{ab+2bc+ac}{c+3b}$

SPECIAL ARTIFICES.

168. The solution of a problem may often be abridged by the use of particular operations, called *Artifices.*

CASE I

169. Uniting terms before clearing of fractions.

1. Given $\frac{4x}{5}+12=\frac{3x}{4}+15$, to find x.

SOLUTION. Transposing the 12 and uniting the terms, we have equation (2); clearing of fractions, we have equation (3); transposing and uniting terms, we have $x=60$.

OPERATION.

$$\frac{4x}{5}+12=\frac{3x}{4}+15 \quad (1)$$
$$\frac{4x}{5}=\frac{3x}{4}+3 \quad (2)$$
$$16x=15x+60 \quad (3)$$
$$x=60$$

EXAMPLES.

2. Given $2x-4=\frac{x}{2}+2$, to find x. *Ans.* $x=4$.

3. Given $\frac{x}{2}-6+\frac{x}{3}=\frac{x}{6}+2$, to find x. *Ans.* $x=12$.

4. Given $\frac{3x}{4}+\frac{4x}{5}-1\frac{5}{6}=\frac{3x}{5}+17\frac{1}{6}$, to find x. *Ans.* $x=20$.

5 Given $\frac{x}{a}-\frac{x-a}{3}-a=2a$, to find x. *Ans.* $x=\frac{8a^2}{3-a}$.

6 Given $\frac{2x}{3}-3\frac{1}{3}+13=13\frac{11}{12}-\frac{3x}{4}$, to find x. *Ans.* $x=3$.

7 Given $\frac{3x}{4}-4\frac{1}{2}+a=a+\frac{x}{5}+1$, to find x. *Ans.* $x=10$.

8 Given $2ax+3m-\frac{1}{3}cx=ax+2m+\frac{2}{3}cx+n$, to find x.
Ans. $x=\frac{n-m}{a-c}$.

9. Given $3ax-2bx-\frac{1}{3}c-\frac{1}{4}mx=\frac{2}{3}c+\frac{3}{4}mx-n-bx+2ax$, to find x.

Ans. $x=\dfrac{c-n}{a-b-m}$.

CASE II.

170. Indicating some of the operations.

1. Given $\frac{x}{3}+\frac{x}{4}+\frac{x}{5}=47$, to find x.

Solution. We multiply both members by 60, the least common multiple of the denominators, indicating the operation in the second member, and obtain (2); reducing, we have (3); dividing by 47, we have (4).

OPERATION.

$$\frac{x}{3}+\frac{x}{4}+\frac{x}{5}=47 \quad (1)$$
$$20x+15x+12x=47\times 60 \quad (2)$$
$$47x=47\times 60 \quad (3)$$
$$x=60 \quad (4)$$

EXAMPLES.

2. Given $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=26$, to find x. *Ans.* $x=24$.

3. Given $\frac{x}{2}+\frac{x}{6}+\frac{x}{12}=45$, to find x. *Ans.* $x=60$.

4. Given $\frac{1}{3}x+\frac{1}{5}x+\frac{1}{6}x=42$, to find x. *Ans.* $x=60$.

5. Given $3x+\frac{2x}{3}+\frac{5x}{6}=54$, to find x. *Ans.* $x=12$.

CASE III.

171. Substituting some other unknown quantity for a common expression.

1. Given $x+2+3(x+2)=\frac{x+2}{4}+15$, to find x.

Solution. Let y represent $x+2$; substituting y for $x+2$, we have equation (2); uniting terms, clearing of fractions, etc., we have the equation $y=4$; but $y=x+2$; hence $x+2=4$, from which we have $x=2$.

OPERATION.

$$x+2+3(x+2)=\frac{x+2}{4}+15 \quad (1)$$
$$y+3y=\frac{y}{4}+15 \quad (2)$$
$$4y=\frac{y}{4}+15 \quad (3)$$
$$y=4 \quad (4)$$
$$\therefore x+2=4 \quad (5)$$
$$x=2 \text{ Ans.} \quad (6)$$

2. Given $x+3+2(x+3)=18$, to find x. *Ans.* $x=3$.

3. Given $\frac{x+4}{2}+\frac{3(x+4)}{4}=\frac{4(x+4)}{5}+4\frac{1}{20}$, to find x. *Ans.* $x=5$.

4. Given $\frac{x-5}{3}+\frac{2(x-5)}{4}=\frac{3}{5}(x-5)+14$, to find x. *Ans.* $x=65$.

5. Given $\frac{2x+4}{3}-\frac{x-3}{4}=\frac{x+2}{3}+3\frac{1}{3}$, to find x. *Ans.* $x=23$.

6. Given $\frac{x+c}{3}-\frac{3(x+c)}{4}=\frac{1}{3}(x+c)-c$, to find x. *Ans.* $x=\frac{c}{3}$.

CASE IV.

172. Separating and uniting terms before clearing of fractions.

1. Given $\frac{5x+4}{4}-\frac{6x-8}{5x-20}=\frac{10x-5}{8}-\frac{3}{8}$, to find x.

OPERATION.

$$\frac{5x+4}{4}-\frac{6x-8}{5x-20}=\frac{10x-5}{8}-\frac{3}{8} \quad (1)$$
$$\frac{5x}{4}+1-\frac{6x-8}{5x-20}=\frac{10x}{8}-\frac{5}{8}-\frac{3}{8} \quad (2)$$
$$2=\frac{6x-8}{5x-20} \quad (3)$$
$$10x-40=6x-8 \quad (4)$$
$$4x=32 \quad (5)$$
$$x=8 \quad (6)$$

Solution.—Separating the terms, we have equation (2); transposing and reducing, we have equation (3); clearing of fractions, we have equation (4); transposing, uniting terms and dividing by 4, we have equation (6).

Note.—Considerable labor is saved by this artifice. Let the pupil solve the problem by the ordinary method and observe the difference.

2. Given $\frac{7-9x}{12}-\frac{12-4x}{5-3x}=\frac{15-6x}{8}-\frac{7}{24}$, to find x. *Ans.* $x=2\frac{3}{7}$.

3 Given $\frac{6x-15}{9}-\frac{10x-17}{15}=\frac{4x-15}{3-2x}+\frac{2}{15}$, to find x. *Ans.* $x=4\frac{7}{8}$.

4. Given $\frac{x-16}{18}-\frac{17-4x}{9}=\frac{5x}{7}-\frac{4-26x}{32-17x}-\frac{3x}{14}$, to find x. *Ans.* $x=4$.

PROBLEMS IN SIMPLE EQUATIONS.

173. A **Problem** is a question requiring some unknown result from things which are known.

174. The **Solution** of a problem is the process of finding the required unknown result.

175. The solution of a problem in Algebra consists of two distinct parts—

1st. The formation of the equation;

2d. The solution of the equation.

176. The **Method of Solving** a problem cannot be stated by any general or precise rule. The following directions may be of some value:

1. *Represent the unknown quantity by one of the final letters of the alphabet.*

2. *Form an equation by indicating the operations necessary to verify the result were it known.*

3. *Solve the equation thus derived.*

NOTE.—The formation of the equation is called the *concrete* part of the solution; the reduction of the equation the *abstract* part. The first part is also called the *statement* of the problem. It is merely a translation of the problem from *common* into *algebraic* language.

PROBLEMS.

CASE I.

1. A farmer bought a cow and a horse for \$375, paying 4 times as much for the horse as for the cow; required the cost of each.

SOLUTION. Let x represent the cost of the cow; then, since he paid 4 times as much for the horse as for the cow, $4x$ will represent the cost of the horse; and since both cost \$375, we have the equation $x+4x=375$; uniting the terms, we have $5x=375$; dividing by 5, we have $x=75$; and multiplying by 4, we have $4x=300$. Hence, the cow cost \$75, and the horse \$300.

OPERATION.

Let $x=$ the cost of the cow.
Then $4x=$ the cost of the horse.

$$x+4x=375 \qquad (1)$$
$$5x=375 \qquad (2)$$
$$x=75, \text{ cost of cow.} \qquad (3)$$
$$4x=300, \text{ cost of horse} \qquad (4)$$

2. The income of A and B for one month was \$1728, and B's income was 3 times A's; required the income of each.

Ans. A's, \$432; B's, \$1296.

3. A tree, 96 feet high, in falling broke into three unequal parts; the longest piece was 5 times the shortest, and the other was twice the shortest; required the length of each piece.

Ans. 1st, 12 ft.; 2d, 24 ft.; 3d, 60 ft.

4. Divide the number represented by a into 3 parts, such that m times the first part shall equal the second part, and n times the second part shall equal the third part.

Ans. $\dfrac{a}{1+m+mn}$; $\dfrac{ma}{1+m+mn}$; $\dfrac{mna}{1+m+mn}$.

CASE II.

1. A boy bought a book and a toy for \$2.25, and the toy cost $\frac{2}{3}$ as much as the book; required the cost of each.

SOLUTION. Let x represent the cost of the book; then will $\frac{2}{3}x$ represent the cost of the toy; and since both cost \$2.25, we have the equation $x+\frac{2}{3}x=225$. Clearing of fractions, we have $3x+2x=675$; uniting the terms, we have $5x=675$, from which $x=135$, and $\frac{2}{3}x=90$. Hence, etc.

OPERATION.

Let $x=$ cost of book.
Then $\frac{2}{3}x=$ cost of toy.

$$x+\tfrac{2}{3}x=225 \qquad (1)$$
$$3x+2x=675 \qquad (2)$$
$$5x=675 \qquad (3)$$
$$x=135, \text{ cost of book.}$$
$$\tfrac{2}{3}x=90, \text{ cost of toy.}$$

2. A watch and chain cost \$350; what was the cost of each if the chain cost $\frac{3}{4}$ as much as the watch?

Ans. Watch, \$200; chain, \$150.

3. Divide \$2782 among Harry, Harvey and Hinkley, so that Harry may have $\frac{2}{3}$ as much as Hinkley, and Harvey $\frac{3}{4}$ as much as Harry.

Ans. Harry, \$856; Harvey, \$642; Hinkley, \$1284.

4. Divide the number a into 2 parts, so that $\dfrac{n}{m}$ of the first part shall equal the second part.

Ans. $\dfrac{ma}{m+n}$; $\dfrac{na}{m+n}$.

CASE III.

1. A man weighs 27 lbs. more than his wife, and the sum of their weights is 313 lbs.; required the weight of each.

Ans. Man, 170 lbs.; wife, 143 lbs.

2. An angler's pole and line measure 28 feet, and $\frac{2}{3}$ of the sum obtained by increasing the length of the pole by 12 feet equals the length of the line; required the length of each.

Ans. Pole, 12 ft.; line, 16 ft.

3. The sum of 3 numbers is 215; the first equals twice the second, increased by 15, and the second equals $\frac{2}{3}$ of the remainder of the third diminished by 20; required the numbers.

Ans. 1st, 95; 2d, 40; 3d, 80.

4. Divide the number a into 2 parts, such that the second part shall equal m times the first part, plus n.

Ans. 1st, $\frac{a-n}{1+m}$; 2d, $\frac{ma+n}{1+m}$.

CASE IV.

1. A gentleman gave 6 cents each to some poor children had he given them 9 cents each, it would have taken 48 cents more; how many children were there?

Solution. Let x equal the number of children; then $6x$ will equal what he gave them, and $9x$ will equal what he would have given them by giving them 9 cents each. Then, by the conditions of the problem, we have the equation $9x-6x=48$; uniting the terms, we have $3x=48$, or $x=16$. Hence, there were 16 children.

OPERATION.

Let $x=$ number of children;
then $6x=$ what he gave them;
and $9x=$ what he would have given

$9x-6x=48$ (1)
$3x=48$ (2)
$x=16$ (3)

2. A man gave some beggars 10 cents each, and had 75 cents remaining; had he given them 15 cents each, it would have taken all his money; required the number of beggars.

Ans. 15 beggars.

3. A lady gave 60 cents to some poor children; to each boy she gave 2 cents, and to each girl 4 cents; how many were there of each, provided there were 3 times as many boys as girls?

Ans. 6 girls; 18 boys.

4. A and B had equal sums of money; A bought sheep at $12 each, and had $40 remaining; B bought twice as many lambs at $8 each, and wanted $40 to pay for them; how much did each invest?

Ans. A, $240; B, $320.

5. A man gave a number of beggars m cents each, and had a cents remaining; had he given them n cents each, he would have had b cents remaining; how many beggars were there, and what was his money? *Ans.* Number, $\frac{b-a}{m-n}$; money, $\frac{bm-an}{m-n}$.

CASE V.

1. A can do a piece of work in 6 days, and B in 8 days; in what time can they together do it?

Solution. Let x equal the time in which they together can do the work; then $\frac{1}{x}$ will equal what they both do in a day; but A does $\frac{1}{6}$ of it, and B $\frac{1}{8}$ of it, in a day; hence, $\frac{1}{6}+\frac{1}{8}=\frac{1}{x}$; from which we find $x=3\frac{3}{7}$ days.

OPERATION.

Let $x=$ the time;
$\frac{1}{x}=$ what both do in 1 day;
$\frac{1}{6}=$ what A does in 1 day;
$\frac{1}{8}=$ what B does in 1 day;
then, $\frac{1}{6}+\frac{1}{8}=\frac{1}{x}$;
$8x+6x=48$;
$x=3\frac{3}{7}$ days.

2. A pound of tea lasted a man and wife 3 months, and the wife alone 4 months; how long will it last the man alone?

Ans. 12 months.

3. A can build a wall in 20 days, B in 30 days, and C in 40 days; in what time can they together build it. *Ans.* $9\frac{3}{13}$ days.

4. A can paper a room in $\frac{1}{2}$ of a day, B in $\frac{1}{3}$ of a day, and C in $\frac{1}{4}$ of a day; in what time will they do it working together?

Ans. $\frac{1}{9}$ of a day.

5. A can do a piece of work in a days, B in b days, C in c days; in what time can they perform it if all work together?

Ans. $\frac{abc}{ab+ac+bc}$ days.

CASE VI.

1. A man receives \$4 a day for his labor, and forfeits \$2 each day he is idle, and at the end of 30 days receives \$60; how many days has he worked?

SOLUTION.

Let x = number of working days,
and $30 - x$ = number of idle days;
then, $4x$ = sum earned,
and $(30 - x)2$ = sum forfeited.
Then, $4x - 2(30 - x) = 60$
whence, $x = 20$, number of days he worked,
and $30 - x = 10$, number of days he was idle.

2. Fannie James agreed to carry 12 dozen eggs to a store for $\frac{1}{4}$ cent each, on condition that she should forfeit $2\frac{1}{4}$ cents for each one she broke; she received 26 cents; how many were broken? *Ans.* 4.

3. Francis receives \$2.50 a day for his labor, and pays 50 cents a day for his board, and at the end of 40 days has saved \$50; how many days was he idle? *Ans.* 12.

4. A man receives \$$a$ a day for his labor, on condition that he forfeits \$$b$ each day he is idle; at the expiration of n days he has received \$$c$; required the number of working and idle days. *Ans.* Working days, $\frac{c + nb}{a + b}$; idle days, $\frac{an - c}{a + b}$.

CASE VII.

1. The head of a fish is 10 inches long, the tail is as long as the head, plus $\frac{1}{2}$ of the body, and the body is as long as the head and tail both; required the length of the fish.

SOLUTION.

Let x = the length of the body,
and $10 + \frac{x}{2}$ = the length of the tail.
Then, $x = 10 + \frac{x}{2} + 10$, by the last condition;
whence, $x = 40$, the length of the body, etc.

2. The head of a whale is $\frac{1}{2}$ as long as the tail, plus 3 feet; the tail is $\frac{1}{4}$ as long as the body, plus 4 feet; and the body is twice as long as the head and tail; what is the length of the whale? *Ans.* 108 ft.

3. The artillery of an army corps consisted of 60 men less than $\frac{1}{3}$ of the cavalry; the cavalry consisted of 2040 men more than the artillery; and $\frac{1}{6}$ of the infantry was 370 men less than the cavalry; how many men were in the corps? *Ans.* 19,500.

4. The head of a fish is a inches long; the tail is as long as the head plus 1-nth of the length of the body; and the body is as long as the head and tail; what is the length of the fish? *Ans.* $\frac{4an}{n-1}$.

CASE VIII.

1. How far may a person ride in a coach, going at the rate of 10 miles an hour, and walking back at the rate of 6 miles an hour, provided he is gone 8 hours?

SOLUTION.

Let x = the distance he goes;
then $\frac{x}{10}$ = time in going,
and $\frac{x}{6}$ = time in returning.
Then $\frac{x}{10} + \frac{x}{6} = 8$, etc.

2. A steamboat, whose propelling rate in still water is 15 miles an hour, descends a river whose current is 3 miles an hour; how far may it go that it may be gone but 10 hours? *Ans.* 72 miles.

3. An equestrian rides 24 miles, going at a certain rate. He walks back at the rate of 3 miles an hour, and is gone 11 hours. At what rate does he ride? *Ans.* 8 miles an hour.

4. How far may a person ride in a stage-coach going at the rate of a miles an hour, provided he returns immediately by railroad at the rate of c miles an hour, and is gone n hours? *Ans.* $\frac{acn}{a+c}$.

5. A steam-packet, whose propelling rate in still water is a miles an hour, descends a river whose current is c miles an hour; how far may it go that it may be gone n hours?

Ans. $\frac{(a^2-c^2)n}{2a}$.

CASE IX.

1. Eight men hire a coach to ride to Lancaster, but by taking in 4 more persons the expense of each is diminished by $\$\frac{3}{4}$; what do they pay for the coach?

SOLUTION.

Let x = the sum to be paid;

then $\frac{x}{8}$ = share of each by 1st condition,

and $\frac{x}{12}$ = share of each by 2d condition.

Hence, $\frac{x}{8}-\frac{x}{12}=\frac{3}{4}$.

2. Fifteen persons engage a yacht, but, before sailing, 3 of the company decline going, by which the expense of each is increased $\$2\frac{1}{2}$; what do they pay for the yacht?

Ans. \$150.

3. A company of 15 persons engage a dinner at a hotel for \$15, but, before paying the bill, a number of them withdraw, by which each person's bill is augmented $\$\frac{1}{2}$; how many withdraw? *Ans.* 5 persons.

4. A number of persons, n, hire a coach to ride, but, by taking in m more persons, the expense of each is diminished a dollars; what do they pay for the coach? *Ans.* $\frac{(n^2+nm)a}{m}$.

5. A number of persons, n, chartered a steamboat for an excursion, for which they were to pay $\$a$; but, before starting, several of the company declined going, by which each person's share of the expense was increased $\$b$; how many persons went and how many remained?

Ans. Went, $\frac{an}{a+bn}$; remained, $\frac{bn^2}{a+bn}$.

CASE X.

1. A, at a game of chess, won \$120, and then lost $\frac{1}{4}$ of what he then had, and then found he had 3 times as much as at first; how much had he at first?

SOLUTION.

Let x = the sum at first;

then $\frac{3}{4}(x+120)=3x$, etc.

2. A person being asked his age, said that if his age were increased by its $\frac{2}{5}$ and $2\frac{3}{5}$ years, the sum would equal 4 times his age 13 years ago; what was his age? *Ans.* 21 yrs.

3. A merchant lost \$1400 of his stock, and the next year gained $\frac{4}{5}$ as much as remained of his stock, and then had $\frac{3}{4}$ as much as at first; what was his original stock? *Ans.* \$2400.

4. A, having a certain sum of money, found a dollars, and then lost 1-nth of what he then had, and then found he had m times as much as he had at first; how much had he at first?

Ans. $\frac{a(n-1)}{mn-n+1}$.

CASE XI.

1. If 80 lbs. of sea water contain 2 lbs. of salt, how much fresh water must be added to these 80 lbs., so that 10 lbs. of the new mixture may contain $\frac{1}{6}$ of a pound of salt?

SOLUTION.

Let x = number of pounds to be added.

Then, $\frac{2}{80+x}=\frac{1}{60}$,

or $x=40$. *Ans.*

2. In a mixture of silver and copper consisting of 60 oz. there are 4 oz. of copper; how much silver must be added that there may be $\frac{2}{3}$ oz. of copper in 12 oz. of the mixture? *Ans.* 12 oz.

3. In a mixture of gold and silver there are 6 oz. of silver; and if 56 oz. of gold be added, there will be 10 oz. of gold to $\frac{2}{3}$ oz. of silver; how much gold was there at first? *Ans.* 94 oz.

NOTE.—In No. 2, let x = no. of oz. to be added; then $4\div(60+x)=\frac{2}{3}\div 12$. In No. 3, let x = no. of oz. of gold at first; then $6\div(x+56)=\frac{2}{3}\div 10$.

4. In a mixture of silver and copper there are 4 oz. of copper; and if 12 oz. of silver be added to the mixture, there will be 12 oz. of the mixture to $\frac{2}{3}$ oz. of copper; how many ounces in the mixture? *Ans.* 60 oz.

NOTE.—Let x = no. of ounces of the mixture; then $4 \div (x+12) = \frac{2}{3} \div 12$.

5. If a lbs. of sea water contain b lbs. of salt, how much salt must be added so that m lbs. of the mixture may contain n lbs. of salt? *Ans.* $\frac{an-bm}{m-n}$.

CASE XII.

1. Two men, A and B, in partnership gain \$300. A owns $\frac{2}{3}$ of the stock, lacking \$40, and gains \$180; required the whole stock and share of each.

SOLUTION.

Let x = the stock;

then, $\frac{2x}{3} - 40$ = A's stock,

and $\frac{x}{3} + 40$ = B's stock.

Then $300 \div x = \frac{300}{x}$ = gain on \$1,

and $120 \div \left(\frac{x}{3}+40\right) = \frac{120}{\frac{x}{3}+40}$ = gain on \$1.

Hence, $$\frac{300}{x} = \frac{120}{\frac{x}{3}+40}$$

from which we find $x = 600$, the entire stock.

2. C and D in partnership gain \$820; C owns \$12,750 of the stock, and D's gain is \$565; required the amount of stock that D owns. *Ans.* \$28,250.

3. Two men engage to build a boat for \$84; the first labors 6 days more than $\frac{1}{3}$ as many as the second, and receives \$48; how many days does each labor? *Ans.* 1st, 8 days; 2d, 6 days.

4. Two men, A and B, in partnership gain \$$a$; A owns 1-$n$th of the stock, lacking \$$b$, and gains \$$c$; required the entire stock and share of each. *Ans.* Stock, $\frac{anb}{a-cn}$.

CASE XIII.

1. What time of day is it, provided $\frac{1}{3}$ of the time past midnight equals the time to noon?

SOLUTION.

Let x = the time past midnight,

and $\frac{x}{3}$ = the time to noon.

Then, $x + \frac{x}{3} = 12$, etc.

2. What is the time of day, provided $\frac{3}{7}$ of the time past midnight equals the time past noon? *Ans.* 9 P. M.

3. What is the hour of day when $\frac{2}{5}$ of the time to noon equals the time past midnight? *Ans.* $4\frac{1}{2}$ A. M.

4. Required the hour of day if $\frac{2}{3}$ of the time past 10 o'clock A. M. equals $\frac{1}{5}$ of the time to midnight. *Ans.* 4 P. M.

5. What time of day is it if $\frac{2}{5}$ of the time past 4 o'clock A. M. equals $\frac{3}{4}$ of the time to 10 o'clock P. M. *Ans.* 2 P. M.

6. What time of day is it if 1-nth of the time past midnight equals the time to noon? *Ans.* $\frac{12n}{n+1}$ A. M.

CASE XIV.

1. A man being asked the time of day said, "It is between 2 and 3 o'clock, and the hour- and minute-hands are together;" what was the time?

SOLUTION.

Let x = the distance the minute-hand goes;

then, $\frac{x}{12}$ = the distance the hour-hand goes.

Then, $x - \frac{x}{12} = 10$, the number of minute-spaces they are apart at 2 o'clock;

whence, $\frac{11x}{12} = 10$, and $x = 10\frac{10}{11}$; ∴ it is $10\frac{10}{11}$ minutes past 2.

2. A man being asked the hour of the day replied, "It is between 3 and 4 o'clock, and the hour- and minute-hands are together;" what was the time? *Ans.* $16\frac{4}{11}$ min. past 3 o'clock.

3. A lady being asked the time of day replied, "It is between 4 and 5 o'clock, and the hands of my watch are 5 minute-spaces apart;" what was the time?

Ans. $16\frac{4}{11}$ min. past 4 o'clock.

4. A companion of the lady also said, "By my watch it is between 4 and 5 o'clock, and the hour- and minute-hands are 5 minutes of time apart;" what was the time by her watch?

Ans. $16\frac{9}{11}$ min. past 4.

5. What is the time of day if it is between m and $m+1$ o'clock, and the hands of the clock are together?

Ans. $5\frac{5}{11}$ m min. past m o'clock.

6. What is the time of day if it is between m and $m+1$ o'clock, and the two hands are n minute-spaces apart?

Ans. $\frac{12(5m-n)}{11}$ min. past m o'clock.

CASE XV.

1. A is 6 years old, and B is 5 times as old; in how many years will B be only 4 times as old as A?

SOLUTION.

Let x = the number of years;
then, $6+x$ = A's age at that time,
and $30+x$ = B's age at that time.
Then, $4(6+x)=30+x$, etc.

2. Jones is 10 years old, and Smith is 3 times as old; how long since Smith was 5 times as old as Jones? *Ans.* 5 yrs.

3. Mary is $\frac{1}{4}$ as old as her aunt, but in 20 years she will be $\frac{1}{2}$ as old; what is the age of each? *Ans.* Mary, 10; aunt, 40.

4. Six years ago B's house was 4 times as old as his barn, but 2 years hence it will be only twice as old; how long has each been built? *Ans.* House, 22 yrs.; barn, 10 yrs.

5. A is a years old, and B is b years old; in what time will A be n times as old as B? *Ans.* $\frac{nb-a}{1-n}$ yr.

6. A is m times as old as B, but in c years he will be n times as old as B; required the age of each at present.

Ans. A, $\frac{mc(n-1)}{m-n}$ yr.; B, $\frac{c(n-1)}{m-n}$ yr.

MISCELLANEOUS PROBLEMS.

1. How many roses and pinks in my garden if there are 70 of both, and the number of roses plus $\frac{1}{2}$ of the number of pinks equals 3 times the number of pinks?

Ans. 50 roses; 20 pinks.

2. Five times a certain number, plus 60, equals 3 times the sum obtained by increasing the number by 60; what is the number? *Ans.* 60.

3. Divide the number 130 into 4 parts, so that each part is greater than the immediately preceding one by its $\frac{1}{2}$.

Ans. 16; 24; 36; 54.

4. In an orchard $\frac{1}{2}$ of the trees bear apples, $\frac{1}{3}$ bear peaches, and the remainder, 24, bear plums; how many trees are there in the orchard? *Ans.* 144.

5. Find a number such that, if we add to it its $\frac{1}{2}$, the sum exceeds 60 by as much as the number itself is less than 65.

Ans. 50.

6. A lady has 2 purses; if she puts $12 in the first, the whole is worth 5 times as much as the second purse; what is the value of each if the first is worth twice as much as the second?

Ans. 1st, $8; 2d, $4.

7. In a mixture of copper and zinc, the copper comprised 6 oz. more than $\frac{1}{2}$ of the mixture, and the zinc 4 oz. more than $\frac{2}{3}$ of the copper; how much was there of each?

Ans. Copper, 48 oz.; zinc, 36 oz.

8. A young man received a fortune from England, and spent $\frac{2}{5}$ of it the first year, and $\frac{3}{5}$ of the remainder the following year, and then had only $6000 remaining; what was the fortune? *Ans.* $25,000.

9. A lady gave $2.10 to her pupils: to each boy she gave 3 cents, and to each girl 5 cents; how many were there of each, provided there were 3 times as many boys as girls?

Ans. 15 girls; 45 boys.

10. A cistern has two supply-pipes, which will singly fill it in 4 and 6 hours respectively, and it has also a leak by which