3. A lady being asked the time of day replied, "It is between 4 and 5 o'clock, and the hands of my watch are 5 minute-spaces apart;" what was the time?

Ans. $16\frac{4}{11}$ min. past 4 o'clock.

4. A companion of the lady also said, "By my watch it is between 4 and 5 o'clock, and the hour- and minute-hands are 5 minutes of time apart;" what was the time by her watch?

Ans. $16\frac{9}{11}$ min. past 4.

5. What is the time of day if it is between m and $m+1$ o'clock, and the hands of the clock are together?

Ans. $5\frac{5}{11}$ m min. past m o'clock.

6. What is the time of day if it is between m and $m+1$ o'clock, and the two hands are n minute-spaces apart?

Ans. $\frac{12(5m-n)}{11}$ min. past m o'clock.

CASE XV.

1. A is 6 years old, and B is 5 times as old; in how many years will B be only 4 times as old as A?

SOLUTION.

Let x = the number of years;
then, $6+x$ = A's age at that time,
and $30+x$ = B's age at that time.
Then, $4(6+x)=30+x$, etc.

2. Jones is 10 years old, and Smith is 3 times as old; how long since Smith was 5 times as old as Jones? *Ans.* 5 yrs.

3. Mary is $\frac{1}{4}$ as old as her aunt, but in 20 years she will be $\frac{1}{2}$ as old; what is the age of each? *Ans.* Mary, 10; aunt, 40.

4. Six years ago B's house was 4 times as old as his barn, but 2 years hence it will be only twice as old; how long has each been built? *Ans.* House, 22 yrs.; barn, 10 yrs.

5. A is a years old, and B is b years old; in what time will A be n times as old as B? *Ans.* $\frac{nb-a}{1-n}$ yr.

6. A is m times as old as B, but in c years he will be n times as old as B; required the age of each at present.

Ans. A, $\frac{mc(n-1)}{m-n}$ yr.; B, $\frac{c(n-1)}{m-n}$ yr.

MISCELLANEOUS PROBLEMS.

1. How many roses and pinks in my garden if there are 70 of both, and the number of roses plus $\frac{1}{2}$ of the number of pinks equals 3 times the number of pinks?

Ans. 50 roses; 20 pinks.

2. Five times a certain number, plus 60, equals 3 times the sum obtained by increasing the number by 60; what is the number? *Ans.* 60.

3. Divide the number 130 into 4 parts, so that each part is greater than the immediately preceding one by its $\frac{1}{2}$.

Ans. 16; 24; 36; 54.

4. In an orchard $\frac{1}{2}$ of the trees bear apples, $\frac{1}{3}$ bear peaches, and the remainder, 24, bear plums; how many trees are there in the orchard? *Ans.* 144.

5. Find a number such that, if we add to it its $\frac{1}{2}$, the sum exceeds 60 by as much as the number itself is less than 65.

Ans. 50.

6. A lady has 2 purses; if she puts $12 in the first, the whole is worth 5 times as much as the second purse; what is the value of each if the first is worth twice as much as the second?

Ans. 1st, $8; 2d, $4.

7. In a mixture of copper and zinc, the copper comprised 6 oz. more than $\frac{1}{2}$ of the mixture, and the zinc 4 oz. more than $\frac{2}{3}$ of the copper; how much was there of each?

Ans. Copper, 48 oz.; zinc, 36 oz.

8. A young man received a fortune from England, and spent $\frac{2}{5}$ of it the first year, and $\frac{3}{5}$ of the remainder the following year, and then had only $6000 remaining; what was the fortune? *Ans.* $25,000.

9. A lady gave $2.10 to her pupils: to each boy she gave 3 cents, and to each girl 5 cents; how many were there of each, provided there were 3 times as many boys as girls?

Ans. 15 girls; 45 boys.

10. A cistern has two supply-pipes, which will singly fill it in 4 and 6 hours respectively, and it has also a leak by which

it would be emptied in 8 hours; in what time will it be filled if all flow together? *Ans.* $3\frac{3}{7}$ hours.

11. An Englishman having bought some nutmegs, said that 3 of them cost as much more than a penny as 4 cost him more than twopence half-penny; required the price of the nutmegs. *Ans.* $1\frac{1}{2}$d. each.

12. Two persons, A and B, having received equal sums of money, A spent \$25 and B \$60, and then it appeared that A had twice as much as B; required the sum each received. *Ans.* \$95.

13. How many cows must a person buy at \$24 each, that, after paying for their keeping at the rate of \$1 for 12, he may gain \$142 by selling them at \$30 each? *Ans.* 24 cows.

14. Find a number which being doubled, and 16 subtracted from the result, the remainder shall exceed 100 as much as the required number is less than 100. *Ans.* 72.

15. There is a number such that the sum of its $\frac{1}{4}$ and $\frac{1}{5}$ exceeds the sum of its $\frac{1}{6}$ and $\frac{1}{8}$ by 19; required the number. *Ans.* 120.

16. Out of a cask of wine, from which $\frac{1}{4}$ had already been taken away, 24 gallons were afterward drawn, and then, being gauged, it was found to be half full; how much did it hold? *Ans.* 96 gals.

17. A and B, traveling with \$500 each, are met by robbers, who take twice as much from A as from B, and leave B with three times as much money as A; how much was taken from each? *Ans.* A, \$400; B, \$200.

18. In a mixture of copper, tin and lead, $\frac{1}{2}$ of the whole, minus 16lbs., was copper; $\frac{1}{3}$ of the whole, minus 12 lbs., tin; $\frac{1}{4}$ of the whole, plus 4 lbs., lead; what quantity of each was there in the composition? *Ans.* 128 lbs.; 84 lbs.; 76 lbs.

19. A person agreed to do a piece of work on condition that he received \$4 for each day he worked, and forfeited \$1 each day he was idle; he worked twice as many days as he was idle, and received \$140; how many days was he idle? *Ans.* 20 days.

20. The sum of \$750 was raised by 4 persons, A, B, C and D; B contributing twice as much as A, C twice as much as A and B, and D twice as much as B and C; what did each contribute? *Ans.* A, \$30; B, \$60; C, \$180; D, \$480.

21. A person borrowed a certain sum of money on interest at 6%: in 12 years the interest received amounted to \$140 less than the sum loaned; what was the sum loaned? *Ans.* \$500.

22. A farmer has 128 animals, consisting of horses, sheep and cows; required the number of each, provided $\frac{1}{2}$ of the number of sheep, plus 12, equals the number of cows, and $\frac{1}{2}$ of the number of cows, plus 12, equals the number of horses. *Ans.* Sheep, 56; cows, 40; horses, 32.

23. Divide the number 90 into 4 such parts that the first increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, may all be equal. *Ans.* 18; 22; 10; 40.

24. A general drawing up his army in the form of a solid square, finds he has 44 men over; then increasing the side of the square by 1 man, he finds he lacks 225 men to complete the square; what was the number of men in the army? *Ans.* 18,000 men.

25. Said Mary to William, "Our purses contain the same sum of money, but if you give me \$60 and I give you \$20, I shall have 3 times as much as you;" how much had each? *Ans.* \$80.

26. A lady bought a number of eggs, half of them at 2 for a penny, and half of them at 3 for a penny; she sold them at the rate of 5 for twopence, and lost a penny by the transaction; what was the number of eggs? *Ans.* 60.

27. There are three sisters, the sum of whose ages is $43\frac{1}{2}$ years, and their birth-days are $2\frac{1}{2}$ years apart, respectively; what is the age of each? *Ans.* 12 yrs.; $14\frac{1}{2}$ yrs.; 17 yrs.

28. At an election $\frac{1}{2}$ of the votes were cast for A, $\frac{1}{3}$ for B, and the remainder for C, and A's majority over C was 800; how many voted for each? *Ans.* A, 1200; B, 800; C, 400.

29. A had twice as much money as B; but after each had spent $\frac{1}{5}$ of his money, and A had paid B \$600, B had twice as much as A; how much had each? *Ans.* A, \$1500; B, \$750.

30. A, B and C can do a piece of work in 24 hours; how long will it take each to do it if A does $\frac{1}{2}$ as much as B, and B $\frac{1}{2}$ as much as C?

Ans. A, 168 hrs.; B, 84 hrs.; C, 42 hrs.

31. If 10 men, 20 women and 30 children receive $424 for a week's work, and 2 men receive as much as 3 women or 5 children, what does each child receive for a day's work?

Ans. 80 cts.

32. Said E to F, My age is 9 years greater than yours; but 12 years ago my age was $\frac{3}{2}$ of what yours will be 7 years hence; what was the age of each? *Ans.* E's, 27 yrs.; F's, 18 yrs.

33. From one end of a line I cut off 3 feet more than $\frac{1}{4}$ of it, and from the other end 6 feet less than $\frac{1}{5}$ of it, and then there remained 25 feet; how long was the line? *Ans.* 40 ft.

34. In a bag containing eagles and dollars, there are 4 times as many eagles as dollars; but if 6 eagles and as many dollars be taken away, there will be left 6 times as many eagles as dollars; how many were there of each?

Ans. Eagles, 60; dollars, 15.

35. A bright young lady being asked her age by a gentleman who was "not very smart at figures," said, "Twice my age $2\frac{1}{2}$ years ago will equal 3 times $\frac{1}{2}$ of my age $2\frac{1}{2}$ years hence;" what was her age? *Ans.* $17\frac{1}{2}$ yrs.

36. An English lady distributed 20 shillings among 20 persons, giving 6 pence each to some, and 16 pence each to the rest; how many persons received 6 pence each?

Ans. 8 persons.

37. A mason receives $450 for building a wall; if he had received $$1\frac{1}{5}$ more a rod he would have received for the entire work $540; how many rods of wall did he build?

Ans. 75 rods.

38. A man loans $8250, part at 5% and part at 6%; how much did he loan at each rate if he receives equal sums of interest for each part? *Ans.* $4500; $3750.

39. An army lost $\frac{1}{8}$ of its number in killed and wounded and 5000 prisoners; it was then reinforced by 10,000 men, but retreating, it lost $\frac{1}{5}$ of its number on the march, when there remained 60,000 men; what was the original force?

Ans. 80,000.

40. Find two consecutive numbers, such that the fifth and the seventh of the first taken together shall equal the sum of the fourth and the twelfth of the second taken together.

Ans. 35 and 36.

41. A person being asked the time of day, replied, "It is between 3 and 4 o'clock, and the hour- and minute-hands of my watch are exactly opposite each other;" what was the time?

Ans. $49\frac{1}{11}$ min. past 3.

42. A colonel forms his regiment into a solid square, and then sending out a picket-guard of 295 men and re-forming the square, finds there were 5 men less on a side; what was the number of men in the regiment at first? *Ans.* 1024.

43. A person being asked the time of day, replied, "The number of minutes it lacks of being 4 o'clock is equal to $\frac{1}{2}$ of the number of minutes it was past 2 o'clock $\frac{3}{4}$ of an hour ago;" what was the time? *Ans.* 25 min. of 4.

44. A regiment consisting of 1296 men can be formed into a hollow square 12 men deep; required the number of men in the outer rank of a side. *Ans.* 39.

45. A boatman who can row at the rate of 12 miles an hour finds that it takes twice as long to run his boat a mile up the river as to run the same distance down the river; what is the rate of the current? *Ans.* 4 miles an hour.

SIMPLE EQUATIONS,

CONTAINING TWO UNKNOWN QUANTITIES.

177. Independent Equations are such as cannot be derived from one another, or be reduced to the same form.

178. The equations $4x+2y=6$ and $6x+3y=9$ are not independent, since both can be reduced to the form $2x+y=3$.

179. Simultaneous equations are those in which the unknown quantities have respectively the same values.

180. To find the value of any unknown quantity in two equations of two unknown quantities, we must derive from them a single equation containing this unknown quantity. The process of doing this is called *Elimination.*

181. Elimination is the process of deducing from two or more simultaneous equations a less number of equations containing a less number of unknown quantities.

182. There are three principal methods of elimination:

1. By Substitution; 2. By Comparison;
3. By Addition and Subtraction.

NOTE.—There is a fourth method of elimination, called the method of *Indeterminate Multipliers*, due to the French mathematician *Bezout.* The three methods given above, however, are all that are generally used in the solution of equations.

CASE I.

ELIMINATION BY SUBSTITUTION.

183. Elimination by Substitution consists in finding an expression for the value of an unknown quantity in one equation, and substituting it in another.

1. Given the equations $2x+3y=12$ and $3x+y=11$, to find the values of x and y.

OPERATION.

$$2x+3y=12 \quad (1)$$
$$3x+y=11 \quad (2)$$
$$x=\frac{12-3y}{2} \quad (3)$$
$$\frac{36-9y}{2}+y=11 \quad (4)$$
$$36-9y+2y=22 \quad (5)$$
$$-7y=-14 \quad (6)$$
$$y=2 \quad (7)$$
$$x=\frac{12-6}{2}=3 \quad (8)$$

SOLUTION. From equation (1), by transposing $3y$ and dividing by 2, we have eq. (3). Substituting this value of x in eq. (2), we have eq. (4). Clearing of fractions, we have eq. (5). Transposing and uniting terms, we have eq. (6). Dividing by -7, we have $y=2$. Substituting this value of y in eq. (3), we have eq. (8). Hence, in the given equations, $x=3$ and $y=2$.

NOTE.—In explaining, the pupil may read the equation instead of giving its number, as above.

Rule.—I. *Find an expression for the value of one of the unknown quantities in either equation.*

II. *Substitute this value for the same unknown quantity in the other equation, and reduce.*

NOTES.—1. Use first the equation which will give the simplest expression for the value of the unknown quantity.

2. This method is especially appropriate when the coefficient of one of the unknown quantities is 1.

EXAMPLES.

2. Given $\begin{cases} x+2y=7 \\ 2x+3y=12 \end{cases}$ to find x and y. *Ans.* $x=3$; $y=2$.

3. Given $\begin{cases} 3x-y=10 \\ x+4y=12 \end{cases}$ to find x and y. *Ans.* $x=4$; $y=2$.

4. Given $\begin{cases} 6x-2y=2 \\ 2x+3y=19 \end{cases}$ to find x and y. *Ans.* $x=2$; $y=5$.

5. Given $\begin{cases} 3y-x=7 \\ y+2x=14 \end{cases}$ to find x and y. *Ans.* $x=5$; $y=4$.

6. Given $\begin{cases} 5x+2y=41 \\ 3x-4y=9 \end{cases}$ to find x and y. *Ans.* $x=7$; $y=3$.

7. Given $\begin{cases} 5x-y=48 \\ x+5y=46 \end{cases}$ to find x and y. *Ans.* $x=11$; $y=7$.

8\. Given $\left\{\begin{array}{l}\frac{1}{2}x+\frac{1}{3}y=8\\ \frac{1}{3}x+\frac{1}{2}y=7\end{array}\right\}$ to find x and y. *Ans.* $x=12$; $y=6$.

9\. Given $\left\{\begin{array}{l}\frac{3x}{4}-\frac{5y}{2}=-9\\ \frac{x}{2}+\frac{y}{3}=6\end{array}\right\}$ to find x and y. *Ans.* $x=8$; $y=6$.

10\. Given $\left\{\begin{array}{l}\frac{4x}{5}+\frac{3z}{4}=21\\ \frac{x}{3}-\frac{2z}{3}=-3\end{array}\right\}$ to find x and z. *Ans.* $x=15$; $z=12$.

CASE II.

ELIMINATION BY COMPARISON.

184. Elimination by Comparison consists in finding an expression for the value of the unknown quantity in each equation, and placing these values equal to each other.

1\. Given $\left\{\begin{array}{l}2x+3y=12\\ 3x+2y=13\end{array}\right\}$ to find x and y.

OPERATION.

$$2x+3y=12 \quad (1)$$
$$3x+2y=13 \quad (2)$$
$$x=\frac{12-3y}{2} \quad (3)$$
$$x=\frac{13-2y}{3} \quad (4)$$
$$\frac{12-3y}{2}=\frac{13-2y}{3} \quad (5)$$
$$36-9y=26-4y \quad (6)$$
$$-9y+4y=26-36 \quad (7)$$
$$-5y=-10 \quad (8)$$
$$y=2. \quad (9)$$
$$x=\frac{12-6}{2} \quad (10)$$
$$x=3. \quad (11)$$

SOLUTION. Transposing $3y$ in equation (1), and dividing by 2, we obtain (3). Transposing $2y$ in (2), and dividing by 3, we obtain (4). Quantities which are equal to the same thing are equal to each other; hence, we have (5). Clearing of fractions, we have (6). Transposing terms, we have (7). Uniting terms, we have (8). Dividing by -5, we have $y=2$. Substituting the value of y in (3), we have (10). Reducing, we have $x=3$.

Rule.—I. *Find an expression for the value of the same unknown quantity in each equation.*

II. *Place these values equal to each other, and reduce the resulting equation.*

EXAMPLES.

2\. Given $\left\{\begin{array}{l}2x+5y=18\\ 3x+4y=20\end{array}\right\}$ to find x and y. *Ans.* $x=4$; $y=2$.

3\. Given $\left\{\begin{array}{l}4x-3y=11\\ 5x+y=28\end{array}\right\}$ to find x and y. *Ans.* $x=5$; $y=3$.

4\. Given $\left\{\begin{array}{l}x+3y=17\\ 2x+5y=31\end{array}\right\}$ to find x and y. *Ans.* $x=8$; $y=3$.

5\. Given $\left\{\begin{array}{l}8x-5y=3\\ 12x-7y=5\end{array}\right\}$ to find x and y. *Ans.* $x=1$; $y=1$.

6\. Given $\left\{\begin{array}{l}7x-5y=-3\\ 3x+7y=81\end{array}\right\}$ to find x and y. *Ans.* $x=6$; $y=9$.

7\. Given $\left\{\begin{array}{l}\frac{x}{3}+\frac{4z}{5}=6\\ \frac{x}{6}+\frac{3z}{5}=4\end{array}\right\}$ to find x and z. *Ans.* $x=6$; $z=5$.

8\. Given $\left\{\begin{array}{l}\frac{2x}{3}-\frac{4y}{5}=-4\\ \frac{3x}{4}-\frac{2y}{3}=-1\end{array}\right\}$ to find x and y. *Ans.* $x=12$; $y=15$

9\. Given $\left\{\begin{array}{l}\frac{3x-2y}{5}+3y=16\\ 2x-\frac{2x-3y}{5}=11\end{array}\right\}$ to find x and y. *Ans.* $x=5$; $y=5$

10\. Given $\left\{\begin{array}{l}\frac{5x-6y}{6}+\frac{5y}{3}=13\\ \frac{5x}{6}-\frac{7x-4y}{3}=7\end{array}\right\}$ to find x and y. *Ans.* $x=6$; $y=12$.

CASE III.

ELIMINATION BY ADDITION AND SUBTRACTION.

185. Elimination by addition and subtraction consists in adding or subtracting the equations when the coefficients of one of the unknown quantities are alike or are made alike.

1. Given $\left\{\begin{array}{l} 2x+3y=12 \\ 3x+2y=13 \end{array}\right\}$ to find x and y.

OPERATION.

$$\begin{array}{rl} 2x+3y=12 & (1) \\ 3x+2y=13 & (2) \\ \hline 6x+9y=36 & (3) \\ 6x+4y=26 & (4) \\ \hline 5y=10 & (5) \\ y=2 & (6) \\ 2x+6=12 & (7) \\ x=3 & (8) \end{array}$$

SOLUTION. Multiplying equation (1) by 3, and equation (2) by 2, to make the coefficients of x alike, we have equations (3) and (4). Subtracting (4) from (3), we have $5y=10$, from which $y=2$. Substituting the value of y in equation (1), we have (7), from which we find $x=3$

Rule.—I. *Multiply or divide one or both equations if necessary, so that the coefficients of one unknown quantity shall be the same in both.*

II. *Add these equations when the signs of the equal coefficients are unlike, and subtract the equations when they are alike.*

NOTES.—1. If the coefficients to be made alike are prime to each other, multiply each equation by the coefficient of that quantity in the other equation.

2. This method is generally preferred in finding the value of one quantity, the value of the other quantity being found by substitution.

EXAMPLES.

2. Given $\left\{\begin{array}{l} 3x+4y=7 \\ 4x+2y=6 \end{array}\right\}$ to find x and y. *Ans.* $x=1$; $y=1$.

3. Given $\left\{\begin{array}{l} 5x-2y=14 \\ 2x+3y=17 \end{array}\right\}$ to find x and y. *Ans.* $x=4$; $y=3$.

4. Given $\left\{\begin{array}{l} 3x+5y=25 \\ 2x+4y=18 \end{array}\right\}$ to find x and y. *Ans.* $x=5$; $y=2$.

5. Given $\left\{\begin{array}{l} 3x+5y=46 \\ 6x-8y=2 \end{array}\right\}$ to find x and y. *Ans.* $x=7$; $y=5$.

6. Given $\left\{\begin{array}{l} 15y-6x=87 \\ 9y+3x=105 \end{array}\right\}$ to find x and y. *Ans.* $x=8$; $y=9$.

7. Given $\left\{\begin{array}{l} x+y=a \\ x-y=b \end{array}\right\}$ to find x and y.

Ans. $x=\dfrac{a+b}{2}$; $y=\dfrac{a-b}{2}$.

8. Given $\left\{\begin{array}{l} ax+by=ab \\ 2ax+3by=\dfrac{5ab}{2} \end{array}\right\}$ to find x and y.

Ans. $x=\dfrac{b}{2}$; $y=\dfrac{a}{2}$

9. Given $\left\{\begin{array}{l} \dfrac{x+8}{4}+6y=21 \\ \dfrac{x+y}{3}=22\frac{1}{3}-5x \end{array}\right\}$ to find x and y. *Ans.* $x=4$; $y=3$.

10. Given $\left\{\begin{array}{l} \dfrac{2x-8}{4}-\dfrac{3x-4y}{6}=2 \\ \dfrac{3x-2y}{3}+\dfrac{2y+6}{3}=14 \end{array}\right\}$ to find x and y. *Ans.* $x=12$; $y=6$.

MISCELLANEOUS EXAMPLES.

186. In the following examples the pupil will exercise his judgment which method to use:

1. Given $\left\{\begin{array}{l} 3x+4y=24 \\ 4x+3y=25 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=4, \\ y=3. \end{array}\right.$

2. Given $\left\{\begin{array}{l} 5x-6y=7 \\ 3x+3y=24 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=5, \\ y=3. \end{array}\right.$

3. Given $\left\{\begin{array}{l} 2x-6y=-18 \\ 6x-7y=1 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=6, \\ y=5. \end{array}\right.$

4. Given $\left\{\begin{array}{l} 6x-5z=1\frac{3}{4} \\ 4z-5x=-1\frac{1}{2} \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=\frac{1}{2}, \\ z=\frac{1}{4}. \end{array}\right.$

5. Given $\left\{\begin{array}{l} x+y=2a \\ x-y=2b \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=a+b, \\ y=a-b. \end{array}\right.$

6. Given $\left\{\begin{array}{l} x+y=a+b \\ x-y=a-b \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=a, \\ y=b. \end{array}\right.$

7. Given $\left\{\begin{array}{l} \frac{1}{2}x+\frac{1}{3}y=7 \\ \frac{1}{3}x+\frac{1}{4}y=5 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=6, \\ y=12. \end{array}\right.$

8. Given $\left\{\begin{array}{l} x+\frac{1}{4}y=14 \\ \frac{1}{3}x+y=12 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=12, \\ y=8. \end{array}\right.$

9. Given $\left\{\begin{array}{l} \dfrac{a}{2}x+\dfrac{b}{3}y=2ab \\ \dfrac{a}{2}x-by=-2ab \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=2b, \\ y=3a. \end{array}\right.$

10. Given $\left\{\begin{array}{l} 6x-7y=42 \\ .7x-.6y=7.5 \end{array}\right\}$. *Ans.* $\left\{\begin{array}{l} x=21, \\ y=12. \end{array}\right.$

11. Given $\left\{\begin{array}{l}\frac{x+1}{y-1}-\frac{x-1}{y}=\frac{6}{y} \\ x-y=1\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=3, \\ y=2.\end{array}\right.$

12. Given $\left\{\begin{array}{l}\frac{x+y}{3}+x=15 \\ \frac{x-y}{5}+y=6\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=10, \\ y=5.\end{array}\right.$

13. Given $\left\{\begin{array}{l}\frac{1}{x}+\frac{1}{y}=a \\ \frac{1}{x}-\frac{1}{y}=b\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=\frac{2}{a+b}, \\ y=\frac{2}{a-b}.\end{array}\right.$

14. Given $\left\{\begin{array}{l}bx+ay=2ab \\ x+y=a+b\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=a, \\ y=b.\end{array}\right.$

15. Given $\left\{\begin{array}{l}ax+by=2m \\ ax-by=2n\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=\frac{m+n}{a}, \\ y=\frac{m-n}{b}.\end{array}\right.$

16. Given $\left\{\begin{array}{l}ax+by=c \\ bx+ay=d\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=\frac{ac-bd}{a^2-b^2}, \\ y=\frac{ad-bc}{a^2-b^2}.\end{array}\right.$

17. Given $\left\{\begin{array}{l}\frac{x}{a}+\frac{y}{b}=2 \\ bx-ay=0\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=a, \\ y=b.\end{array}\right.$

18. Given $\left\{\begin{array}{l}ax+by=a \\ bx-ay=b\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=1. \\ y=0.\end{array}\right.$

19. Given $\left\{\begin{array}{l}\frac{x}{a}+\frac{y}{b}=1 \\ \frac{x}{b}+\frac{y}{a}=1\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=\frac{ab}{a+b}, \\ y=\frac{ab}{a+b}.\end{array}\right.$

20. Given $\left\{\begin{array}{l}\frac{a}{x}+\frac{b}{y}=m \\ \frac{c}{x}+\frac{d}{y}=n\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=\frac{bc-ad}{nb-md}, \\ y=\frac{bc-ad}{mc-na}\end{array}\right.$

21. Given $\left\{\begin{array}{l}(a+c)x-by=bc \\ x+y=a+b\end{array}\right\}$. *Ans.* $\left\{\begin{array}{l}x=b \\ y=a\end{array}\right.$

PRACTICAL PROBLEMS.

187. Several of the problems in Art. 176 contained more than one unknown quantity, but the conditions were such that they were readily solved with one unknown quantity.

188. In the following problems the solution is most readily effected by using a separate symbol for each unknown quantity.

189. In such problems the conditions must give as many independent equations as there are unknown quantities.

CASE I.

1. A drover sold 6 lambs and 7 sheep for \$71, and at the same price 4 lambs and 8 sheep for \$64; what was the price of each?

Solution. Let x = the price of the lambs, and y = the price of the sheep; then, by the first condition we have equation (1), and by the second condition we have eq. (2). Multiplying eq. (1) by 2, we have (3). Multiplying (2) by 3, we have (4). Subtracting (3) from (4), we have (5). Dividing by 10, we have (6). From which we find $x=6$.

OPERATION.

Let x = the price of lambs, and y = the price of sheep.

$6x+7y=71$	(1)
$4x+8y=64$	(2)
$12x+14y=142$	(3)
$12x+24y=192$	(4)
$10y=50$	(5)
$y=5$	(6)
$6x+35=71$	(7)
$x=6$	(8)

2. A farmer hired 8 men and 6 boys one day for \$36, and at the same rate next day he hired 6 men and 11 boys for \$40; what did he pay each per day? *Ans.* Men, \$3; boys, \$2.

3. A man paid \$1.14 for 12 oranges and 13 lemons; but the price falling $\frac{1}{3}$, he received only 62 cents for 9 oranges and 11 lemons; what was the price paid for each?

Ans. Oranges, 3 cts.; lemons, 6 cts.

4. A man hired a men and b boys for \$$m$, and at the same price c men and d boys for \$$n$; what price did he pay each?

Ans. Men, \$$\frac{md-nb}{ad-bc}$; boys, \$$\frac{an-mc}{ad-bc}$.

CASE II.

1. There is a fraction such that 1 added to its numerator will make it $\frac{1}{2}$, and 1 added to its denominator will make it $\frac{1}{3}$; what is the fraction?

SOLUTION.

Let $\frac{x}{y}=$ the fraction.

Then, by the 1st condition, $\frac{x+1}{y}=\frac{1}{2}$, (1)

and by the 2d condition, $\frac{x}{y+1}=\frac{1}{3}$. (2)

Clearing (1) of fractions, $2x+2=y$ (3)

clearing (2) of fractions, $3x=y+1$. (4)

Subtracting (3) from (4), $x-2=1$;

transposing, $x=3$;

substituting in (3) and reducing, $y=8$;

hence the fraction is $\frac{x}{y}=\frac{3}{8}$. *Ans.*

2. Find a fraction such that if 2 be subtracted from the numerator the fraction will equal $\frac{1}{4}$, or if 2 be subtracted from the denominator the fraction will equal $\frac{1}{2}$. *Ans.* $\frac{5}{12}$.

3. If 4 be subtracted from both terms of a fraction, the value will be $\frac{1}{3}$, and if 5 be added to both terms the value will be $\frac{5}{8}$; what is the fraction? *Ans.* $\frac{5}{7}$.

4. If 1 be added to both terms of a fraction, its value will be $\frac{1}{2}$; and if the denominator be increased by the numerator, and the numerator be diminished by 1, the value will be $\frac{1}{4}$; required the fraction. *Ans.* $\frac{5}{11}$.

5. Required the fraction such that if the numerator be increased by a the result will equal $\frac{m}{n}$, and if the denominator be increased by a the result will be $\frac{n}{m}$. *Ans.* $\frac{an\,(m-n)}{an\,(m-n)}$.

CASE III.

1. Required the number of two figures which, added to the number obtained by changing the place of the digits, gives 77, and subtracted from it leaves 27.

SOLUTION.

Let $x=$ the tens' digit,

and $y=$ the units' digit;

then $10x+y=$ the number,

and $10y+x=$ the number with digits inverted.

Then, by 1st condition, $10x+y+10y+x=77$, (1)

and by 2d condition, $10y+x-(10x+y)=27$; (2)

uniting terms in (1), $11x+11y=77$; (3)

dividing by 11, $x+y=7$; (4)

uniting terms of (2), $9y-9x=27$; (5)

dividing by 9, $y-x=3$; (6)

from (4) and (6), $x=2$, and $y=5$;

hence the number is 2 tens and 5 units, or 25.

2. Required a number of two figures which, increased by the number obtained by inverting the figures, will equal 132, and which diminished by that number will equal 18. *Ans.* 75.

3. A number consisting of two places being divided by the sum of its digits, the quotient is 4; and if 36 be added to it, the digits will be inverted; required the number. *Ans.* 48.

4. There is a number which equals 5 times the sum of its two digits; and if three times the sum of the digits, plus 9, be subtracted from twice the number, the digits will be inverted; required the number. *Ans.* 45.

CASE IV.

1. The amount of a certain principal at simple interest, for a certain time, at 5%, is \$260; and the amount for the same time, at 8%, is \$296; required the principal and time.

SOLUTION.

Let x = the principal,
and y = the time.

Then, by the 1st condition, $xy\times\frac{5}{100}+x=260$ (1)

By the 2d condition, $xy\times\frac{8}{100}+x=296$ (2)

Whence, $x=200$, the principal, and $y=6$, the time.

2. A certain sum of money on simple interest amounts in a certain time, at 6%, to \$310, and at 10%, for the same time, to \$350; required the time and principal.

Ans. Prin., \$250; time, 4 yrs.

3. The amount of a certain principal for 7 years, at a certain rate per cent., is \$810, and for 12 years, at the same rate, is \$960; required the principal and rate.

Ans. Prin., \$600; rate, 5%.

4. A certain sum of money, put out at simple interest, amounts in m years to a dollars, and at the same rate, in n years, to b dollars; required the sum and rate per cent.

Ans. Prin., $\frac{an-bm}{n-m}$; rate, $\frac{100(b-a)}{an-bm}$.

CASE V.

1. There are two numbers whose sum equals twice their product, and whose difference equals once their product; required the numbers.

SOLUTION.

Let x = the greater number,
and y = the less number.

Then, by the 1st condition, $x+y=2xy$ (1)
and by the 2d condition, $x-y=xy$ (2)
Adding (1) and (2), $2x=3xy$.
Dividing by x, $2=3y$
Whence, $y=\frac{2}{3}$
and $x=2$.

2. There are two numbers whose sum equals three times their product, and whose difference equals once their product; what are the numbers? *Ans.* 1 and $\frac{1}{2}$.

3. There are two numbers such that twice their sum equals 3 times their product, and twice their difference equals once their product; what are the numbers? *Ans.* 2 and 1.

4. There are two numbers such that their sum equals 4 times their quotient, and their difference equals twice their quotient; what are the numbers? *Ans.* 9 and 3.

5. There are two numbers whose sum equals a times their product, and whose difference equals b times their product; what are their values? *Ans.* $\frac{2}{a-b}$; $\frac{2}{a+b}$.

MISCELLANEOUS PROBLEMS.

1. A person buys 8 lbs. of tea and 3 lbs. of sugar for \$2.64, and at another time, at the same rates, 5 lbs. of tea and 4 lbs. of sugar for \$1.82; find the price per pound of the tea and sugar. *Ans.* Tea, 30 cts.; sugar, 8 cts.

2. If the greater of two numbers be added to $\frac{1}{3}$ of the less, the sum will be 30, but if the less be divided by $\frac{1}{5}$ of the greater, the quotient will be 3; what are the numbers? *Ans.* 25; 15.

3. A said to B, "Give me \$200 and I shall have 3 times as much as you;" but B replied, "Give me \$200 and I shall have twice as much as you;" how much had each?

Ans. A, \$520; B, \$440.

4. A lady having two watches bought a chain for \$20: if the chain be put on the silver watch, their value will equal $\frac{1}{3}$ of the gold watch, but if it be put on the gold watch, they will be worth 7 times as much as the silver watch; required the value of each watch. *Ans.* Gold, \$120; silver, \$20.

5. Find the fraction which becomes equal to $\frac{1}{3}$ when the numerator is increased by 1, and equal to $\frac{1}{4}$ when the denominator is increased by 1. *Ans.* $\frac{4}{15}$.

6. Mary and Jane have a certain number of plums: if Mary had 4 more she would have 3 times as many as Jane, but if she

had 4 less she would have $\frac{1}{3}$ as many as Jane; how many has each? *Ans.* Mary, 5; Jane, 3.

7. A bill of £120 was paid in guineas (21s.) and moidores (27s.), and the number of pieces of both kinds was just 100; required the number of pieces of each kind. *Ans.* 50.

8. A boy expends 30 cents for apples and pears, buying his apples at 4 and his pears at 5 for a cent, and afterward accommodates his friend with half his apples and one-third of his pears for 13 cents at the price paid; how many did he buy of each? *Ans.* Apples, 72; pears, 60.

9. A person rents 25 acres of land for £7 12s. per annum: for the better part he receives 8s. an acre, and for the poorer part, 5s. an acre; required the number of acres of each sort. *Ans.* 9; 16.

10. If 9 be added to a number consisting of two digits, the two digits will change places, and the sum of the two numbers will be 33; what is the number? *Ans.* 12.

11. Said A to B, "If you will give me $20 of your money, I will have twice as much as you have left;" but says B to A, "If you will give me $20 of your money, I will have thrice as much as you have left;" how much had each? *Ans.* A, $44; B, $52.

12. A and B laid a wager of $20: if A loses, he will have as much as B will then have; if B loses, he will have half of what A will then have; find the money of each. *Ans.* A, $140; B, $100.

13. There is a number consisting of two figures which is double the sum of its digits; and if 9 be subtracted from 5 times the number, the digits will be inverted; what is the number? *Ans.* 18.

14. Several persons engaged a boat for sailing: if there had been 3 more, they would have paid $1 each less than they did; if there had been 2 less, they would have paid $1 each more than they did; required the number of persons and what each paid. *Ans.* 12 persons; $5 each.

15. A and B ran a race which lasted five minutes: B had a start of 20 yards, but A ran 3 yards while B was running 2, and won by 30 yards; find the length of the course and the speed of each. *Ans.* 150 yds.; A 30 yds. and B 20 yds. a min.

16. If a certain rectangular field were 10 feet longer and 5 feet broader, it would contain 400 square feet more; but if it were 5 feet longer and 10 feet broader, it would contain 450 square feet more; required its length and breadth. *Ans.* 30 ft.; 20 ft.

17. A market-woman bought eggs—some at 3 for 7 cents, and some at two for 5 cents, paying $2.62 for the whole; she afterward sold them at 36 cents a dozen, clearing 62 cents; how many of each kind did she buy? *Ans.* 48; 60.

18. A and B ran a mile, A giving B a start of 20 yards at the first heat, and beating him 30 seconds; at the second heat A gives B a start of 32 seconds, and beats him $9\frac{5}{11}$ yards; at what rate per hour does A run? *Ans.* 12 miles.

SIMPLE EQUATIONS,

CONTAINING THREE OR MORE UNKNOWN QUANTITIES.

190. Equations containing three or more unknown quantities may be solved by either of the methods of elimination already explained.

1. Given $\left\{\begin{array}{r} x+y+z=6 \\ x+2y+3z=14 \\ 2x+3y+z=11 \end{array}\right\}$ to find x, y and z.

OPERATION.

$$\begin{array}{rl} x+y+z=6 & (1) \\ x+2y+3z=14 & (2) \\ 2x+3y+z=11 & (3) \\ \hline y+2z=8 & (4) \\ y-z=-1 & (5) \\ \hline 3z=9 & (6) \\ z=3 & (7) \\ y+6=8 & (8) \\ y=2 & (9) \\ x+2+3=6 & (10) \\ x=1 & (11) \end{array}$$

SOLUTION. Subtracting equation (1) from equation (2), we have equation (4). Multiplying (1) by 2, and subtracting from (3), we have (5); subtracting (5) from (4), we have (6); dividing (6) by 3, we have $z=3$; substituting the value of z in (4), we have 8. Transposing, we have $y=2$; substituting the values of z and y in (1), we have (10): from which we have $x=1$.