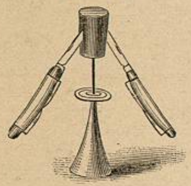
the cork and two knives together form a connected body whose center of gravity is outside, just beneath the needle. By pushing either knife a few oscillations are produced, but a position of rest is soon recovered. Any movement of the toy shown in Fig. 31 tends to raise the center of gravity, and it returns quickly to a state of rest.

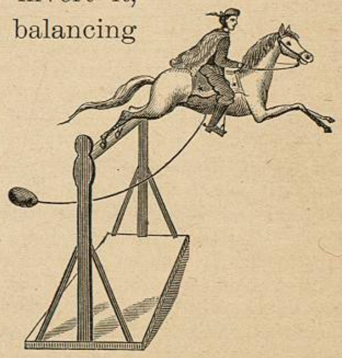
Fig. 30.



Stable Equilibrium.

2d. A body is said to be in *unstable equilibrium* when the center of gravity is above the point of support, or when any movement tends to lower the center of gravity. If we take the cork as arranged with the knives in Fig. 30, and invert it, we shall have difficulty in balancing the needle; and, if we succeed, it will readily topple off, as the least motion tends to lower the center of gravity.

Fig. 31.



Stable Equilibrium.

3d. A body is said to be in *indifferent equilibrium* when the center of gravity is at the point of support, or when any movement tends neither to elevate nor lower the center of gravity. A ball of uniform density on a level surface will rest in any position, because the center of gravity moves in a line parallel to the floor.

(2.) General Principles.—(*a.*) The center of gravity tends to seek the lowest point.

(*b.*) A body will not tip over while the line of direction falls within the base, but will as soon as it falls without.*

(*c.*) In general, narrowness of base combined with height of center of gravity, tends to instability;† breadth of base and lowness of center of gravity, produce stability.

(3.) Physiological Facts.—Our feet and the space between them form the base on which we stand. By turning our toes outward, we increase its breadth.

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* The Leaning Tower of Pisa, in Italy, beautifully illustrates this principle. It is about 188 feet high, and its top leans 15 feet, yet the line of direction falls so far within the base that it is perfectly stable, having stood for seven centuries. The feeling experienced by a person who for the first time looks down from the lower side of the top of this apparently impending structure is startling indeed.

† "This is shown by the difficulty in learning to walk upon stilts. The art of balancing one's self may, however, be acquired by practice, as is seen in the Landes of south-western France. During a portion of the year these sandy plains are half covered with water, and in the remainder are still very bad walking. The natives accordingly double the length of their legs by stilts. Mounted on these wooden poles, which are put on and off as regularly as the other parts of their dress, they appear to strangers as a new and extraordinary race, marching with steps of six feet in length, and with the speed of a trotting-horse. While watching their flocks, they support themselves by a third staff behind, and then with their rough sheep-skin cloaks and caps, like thatched roofs, seem to be little watch-towers, or singular lofty tripods, scattered over the country."—Arnott.

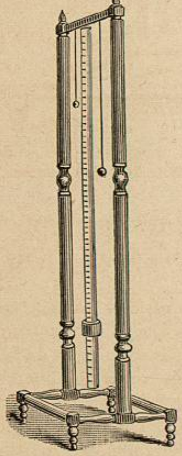
Fig. 32.

Walking on Stilts.

When we stand on one foot, we bend over so as to bring the line of direction within this narrower base. When we walk, we incline to the right and the left alternately. When we walk up hill we lean forward, and in going down hill we incline backward, in unconscious obedience to the laws of gravity. We bend forward when we wish to rise from a chair, in order to bring the center of gravity over our feet. In walking we lean forward, so as bring the center of gravity as far in front as possible. Thus, walking is a process of falling forward and then checking the fall. When we run, we lean farther forward, and so fall faster. ("Hygienic Physiology," p. 37.)

12. The Pendulum consists of a weight so suspended as to swing freely. Its movements to and fro are termed *vibrations* or *oscillations*. The path through which it passes is called the *arc*. The extent to which it goes in either direction from the lowest point is styled its *amplitude*. Vibrations performed in equal times are termed *i-soch'ro-nous* (*isos*, equal; *chronos*, time).

FIG. 33.

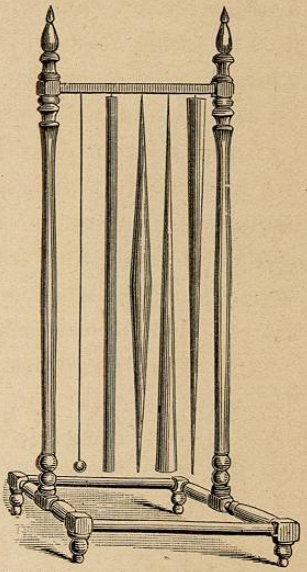


Pendulums.

(1.) THREE LAWS.—I. *In the same pendulum, all vibrations of small amplitude are isochronous.* If we let one of the balls represented in Fig. 33 swing through a short arc, and then through a longer one, on counting the number of oscillations per minute, we shall find them very uniform.

II. *The times of the vibrations of different pendulums are proportional to the square roots of their respective lengths.—Example:* A pendulum $\frac{1}{9}$ the length of another, will vibrate three times as fast.* Conversely, the lengths of different pendulums are proportional to the squares of their times of vibration.

FIG. 34.



Pendulums of apparently the same length, but really different lengths.

III. *The time of the vibration of the same pendulum will vary at different places,* since it decreases as the square root of the number expressing the acceleration of gravity increases. At the equator a pendulum vibrates most slowly. The length of a seconds-pendulum at New York is about $39\frac{1}{10}$ inches.

(2.) CENTER OF OSCILLATION.—The upper part of a pendulum tends to move faster than the lower part,

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* A pendulum which vibrates seconds must be four times as long as one which vibrates half-seconds. The apparatus represented in Figs. 33 and 34 can be made by any carpenter or ingenious pupil, and will serve excellently to illustrate the three laws of the pendulum. The law of the pendulum may be conveniently expressed in symbols. If t be the time of a single vibration in seconds, l the length of the pendulum, g the acceleration of gravity, l and g being expressed in feet, or in meters, and if π be the ratio of the circumference to the diameter of a circle, then

$$t = \pi\sqrt{\frac{l}{g}}.$$

This formula is convenient for use in solving problems.

and so hastens the speed. The lower part of a pendulum tends to move slower than the upper part, and so retards the speed. Between these extremes is a point which is neither quickened nor impeded by the rest, but moves in the same time that it would if it were a particle swinging by an imaginary line. This point is called the *center of oscillation.* It lies a little below the center of gravity.* In Fig. 34 is shown an apparatus containing pendulums of different shapes, but of the same length. If they are started together, they will immediately diverge, no two vibrating in the same time. As pendulums, they are not of the same length.

(3.) THE CENTER OF OSCILLATION IS FOUND BY TRIAL.†—Huyghens discovered that the point of suspension and the center of oscillation are interchangeable. If, therefore, a pendulum be inverted, and a

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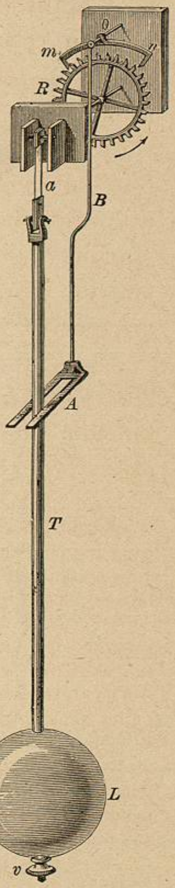
* This determines the real length of a pendulum, which is the distance from the point of support to the center of oscillation. The imaginary pendulum above described is known in Physics as the *Simple Pendulum.*—39.1 inches = 993.3 mm.

† "Take a flat board of any form and drive a piece of wire through it near its edge, and allow it to hang in a vertical plane, holding the ends of the wire by the finger and thumb. Take a small bullet, fasten it to the end of a thread, and allow the thread to pass over the wire so that the bullet hangs close to the board. Move the hand by which you hold the wire horizontally in the plane of the board, and observe whether the board moves forward or backward with respect to the bullet. If it moves forward, lengthen the string; if backward, shorten it till the bullet and the board move together. Now mark the point of the board opposite the center of the bullet, and fasten the string to the wire. You will find that, if you hold the wire by the ends and move it in any manner, however sudden and irregular, in the plane of the board, the bullet will never quit the marked spot on the board. Hence this spot is called the center of oscillation, because, when the board is oscillating about the wire when fixed, it oscillates as if it consisted of a single particle placed at the spot. It is also called the center of percussion, because, if the board is at rest and the wire is

point found at which it will vibrate in the same time as before, this is the former center of oscillation; while the old point of suspension becomes the new center of oscillation.*

(4.) THE PENDULUM AS A TIME-KEEPER.—The friction at the point of suspension, and the resistance of the air, soon destroy the motion of the pendulum. The clock is a machine for keeping up the vibration of the pendulum, and counting its beats. In Fig. 35, *R* is the scape-wheel driven by the force of the clock-weight or spring, and *mn* the escapement, moved by the forked arm, *AB*, so that only one cog of the wheel can pass at each double vibration of the pendulum. Thus the oscillations are counted by the cogs on the wheel, while the friction and the resistance of the air are overcome by the action of the weight or spring.† As "heat expands and cold contracts,"

FIG. 35.



Clock Pendulum.

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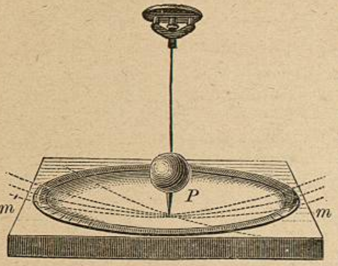
suddenly moved horizontally, the board will at first begin to rotate about the spot as a center."—J. CLERK MAXWELL, on "Matter and Motion," p. 104.

* The center of oscillation is the same as the *center of percussion.* The latter is the point where we must strike a suspended body, if we wish it to revolve about its axis without any strain. If we do not hit a ball on the bat's center of percussion, our hands "sting" with the jar.

† The action of a clock is clearly seen by procuring the works of an old clock and watching the movements of the various parts.

a pendulum lengthens in summer and shortens in winter. A clock, therefore, tends to lose time in summer and gain in winter. To regulate a clock, we raise or lower the pendulum-bob, *L*, by the nut *v*.

FIG. 36.



Foucault's Method.

(5.) OTHER USES OF THE PENDULUM.—(*a*.) Since the time of vibration of a pendulum indicates the force of gravity, and the force of gravity decreases as the square of the distance from the center of the earth increases, we may thus find the semi-diameter of the earth at various places, and ascertain the figure of our globe. (*b*.) Knowing the force of gravity at any point, the velocity of a falling body can be determined. (*c*.) The pendulum may be used as a standard of measures. (*d*.) Foucault devised a method of showing the rotation of the earth on its axis, founded upon the fact that the pendulum vibrates constantly in one plane.* (*e*.) By observing the difference in the length of a seconds-pendulum at the top of a mountain and at the level of the sea, the density of the earth may be estimated.

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* A pendulum 220 feet in length was suspended from the dome of the Pantheon in Paris. The lower end of the pendulum traced its vibrations north and south upon a table beneath, sprinkled with fine sand. These paths did not coincide, but at each return to the outside, the pendulum marked a point to the right. At the poles of the earth the pendulum, constantly vibrating in the same vertical plane, would perform a complete revolution in twenty-four hours, making thus a kind of clock. At the equator it would not change east or west, as the plane of vibration would go forward with the diurnal rotation of the earth. The shifting of the plane would increase as the pendulum was carried north or south from the equator.

PRACTICAL QUESTIONS.

1. When an apple falls to the ground, does the earth rise to meet it?
2. Will a body weigh more in a valley than on a mountain?
3. Will a pound weight fall more slowly than a two-pound weight?
4. How deep is a well if it takes three seconds for a stone to fall to the bottom?
5. Is the center of gravity always within a body—as, for example, a pair of tongs?
6. In a ball of equal density throughout, where is the center of gravity?
7. Why does a ball roll down hill?
8. Why is it easier to roll a round body than a square one?
9. Why is it easier to tip over a load of hay than one of stone?
10. Why is a pyramid such a stable structure?
11. When a hammer is thrown, on which end does it most often strike?
12. Why does a rope-walker carry a heavy balancing-pole?
13. What would become of a ball if dropped into a hole bored through the center of the earth?
14. Would a clock lose or gain time if carried to the top of a mountain? If carried to the North Pole?
15. In the winter, would you raise or lower the pendulum-bob of your clock?
16. Why is the pendulum-bob generally made flat?
17. What "beats-off" the time in a watch?
18. What should be the length of a pendulum to vibrate minutes at the latitude of New York? *Solution:* $(1 \text{ sec.})^2 : (60 \text{ sec.})^2 :: 39.1 \text{ in.} : x = 2.2$ + miles.
19. What should be the length of the above to vibrate half-seconds? Quarter-seconds? Hours?
20. What is the proportionate time of vibration of two pendulums, respectively 16 and 64 inches long?
21. Why, when you are standing erect against a wall, and a piece of money is placed between your feet, can you not stoop forward and pick it up?
22. If a tower were 198 ft. high, with what velocity would a stone, dropped from the summit, strike the ground? (In these problems on falling bodies we may disregard the resistance of the air.)
23. A body falls in 5 seconds; with what velocity does it strike the ground?

24. How far will a body fall in 10 seconds? With what velocity will it strike the ground?

25. A body is thrown upward with a velocity of 192 ft. the first second; to what height will it rise?

26. A ball is shot upward with a velocity of 256 ft.; to what height will it rise? How long will it continue to ascend?

27. Why do not drops of water, falling from the clouds, strike with a force equal to that calculated according to the laws of falling bodies? Because the mass of each drop is so small in proportion to its surface that the resistance of the air soon balances the acceleration of gravity, so that they fall with uniform velocity instead of accelerated velocity.

28. Are any two plumb-lines parallel?

29. A stone let fall from a bridge strikes the water in 3 seconds. What is the height?

30. A stone falls from a church-steeple in 4 seconds. What is the height of the steeple?

31. How far would a body fall in the first second at a distance of 12,000 miles above the earth's surface?

32. A body at the surface of the earth weighs 100 tons; what would be its weight 1,000 miles above?

33. A boy wishing to find the height of a steeple, lets fly an arrow that just reaches the top and then falls to the ground. It is in the air 6 seconds. Required the height.

34. An object let fall from a balloon reaches the ground in 10 seconds. Required the distance.

35. In what time will a pendulum 40 ft. long make a vibration?

36. Two bodies in space are 12 miles apart. Their masses are, respectively, 100 and 200 lbs. If they should fall together by their mutual attraction, what portion of the distance would be passed over by each body?

37. If a body weighs 2,000 lbs. upon the surface of the earth, what would it weigh 2,000 miles above? 500 miles above?

38. At what distance above the earth will a body fall, the first second, 21⅓ inches?

39. How far will a body fall in 8 seconds? In the 8th second? In 10 seconds? In the 30th second?

40. How long would it take for a pendulum one mile in length to make a vibration?

41. What would be the time of vibration of a pendulum 64 meters long?

42. A ball is dropped from a height of 64 ft. At the same moment a second ball is thrown upward with sufficient velocity to reach the same point. How far from the ground will the two balls pass each other?

43. Explain the following fact: A straight stick loaded with lead at one end, can be more easily balanced vertically on the finger when the loaded end is upward than when it is downward.

44. If a body weighing a pound on the earth were carried to the sun it would weigh about 27 lbs. How much would it then attract the sun?

45. Why does watery vapor float and rain fall?

46. If a body weighs 10 kilos. on the surface of the earth, what would it weigh 1,000 kilometers above (the earth's radius being 6,366 km.)?

47. A body is thrown vertically upward with a velocity of 100 meters; how long before it will return to its original position?

48. Required the time needed for a body to fall a distance of 2,000 meters.

49. What would be the time of vibration of a pendulum 39.1 inches long at the surface of the moon, where the acceleration of gravity is only 4.8 ft.?

50. What would be the time of vibration for the same pendulum at the surface of the sun, where the acceleration of gravity is 27 times what it is at the earth's surface?

51. How many vibrations per minute would be made at the surface of the moon by a pendulum 40 ft. long?

52. A pendulum vibrates 200 times in 15 minutes. What is its length?

53. For a certain clock in New York the pendulum was made 500 lbs. in weight. What was the object in making it so heavy?

54. Pendulums are often supported by knife-edges of steel resting on plates of agate. Why?

55. The acceleration of gravity at the equator is 32.088 ft.; at the pole, 32.253 ft. If a pendulum vibrates 3,600 times an hour at the equator, how many times an hour will it vibrate at the pole?

SUMMARY.

There are certain forces residing in molecules and acting only at insensible distances, which are known as the Molecular Forces. The one which ties together molecules of the same kind is styled cohesion. The relation between this force and that of heat chiefly determines whether a body is solid, liquid, or gaseous. Under the action of cohesion, liquids tend to form spheres; and many solids, crystals. The processes of welding and tempering, and the annealing of iron and glass, illustrate curious modifications of the cohesive force. Molecules of different kinds are held together by adhesion. Its action is seen in the use of cement, paste, etc., in the solution of solids, in capillarity, diffusion of gases, and osmose.

Gravitation, though weak,* compared with cohesion, acts

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* As the attraction of gravitation acts so commonly upon great masses of matter, we are apt to consider it a tremendous force. We, however, readily detect its relative feebleness when we compare the weight of bodies

universally. Its force is directly as the product of the attracting and attracted masses, and inversely as the square of their distance apart. Gravity makes a stone fall to the ground. The earth and a kilogram of iron in mid-air attract each other equally, but the mass of the former is so much greater that they move toward each other with unequal velocity, and the motion of the earth is imperceptible. Weight is the measure of the attraction of the earth. At the center of the earth the weight of a body would be nothing; at the poles it would be greatest, and at the equator least. Increase of distance above or far below the surface of the earth will diminish weight. Were the resistance of the air removed, all bodies would fall with equal rapidity. The laws of falling bodies may be studied with the aid of Atwood's Machine. The first second a body falls 16 ft. (4.9 meters), and gains a velocity of 32 ft. (9.8 meters). In general, the final velocity of a falling body is 32 ft., multiplied by the number corresponding to the second, and the distance is 16 ft. multiplied by the square of the number expressing the seconds. The center of gravity is the point about which the weights of all the particles composing a body will balance one another, *i. e.*, be in equilibrium. There are three states of equilibrium—stable, unstable, and indifferent—according as the point of support in a body is above, below, or at the center of gravity. As the center of gravity tends to seek the lowest point, its position determines the stability of a body. A body suspended so as to swing freely is a pendulum. The time of a pendulum's vibration is independent of its material, proportional to the square root of its length and variable according to the latitude. The pendulum is our time-keeper and useful in many scientific investigations.

We are so accustomed to see all the objects around us possess weight, that we can hardly conceive of a body deprived of a property which we are apt to consider as an essential attribute of matter. Nothing is more natural, apparently, than the falling of a stone to the ground. "Yet," says D'Alembert, "it is not without reason that philosophers are astonished to see a stone fall, and those who laugh at their astonishment would

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with their tenacity.—*Example:* Think how much easier it is to lift an iron wire against gravity than to pull it to pieces against cohesion.

soon share it themselves, if they would reflect on the subject." Gravity is constantly at work about us, at one moment producing equilibrium or rest, and at another, motion. When it seems to be destroyed, it is only counterbalanced for a time, and remains, apparently, as indestructible as matter itself. The stability and the incessant changes of nature are alike due to its action. Not only do rivers flow, snows fall, tides rise, and mountains stand in obedience to gravitation, but smoke ascends and clouds float through the combined influence of heat and weight.

HISTORICAL SKETCH.

The latter part of the sixteenth century witnessed the establishment of the principles of falling bodies. Galileo, while sitting in the cathedral at Pisa and watching the swinging of an immense chandelier which hung from its lofty ceiling, noticed that its vibrations were isochronous. This was the germ-thought of the pendulum and the clock. Up to his time it had been taught that a 4-lb. weight would fall twice as fast as a 2-lb. one. He proved the fallacy of this view by dropping from the Leaning Tower of Pisa balls of different metals—gold, copper, and lead. They all reached the ground at nearly the same moment. The slight variation he correctly accounted for by the resistance of the air, which was not the same for all.

Newton and his immediate predecessors knew the law of terrestrial gravity as manifested in falling bodies. When quite a young man, Newton entertained the idea that the attraction which draws bodies downward at the earth's surface must exist also between masses widely separated in space, such as the earth and the moon. To test this, he calculated how far the moon bends from a straight line, *i. e.*, falls toward the earth every second. Knowing the distance a body falls in a second at the surface of the earth, he endeavored to see how far it would fall at the distance of the moon. For years he toiled over this problem, but an erroneous estimate of the earth's diameter then accepted by physicists prevented his obtaining a correct result. Finally, a more accurate measurement having been made, he inserted this in his calculations. Finding the result was likely to verify his conjecture, his hand faltered with the excitement, and he was forced to ask a friend to complete the task. The truth was reached at last, and the grand law of gravitation discovered (1682).

The sun-dial was doubtless the earliest device for keeping time. The clepsydra was afterward employed. This consisted of a vessel containing water, which slowly escaped into a dish below, in which was a float that by its height indicated the lapse of time. King Alfred used candles of a uniform size, six of which lasted a day. The first clock erected in England, about 1288, was considered of so much importance that a high official was appointed to take charge of it. The clocks of the middle ages were extremely elaborate. They indicated the motions of the heavenly bodies; birds came out and sang songs, cocks crowed, and trumpeters blew their horns; chimes of bells were sounded, and processions of dignitaries and military officers, in fantastic dress, marched in front of the dial and gravely announced the time of day. Watches were made at Nuremberg in the fifteenth century. They were styled Nuremberg eggs. Many were as small as the watches of the present day, while others were as large as a dessert-plate. They had no minute or second hand, and required winding twice per day.

On Attraction, as well as on subsequent topics treated in this book, consult Guillemin's "Forces of Nature;" Atkinson's "Ganot's Physics"; Arnott's "Elements of Physics"; Snell's "Olmstead's Natural Philosophy"; Stewart's "Elementary Physics"; Silliman's "Physics"; Everett's "Text-book of Physics"; Young's "Lectures on Natural Philosophy"; "Appleton's Cyclopedia," articles on Clocks and Watches, Weights and Measures, Gravitation, Mechanics, etc.; Peck's "Ganot's Natural Philosophy"; Miller's "Chemical Physics," Chap. III., on Molecular Force; Weinhold's "Experimental Physics"; Pickering's "Elementary Physical Manipulation"; "Fourteen Weeks in Astronomy," sections on Galileo and Newton, pp. 29–34.

The current numbers of "Harper's Magazine," "The Century Magazine," "Scribner's Magazine," "Popular Science Monthly," "Boston Journal of Chemistry," "Scientific American," "Knowledge," and "Nature," contain the latest phases of science.

IV.

ELEMENTS OF MACHINES.

NATURE is a reservoir of power. Tremendous forces are all about us, but they are not adapted to our use. We need to remold the energy to fit our wants. A water-fall can not grind corn nor the wind draw water. Yet a machine will gather up these wasted forces, and turn a grist-mill or work a pump. A kettle of boiling water has little of promise; but husband its energy in the steam-engine, and it will weave cloth, forge an anchor, or bear our burdens along the iron track.

"The hero in the fairy tale had a servant who could eat granite rocks, another who could hear the grass grow, and a third who could run a hundred leagues in half an hour. So man in nature is surrounded by a gang of friendly giants who can accept harder stints than these. There is no porter like gravitation, who will bring down any weight you can not carry, and if he wants aid, knows how to get it from his fellow-laborers. Water sets his irresistible shoulder to your mill, or to your ship, or transports vast bowlders of rock, neatly packed in his iceberg, a thousand miles."

EMERSON.