end of the reflector. On turning the cylinder we have the curve peculiar to this note, and at the same time we have the sinuous line produced by the tuning-fork. Let us next count the number of vibrations made by the voice for any given length of time, and suppose we find that the voice makes one hundred and eighty sinuosities while the fork makes seventy. What is the frequency of the note sung, that of the fork being one hundred? When the fork makes seventy vibrations, the voice makes one hundred and eighty; when the fork executes one hundred vibrations, the voice executes x vibrations. Putting this in the form of a proportion, we have, $70 : 180 :: 100 : x$, from which we find the value of x to be $257\frac{1}{7}$, which corresponds almost exactly with middle C of the pianoforte.

To give you an idea of the variety and beauty of the tracings obtainable, I will project on the screen[1] a number of them as produced by the various notes and combinations of notes of organ-pipes of different frequencies.

The upper sinuous line (Fig. 28) in each pair of undulating tracings was inscribed by a tuning-fork making two hundred and fifty-six vibrations per second. The numbers at the left hand of the figures indicate the relative frequencies of the notes used. Thus the second sinuous line was produced by the joint action on the membrane of the phonautograph of two notes whose relative frequencies were as 4 : 5. Near the middle of the figure is a curve resulting from the combination of three notes, whose relative frequencies were as 4 : 5 : 6. The lowest curve was generated by the sonorous pulses proceeding simultaneously from four organ-pipes whose relative frequencies were 4 : 5 : 6 : 8.

After some familiarity with these and similar curves, one can see at a glance whether the sounds that produce them are simple or complex. Not only this, one can also tell how the constituents of complex sounds are related to each other, and discover, with equal readiness, their comparative intensities.

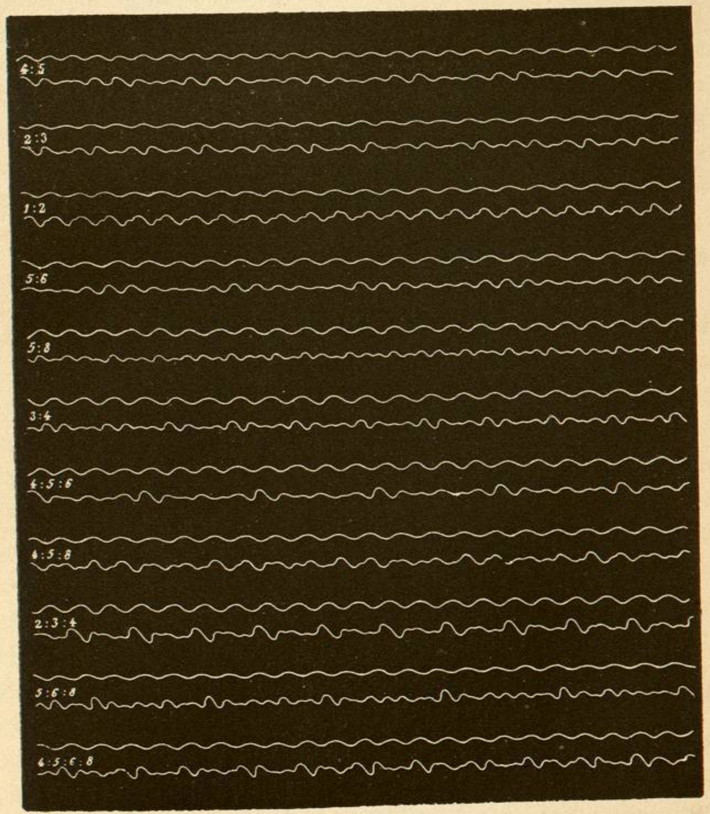

Fig. 28.

The experiments just made have familiarized you with some of the principal methods employed by physicists for determining the pitch of sounds. There are indeed many others, some of which are more difficult and complicated than those just illustrated, but we have not time to consider them now. And even if we had the time, some of them are of such a character as to preclude

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[1] Professor Mayer uses an excellent and ingenious means of obtaining traces from a rotating cylinder on a transparent surface. He takes a band of *thin mica*, and binding it around the cylinder, fastens it down with rubber bands. It is then smoked with camphor smoke. The trace having been made on it, it is taken from the cylinder, and thin *white* negative varnish is flowed over the lampblack. It may then be mounted between plates of glass for the lantern projection.

the possibility of their being introduced in lecture experiments.

In his very exact determination of the pitch of the Diapason Normal, which Lissajous intended should give 435 vibrations, at 15° C., Koenig used a large fork connected with clockwork, the whole acting as a single system. This clock-fork, as Koenig calls it — and a most elaborate apparatus it is — was kept vibrating in a practically constant temperature for many hours at a time, and the experiments extended over a period of several months. The result obtained is probably as near an approximation to the truth as it would be possible to obtain. By this means it was found that the Diapason Normal at 15° C., or 59° F., executed 435.45 instead of 435 vibrations per second. This is a very slight difference, you will say; but it is only one among many instances of the accuracy with which modern scientific apparatus is constructed, especially by such a mechanician as Koenig. Of the large number of forks made by him, which you see here, all are, I dare say, tuned with equal care, and all will give exactly the number of vibrations which his stamp, affixed to each fork, says they will give.

Indeed, nothing is better or more accurate for determining pitch than a carefully constructed set of tuning-forks. And strange though it may seem, the first one to propose and construct a tuning-fork tonometer — an instrument for determining pitch — was a silk manufacturer, J. Heinrich Scheibler, of Crefeld, Germany. In one of the tonometers constructed by him, there were as many as fifty-six tuning-forks, all tuned with the utmost care and accuracy. The tonometers made by Scheibler were long used as the standard for similar sets, and nothing comparable to them was attempted until some decades later, about 1860, when Dr. Koenig began his marvellous career as an acoustic mechanician.

Dr. Koenig has made many sets of tuning-forks similar to those designed by Scheibler, having in his larger tonometers as many as sixty-seven forks. But his most wonderful work is a tonometer commenced in 1877, and now very near completion. It consists of one hundred and fifty forks of exquisite workmanship, and tuned with infinite care and skill. It embraces the entire range of audible sounds, and extends from 16 to 21845.3 vibrations per second. For the compass of sounds employed in music, the forks are so adjusted that no fork differs from the one that precedes or succeeds it by more than four vibrations. For the lower sounds the difference is only one half of a simple vibration.[1]

By means of this extraordinary instrument, the frequency of any note can at once be determined with absolute accuracy. No such work has ever been essayed by any one before, and it is quite safe to assert that no such herculean task will ever again be undertaken by any one else. Only untiring patience, exceptional skill, and a phenomenal love for his work could ever have enabled Dr. Koenig to accomplish a task demanding such care and time and labor. It is a most remarkable achievement of industry and genius, and a monument of which any man might be proud.

If now it can only be secured by some musical organization that will take proper care of it, it could be used for a long time as a standard about whose accuracy there could be no question. And furthermore, if the musicians of the world would only agree to take this for an international standard, it would be a happy solution of many difficulties that have beset musical composers, performers, and manufacturers of musical instruments for several generations past. Nothing better could be desired, and certainly nothing more complete has ever been carried into execution. The musical world has no standard of pitch,[2] and this marvellous tonometer would answer the purpose admirably, and with due care would last for all time to come. The standards of weight and measure of France and England have been worked out with all the nicety and delicacy that

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[1] The largest forks, which are about five feet in length, are provided with great cylindrical resonators of copper. The largest resonator is an immense affair. It is twenty inches in diameter, and nearly eight feet in length. All the resonators are adjustable in length, so as to be used for notes of different pitch.

[2] See note on following page.

human ingenuity could suggest; but I do not think they are any more exact in their sphere than is the *grand tonomètre universal* on which Dr. Rudolph Koenig has spent so many of the best years of his life.

I have spoken of the desirability of having a standard of pitch that would be universally recognized. One would imagine that such a standard would have been agreed upon long ago; but when one thinks of the various and often imperfect standards of measurement that obtain in other branches of science and art, one is not surprised that musicians also are behindhand in this respect. The French, it is true, have done something in this direction, for in the Diapason Normal, already referred to, they have a national standard. But even in France this standard is not universally employed. The Government has no power to enforce it except in the schools, theatres, and conservatories which it subsidizes. In churches, private theatres, conservatories, concerts, and orchestras, the pitch of the instruments used is far from being uniform.

So it is elsewhere. The pitch varies, not only in different countries, but in different cities of the same country, and even in the different theatres of the same city.

But more than this. The pitch varies not only in place, but also in time. It is quite different now from what it was a century ago. Then it was comparatively low. Since then it has been growing higher and higher, until the opinion begins to prevail, almost everywhere, that it is time to call a halt. And to avoid the constant fluctations of pitch that have obtained so long and so extensively, it is felt now more than ever that an international standard of pitch is almost, if not quite, a necessity. The first step in this direction was made by an international conference of musicians held in Vienna in 1885, when the French pitch was unanimously adopted.[1] So far, however, this adoption has amounted to nothing more than an acceptance, in theory, that the French pitch is desirable, and should therefore be adopted. As yet little or nothing has been done towards carrying out in practice what the conference deemed not only advisable but necessary.[2]

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[1] The French pitch was adopted by Russia in 1860, by Spain in 1879, and by Belgium in 1885. The Royal Academy of England accepted it as the standard, June 20, 1885, and a few months subsequently, Feb. 12, 1886, it was formally adopted by the English Society of Arts. Italy, having sent representatives to the Congress of Vienna, adopted French pitch in 1885.

The French standard of pitch, the Diapason Normal, which is preserved in the Musée du Conservatoire in Paris, was designed to make 435 vibrations per second, but actually makes, as we have seen, 435.45.

The starting-point for pitch in music is the second open string of the violin, which gives the tuning note for orchestras. It corresponds to A_3 above middle C of the pianoforte, which in musical notation is written . A_3, of a vibration-number of 435, was chosen in 1859 as the result of a report made by a special commission appointed to determine a standard pitch. Previously to this date, in 1834, the German Society of Physicists, assembled as Stuttgart, had adopted as a standard of pitch a note which had a frequency of 444. Physicists employ a fork whose frequency is much lower, — their A_3 having a vibration-number as low as 426.6. This is very near the frequency of the A_3 fork used by Handel in 1751; its vibration-number was 422.5. Mozart's pitch was a little less, being A_3 421.6. The lowest church pitch, in Mersenne's time (1648), was A_3 373.7. The so-called chamber pitch, at that date, according to Mersenne, was A_3 402.9.

Since Mersenne's time, as is apparent from the foregoing numbers, the rise in pitch has been very great indeed. But without going back any farther than the days of Mozart and Handel, we find that the rise in pitch has

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[2] Since the above was written, "The Piano Manufacturers' Association" of New York and vicinity have unanimously selected the French pitch as the standard for all instruments made by the members of the Association. At a meeting held by this body Nov. 6, 1891, it was "Resolved, that the standard musical pitch adopted by the piano manufacturers of the United States, giving that A which vibrates 435 double vibrations in a second of time at 68° Fahr., shall be known as the 'International Pitch.'" It was further decided that this resolution should go into effect July 1, 1892.

been so great as frequently to make it difficult to sing and play the works of these great masters with proper effect. In England, for instance, in spite of all the efforts that have been made to keep it down to A_3 444, orchestra and pianoforte pitch has risen higher and higher, until it now runs from A_3 449.7 to A_3 454.7. In some parts of the United States, especially in New York, pitch, in some instances, has gone up as high as A_3 460.8. A Chickering piano is tuned by a standard fork which gives A_3 451.7, and a piano by Steinway is tuned to A_3 458. Between Mozart's pitch and that used by Chickering and Steinway there is, therefore, a difference of between thirty and thirty-one vibrations, amounting practically to three fourths of a tone.

The disadvantages, especially to vocalists, consequent on such a rise of pitch, are apparent. Music written by Mozart, Handel, Beethoven, and Haydn must be sung more than a semitone higher than it was intended to be sung. For the higher notes this is often difficult without straining the voice. Besides, the effect produced by this elevation of pitch is often entirely different from that which was aimed at by the composer, and which would be secured if the music were sung at the pitch for which it was written.

Orchestras and military bands are, in the main, responsible for this undue elevation of pitch. Wind instruments especially have more brilliancy of tone when tuned to this high pitch, and, for this reason, popular taste has demanded from the manufacturers of such instruments that they should give them the high pitch which now prevails.

The "tuning note" for orchestras, as above stated, is A_3 of the frequencies already given. For pianos, however, the tuning note is the first C above A_3, namely, C_4. This C_4, according to the old theoretical pitch, which is that now used by physicists, has a frequency of 512. In French equal temperament, with A_3 435, C_4 has a frequency of 517.3. The pitch of C_4 of the English Society of Arts,

based on the German standard pitch, A_3 440, is 528. In modern concert pitch the frequency of C_4 is still higher, being 540.

The frequency, 512, of the standard C_4 of the physicist, is the ninth power of 2, and gives, consequently, vibration-numbers to all the C's which are powers of 2. This number was proposed by the distinguished acoustician Sauveur, and subsequently, in 1830, adopted by Chladni. It has above all others the advantage of simplicity. For this reason, although considerably lower than other pitches in use, it is almost universally employed by physicists and acousticians. All the forks that we shall employ in our experiments, unless otherwise specified, are tuned to this standard of pitch. In musical notation this note C_4, of 512 vibrations, would be written .

To get C_5, an octave above this, it is only necessary to multiply 512 by 2, which gives 1024. Dividing 512 by 2 will give C_3, the octave below having 256 vibrations. In general, by doubling the number of vibrations corresponding to any given note we obtain a note an octave higher, and by halving it we get a note an octave lower. Having the notes, then, of one octave of what is called the diatonic scale, we can readily obtain all the others used in music by simply multiplying or dividing by 2 or a multiple of 2.

The notes of the gamut are variously designated in different countries. In France the first six notes still bear the names given them by the monk Guy of Arezzo in 1026. They are the beginnings of words which occur in a hymn to Saint John the Baptist,[1] and are as follows: *ut, re, mi, fa, sol, la.* The seventh syllable, *si*, was added in 1684 by Lemaire. In Italy *do* has been substituted in place of *ut*, because more easily pronounced in singing. In England the notes are named after the first letters of the

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[1] The words are, —

"*Ut* queant laxis *re*sonare fibris
*Mi*ra gestorum *fa*muli tuorum,
*Sol*ve polluti *la*bii reatum,
Sancte Joannes."

alphabet, and are called C, D, E, F, G, A, B. In Germany H is substituted for B.

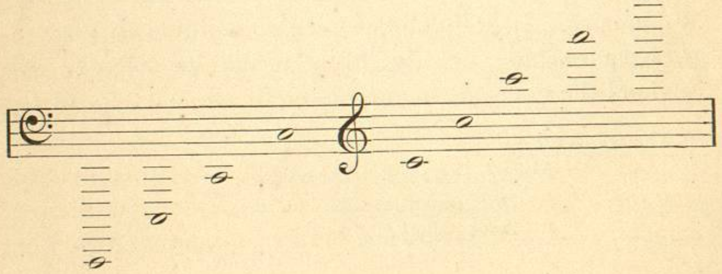
But we must go farther. The letters and syllables just given distinguish the notes of an octave from each other, but it is necessary besides to have a means of designating the different octaves of any musical instrument. In Germany and England this is ordinarily done by using capital letters, one unaccented, and the others variously under-accented or under-lined for the lower octaves, and small letters, one likewise unaccented and the others over-accented or over-lined, for the higher octaves. The C's of the eight octaves of the organ, when accented or lined, are usually written as follows: —

$C_{\prime\prime}$, C' C c c' c'' c''' c'''' c'''''

$\underline{\underline{C}}$ $\underline{C}$ C c $\overline{c}$ $\overline{\overline{c}}$ $\overline{\overline{\overline{c}}}$ $\overline{\overline{\overline{\overline{c}}}}$ $\overline{\overline{\overline{\overline{\overline{c}}}}}$

The French method of designating the same notes is the following: —

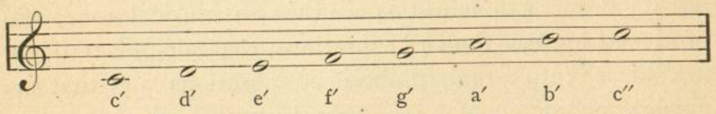
Ut_{-2}, Ut_{-1}, Ut_1, Ut_2, Ut_3, Ut_4, Ut_5, Ut_6, Ut_7.

The tuning-forks that we shall use give for these notes the following frequencies, unless stated otherwise: 16, 32, 64, 128, 256, 512, 1024, 2048, 4096. As we shall have occasion to refer frequently to these notes, octaves, and vibration-numbers, and as it is important that we should be able to locate them at once, it is desirable to give them as written in musical notation, together with their names and frequencies.

In this manner we may recognize them at a glance.

German:	$C_{\prime\prime}$	$C_{\prime}$	C	c	c'	c''	c'''	c''''	c'''''
French:	Ut_{-2}	Ut_{-1}	Ut_1	Ut_2	Ut_3	Ut_4	Ut_5	Ut_6	Ut_7
Frequency:	16	32	64	128	256	512	1024	2048	4096
	C_{-2}	C_{-1}	C_1	C_2	C_3	C_4	C_5	C_6	C_7

The last line is a partial combination of the French and the German systems, and is, in many respects, more convenient than either. For this reason we shall adopt it in preference to either of the other two. I shall frequently have occasion to speak of higher notes and higher octaves than those used in any musical instrument, and with this last system one can indicate any given note with the greatest facility and accuracy. Thus C_9, one of the forks of a series on the table, gives a note just two octaves above C_7, the highest note of the organ. F_9 designates a fork of the same series four notes higher. G_{10}, the highest note for which any tuning-fork has yet been made, is full three and a half octaves above the highest note used in music. Its relation to a corresponding note of any of the lower octaves is seen as soon as one knows the frequency of the fork.

The notes of the diatonic scale, with their corresponding vibration-numbers, for what is known as the "two-foot" or "one-stroked" octave, are in musical notation as follows:



c'	d'	e'	f'	g'	a'	b'	c''
C_3	D_3	E_3	F_3	G_3	A_3	B_3	C_4
256,	288,	320,	341.3,	384,	426.6,	480,	512.

By multiplying or dividing the vibration-numbers of the notes of this octave by 2, or some power of 2, we can, as just stated, readily determine the frequency of any note of any octave, high or low.

The subject we have been studying has prepared the way for a question of considerable experimental interest; namely, what are the limits of audible vibrations? Very few