persons, I fancy, have any idea of the many and apparently conflicting answers given to this question.

According to Sauveur's experiments with organ-pipes the lowest audible sound corresponds to $12\frac{1}{2}$ vibrations per second. Biot's and Chladni's experiments with strings raised the number to 16. Savart in his investigations used a rotating rod striking through a narrow slit, and came to the conclusion that 8 vibrations were sufficient to produce a continuous sound. By means of a specially constructed sounding-box, over which a string was stretched, Helmholtz finds that the lowest limit for grave tones is about 30 vibrations. "At B_{-26}," he says, "with $29\frac{1}{3}$ vibrations in a second, there was scarcely anything audible left;" and he concludes with the statement that "although tones of 24 to 28 vibrations have been heard, notes do not begin to have a definite pitch until about 40 vibrations are performed in a second."[1] According to Mr. Ellis's observations with a large tuning-fork, the lowest audible sound was in the neighborhood of 30 vibrations. "For 30 vibrations," he observes, "I could still hear a weak drone; for 28 scarcely a trace." With the same instrument Prof. W. Preyer, of Vienna, was able to hear a continuous sound at 24 vibrations. But by using specially loaded tongues in reed pipes made by the acoustician Herr Appunn, of Hanau, he declares that he was able to hear tones as low as 15 vibrations. Preyer's conclusions are that no musical tones are produced by less than 15 vibrations, that air-pulses begin to coalesce into a tone at about 20 vibrations, and that the musical character of bass tones is perceived only when their frequencies exceed 24 vibrations per second. According to Despretz, the lowest limit of audible sounds is 16 vibrations, — the note that is given, or supposed to be given, by the thirty-two foot organ-pipe. There is grave doubt among experimenters whether the thirty-two foot organ-pipe actually gives a continuous note of 16 vibrations, or whether the sound heard is not in reality due to

[1] See Mr. A. J. Ellis's admirable translation of Helmholtz's "Tonempfindungen," chap. ix.

what are called "upper partials," which we shall consider in the sequel. According to Helmholtz, the vibrations of this pipe can always be heard as separate pulses, and never blend into a continuous sound. Its value, then, depends entirely on its power of reinforcing the notes of the octaves above it, and on the so-called upper partials, which are always produced simultaneously with the note having 16 vibrations, which it is supposed to produce.

Regarding the limits of audibility of acute sounds, there is the same diversity of opinion. Sixty-four hundred vibrations, according to Sauveur, constitute the highest perceptible note. Chladni made the number 8,192, and Walloston 25,000. By means of a large toothed wheel, similar to the one used here, Savart showed that it was possible to hear a sound corresponding to 24,000 vibrations. Using tuning-forks tuned for him by Marloye, Despretz was able to obtain sounds whose frequencies were over 36,000 vibrations. Employing a very large Seebeck siren, Preyer heard a sound produced by 24,000 vibrations, although this sound was quite inaudible to other persons present. The highest tuning-fork made by Koenig gives 21,845 vibrations, but he makes a set of short steel cylinders, of which the shortest is calculated to give 32,768 vibrations per second. This note corresponds to C_{10}. Herr Appunn makes a set of thirty-one tuning-forks, the highest of which is G_{10}, which makes 49,152 vibrations per second. One of these G_{10} forks I hold in my hand. As you see, it is exceedingly diminutive. It is about half an inch long, two fifths of an inch wide, and its tines are but one eighth of an inch thick. Many persons have been able to hear the note yielded by this fork; but a question may arise whether it really gives a note of the high pitch claimed for it. Without here entering into an explanation of the manner in which the pitch of such forks is determined, I may observe that Herr Appunn, in a letter to me about this and other forks of very high pitch which he furnished me, states that he can guarantee that the frequencies of the forks correspond absolutely with the numbers stamped on them. No one can doubt the skill of Herr Appunn as a mechanician, and the delicacy of his ear for very acute sounds is, according to the testimony of all who are acquainted with him, something quite astonishing. It would probably be impossible for one with a less delicate ear to tune such a fork, even if he were familiar with the method of tuning employed in such cases. We are consequently, by the very necessities of the case, compelled to accept Herr Appunn's estimate as that of an expert and that he is an expert in his specialty no one can gainsay.

Only ears that are specially sensitive to acute sounds are capable of perceiving the notes of such tiny forks. For most persons, especially those advanced in years, the limit of audition is ordinarily below C_9, giving 16,384 vibrations per second.

Such acute sounds, however, are anything but agreeable to the ear. They have a peculiar grating, rasping effect that, at times, becomes extremely painful. In other cases they produce a peculiar indefinable feeling of discomfort, which persists for some time. Dr. Koenig has often told me that he does not like to experiment with these high notes, as they frequently continue to ring in his ears for days, and even weeks afterwards. Prof. W. Preyer, who has distinguished himself by his experiments on the limits of the perception of tone, speaks also of the disagreeable character of the higher notes. In describing his experience with the notes above C_9, he says they affected him "as if a thin wire were drawn through both ears towards the middle, and thence towards the top of the head."

Permit me now to illustrate experimentally the subject we have been discussing. As you already understand how Savart's wheel and Seebeck's siren can be used to determine the limits of perceptible sounds, whether grave or acute, — having seen these instruments used in other experiments, — we shall have recourse to other and more exact instruments.

For investigating the limits of grave sounds we have here a very large fork (Fig. 29), made by Koenig, and similar

to the ones used by Mr. Ellis in his researches. It is mounted on a heavy cast-iron base, and to its prongs, which are nearly thirty inches long, are attached two sliding weights about three inches in diameter, by means of which the pitch of the fork can be raised or lowered. The range of the fork is from C_{-2} to G_{-2}; that is, from 16 to 24 vibrations. There is a scale along each prong to show where the weights are to be adjusted in order to produce the different vibrations marked thereon.

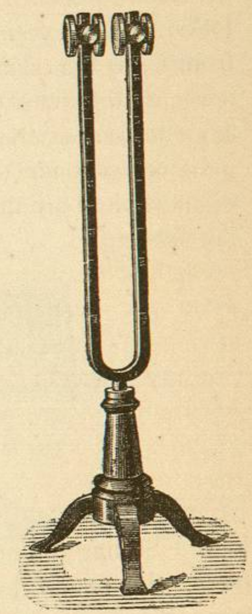


Fig. 29.

Clamping the weights at the top of the prongs, at the place marked for 16 vibrations, we now cause the fork to vibrate; but although you can see that the prongs are in motion, I am quite sure that no one present is able to perceive the sound corresponding to 16 vibrations. You may, when the fork is first excited, hear a deep musical note; but this is one of the upper partial tones to which allusion has been made. It is an octave higher than C_{-2}, and is not, therefore, the note for which we are seeking. In working with the fork, the ear is brought as close as possible to one of the sliding weights, which, on account of their great surface, act as an aid to hearing. But notwithstanding numerous experiments which I have made with many persons having an acute ear for musical sounds, I have never yet been able to find even one who could detect what would be denominated a pure musical note.

I bring the weights down to the bottom of the prongs, to the tone marked 24 vibrations, and again agitate the fork. The result is practically the same as before. You can see the vibratory motion of the fork, but you cannot hear the note G_{-1}, that answers to the 24 vibrations. If, however, the ear is placed as close as possible to one of the sliding disks, it is possible for some persons to hear a kind of low drone, probably the nearest approach to a musical note, — at least with such apparatus.

Tuning-forks are also the best means for determining the limit of acute sounds. Reeds have been used, as well as sirens of various kinds; but the results obtained by these means are not so trustworthy as those given by well-tuned forks. Before you is a superb series of forks for the notes from C_7 to F_9. They are so arranged on a support that it is easy to excite them in succession by merely drawing a violin-bow across their prongs.

C_7 corresponds to the highest note of the organ, and when excited by the bow, its tone comes out clear and

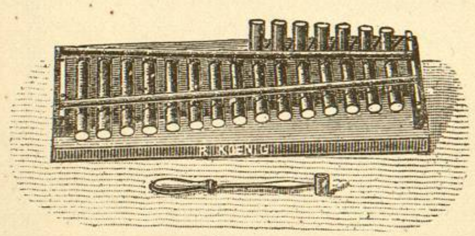


FIG. 30.

loud. Setting the others in vibration in the order of their pitch, the sound becomes correspondingly higher and more piercing. As we approach C_9 the sounds will die out for some of you, while they will remain unpleasantly painful for others. The nearer the forks are to the ear, the more these effects are intensified. Above C_9 the forks will, I think, be inaudible to most of you, no matter how vigorously I draw the bow across them, or how near you may be to them. We have reached the border-land of acute sounds, and by the most perfect means that science has thus far at its command.

Koenig employs another method for showing the limit of perceptibility for acute sounds, which may be illustrated here, as it possesses considerable interest and is capable of giving quite reliable results.

The instrument used for this purpose consists of twenty-two cylindrical steel rods (Fig. 30), giving notes as high as C_{10}, making 32,768 vibrations per second. The lowest note, C_9, is given by the longest cylinder, and the notes become higher in proportion as the cylinders are made shorter. Striking the longest cylinder with an ivory hammer, made for the purpose, you at once perceive a clear, penetrating sound that almost quenches the sound of the hammer itself. When we strike in succession the shorter bars, the musical note becomes more acute, and the shock of the hammer comparatively louder. With the shortest rods the notes due to their vibrations become almost, if not entirely, inaudible, and one hears only the sound of percussion when the hammer comes in contact with the cylinders. With such cylindrical rods, G_8 is heard with difficulty by ordinary ears, and C_9 marks the limits of audition for elderly persons generally, while even the most sensitive ear scarcely ever reaches G_9.

By means of a small whistle (Fig. 31) Captain Douglas Galton has been able to obtain sounds which are said to be as acute as any of those we have been considering, if indeed they are not more acute. Such a whistle I hold in my hand. The air is supplied by a little rubber bulb, and the vibrating column of air, by means of a scale attached to the whistle, can be accurately shortened by such a small amount as the $\frac{1}{250}$th of an inch. When the whistle is made to sound, you hear a very high note, resembling somewhat that emitted by some of the smaller mammalia. The sharp, attenuated notes of white mice, which probably many of you have heard, are not unlike some of the notes we can evoke from this whistle when it is properly adjusted.

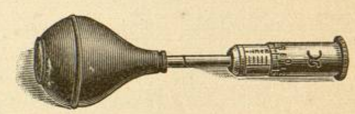


FIG. 31.

By means of this simple little contrivance, one may readily estimate the pitch of very acute sounds. With it I have been able to determine the pitch of a creaking door, which yielded a note full two octaves above the highest note used in music.

From the foregoing we learn that the range of audition extends somewhat over eleven octaves.

Light, like sound, is due to a mode of vibratory motion, and the various colors, like notes of different pitch, have their origin in different rates of vibration. The extreme red of the spectrum corresponds to the gravest musical notes, while the more acute sounds correspond to the extreme violet. Intervening colors correspond to the notes between the most grave and the most acute. But the range of perception for the different rates of vibration is much less for the eye than it is for the ear. For the former it is at most an octave and a half; for the latter it is nearly eight times as much. The frequency of the extreme violet is never more than three times that of the lowest red, — ordinarily it is not much more than twice as great, — whereas the frequency of G_{10} of Appunn's fork is over three thousand times that of the note of the thirty-two foot organ-pipe.[1]

Only an exceptional ear, as we have seen, has a perception extending over the entire eleven octaves of sound. But no human ear, however acute or well trained, is able to separate all audible sounds from each other. It requires a good ear to distinguish the lower notes from each other, but it is a far more difficult matter to discriminate the higher notes from each other after they rise above C_6.

Experimentally we have been dealing with sounds hav-

[1] "Assuming, then," — I quote from Ellis, — "that the yellow of the spectrum answers to the tenor C in music, and Frauenhofer's 'line A' corresponds to the G below it, Professor Hemholtz, in his 'Physiological Optics,' gives the following analogies between the notes of the piano and the colors of the spectrum: —

F♯, End of the Red,	*c*, Yellow,	*f*♯, Violet,
G, Red,	*c*♯, Green,	*g*, Ultra-violet,
G♯, Red,	*d*, Greenish-blue,	*g*♯, Ultra-violet,
A, Red,	*d*♯, Cyanogen-blue,	*a*, Ultra-violet,
A♯, Orange-red,	*e*, Indigo-blue,	*a*♯, Ultra-violet,
B, Orange,	*f*, Violet,	*b*, End of the Solar Spectrum."

ing a compass of something over eleven octaves. The largest organ never has more than eight octaves, the ordinary small organ never more than seven; the piano, as usually constructed, embraces from seven to seven and one half octaves. The violin has three and one half octaves, and the compass of some other instruments is still less. On the organ and the piano, neither the lowest nor the highest octave, as compared with the intervening ones, is much used; this reduces the practical range of these instruments to about five octaves.

In tempered instruments, like the organ, piano, and harmonium, for instance, the number of notes available for each octave is also limited. Counting white and black keys, there are only twelve notes to each octave. This gives for the five octaves employed in ordinary music only sixty different notes, — sixty notes out of the fifty thousand different rates of vibration which we have been considering! In the case of the violin only about forty different notes are used, — less than the $\frac{1}{1000}$th part of the number with which the acoustician deals.

In the human voice the range is much less than that of any of the musical instruments just named. For the ordinary voice the compass, or register, as it is called, is about two octaves. In extraordinary cases the register may embrace two and a half octaves, and in a few phenomenal instances an octave more.

The average human voice, therefore, in singing a solo, in which the key remains unchanged, does not ordinarily use more than twelve or fifteen different notes; and yet, with these few notes, it is able to execute those marvels of melody that so charm the ear.

The human voice has well been compared to the viol family, which embraces four different instruments, — bass, tenor, alto, and soprano. Besides these there are also two intermediate voices, baritone, between bass and tenor, and mezzo-soprano, between soprano and alto.

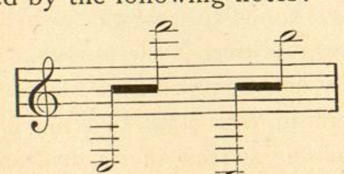
Male voices are known as bass, baritone, and tenor, and female voices are classed as alto, mezzo-soprano, and soprano. The ordinary compass of these various voices is indicated in the following musical notation: —

Bass. Baritone. Tenor. Alto. Mezzo-Soprano. Soprano.

Amongst phenomenal voices were those of Gassner and the brothers Fischer, who were at the court of Bavaria in the sixteenth century, all of whom were able to sing as low as F_{-1}. The voice of Forster, the Dane, had a compass of three octaves. The highest voice on record was undoubtedly that of Lucrezia Ajugari, who sang for Mozart in Parma in 1770. She could sing as high as C_6, and descend as low as G_2, and had therefore the marvellous compass of three and one half octaves. But with all this, her voice, even in its highest tones, remained, according to the testimony of Mozart's father, as pure as a flute. Nilsson and Patti have also attained marvellous heights. The voices of Catalani, Farinelli, and the younger of the sisters Sessi were extraordinary for their depth and compass, having in each case a range of three and a half octaves.

The greatest observed compass of the human voice, from the lowest bass to the highest soprano, is, then, fully five and a half octaves, extending from F_{-1}, of 43, to C_6 of 2,048 vibrations per second. This range, expressed in notes, is as follows: —

The compass of the wonderful voices of Sessi and Farinelli is indicated by the following notes: —



Sessi. Farinelli.

The lowest note used in the orchestra is C_{-1}, and is given by the double bass. As usually made, it gives 33 vibrations per second. The highest note employed in orchestral music is the D_7 of the piccolo-flute, giving, according to the physicist's pitch, 4,608 vibrations per second, but a much higher frequency according to the standard of pitch at present in use. The lowest note, A_{-2}, on a grand piano, is made to give about $27\frac{1}{2}$ vibrations per second. The highest note, C_7, has a frequency of about 4,200 vibrations per second.

In our experiments we have discovered that the range of hearing varies greatly with different persons. One will be astonished at the extent of this variation when he comes to examine the matter with a little attention; he will find, to his surprise, that there are many sounds in nature that are very unpleasant to some, but are entirely beyond the perception of other ears. There are many to whom the multifarious sounds of insect life are inaudible, while these same sounds are disagreeably shrill and piercing to others. There are those, even, who are capable of enjoying music, who cannot hear the upper notes of the piano or organ, or distinguish in the lowest octave one note from another.

Some savages have remarkably acute powers of hearing, but only for certain sounds. Their range of audition is frequently as limited as their perception of some sounds is acute.

Then, again, the sensitiveness of the ear varies greatly for the different notes. It is not as marked for high or low notes as it is for those which are intermediate. Strike in succession the notes of the highest or the lowest octaves of the piano, and you will find that there is not by any means such a marked difference in pitch between the consecutive notes as there is in the intervening octaves. I have known a piano-tuner, for instance, who was an expert in tuning all the octaves except the lowest. In this octave he lost completely his perception of pitch and intervals, and — what was more remarkable, in his case — he was utterly unconscious of his lack of musical appreciation in this part of the scale, and could not be brought to believe that his ear was less sensitive to low than to high notes.

But notwithstanding all this, the ear is a wonderfully comprehensive instrument. As compared with the eye, it is vastly superior in the extent of the sensations it is capable of experiencing. The eye possesses barely an octave and a half of sensations, whereas the average ear, as we have seen, has a range of six or seven, while more acute ears have a compass of fully eleven octaves.

And then the ear is a wonderfully accurate instrument, and capable of appreciating minute differences that would be wholly impossible in the case of the eye. According to Dr. W. H. Stone, "an architect or draughtsman who, between two lines neither parallel nor in one plane, made an error of estimation by eye not exceeding one thirtieth, would gain credit for unusual precision. But in the ear one thirtieth amounts to a quarter of a tone, and by ear one forty-fifth of a tone is easily determined." A skilful pianoforte-tuner can do much more. He is called upon, for instance, to distinguish between a true and an equally tempered fifth, where the difference is only the one hundredth of a tone. He should, accordingly, be able to recognize at least six hundred different sounds in an octave. More than this, according to the investigations of Professor Mayer, it is possible, under specially favorable conditions, and for sounds whose pitch is near that of C_3, to distinguish from each other notes which do not differ by more than the $\frac{1}{120}$th of a semitone.

In the rapidity of its appreciation the ear is equally remarkable. In a fraction of a second it can accurately refer any note to its place in the scale, and can just as easily and as quickly separate from each other several widely different notes. According to recent investigations, the ear is capable of hearing a sound when only two vibrations are made. It should therefore hear the middle notes of the pianoforte in the two or three hundredth part of a second. It requires more time, however, for the ear to distinguish the full characteristic of a note. To do this,

according to the experiments of Exner, Auerbach, and W. Kohlrausch, from 2 to 20 vibrations are necessary.

With proper training and practice the organ of hearing can be rendered remarkably sensitive and accurate. There is rarely any physical defect in the ear itself. The defects ordinarily noticed and spoken of are such as can be easily remedied by cultivation. It may, it is true, never be able to attain the remarkable range of audition we have spoken of above, it may never become so "apprehensive and discriminant" as the ear of Mozart; but its delicacy can be increased and its general appreciation of musical sounds wonderfully improved. This is especially true if the work of instruction is begun in childhood, when the organ of hearing is naturally most sensitive and most readily susceptible of cultivation.

In making experiments with rods and tuning-forks giving very acute sounds, I have frequently been struck with the very great difference in the ability to perceive such sounds as manifested by young and old persons. Even when the latter were trained musicians they were incapable of hearing sounds that were quite audible to children who had no musical training whatever. This fact, like many others that might be adduced, is a striking commentary on the necessity of beginning early the training of the young, when eye and ear, not to speak of the other senses, are ever on the alert, and quick to detect sounds and forms and colors which at a later period would entirely escape their observation, or that of one who had never been taught the wonderful powers and capabilities of the five senses when properly educated.