mathematician, and to defy all attempt to bring them within the range of mathematical analysis. In the instance last given the magnitude of the volume of matter set in motion by a tiny, insignificant insect is something calculated to excite our astonishment. But more wonderful still, when we come to think of it, is the fact that notwithstanding the small amplitude of movement of the air particles a mile distant from the stridulating locust, the vibratory motion excited by this insignificant little insect is still competent to excite the sensation of sound. We know, indeed, that very slight, almost infinitesimal, periodic tremors are sufficient to generate sonorous pulses. Lord Rayleigh has shown that sound-vibrations may be produced when the amplitude of movement is not more than the $\frac{1}{25000000}$th part of an inch. But such reflections and calculations, far from detracting from the marvellous in the case we are considering, tend only to enhance it and to place it in a brighter light. Nothing could give us a better idea of the transcendent delicacy of the ear, nor could we have a better example of the perfect conservation and correlation of force, than that afforded by the illustration just given. But here I must close. We have again come into contact with more of those innumerable mysteries of the natural order which hitherto have baffled all attempts at their solution, and which will, most likely, ever remain as they are at present, — fascinating, yet inscrutable.

CHAPTER IV.

MUSICAL STRINGS.

REVIEWING the ground over which we have thus far travelled, we shall find that we have been dealing with only the more general laws and phenomena of sound. We are now prepared to consider, in greater detail, the laws and phenomena that are observed in connection with special forms of sonorous bodies. Most of our attention will, naturally, be given to such vibrating bodies as are used in music. Chief among these are strings, wires, reeds, bars, plates, bells, membranes, and various forms of sonorous tubes.

To-day we shall occupy ourselves in studying the very interesting phenomena which characterize the vibration of wires and strings. By the term *string*, in acoustics, we mean "a perfectly uniform and flexible filament of solid matter stretched between two fixed points." It thus includes wires as well as strings properly so-called. An acoustic string, however, is quite ideal, as no string is perfectly elastic or perfectly uniform. The most that is ever realized in the strings employed in musical instruments is a more or less close approximation to the ideal string which the mathematician has in view in all of his calculations.

From the earliest times strings have been used in the construction of musical instruments, for we have records of them that date back to the twilight of fable. Figures of what are evidently primitive forms of the harp and the lute are to be found on Egyptian monuments all along the Nile valley. Similar instruments were used by the earliest inhabitants of western Asia, as is evidenced by inscriptions found among the ruins of the great cities that once graced the plains of Chaldea and Mesopotamia. The favorite instrument of the Hindoos — the vina, resembling somewhat the guitar — was given to mankind, we are told, by Sarasvati, the benevolent consort of Brahma. And then we must recall other stringed instruments scarcely less ancient, — the kinnor and hasur and psaltery of the Israelites, and the lyre and cithar of the Greeks, not to speak of many similar instruments employed by other nations of antiquity. According to the Greeks, the lyre was invented by Apollo, while the Hebrews tell us that a similar instrument was devised by Jubal. The Egyptians attribute the glory of a like invention to Mercury. "The Nile," says Apollodorus, "after having overflowed the whole country of Egypt, when it returned within its natural bounds left on the shore a great number of dead animals of various kinds. Among the rest was a tortoise, the flesh of which being dried and wasted by the sun, nothing was left within the shell but nerves and cartilages. These, being braced and contracted by dessication, were rendered sonorous. Mercury, in walking along the banks of the Nile, happening to strike his foot against the shell of this tortoise, was so pleased with the sound it produced that it suggested to him the first idea of the lyre. This he afterwards constructed in the form of a tortoise, and strung it with the sinews of dead animals."

No attempt was made to inquire into the scientific basis of music until the time of Pythagoras, the seventh century B. C. Of this distinguished philosopher and mathematician it is said that —

> "A stream
> Of song divine stole on his raptured ears,
> And round him burst the music of the spheres."

To illustrate his theory of musical harmony, as based on numbers, he invented the monochord, — an instrument that is still employed in many investigations regarding the nature and mysteries of the tonal art.

A modified form of the instrument devised by the Greek sage is before you. It is known as the differential sonometer of Marloye, who invented it, and it differs from the monochord in that it has two strings instead of one, and is available for a greater number of experiments. We shall have frequent occasion to use it during the course of this lecture, as with it can be illustrated all the leading laws of vibrating strings. As you will observe (Fig. 42), it is constructed of a long resonant case of fir, *M N*, on which are stretched two wires. One of the wires, *a d*, is stretched between two pins, by means of a piano key, *p*, the other, *b*, *R*, passes over a movable pulley, and is stretched by a weight, *P*, which can be varied at pleasure. Near their

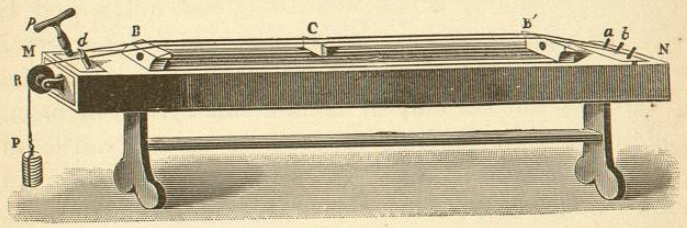

FIG. 42.

extremities these wires rest upon fixed bridges, *B* and *B'*. A movable bridge, *C*, rests under the wires, and permits a variation in the lengths of the vibrating parts. A scale divided into millimeters is fixed on the top of the box, and gives the length of the strings between the bridges.

I now pluck one of the strings and cause it to vibrate as a whole. It gives the lowest note it is capable of yielding with the tension to which it is at present subjected. The lowest note emitted by any sonorous body musicians call the *fundamental*, or the *prime*. By means of the movable bridge, the string is now divided into two exactly equal parts. Plucking either one of the halves, we get a note that the musicians present will recognize as the octave of the fundamental. That a string sounding an octave is only one half the length of a string emitting its fundamental, is one of the first discoveries made by Pythagoras with the monochord. By placing the movable bridge so that the string is divided into two parts whose lengths are as the numbers 2 : 3, he found that the sounds yielded are those that are separated by the interval known as a fifth. Thus, if the longer string gives the note C_3, the shorter one will give G_3. Continuing his experiments, he divided the string in such a way that the relative lengths of the two parts were as the numbers 3 : 4. Such strings he found gave the interval known as the fourth. As before, if the longer string were to sound C_3, the shorter would emit F_3.

Although the Greek philosopher made many other observations with his monochord and designed several other instruments, among which was his famous tetrachord, he does not appear to have discovered any other intervals used in modern music. If he did make such discoveries, he certainly did not employ them in his system of music. One of the most pleasing intervals in modern music was not introduced till the fifteenth or sixteenth century. It is known as the *major third.* It is obtained on the monochord by causing two strings to vibrate whose lengths are to each other as 4 : 5. In this instance if the longer string yields C_3, the shorter will give E_3. In lieu of this interval, Pythagoras employed one much more complicated. It is called, after its inventor, the *Pythagorean third.* The relative lengths of the vibrating strings in this interval are 64 : 81. True, the difference between this interval and our major third is small, — so small as scarcely to be recognized in ordinary music.

As a result of his investigations, Pythagoras discovered the law that *the simpler the ratio of the two parts into which the vibrating string is divided, the more perfect is the consonance of the two sounds.* But no explanation of this relation of simple whole numbers to musical harmony was given until the appearance of Helmholtz's great work, "Die Lehre von den Tonempfindungen," in 1859. The school of Pythagoras made the fact simply the basis of fantastic mathematical and philosophic speculations, the

most famous of which was their theory regarding "the harmony of the spheres."

Full two thousand years elapsed after the time of Pythagoras before any other notable advance was made in the science of musical harmony. The subject was then taken up by one of the ablest experimenters and most profound thinkers of modern times. I allude to the illustrious Franciscan friar, Père Mersenne, who has justly been called the "Father of Acoustics."

Taking up the investigations of Pythagoras, he found that the simple harmonic intervals above mentioned demanded not only that the lengths of the strings should bear a simple ratio to each other, but also that *the ratio of vibration of these strings should be equally simple.* Thus he found that the octave vibrated with twice the rapidity of its fundamental;[1] that the fifth vibrated three times, while its fundamental vibrated twice; and that the rate of vibration of the fourth to its fundamental was as 4 to 3. In a similar manner he discovered that the same law held good for all intervals whatever. In other words, he first laid down the all-important law that *the number of vibrations in any case is inversely proportional to the length of the string.* Moreover, he was the first to demonstrate the fact that *pitch depends on the rate of vibration; that the greater the number of vibrations per second, the higher the pitch.* Going still farther, he proved that this law regarding pitch applied not only to vibrating strings, but to all sonorous bodies whatever. This was indeed a gigantic step forward, and threw new light on the mystical numbers of the Pythagoreans.

Pythagoras had made some observations on the effects of tension on vibrating strings, but does not seem to have arrived at any definite results. Mersenne took up the problem and determined the law as it now stands; namely, *the number of vibrations per second of a string is proportional to the square root of its tension.*[2] This means that if a string stretched by a weight of one pound

[1] *Op. cit.*, lib. i. Prop. 15; and lib. ii. Prop. 6. [2] *Op. cit.*, lib. ii. Prop. 8.

gives forth a certain note, it will yield a note an octave higher if the weight be four pounds. By making the weight nine pounds, the string will execute three times the number of vibrations, and the note produced will be the fifth of the second octave above the note emitted by the string stretched by a weight of one pound. Similarly, a weight of sixteen pounds would cause the string to vibrate four times as fast, and the resultant note would be two octaves higher than the first. Thus, if the same string be successively under a tension of one, four, nine, and sixteen pounds, and the note given by the string with one pound be C_2, the note given by the string with the other weights will be C_3, G_3, and C_4. This means that if C_2 give 128 vibrations per second, C_3, G_3, and C_4 will give 256, 384, and 512 vibrations respectively.

Continuing his experiments with strings of different thicknesses, but of the same tension, Mersenne found that any given string must be twice as thick as another in order that the thicker string may yield a note an octave lower than that emitted by the thinner one.[1]

From this and similar observations is deduced the law that *the number of vibrations varies inversely as the thickness of the string*. Thus, if two strings of the same material, length, and tension have diameters which are to each other as 2 to 1, the thicker string will execute one half the number of vibrations of the thinner one. If one string be three or four times as thick as another, it will vibrate three or four times more slowly than the one of smaller diameter.

A fourth law, which the preceding seems, in a measure, to indicate, is that, the length, thickness, and tension being the same, *the number of vibrations of a string is inversely proportional to the square root of its density*. Thus, other things being equal, if two strings, A and B, whose densities are respectively as 1 : 4, be set in vibration, A will execute twice the number of vibrations made by B. If the ratio of the densities of the two strings be 1 : 9, the

[1] Lib. ii. Prop. 7.

lighter one will vibrate with three times the rapidity of the heavier one. The specific gravities of aluminum and copper are respectively 2.6 and 8.9, and hence their relative densities are as 1 : 3.46, nearly as 1 : 4. If, therefore, two wires, one of aluminum and one of copper, be caused to vibrate, the aluminum wire will vibrate with very nearly twice the rapidity of the copper one. Catgut and brass have specific gravities that are to each other approximately as 1 : 9. Hence, the relative frequencies of the notes yielded by two strings, one of catgut and one of brass, both being of the same length, diameter, and tension, will be as 3 : 1, — the catgut string vibrating three times as rapidly as the one of brass.

All the foregoing laws can be roughly illustrated by any stringed instrument. For their more exact verification an instrument like the sonometer is necessary. With such an apparatus, we can regulate the length and tension of the strings with the greatest ease and accuracy.

The laws of vibrating strings have been determined mathematically as well as experimentally. The first one to attempt a mathematical solution of the problem involved was the English mathematician, Brook Taylor, in 1715. His solution, however, was incomplete. Later the problem was attacked, in turn, by the ablest mathematicians in Europe. Among these were John and Daniel Bernouilli, D'Alembert, and Euler. The celebrated mathematician Lagrange eventually completed the work at which the others had so indefatigably labored.

But it was soon discovered that the results of theory and experiment did not agree. As early as 1736 Mersenne recognized the existence of this discrepancy.[1] Thus, when a string is divided into two parts, each part does not when set in motion give *exactly* the higher octave of the note emitted by it when vibrating as a whole. The higher note is *flat* by about a quarter of a tone. And the shorter the string, and the greater the diameter, the more pronounced the difference between theory and experiment.

[1] Lib. ii. Prop. 8.