When one string is twice the thickness of another, this discrepancy for the law of diameters may amount to as much as a half tone. N. and F. Savart attempted a solution of the difficulty; but although they employed every refinement in their experiments that ingenuity could suggest, and devised many special forms of apparatus in the course of their investigations, they were utterly unable to make the results of their experiments agree with the formula of the mathematician. Finally, it was pointed out that it was impossible for the experimenter to have such a string as the mathematician assumed in his calculations, — one, namely, that is *perfectly flexible*. All strings used by the experimenter are more or less *rigid*, and their rigidity, and this alone, supposing the strings to have the same diameter and homogeneity throughout, accounts for the differences observed between experiment and theory. When, however, the experiments are made with the requisite amount of care, these differences are ordinarily so small as to be scarcely recognizable.

This instance, and it is only one among many, well illustrates the difficulties, inherent in the nature of the materials at his disposal, that the man of science meets with in his investigations, and in his attempts to make the results of his experiments agree with those of calculation. Our conclusions, indeed, when based solely on experiments, are in many cases only approximately true at best, and it is impossible in the nature of things to make them other than approximate. In such cases as those under consideration our experimental results approach more nearly to truth just in proportion as they more nearly coincide with the demands of theory. If the experimenter could have at his disposal a perfectly flexible, uniform, and homogeneous string, he could without doubt make his observations conform with the formula of Lagrange, but not otherwise.

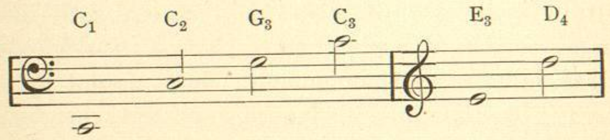
All the foregoing laws of vibrating strings are applied in the construction of the various forms of stringed instruments. Thus, in instruments like the harp and pianoforte,

the strings designed to give acute sounds are short, light, and thin. Those calculated to produce graver tones are proportionately longer, heavier, and thicker. To avoid increasing the thickness of a string or making it inconveniently long, it is sometimes loaded. In the violin, for example, three of the strings are of catgut of different thicknesses, and subject ordinarily to different tensions. The fourth string is rendered heavier by a spiral of silver wire. This device obviates rigidity, and at the same time renders the notes emitted as grave as may be required. Similarly, those strings of the harp and pianoforte that are designed to sound the lower notes are weighted by being wrapped with wire.

The tensile force with which the wires of a modern pianoforte are stretched is quite surprising. It varies all the way from one to five hundred pounds. The aggregate tension of a Broadwood instrument is equal to about eighteen tons weight, while the total stress of a Steinway Grand is fully twice that amount. It is found by experiment that the greater the tension and the longer the string, within certain limits, the richer and more harmonious the tones produced. This great increase of tension necessitates a heavy framework, and hence the massiveness of our modern pianofortes.

In the pianoforte there is a separate string, or group of strings, for each note. In the harp the notes are arranged to yield the notes of the diatonic scale. These, however, can be sharpened or flattened by means of pedals. In the banjo and guitar there are only a few strings, but as the length of these may be varied by pressing them against the frets with which such instruments are provided, a comparatively large number of notes may be elicited from them. The violin and violoncello have only four strings, and yet their compass is remarkably great. In such instruments the performer must be guided by his judgment and his ear and by practice as to the amount by which a string is to be shortened for the production of any given note. He has no mechanical aids like those afforded by the guitar and the banjo, whose frets serve as a guide as to how much a string is to be shortened for any determinate case.

Besides the note which a string of any given length emits when vibrating as a whole, and which is called its fundamental note, it also gives forth certain superior tones, which are sometimes called natural harmonics. Mersenne makes special mention of them in his great work.[1] He tells us that he was able to perceive tones corresponding not only to the first and second octave above the fundamental, but also the fifth of the second octave, the major third of the third octave, and the major second of the fourth. Supposing that the string gave as its fundamental the note C, the superior tones, or natural harmonics, heard along with their fundamental by Mersenne, would in musical notation be written as follows: —



We now know — what was unknown but probably suspected in the time of Mersenne — that a string emitting a musical note rarely, if ever, vibrates as a whole without at the same time vibrating in segments which are aliquot parts of the whole. The motion of these segments is usually superposed on that of the string vibrating as a whole. For this reason *harmonics* — a term introduced by Sauveur — are more properly called *upper partial tones*, or, considering the compound nature of the note composed of the fundamental and upper partial tones, as simply *partial tones*. In this case the fundamental would be the first partial of the compound tone, the octave the second, the fifth of the second octave the third, and so on. The fundamental is, of course, ordinarily more prominent than are any of the other partial tones. Nevertheless, in certain exceptional cases some of the upper

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[1] Harm., lib. ix. Prop. 33.

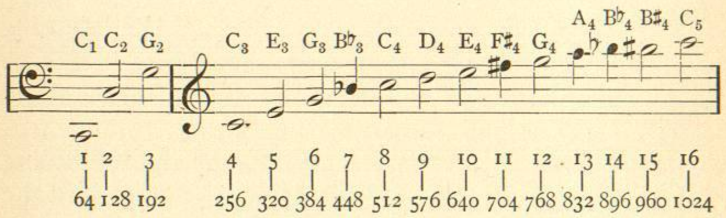
partials may sound louder than the prime. The pitch of the compound note heard is gauged by that of its fundamental; the quality of the tone is determined, as we shall see later on, by the number and relative intensity of the concomitant upper partials.

Mersenne's observations have been verified and explained by a number of subsequent investigators, chief among whom are D. Bernouilli, Riccati, Rameau, Sauveur, and Chladni. Rameau in 1722 attached so much importance to upper partial tones that he made them the basis of his system of musical harmony. Chladni gives a detailed explanation of them and shows that they are found in nearly all sonorous bodies, and that they are especially marked in organ-pipes, wind instruments, and bells.

Sauveur in 1701 appears to be the first to give a satisfactory explanation of the existence of upper partials. He attributes them to the string vibrating in parts, while at the same time vibrating as a whole. After showing how this can take place, he declares that "each half, each third, each fourth part of a string has its own special vibrations, while at the same time the string vibrates as a whole." And then, after enumerating the successive partials that accompany the fundamental note of a string, he observes: "It seems, therefore, that whenever Nature makes for herself, so to speak, a musical system, she employs only such sounds. Nevertheless, they have so far not been received in musical theory."

While speaking of Sauveur, I must not fail to mention what you will surely regard as a striking circumstance. He is justly regarded as one of the founders of the science of acoustics. He first applied the word *acoustics* to designate the science of sound. And yet he was mute until the age of five years, and remained almost deaf during his entire life. Hüber was blind when he carried on his wonderful investigations regarding the nature and habits of bees, which have made him one of the greatest authorities on the subject treated. Plateau, who was so distinguished for his wonderful discoveries in optics and molecular mechanics, did most of his work while deprived of sight. But even their achievements, astonishing as they are, seem to pale before those of Sauveur, who, although deprived almost entirely of the sense of hearing, was yet able to contribute more to the science of sound than any one of his age, and to detect the existence of tones that even cultivated musical ears did not recognize.

The number of upper partials that may accompany any given tone depends upon circumstances and upon the nature of the sonorous body itself. Sometimes only three or four may be detected; occasionally we may be able to demonstrate the existence of fifteen or twenty. They sometimes occur in the order of all the natural numbers, 1, 2, 3, 4, 5, etc.; at other times in the order of the odd numbers only. In the former case the first sixteen partials, beginning with C_1, as a fundamental, succeed each other as follows: —



The seventh, eleventh, thirteenth, and fourteenth partials are, as will be observed, indicated by crotchets, while the positions of the others are shown by minims. The former do not occupy exactly the position marked, as they have not the pitch of any note used in music. Their positions, therefore, can be indicated only approximately in the ordinary musical notation.

The second row of figures below the staff shows the frequencies of the corresponding partials. Inspection will show that the frequency of each partial is some multiple of that of the fundamental.

Only the lower partials, however, are usually considered in music. Ordinarily no importance is attached to those above the fifth or sixth. As a rule they diminish in

intensity as they ascend. Nevertheless, as we shall see, in some special cases, particularly in bars, bells, and tuning-forks, the higher upper partials may be so loud as to be unpleasantly sharp and piercing.

The first six partials, counting the fundamental as one, constitute in ordinary musical instruments a compound tone that is perfectly harmonious. To these may be added the eighth, tenth, twelfth, fifteenth, and sixteenth, without impairing in the least the harmony that characterizes the tone due to the combination of the six first. The seventh, eleventh, thirteenth, and fourteenth do not, as we have seen, belong to the musical scale. The eleventh and the thirteenth, together with the ninth, — D_4, — are discordant, and cannot be combined with the first-mentioned partials without marring the harmony which these latter yield alone. In an instrument tuned in pure intonation, — a harmonium, for instance, — the seventh and fourteenth partials, contrary to what musicians usually maintain, may be added to the six first partials, and give a compound tone of superior richness, brilliancy, and harmony.

The relative intensity of the various partials constituting a compound tone depends chiefly on the nature of the stroke, the point struck, and the rigidity, density, and elasticity of the string.

The same string will give a different sound according as it is struck or plucked or bowed. The harp and the guitar are plucked with the finger, and give a sound that is characterized by softness, richness, and the predominance of the lower partials. The zither and the mandolin are plucked with a point of wood or metal very much in the same manner as the ancient varieties of the harp were excited by the plectrum. The tones of these instruments are distinguished from those of the guitar and harp by the number and intensity of their upper partials; the sound is therefore shriller and more tinkling in character. The strings of the pianoforte are struck with soft elastic hammers of felt, and yield the pure, rich tones that contribute much to make this instrument so popular. In the best instruments, particularly when new, the first six partials predominate, to the exclusion, almost, of all higher ones. The strings of the violin family are bowed. This method of exciting vibration brings out a large number of partials, both high and low, and we have, in consequence, the sharp, full, brilliant tone of the "most perfect" of musical instruments.

The point struck or bowed always determines the presence or absence of a certain number of partials. In the pianoforte the string is struck in such a manner as to allow the formation of the first six partials, and to exclude or weaken those which are higher, — especially the seventh and the ninth, the latter of which is very discordant. To secure this result, the hammers are made to strike the string at from one seventh to one ninth — preferably one ninth — the distance from the end of the vibrating length of the string. The reason for this we shall see presently. Eliminating or weakening all partials above the sixth, there are left only such tones as enter into the formation of the major chord, because in the first six partials we have only octaves, fifths, and major thirds of the fundamental.

Again, the force and number of upper partials are greatly modified by the thickness and material of the string. Thick strings, by reason of their rigidity, do not permit the formation of very high partials, while very thin strings yield them quite readily and in great numbers. On a string of very fine iron wire Helmholtz was able to isolate the eighteenth partial tone. These high partials, however, form a series of very dissonant tones. The reason of this is because they lie so close to each other in the scale that the intervals formed are highly inharmonious. Above the eighth they are less than a whole tone apart, and above the fifteenth they are separated by an interval which is less than a semitone.

Every one has observed the difference in quality of tones emitted by metal and catgut strings. Other things being equal, a string of catgut, on account of its greater lightness, should produce higher partial tones than one of metal. But by reason of the inferior elasticity of the former, its higher partials are sooner damped than those of the latter. Hence the acute tinkling sounds that frequently characterize thin metal strings, as in the mandolin, and the comparative softness of the tones of strings of catgut, as in the harp or violin.

All the phenomena we have been discussing can be beautifully illustrated by the sonometer. But before going farther, we must examine more particularly the manner in which strings vibrate, and the way in which they subdivide so as to yield the partial tones we have been considering.

Mersenne had observed that when a string was set in vibration, a neighboring string in unison with it would also vibrate, although it might not have been touched. And he found this to be the case not only when the strings were in unison, but also when the second is an octave or a twelfth below the first. The same observation was afterwards made by Noble and Pigott at Oxford, and communicated by Wallis to the Royal Society in 1674. They showed that when the second string was two or three times the length of the first, it was divided into two or three equal vibrating segments, each segment being separated from the one adjacent by a point at rest, and each being of the same length as the vibrating portion of the first string. The existence of these points of rest was cleverly shown by placing paper riders along the string. Those on the vibrating segments were instantly thrown off, whereas those on the points of rest remained undisturbed. The tones excited by the first string in the second one are what are known as sympathetic tones, and we shall learn more of them later. What we are now more particularly interested in is the formation of the vibrating segments, and the points of rest discovered by Noble and Pigott.

In 1701 Sauveur, without any knowledge of the discoveries of the English investigators, made the same experi-