ment in a somewhat different way. Instead of using two strings he employed but one. Bowing a string so as to cause it to emit its fundamental, and to vibrate, therefore, as a whole, he touched it with a feather at points one half, one third, and one fourth its length from one of the extremities, and he then heard the octave, the twelfth, and the second octave of the fundamental. He made evident to the eye the subdivisions of the string by means of riders, some of which were white, others black. The former were placed at the points of rest, which Sauveur called *nodes*, and the latter on the vibrating segments which he named *ventres*.

All the laws and phenomena that we have been discussing can be clearly and accurately illustrated by the sonometer.

To repeat Sauveur's experiment exhibiting the nodes and ventres, I damp the middle of one of the strings by pressing a feather gently against it, and draw the bow across one of the halves. Immediately the other half is set in vibration, as is evidenced by the paper rider being at once thrown off. In this instance we have two *ventres*, or *ventral segments*, as we shall call them, and one node which is an immovable point in the centre of the string.

Damping the string at one third of its length from one of its extremities, and exciting it as before, we have formed three ventral segments separated by two nodal points. Placing red riders on the ventral segments, and white ones on the nodes, and bowing the string as before, the red riders are cast off, while the white ones remain undisturbed.

When the string is damped at one fourth of its length from one of the fixed bridges, and the shorter segment is set in vibration, the longer one is immediately divided into three equal parts, separated by two nodes. The whole string is now made up of four ventral segments separated from each other by three nodal points. As before, the red riders on the ventral segments are all rejected, while the white ones on the quiescent nodes retain their places.

In like manner the string might be divided into five, six, or more segments, separated from each other by nodal points, and the existence of both segments and nodes could be shown in the same way as before.

If the string is sufficiently tense, and we listen to the notes emitted by these successive subdivisions, we shall find that they are the upper partials of which we have been speaking. Thus, one half of the string yields the second partial, or the octave of the fundamental, one third of the string gives the twelfth, one fourth the second octave, while the fifth and sixth subdivisions give the major third and the fifth of the second octave. More minute subdivisions would of course give higher partials.

By simply striking the string with a pencil or a small metal bar, we can evoke all these partials in such a manner as to be distinctly audible to all who are near the instrument. In this case, however, we have not only the individual partials distinct and alone, but also their fundamentals, and a number of other tones due to various subdivisions of the string. Striking the string in succession at the centres of the ventral segments corresponding to the various partials, we elicit the corresponding notes with the greatest ease. You hear others, it is true, but the partials specially excited come out with greater purity and force than any of the others, except, it may be, the fundamental, which is always present with considerable power. If the string is struck sharply at points one half, one third, one fourth, one fifth, one sixth, and one seventh of its length from one of its extremities, and then in the inverse order at the same points, the notes referred to come out from the general mass of sound in a way that is quite surprising. With a little practice, one could thus play a simple melody on a single string, without changing its tension or its length.

A series of experiments made by Young in 1800 enables us to account for the absence or the prominence of the partial tones that we have been considering. He demonstrates that if a string be excited at its middle point, the octave and all the evenly numbered partials that have nodes at this point vanish from the compound tone that is emitted. This results in a note that is hollow and nasal in character. In like manner, when the string is excited at a distance of one third its length from either of its points of attachment, the third partial is quenched, as are also the sixth, ninth, and higher multiples of the third. The tone is still hollow, but less so than when the even partials were absent. The more nearly the point of excitation approaches the end of the string, the more pronounced become the higher partials, and the poorer and more tinkling the quality of the sound produced. In general, according to Young, there are always wanting in any given compound sound, all those upper partials that have their nodes at the point of excitement. According to the researches of Ellis and Hipkins, however, this principle enunciated by Young seems to require some modification, at least for the higher partials. They found that when a pianoforte string is struck by a soft or hard hammer at a node, the corresponding partial, especially if it be one of the higher ones,—the eighth, for instance,—is not necessarily extinguished. So far, no one seems to have offered a satisfactory explanation of this singular phenomenon.[1]

In speaking of the upper partial tones as existing in any compound sound, I have spoken of them as having frequencies related to the fundamental, as the natural numbers 2, 3, 4, 5, etc., are to 1. This is the general impression both among musicians and acousticians. It was because he believed they were so related that Sauveur gave to these partials the name they still bear, *harmonics*. But it is quite rarely that the upper partials bear such relations to their fundamental. Instead of having their frequencies related to that of the fundamental as the whole numbers 2, 3, 4, 5, etc., are to 1, upper partials have, in the majority of cases, frequencies whose ratios to that of the fundamental cannot be expressed in whole numbers. This is particularly true of bars, plates, and bells, as we

[1] Sensations of Tone, pp. 545, 546.

shall learn in our next lecture. When the succession of partials as to their frequencies differs from the order of the natural numbers, we have what are called *inharmonic partials*, as contradistinguished from *harmonic partials*, whose frequencies are to those of the fundamental exactly as 2, 3, 4, 5, etc., are to 1.

In organ-pipes, open or stopped, and in vibrating strings, although the upper partials are not so inharmonic as those formed in bars, plates, and bells, they are far from being perfect.

Wertheim found, during his researches, that the partial tones of pipes are higher than the theoretical harmonics. Koenig, in experimenting with a certain open organ-pipe, discovered that the eighth partial tone — he calls it a *sound of subdivision* — was nearly a whole major tone higher than the true harmonic, and that, consequently, it almost coincided with the theoretical ninth harmonic partial.

In the case of strings, the upper partials would correspond to true harmonic partials, if we could have a string uniform in thickness and homogeneous in texture, and entirely devoid of rigidity. But such an ideal string cannot be obtained. In the catgut strings of a violin the irregularities in form and density are so great that one often finds a difference of a semitone, or even of a whole tone, in the notes emitted by the halves of the same string.

The difference between harmonic and inharmonic partials cannot always be detected by the ear, especially when these differences are very small; but by means of the graphical method of registering vibrations, it can always be shown to have a real existence in fact. Koenig, in Fig. 43, gives a graphic trace furnished by a steel wire in which had been simultaneously excited the fundamental and its octave. These two tones were but very slightly separated from the true interval of an octave, but though the difference was but slight, it was indicated by the continually changing form of the successive waves. The upper partial, as disclosed by the trace, was sharper, by one wave in 180, than a perfect harmonic octave. The record is divided into five parts, in order to be more easily inscribed in the text. Had the two notes constituted the interval of a theoretic harmonic octave, the waves would have remained unchanged, and the wavelet due to the upper partial would have retained the same position on the larger wave denoting the fundamental, from one end of the record to the other.

By the graphical method just illustrated, we have the means of making clear to the eye the amount by which the interval in question deviates from a true interval, while it would be difficult, if not impossible, to appreciate such difference by the ear.

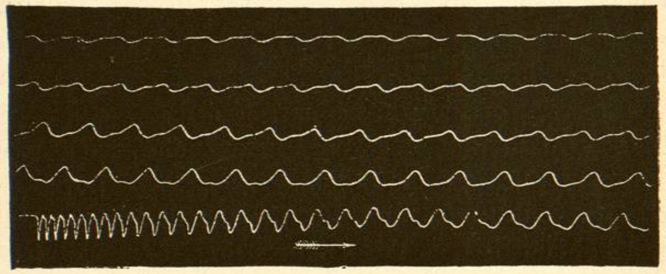


FIG. 43.

Free vibrating reeds, unlike strings and organ-pipes, may, and in most cases do, generate true harmonic partials. In the case of reeds, however, we have not such subdivisions as occur in strings and other sonorous bodies. Reeds vibrate as a whole, and their vibrations, so far as the most careful observation can determine, are perfectly simple and pendular. But strange as it may appear, these same simple pendular vibrations have the power of exciting the air in such wise as to generate compound tones. Such compound tones, as G. S. Ohm has demonstrated, the ear has the power of analyzing and resolving into a series of simple tones, each simple tone corresponding to a simple pendular vibration of the air transmitting the sound.

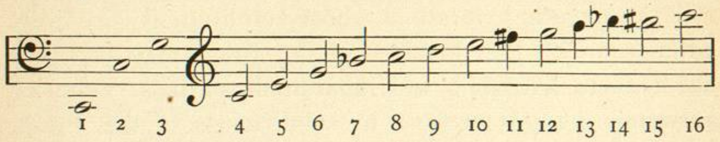
Some sonorous bodies — tuning-forks, for instance, which have long, thin prongs, and execute vibrations of great amplitude — may generate both harmonic and inharmonic partials. Koenig has recognized as many as five harmonic partials in tuning-forks of this kind, in addition to the usual inharmonic partials found in all forks. Long, thin strings, executing vibrations whose amplitude is very great compared with the thickness of the string, may also give rise both to harmonic and inharmonic partials. The inharmonic partials are due to subdivisions of the string, whence the name Koenig gives them of sounds of subdivision. The harmonic partials are constituents of a compound tone corresponding to a compound vibration made up of a certain number of pendular vibrations. The fact, then, that there is such a marked difference between these two kinds of tones, and the further fact that they frequently coexist in the same sonorous body and accompany the same fundamental, show clearly the necessity of carefully discriminating between them. The majority of sounds, then, employed in music have partials which, instead of being harmonic, are inharmonic. This is contrary to what is usually supposed and taught by those who are eminent both in the art and science of music, and by those, too, who are distinguished as teachers of the science of acoustics.[1]

And while speaking of this subject, it must be observed that harmonic partials, or harmonics, — as the term was understood by Sauveur, and as it is generally employed in acoustics, — are not identical with harmonics as ordinarily designated in music. In acoustics, the term *harmonics* is used to designate simple tones only, — tones, namely, that enter into the composition of a determinate compound tone. In music, on the contrary, harmonics are in nearly all cases compound tones. In the violin and harp, for example, the notes yielded when some of the aliquot parts of a string vibrate, are said to be harmonics of the notes emitted by the same string vibrating as a whole.

[1] Compare Koenig, Quelques Expériences d'Acoustique, pp. 218 *et seq.*

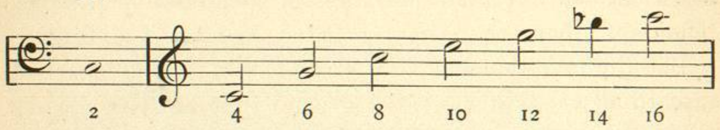
Let me illustrate. Suppose that under a certain tension the string on this sonometer gives the note C_1 as a fundamental. Its first sixteen partials, including its fundamental, will, as we have learned, be written in musical notation as follows: —

I.



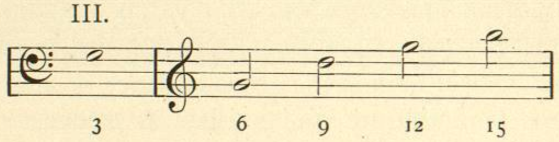
Taking one half the string, its partials, with fundamental, would be written thus: —

II.



Proceeding in the same manner with one third the string, we should obtain: —

III.



One fourth the string, for a similar reason, would give:

IV.

Inspection of I. and II. of the preceding diagrams will show that when only one half of a given string is set in vibration, its partials correspond only with the even partials yielded by the string vibrating as a whole. Thus, of the sixteen partials entering into the composition of the compound tone (tone emitted by the string vibrating as a whole), only those, as indicated, which in I. are numbered

2, 4, 6, 8, 10, 12, 14, 16, are found in II. When one third the string is caused to vibrate, only the odd partials of 1, as shown in III., constitute the components of the resulting compound tone. When one fourth of the string vibrates, we have, as seen in IV., only half of the evenly numbered partials of I., namely, 4, 8, 12, and 16. IV., as will be observed, bears the same relation to 2, as the latter bears to I. I. has sixteen partials, II. has eight, III. has five, and IV. has four. Each note in I., II., III., IV., answers, as you remember, to what we have called a partial tone; or, to distinguish it from an inharmonic partial, each note is an harmonic partial. And all the upper partials, that is, all the partials exclusive of the fundamental, are what Sauveur called *harmonics*, and what many acousticians still denominate *harmonics*.

But the musicians' harmonics are quite different. They discard all partials, and consider only fundamentals. Thus C_2, in II.. is the first harmonic of I., G_2 is the second harmonic, and C_3 the third harmonic. But as in these cases we cannot separate the partials from their fundamentals, the harmonics musicians actually refer to are compound tones, and not simple tones, as their language would seem to imply. Such being the case, the first harmonic of C_1 would not be simply C_2, but all the partials, as seen in II., which accompany this note. For a similar reason, the second harmonic would be G_2 and its four upper partials; and the fourth harmonic would be C_3 and its three upper partials. For the sake of simplicity, I have taken no account of any sounds that might be due to subdivisions of the string. These, as is obvious, would simply add other partials to the harmonics of which the musician speaks, and make them proportionally more complex.

To avoid all confusion, I shall therefore adhere to the terms *harmonic partials* and *inharmonic partials*, as already defined. This is important, as we shall avoid many errors and misconceptions that arise from confounding the two meanings so frequently given to the unqualified term *harmonic*.