## CHAPTER V.

### VIBRATION OF RODS, PLATES, AND BELLS.

IN our last lecture we considered the laws which govern, and the phenomena which characterize, the vibrations of strings. To-day we shall devote the time at our disposal to the discussion and illustration of the vibrations of rods, plates, and bells. And as the subject is so very comprehensive, we shall be obliged to confine ourselves chiefly to the examination of such matters as are connected directly or indirectly with the science of music, and of special interest, therefore, not only to those who are interested in the science of music, but also to those who desire information regarding certain mysterious points bearing on the art of music.

It is still a moot point as to which were the first forms of musical instruments used by our race. Stringed instruments, as we saw in our last lecture, were employed at a very early period. Playing on pipes probably antedated the use of stringed instruments. Nevertheless, if we may judge by analogy based on the customs of the barbarous nations of our own time, we should infer that instruments of percussion were first introduced; that these were followed by wind instruments; and that stringed instruments were the last in the order of time with every people, whether ancient or modern.

Pieces of bone and bars of wood and metal readily lend themselves to the production of musical notes, and for this reason the first musical instruments invented by prehistoric man were probably not unlike the various rude harmonicas which are in vogue in our own day in different parts of Asia and Africa. In some places pieces of bone



or rock are employed, in others pieces of wood or metal. In Java the principal music of the natives is produced by various forms of harmonicas and gongs. When the rods of such harmonicas are made of the outer silicious layers of the bamboo, and reinforced by resonators, as is frequently the case, the notes yielded are wonderfully full and pure. And what is true of Java applies in great measure to Siam, China, and Japan. Here instruments of percussion predominate over all other kinds. Tom-toms of all forms and sizes, cymbals, wooden clappers, bars and plates of wood or metal, and gongs of every shape or design, are the chief instruments that go to make up the ordinary orchestra of these semi-barbarous nations of the Orient. The deep booming thunder of their large drums, and the sharp rattle of their smaller ones, seem to possess a charm for Chinese and Japanese ears which makes them prefer instruments of percussion to either stringed or wind instruments.

The vibrations of rods, like those of strings, have been very carefully studied both mathematically and experimentally. D. Bernouilli in 1741 was the first to attempt a mathematical solution of the problem involved in the transverse vibrations of rods. Nevertheless it was reserved for the illustrious mathematician Euler to give the first satisfactory answer to the difficulty propounded. Later on, the problem was taken up and further developed by other mathematicians scarcely less eminent. Among those whose mathematical contributions to the subject are especially notable were Riccati, Poisson, Cauchy, Strehlke, Lissajous, and A. Seebeck.

The first one to attack the problem experimentally with any success was one who has immortalized himself by his experiments and researches in the domain of acoustics, — Ernst Florens Friedrich Chladni, a German physicist, whose work, "Die Akustik," published in 1802, is still justly regarded as a classic. We shall learn more of the character of his achievements when we come to study the nature of vibrating plates, to which, for many years, Chladni gave especial attention.

In studying the laws governing the transverse vibrations of rods, we must carefully distinguish the various ways in which such rods may be held or supported. There are six cases, all told, which may present themselves. Either one end may be fixed and the other free; or one end may be supported and the other fixed; or one end supported and the other free. Or, again, both extremities of the rod may be supported or fixed in a holder of some sort, or free. Of these six cases, however, we shall consider only the first and the last; namely, that of rods having one extremity free, and the other fastened to a support, or that of a rod having both extremities free. I choose these two cases, as they are the only ones that have been utilized in practical music.

The first case, then, that shall occupy our attention is that of a rod fixed at one end and free at the other. In a vice we have strongly clamped a rod like the one used in our first lecture (Fig. 1) to illustrate the nature of vibratory motion. I draw the free end of the rod from its position of equilibrium to the point *a*. On being liberated it oscillates about its former position of equilibrium, and executes a series of perfectly isochronous vibrations of gradually diminishing amplitude. The vibrations in this case, unlike those of strings, are not sustained by external tension applied to the rod, but by the elasticity of the material of the rod itself. The rod is now so long that the vibrations it executes may easily be counted. As they are only three or four per second, they are of course inaudible. But if the length of the vibrating portion of the rod is diminished, the number of oscillations is augmented. They are now sufficiently numerous to yield a distinctly audible sound. By means of the graphical method of registering the number of vibrations, or by means of the siren or a tuning-fork, we could determine exactly the number of oscillations the rod is now executing. Let us suppose that the number is 32, corresponding to the note $C_{-1}$. If, now, we diminish the length of the rod by one half, and again excite it, you observe that a note much higher in



pitch than the last is the result. If we were to determine the rate of vibration of the rod by any of the methods just mentioned, we should find that it is now making four times as many vibrations as it did before. The note yielded is consequently the second octave above $C_{-1}$, and corresponds to $C_2$, 128 vibrations per second. Making the rod one third as long as it was when it gave the note $C_{-1}$, the number of vibrations is rendered ninefold greater. The note now emitted is $D_3$, of 288 vibrations. If one fourth the rod were caused to vibrate, it would execute sixteen times as many vibrations as before; if one fifth entered into vibration, the number of oscillations would be twenty-five times greater than it was when it emitted the first note.

According to theory, the number of vibrations per second is inversely proportional to the square of the length of the vibrating part of the rod. Acting on the supposition that theory and experiment agreed in this case, Chladni constructed a tonometer made of bars, whose rates of vibration were determined as above. With this he hoped to be able to determine the rate of vibration of any sonorous body whatever. More exact investigations, however, have shown that the results given by experiment only approximate those demanded by theory. Chladni's tonometer, therefore, could not be relied upon when it was desirable to make anything like exact measurements.

In another vice near the one I have been using, there is fastened a strip of steel terminating in a disk at its free end, which beautifully illustrates the principle of Chladni's tonometer. It was made for me by Herr Appun, of Hanau, and is designed for determining the lowest audible number of vibrations. The vibration-numbers marked on the strip run from 4 to 24. But the vibrations corresponding to any given length of the strip were determined by means of some of the exact measurements above indicated, and not by the method proposed by Chladni. The results afforded by this little strip are as satisfactory

as the instrument is simple. For the purpose of determining the lowest limit of audible sounds it replaces admirably the more costly and complicated apparatus which were employed in the second lecture.

Small rods, fastened at one end and free at the other, are used in the construction of the so-called nail-fiddle, or *violon de fer*. Such an instrument is on the table before you. As you observe, it is composed of a number of rods of steel arranged in the form of a semicircle on a resonant case. Their lengths are so regulated, according to the law just enunciated, that when excited by a bow they give the notes of the gamut. The tones emitted are far from being disagreeable, and with a little practice one could make this homely little instrument yield fairly good music.

Music-boxes are constructed on the same principle. In them, however, the rods of the *violon de fer* are replaced by plates or tongues of steel. These are placed side by side like the teeth of a comb on a common base, and are of various lengths, according to the notes they are designed to produce. Those which yield the lowest notes are loaded with some extraneous material, so that they may thereby vibrate more slowly. A cylinder provided with teeth suitably arranged is kept in motion by clockwork. Each tooth raises one of the steel tongues and sets it in vibration. The air played will obviously depend on the manner in which the teeth are distributed on the cylinder's surface.

The reeds used in harmonicas, concertinas, mouth-harmonicas, accordions, organs, and other instruments of music, operate in essentially the same manner as the tongues of the music-box and the rods of the nail-fiddle. In all these instruments the vibrating element is free at one end and fixed at the other. They vary simply in length, thickness, and the manner in which they are set in vibration.

The *guimbarde*, or "jews-harp," instead of a reed or tongue, has a long spring, which is set in vibration by



striking its free end with the forefinger. The fundmental note it yields is modified by the various forms assumed by the cavity of the mouth; hence the peculiar variations of tone which characterize the instrument.

In none of the instruments thus far spoken of has any reference been made to any other notes than the fundamental. True, like the notes of most other sonorous bodies, the tones of the rods, tongues, and reeds spoken of are more or less compound tones; but in all cases it is the fundamental that determines the pitch of the note heard.

It is now time for us to turn our attention more directly to the consideration of the upper partial tones that rods and bars are competent to produce. Our first experience will show us that there is a very marked difference between these tones and those afforded by vibrating strings. In the case of strings, as you remember, the order of the partial tones was practically that of the natural numbers 1, 2, 3, 4, 5, 6, etc., and for this reason, as was stated, they are called harmonic partials. The upper partials of vibrating rods follow quite a different order, and have anything but a harmonious relation to their fundamental. Hence, as said before, they are termed inharmonic partials.

The frequency of a rod vibrating as a whole as compared with that of its first subdivision — that is, the frequency of its fundamental as compared with its first upper partial — is very nearly as the square of 2 is to the square of 5, or as 4 : 25. After the first subdivision of the rod, the rates of vibration, and consequently the frequencies of the notes produced, are approximately as the squares of the odd numbers 3, 5, 7, 9, 11, etc. For this reason the pitch of the upper partials in rods rises far more rapidly than does that of the partials of vibrating strings.

Supposing the rod vibrating as a whole to yield the note $C_{-1}$, Chladni gives for the first six partials, including the fundamental, the following series of notes, together with their relative rates of vibration and the order in which they occur: —

| | | | | | |
|---|---|---|---|---|---|
| $C_{-1}$ | $G\sharp_2$ | $D_4$ | $D_{5-}$ | $B\flat_5$ | $F_{6+}$ |
| 1 | $6\frac{1}{4}$ | $17\frac{1}{2}$ | $34\frac{1}{4}$ | $56\frac{1}{2}$ | 84 |
| $(1.2)^2$ | $3^2$ | $5^2$ | $7^2$ | $9^2$ | $11^2$ |

According to theory, partial tones, commencing with the third, succeed each other exactly in the order of the squares of the odd numbers, the relative frequency of the fundamental being $(1.194)^2$, and that of the second being $(2.989)^2$.

The minus sign after $D_5$, and the plus sign after $F_6$, indicate that the number of vibrations in these two cases does not correspond exactly with any fixed musical notes. In the former case the number of vibrations is less than $C_5$, and in the latter greater than $F_6$. A little arithmetical computation will show that the theoretic and observed values above given are by no means identical. In some instances, indeed, they differ by quite an appreciable quantity. But this should not surprise us, as we have found in other similar instances how difficult it is to get the results of experiment to coincide with those required by theory.

Chladni viewed the tuning-fork as vibrating like an ordinary bar free at both ends. The only difference between the two, in his estimation, was that the former was bent, the latter straight. But the law of succession of the upper partial tones and the absolute number of vibrations of the fundamental of a tuning-fork show that its mode of vibration resembles rather that of a rod fixed at one end and free at the other. When the fork vibrates so as to emit its fundamental tone, it forms two nodes, one at the base of each branch, as shown in Fig. 51. Each of these nodes represents exactly the point of attachment of a rod fixed at one end and free at the other. The part of the fork intermediate between the two nodes, to which the stem is attached, vibrates in unison with the two branches, and when fixed to a resonant case sets it in vibration also. When a fork emits its first upper partial, whose frequency, as in a rod fixed at one end, is $6\frac{1}{4}$ times that of the fundamental, it has four nodes, as shown in the second diagram



of the subjoined figure. But when the fork vibrates so as to yield its third partial, it possesses six nodes, and the frequency of the partial, as indicated in the figure exhibiting its mode of division, is $17\frac{1}{2}$ times that of its fundamental. The nodes and ventral segments in all the three cases illustrated in the figures are precisely the same as those of simple vibrating rods yielding the same partials. The upper partials of a tuning-fork, then, succeed each other according to the law which governs the same notes in the case of a simple fixed rod, and the frequencies of these upper partials, in both cases, have the same ratios to their primes.

It would nevertheless be a mistake to infer from what has been said that the upper partials of tuning-forks have

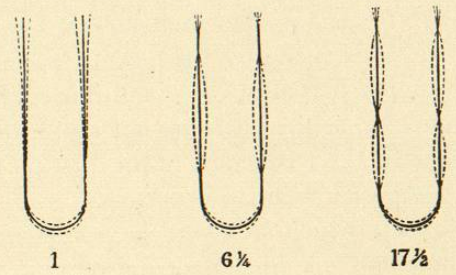


FIG. 51.

always the same pitch as compared with their fundamentals. Such is by no means the case. In a number of forks examined by Helmholtz, the first inharmonic partial executed between 5.8 and 6.6 as many vibrations in a given time as the fundamental. The number will vary slightly, according to the form of the fork and the material of which it is made.

Before you are three tuning-forks mounted on resonant cases. The largest one, which we shall here regard as the fundamental, is $C_2$, executing 128 vibrations per second; the second one is $G_4$, the sixth partial of $C_2$, making 768 vibrations per second; and the third one corresponds to the seventh partial, and executes 896 vibrations per second.

The larger fork is now struck so as to elicit its first upper partial. You hear it loud and clear. The second fork, $G_4$, is next excited, so as to yield its prime tone, and you will perceive that its pitch is very nearly that of the upper partial of the large fork. But there is a difference of several vibrations, as is disclosed by the beats that are heard. The prime tone of the third fork is compared in a similar manner with the upper partial of the first fork, and, as before, we get very distinct beats. But now they are much more numerous than with the other fork. This shows that the first upper partial of the large fork more nearly approaches the note emitted by the fork $G_4$ than that yielded by the fork giving the seventh harmonic partial of $C_2$. In other words, the frequency of the first upper partial of the large fork is more nearly six than seven times the frequency of its fundamental. By counting the number of beats made by the fork $G_4$ when sounding with the first upper partial of $C_2$, — and this could be done with little difficulty, as the beats are not rapid, — we could determine exactly the frequency of the first partial of the fork $C_2$ as compared with its prime.

Again, two forks which are identical in appearance, and whose fundamentals are in perfect unison, may, and generally do, give rise to beats when the same upper partials are educed. Here are two forks, each making exactly 512 vibrations per second, and therefore in perfect unison. If we excite the forks in such a way as to bring out their first upper partials, beats are at once heard, due to a want of unison on the part of these upper partials. The second set of upper partials might be excited in a similar manner, and the results would be the same.

Ordinarily when a tuning-fork is set in vibration, one hears in addition to the prime tone, one or more of its upper partials. But these are in most instances very evanescent as compared with the fundamental. If, however, the fork is excited at a point near the centre of the ventral segment corresponding to the first upper partial, this partial will be generated with exceeding purity and in-



tensity. In like manner the second partial may be brought out so as almost to quench all other tones. By means of the graphical method it is easy to show the co-existence

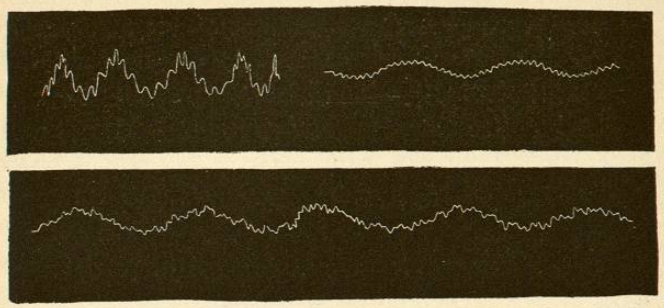

FIG. 52.

of these partial tones of a tuning-fork. In Fig. 52 we have traces corresponding to the fundamental and its first upper partial, as also to the fundamental and its first two upper partials.

The second case in which bars and rods are used in music is where they are free at both extremities. Fig. 53 shows how such bars may be supported. As will be noticed, the bar $CD$ rests on two triangular pieces, $A$ and $B$, which are ordinarily of wood or cork. The simplest division of the bar, corresponding to its gravest note, — its fundamental, — is here

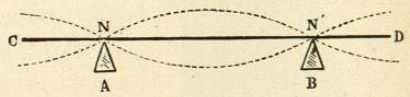


FIG. 53.

represented. The two nodes, $N$ and $N'$, are situated at the points of contact of the two supports. Dotted lines indicate the ventral segments of the bar when in a state of vibration. As will be observed, the bar, when vibrating, divides itself into three segments of unequal lengths, the two extremities being a little less than one half as long as the middle segment, or 2 : 5. The two ends vibrate about the nodal points, $N$ and $N'$. The intervening portion executes a movement of totality, as would a cord if attached at the points $N$ and $N'$. Besides the one indicated, the

bar can also assume other subdivisions, to each of which will correspond a higher partial tone. The number of nodes in such a vibrating bar, beginning with the fundamental note, are in the order of the numbers 2, 3, 4, 5, 6, 7, etc. Chladni was the first to determine the musical relation of the partial tones corresponding to the different modes of subdivision of a bar or rod. The following table gives the result of his investigations: —

| | | | | | | |
|---|---|---|---|---|---|---|
| Number of nodes, | 2 | 3 | 4 | 5 | 6 | 7 |
| Frequencies of the notes emitted — corresponding approximately to the squares of the odd numbers . . . . . | $(3)^2$ | $(5)^2$ | $(7)^2$ | $(9)^9$ | $(11)^2$ | $(13)^2$ |

The laws of vibration of a rod free at both ends, and of one free at one end and fixed at the other, are identical for all the partial tones except the fundamental. The prime of a rod free at both ends is higher than that of the same rod fixed at one end, in the ratio of 25 : 4.

Rods or bars free at both ends are used in the construction of an instrument called by the French a *claque-bois*. It is also known as a xylophone. Such an instrument (Fig. 54) is before you. It is, as you see, composed of a series of bars, *a b* to *a′ b′*, of wood of different lengths and thicknesses, and so tuned as to yield the notes of the gamut. The bars are held together by two cords, *c d* and *c′ d′*, passing through their two nodes. The notes which the bars are capable of yielding are educed by striking them with a small hammer.

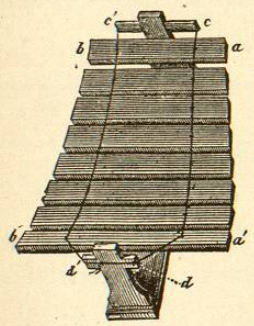

FIG. 54.

Near by is a larger and more elaborate instrument, made of pieces of harder and more resonant wood. In this case the billets of wood are supported at their nodes on ropes made of straw, whence the name "straw-fiddle," which the instrument sometimes bears. Metal and glass rods and



strips, and even bars of slate or other compact varieties of rock, are occasionally substituted for bars of wood. On the table are two instruments, in which the sonorous bars are made of steel. They are known as metallophones, and are sometimes employed to give brilliancy and color to band and orchestral music. When pieces of glass or rock are used as the vibrating material, the instruments are called glass or rock harmonicas.

The xylophone is apparently becoming more popular daily. It is frequently employed in orchestras for short solos with pleasing effect. Mozart introduced it into his opera of "Die Zauberflöte" to imitate the sound of bells. The metallophone, unlike the claque-bois, on account of the intensity and piercing character of its upper partials, could never be used alone; but when it is used with other instruments, these penetrating tones are so far quenched as to be no longer disagreeable, and the fundamental note, which has a bright, clear, bell-like tone, often contributes materially to the beauty and richness of the general mass of sound.

The partial tones of rods and bars are, as we have learned, of the kind denominated inharmonic. They do not all, by any means, form discordant intervals, but, unlike true harmonic intervals, their rates of vibration do not rise in the order of the natural series of whole numbers. Many of the intervals, it is true, are eminently discordant, and hence the unfitness of rods and bars for use in musical instruments, especially when played alone. But it must not be forgotten that tuning-forks, although vibrating as a bar fixed at one end, yield, as has been before stated, not only inharmonic partials, but also harmonic ones. So far, tuning-forks have never been used as musical instruments, although they may, as you know, be made to emit tones of exceeding purity and volume.

Let us now pass from the transverse to the longitudinal vibrations of a rod. An apparatus devised by Koenig (Fig. 55) enables us to demonstrate in a most striking manner the existence of longitudinal vibrations. A rod

of brass mounted on a support is clamped at its middle point, and from the support an ivory ball is so suspended as just to touch the end of the rod. I now set in vibration the half of the rod farthest away from the ball, by rubbing it with a piece of resined leather. The point at the clamp is a node; but the vibrations imparted at the half of the rod which is being rubbed are at once communicated to the other end, as is evidenced by the tremulous motion of the ball. Rubbing the rod more vigorously, the vibrations become so intense that the ball is repelled violently whenever it touches the end of the rod.

By being clamped, the middle point of the rod is made a node. Here all the molecules are at rest. At the ex-

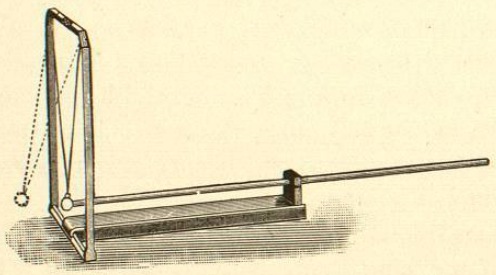

FIG. 55.

tremities of the rod, on the contrary, the molecules have great amplitude of motion, as is attested by the experiment just made. By means of a spherometer, Savart measured the amount of elongation of a rod of brass, about an inch and a half in diameter and four feet long, under the influence of longitudinal vibrations. The strain, he found, was equivalent to that of a tensile force of over eighteen tons. The relatively feeble impulses thus communicated to the molecules of the rod may thus develop an enormous force. This is explained by the cumulative character of the motions imparted. A number of feeble impulses, properly timed, may, therefore, produce effects that a much superior force applied once could not effect. The elongations due to this vibratory motion frequently become so great as to cause the rupture of the strongest



materials. Engineers and architects must take this fact into consideration in calculating the strength of materials. The cables of bridges are sometimes snapped by the longitudinal vibrations produced by the measured tread of soldiers crossing them. An accident of this kind befell a regiment of soldiers while crossing a bridge in France some years ago.

FIG. 56.

We owe to Savart an experiment which illustrates in a most striking manner the nature and intensity of the force developed by longitudinal vibrations. By clasping a glass tube with one hand, and rubbing it with a wetted cloth held in the other (Fig. 56), it is possible to develop such amplitude of motion in the molecules of the tube as to shatter its lower portion into fragments. The forms of the fragments are, as might be inferred from the character of the vibrations producing them, always annular, and the line of fracture is at right angles to the axis of the tube.

If a rod, *a b*, is held at its middle point, *B*, as in Fig. 57, and caused to vibrate by rubbing, as at *A*, one of its halves, it will emit its fundamental note. The rod in this case has a node at its centre, while the points of maximum vibration are at its extremities. If the same

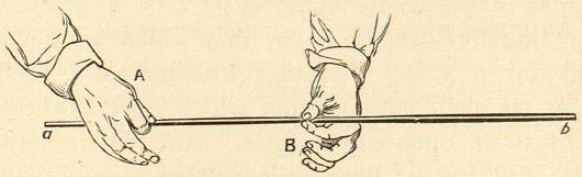


FIG. 57

rod be held at *N*, I (Fig. 58), at a fourth of its length from the end *A*, and if the part *A N* be then excited, a node is spontaneously formed at *N'*, at a point such that $N' B = A N$. $N' N$, therefore, equals $A N$, and $N' B$ equals $\frac{A B}{2}$. The rod is thus divided into one whole ven-