tral segment, $N'N$, and into two half ones, AN, NB. The note emitted is now an octave higher than it was before, and the number of vibrations is double what it was when there was only one node. The arrows in the figure indicate the direction of motion of the direct and reflected pulses.

If the rod be now fixed at N, II, of the adjoining figure, and caused to vibrate as before, it at once forms two other nodes, one at N and one at N'', — points so situated that $N''B = AN$, and that $N''N' = N''N = AN + N''B = \frac{AB}{3}$. These four divisions vibrate in unison, and constitute three complete ventral segments of equal lengths. The number of vibrations executed in this case is three times that corresponding to the fundamental when the rod is fixed at the centre. If the rod be divided into four complete ventral segments, it will produce a note whose pitch is four times that of the fundamental, and so on for higher subdivisions. Hence the notes emitted by a rod vary directly as the number of complete ventral segments, and inversely as the length of these segments. The frequencies of the notes yielded follow each other in the order of the harmonic partials and according to the series of the whole numbers 1, 2, 3, 4, 5, etc. The law is, therefore, the same as that which we have seen obtains for a string vibrating longitudinally, and the same, as we shall learn, as that which governs the vibrations of air in open organ-pipes. Another similarity between a rod free at both ends and an open organ-pipe is that in both cases the nodes occupy the same relative positions.

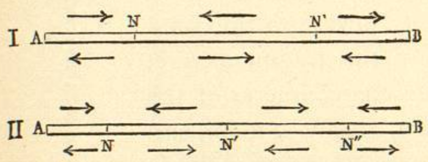

Fig. 58.

When the rod just used is fixed at one end and free at the other, the number of vibrations that it will execute in a given time is different, as is also the order of occurrence of the upper partials which may be produced.

Suppose the rod AB (Fig. 59), fixed at A, and free at B. When vibrating in its simplest way, so as to yield its prime tone, there is necessarily a node at A, and the centre of a ventral segment at B. I say the centre of a ventral segment because the rod, when vibrating so as to emit its fundamental note, is only a half ventral segment in length. Such rods, like those which are free at both ends, execute vibrations whose frequencies are inversely proportional to their lengths. A rod fixed at one end, and yielding its fundamental note, is different in length from a rod of the same length and material when free at both ends and emitting its prime tone. The note yielded by the former is an octave lower than that produced by the latter. In order that the notes may be in unison, the rod free at both ends should have twice the length of the one free at only onè extremity.

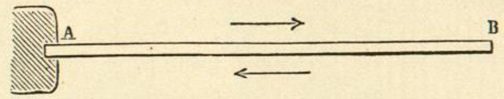

Fig. 59.

Like rods free at both ends, those fixed at one end admit of subdivisions into segments while under the influence of vibratory motion. These divisions must always take place in such a manner that the fixed point is a node, and the free extremity the centre of a ventral segment.

When but one node is formed in the rod, it exists at N (Fig. 60), I, and divides the rod into two vibrating parts such that NB is one half AN, and one third AB. We have in this case a half ventral segment, NB, and a complete one, AN. The number of vibrations corresponding to the note emitted in this case is three times that executed by the rod when emitting its prime.

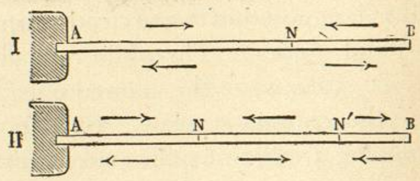

Fig. 60.

When two nodes are formed in the rod II of Fig. 60, the vibrating part $N' B$ is one fifth the length of $A B$. The rod is now divided into three vibrating parts, one half-ventral segment, $N' B$, and two whole ones, $A N$ and $N' N'$. The number of vibrations now executed is five times as great as when the rod sounds its fundamental.

From the foregoing it will be seen that the order of the notes developed in a rod fixed at one end is that of the unevenly numbered harmonic partials, — that is, they succeed each other as the odd numbers 1, 3, 5, 7, etc. The same law, as we shall learn later on, applies in the case of notes yielded by a stopped organ-pipe. Furthermore, rods fixed at one end and stopped organ-pipes have their nodes in the same relative position. The only instrument in music based on the longitudinal vibrations of bars is one devised by Marloye. Such an instrument (Fig. 61) is before you. It is, indeed, more of an acoustical curiosity than anything else. It is composed of twenty rods, firmly fixed at one end on a solid support. The white rods yield the notes of the diatonic scale, while the colored ones answer to the semitones of the chromatic scale. By rubbing them with resined fingers, a series of quite pure, sweet tones may be educed, and a simple melody might be played on them which the ear would find quite agreeable. Substituting rods of glass for those of wood, the smoothness and volume of the tones elicited would be considerably enhanced.

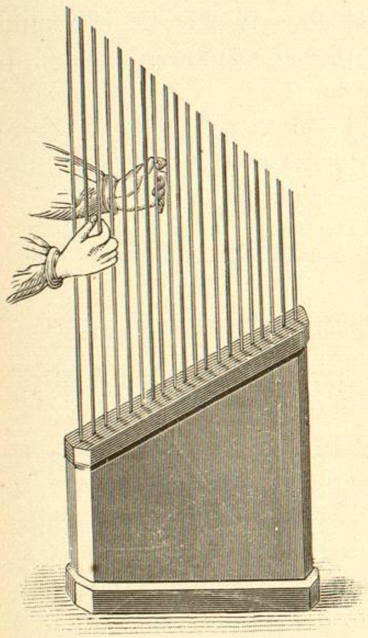

FIG. 61.

In elastic rods the number of longitudinal vibrations varies, as we have seen, inversely as the length of the rods, or the vibrating segments. The diameter and form of their transverse section have no effect on the number of vibrations executed by rods of the same length and material, provided their length is very great in comparison with their width and thickness. This is easily shown by experiment.

On a suitable support fixed to the table are clamped two steel rods (Fig. 62), each being one metre in length.

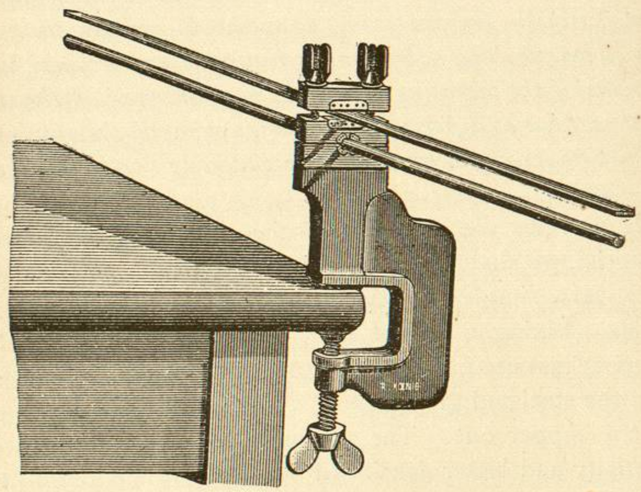

FIG. 62.

The lower one is cylindrical, the upper one prismatic. Passing a resined piece of leather in succession over the two, the same notes, as you hear, are elicited from both. I now replace the prismatic rod by a cylindrical one of greater diameter than that just used. We have now two cylindrical rods of quite different diameters, and yet, when they are thrown into vibration by rubbing them, they both emit the same note. Substituting a rod of one half metre in length for either one of those now clamped to the support, we have two rods, one of which is just twice the length of the other. Passing the resined leather over both of them, we find, as we have already learned should be the case, that the shorter rod yields a note which is just an octave higher than that produced by the longer one. Taking in place of the rod one half metre in length another one measuring one third of a metre, and causing it to vibrate with the rod below it, which is three times its length, we obtain notes whose frequencies are as 3 : 1. The short rod, as was to be expected, emits a note which is exactly a twelfth above that sounded by the longer one.

These experiments beautifully corroborate the results already obtained by causing rods to vibrate in segments and verify the law previously enunciated; namely, *the number of longitudinal vibrations is inversely proportional to the lengths of the vibrating segments, or, when rods of the same material but of different lengths are employed, the number of vibrations executed per second is inversely as the lengths of the rods.*

If now we use rods not of the same, but of different material, we shall find ourselves in a position to determine in a very simple way the velocity of sound in different solids. Fixing a rod of steel and one of copper in the support just used, and causing them to vibrate, you notice that the steel rod gives a more acute sound than that given by the copper one. The reason is found in the superior elasticity and lesser density of the steel, which permit the sound-pulse to travel through it more rapidly than it does in copper. If, instead of having rods of equal lengths as we now have, we were to use a steel rod seventeen inches long, and a copper one eleven inches long, we should, on causing them to vibrate, obtain notes that have approximately the same pitch. But the lengths of the rods employed are to each other very nearly as the velocities of sound in the two metals. The velocity of sound in steel and in copper is, in round numbers, 17,000 and 11,000 feet respectively. By simply making the rods of different materials of such lengths that they will yield the same note, we at once have an approximation to the relative velocities of sound in these materials, and knowing the

velocity of sound in air, we can easily determine their absolute velocities.

Instead of steel and copper, let us take oak and fir. Cutting the rods to such a length that they both emit the same note, we find that the lengths are twenty-five inches for the oak, and thirty inches for the fir rod. But the ratio of the lengths of these rods, 25 : 30, = 12.5 : 15, is very nearly that of the relative velocities of sound in oak and fir. In the former the velocity of sound is a little more than 12,500 feet, and in the latter it is slightly in excess of 16,000 feet per second.

This method of determining the velocity of sound in solids was first suggested and applied by Chladni. The results he obtained for various substances correspond very closely with those arrived at by more refined methods of measurement. Its simplicity certainly commends it to the investigator who desires only approximate values.[1] It is applicable to all solids which can be fashioned into rods capable of executing longitudinal vibrations competent to yield a definite musical tone. Measuring the length of the sonorous rod, and estimating its pitch, both of which are exceedingly easy, are all that is required to enable one to calculate with a fair degree of approximation the velocity of sound in any given material.

A beautiful experiment, due to Biot, enables us to investigate, better than any other means at our disposal, the conditions of the molecules in various parts of a bar or rod when in a state of longitudinal vibration. It has been stated that the particles constituting the nodes of any vibrating body are quiescent, while those which compose the ventral segments are always in a condition of greater or less vibratory motion. In a rod free at both ends and emitting its prime tone, there is, as we have learned, but one node, which is at the centre, while on either side of the node there is a semi-ventral segment. In this case the

[1] According to recent investigations by Prof. A. M. Mayer, as yet unpublished, Chladni's method of determining the velocity of sound in solids is capable of giving more exact results than any other known method.

molecules that have the greatest amplitude of motion are at the extremities of the rod. At the node there can be no motion, because here the opposite sonorous pulses meet. There are, however, alternations of strain and pressure, and hence alternations in density. While, therefore, the node is characterized by absence of movement, and by variations of density due to pulses of condensation and rarefaction, which alternately meet at this point, the ends of the rod, corresponding to centres of ventral segments, are distinguished for great amplitude of movement, while the density remains always the same.

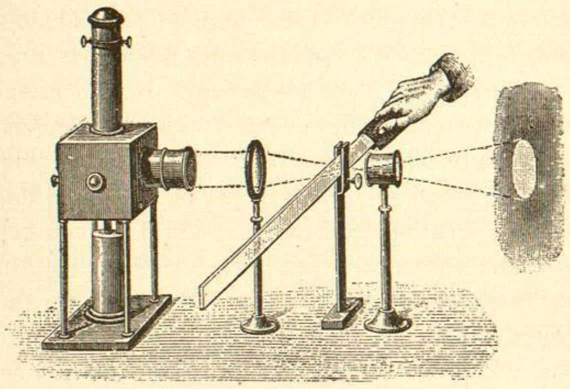

FIG. 63.

To the front of our lantern (Fig. 63) is attached a Nicol prism that gives a beam of polarized light. If now a second Nicol be placed in front of the first, in such a manner that the directions of vibrations in both are parallel, the beam will pass through the second prism also, as is evidenced by the luminous disk on the screen. But if the two prisms are so adjusted that their directions of vibration are at right angles to each other, the polarized beam from the first Nicol will not pass through the second, although both prisms are perfectly transparent. The light is quenched almost as completely as if it had been intercepted by a body perfectly opaque.

Light thus polarized is remarkable for its behavior with

respect to bodies in a condition of strain or compression. I take a narrow strip of plate glass and introduce it between the two prisms so that it is inclined to their direction of vibration. There is as yet no change on the screen. As soon, however, as the glass strip is bent, there is induced a condition of pressure on the concave and one of strain on the convex surface. The moment such change is effected, the light flashes out on the screen, If a similar condition of stress be caused by other means, by heat, for instance, or by sound-pulses excited in the molecules of the glass, a like result is obtained.

Adjusting the strip of glass in a vice in such a manner that the ray of polarized light can pass through its node, and sweeping over one of its halves a moist woollen rag, there is heard an acute note due to the longitudinal vibrations of the glass. Simultaneously with the production of the sound a brilliantly illuminated disk flashes out upon the screen. When the vibrations cease, the light is extinguished. But each time the cloth is passed over the glass the luminous disk is restored. Here, as is evidenced by the flashes of light on the screen, we induce changes of density — alternate states of condensation and rarefaction — in the node of the glass strip, precisely like those developed by heat or mechanical stress of any kind.

If now the glass strip is so placed that the beam of polarized light passes through it near either of its extremities, and it is thrown into vibration as before, no effect whatever is produced. The reason is that at these points of the glass bar there is no variation of density, due to alternations of strain and pressure, although the width of swing, or amplitude of movement, of the oscillating molecules is here at its maximum.

Like strings, rods may also execute torsional vibrations. If a rod be clamped at one end in a vice, and a violin bow be drawn around it, it will be caused to twist and untwist itself around its axis so as to execute vibrations that are as isochronous as transverse or longitudinal vibrations. According to Chladni, the pitch of a note due to the torsional vibration of a rod is about one fifth lower than that of a note produced by the longitudinal vibrations of the same rod having the same number of segmental divisions.

Like strings, rods may also execute very complex vibrations, in which transverse or torsional vibrations, or both, are compounded with longitudinal vibrations.

Savart was the first to elicit simultaneously from the same rod two notes, one of which is due to transverse, and the other to longitudinal vibrations. Since his time Terquem[1] and Koenig have studied these joint vibrations more closely, and, thanks to their investigations, we now know not only the laws which govern such compound vibrations, but also under what circumstances they may most easily be produced.

Clamping this steel rod, one metre long, in the support which we have just been using, I rub one of its halves vigorously with a piece of resinous leather. The rod is thrown into longitudinal vibration as in the preceding experiment, and a loud, clear note is the result. But in addition to the fundamental tone of the rod, you hear another note equally pure, and almost equally loud, which is exactly an octave lower. This is due to the transverse vibrations, which are developed simultaneously with and by those which are longitudinal. Such a grave tone is called by the French *son rauque*, — a raucous sound, — and, as Terquem has shown, is produced only when the rod is of such a length that the note it emits when vibrating transversely is sensibly identical with a note that is an octave lower than that yielded when the rod vibrates longitudinally. Koenig has further found that the first upper harmonic partial due to longitudinal vibrations may, like the prime tone, excite transverse vibrations that will yield a note an octave lower than such partial. The vibrations thus developed in rods are, therefore, quite analogous to those which we have witnessed in Melde's experiments, in which a tuning-fork vibrating in the direc-

[1] See his "Étude de Vibrations longitudinales des Verges prismatiques libres aux deux Extrémités."

tion of the length of a string causes the string to execute transverse vibrations whose number in a given time is just one half that executed by the fork itself.

We are now prepared to pass to the vibrations of plates. They are far more complex than those of rods, but at the same time they are, by reason of the figures to which they may give rise, far more interesting. Chladni was the first to study experimentally the modes of subdivision of plates when under the influence of vibratory motion, and to him and F. Savart we owe most of our knowledge concerning the experimental part of this subject. Napoleon Bonaparte,[1] who had witnessed some of the experiments of the German philosopher, was so impressed by them that he had the French Institute offer a prize to the one who would offer a satisfactory theory of the phenomena observed. A lady mathematician, Mademoiselle Sophie Germain, gave a solution of the problem involved, for which she was especially honored by the Academy. Subsequently the theory of vibrating plates was discussed by the ablest mathematicians in Europe. Chief among these were Lagrange, Poisson, Cauchy, and Kirchhoff. And yet, notwithstanding the great work accomplished by these eminent analysts, much yet remains to be learned regarding the mode of vibration of plates, especially square plates whose edges are free. In the case of circular plates, theory and experiment are more concordant. The vibratory motions of such plates have been analyzed so thoroughly that the mathematician can now determine in almost any given case the number and kind of nodal lines, and calculate with the greatest exactness the series of sounds that will be produced.

By means of the vertical lantern and suitable plates, I shall now give some illustrations of the character of this vibratory motion. A square glass plate is clamped above the condensing lens of the lantern, and then strewn with

[1] On the dedicatory page of the French edition, "Traité d'Acoustique," of Chladni's great work is written, "Napoléon le Grand a daigné agréer la dédicace de cet ouvrage après en avoir vu les expériences fondamentales."

fine sand. The image of plate and sand is now distinctly focused on the screen. Placing my finger at the middle point of one of the edges of the plate, so as to form a node there, and drawing the bow along the edge near one corner, the sand immediately begins to dance about on the plate, and arrange itself along two nodal lines, which are at right angles to each other, parallel to the sides of the plate, and intersecting each other in the

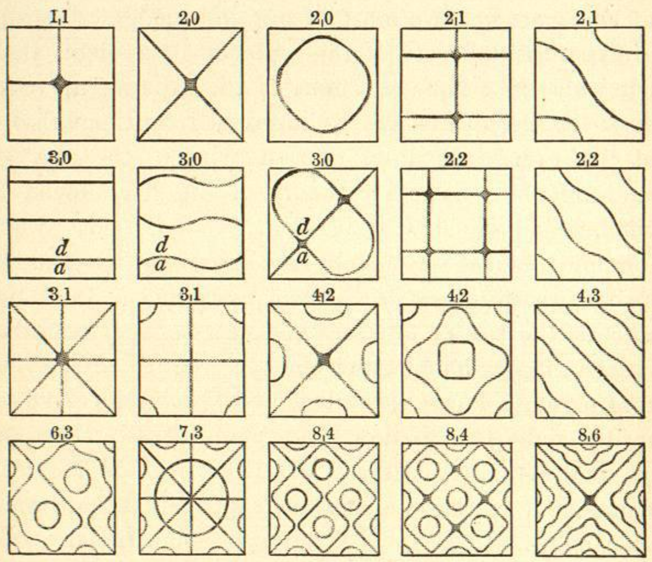

FIG. 64.

centre. These lines, in fact, constitute a cross, 1,1 (Fig. 64), dividing the plate into four equal rectangular segments. Placing my finger at the corner of the plate, and drawing the bow across the middle point of the edge, two nodal lines are formed as before, but their position is different, being along the diagonals of the plate as in 2,0 of the adjoining figure. Clamping the plate about midway between the centre and one of the edges, and bowing it at the proper point, we obtain a still different design, like 2,1 in the figure, composed of two parallel straight lines intersected by a third straight line at right angles.

These figures are named after their discoverer, and are known as Chladni's figures. They have been of invaluable service in studying the nature of vibratory motion in solid bodies, as they reveal at once the positions occupied by nodes and ventral segments. An almost indefinite number and variety of designs can be obtained from one and the same plate, and to most of these designs correspond sounds of different pitch.

A square plate yields its fundamental tone when it is divided into four equal squares, as in 1,1. The notes corresponding to 2,0 and 2,1 of Fig. 64 have a higher pitch. If the prime tone of the plate be C_1, the notes corresponding to the two diagonals will be a fifth higher, that is G_1, while the note corresponding to the third figure, 2,1, will be a major third of the octave above the fundamental, namely, E_2.

The pitch of the note emitted by a vibrating plate increases with the number of nodal lines formed, and the complexity of the figures developed. The designs in the accompanying diagram (Fig. 64) are a few of the multitudinous patterns that may be produced. Experiment shows that for plates of the same material, shape, and dimensions, the same figure always answers to the same sound. Different figures, however, under certain circumstances, may correspond to the same sound. With a little practice one can locate the position of the nodes, and determine the form of the figure that will be produced, with comparative ease and precision.

Wheatstone in 1833 was the first to give an explanation of these curious figures as formed on square plates. Koenig subsequently took Wheatstone's theory up and applied it to rectangular plates. Our knowledge of the transverse vibrations of rods will now be of use to us.

Suppose we have (Fig. 65) two rectangular plates of the same material and thickness, one having the length *a b c d*, the other the length *e f g h k*; and let us further suppose that these are in unison when the former has two nodes, *b* and *c*, and the latter three nodes, *f*, *g*, and *h*.

If we now superpose one on the other, we shall have a plate with a width *a b c d*, and a length *e f g h k*. Such a compound plate will admit both systems of nodes given by the plates separately, because the nodes are independent of the width of the plates, and will, while having the same system of nodes, emit the same sound. Knowing, then, the number and direction of the nodes given by two distinct plates, we can foresee what figures would result from

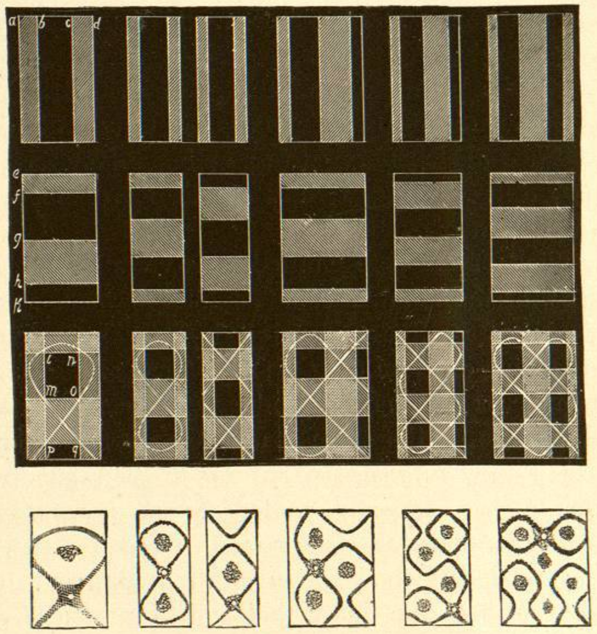

Fig. 65.

their superposition. The segments on the opposite sides of a nodal line, as is evident, must move in different directions, otherwise the formation of nodal lines would be impossible.

The parts of the plate that move upward are considered *positive*, those that have a downward motion, and are below the average position of the plate when at rest, are called *negative*. In the adjoining figure the negative parts are represented by dark spaces, while the positive ones

are indicated by cross lines. In the first and second horizontal series of the figure are shown plates of various sizes and of different systems of nodal lines. In the third series are shown the nodal lines that theoretically should result from a superposition of the corresponding plates of the first two series. A little reflection will make it apparent that when the first two plates of the two upper series are superposed, the resultant nodal curve must pass through the points *l, m, n, o, p, q*, which are the points of intersection of the nodal lines of the plates taken separately. At these points only do the positive vibrations of one system neutralize the negative ones of the other system, and induce the condition of rest indicated by the nodal curve, — a condition that can result only from movements or vibrations which are equal and opposite in direction. In the various figures of the fourth horizontal series are exhibited some of the sand figures obtained by Koenig, showing the perfect agreement of theory and experiment.

Let us now study the effect of vibratory motion in circular plates. And in order to make the Chladni figures visible to all of you, I will, as before, project them by means of the vertical lantern. Clamping a glass circular plate above the condenser, and strewing it with sand, we throw it into vibration by bowing it. Damping any given point of the edge by touching it, and drawing the bow across the edge at a point forty-five degrees from the finger, two rectilinear nodal lines are formed, at right angles to each other and intersecting at the centre of the plate. There are now four equal segments, and the note emitted is the lowest note the plate is capable of yielding. Drawing the bow across the edge thirty degrees from the point damped, six vibrating sectors are formed, separated by as many nodal lines. Agitating the plate at points gradually approaching the one damped, we obtain in succession eight, ten, twelve, and more vibrating sectors, the number of sectors in all cases being an even one.

As in the case of square plates, the pitch of the notes evoked increases with the number of nodal lines that are