produced. When these nodal lines are all rectilinear and intersect each other at the centre of the plate, thus making lines which are diametrical, the pitch of the notes emitted varies directly as the square of the number of diameters produced. Thus with 2, 3, 4, or 5 diameters, the corresponding notes would have frequencies represented by 2^2, 3^2, 4^2, 5^2. If then the prime tone of the plate corresponding to two diameters be C, that for 3, 4, and 5 diameters will be respectively D_2, C_3, $G\sharp_3$.

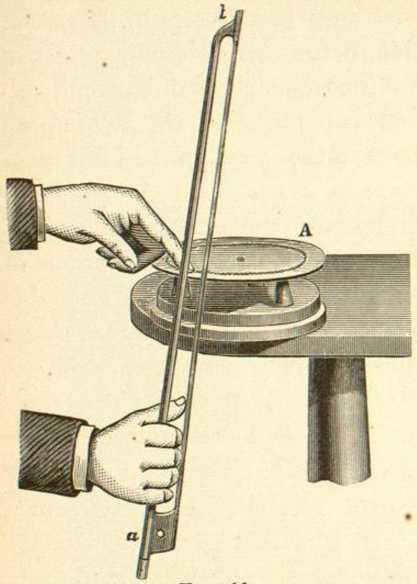


FIG. 66.

By supporting a plate at three points equidistant from the centre, as in Fig. 66, and drawing the bow across the edge, we get a single nodal curve, which in the present instance is a circle. Exciting the plate *A B* by drawing a resined string *b c* through its centre, we obtain two circles, as in Fig. 67. Supporting a plate as in Fig. 68, and damping and bowing it at appropriate points, we elicit a much more complex figure, composed both of diameters and circles, as *p, i, c; g, n, i;* and *p, g, a; m, n, b.*

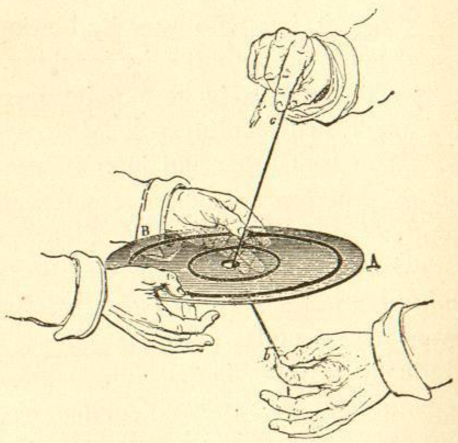


FIG. 67.

If the fundamental note of the plate, corresponding to its division into two diameters, be C_1, theory gives for a figure answering to one circle and no diameter $G\sharp_{1+}$. A circle with one diameter yields B_{2-}, with two diameters $G\sharp_{3+}$, and with three diameters D_{4+}. The signs + and − indicate, as previously, that the results given do not correspond exactly with any musical notes, + or − showing that they are to be slightly sharped or flatted. Two

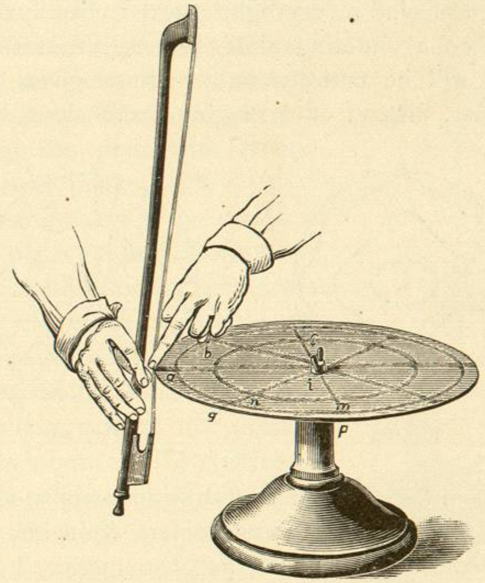


FIG. 68.

circles with no diameters would, under the same circumstances, give $G\sharp_{3+}$.

The pitch demanded by theory, and that obtained by Chaldni for the different figures, approximate very closely. But, as will appear on inspection, many of the partials are inharmonic, and hence the discordant character of the sounds of cymbals, tom-toms, and different kinds of plates.

Damping the plate at certain points in the circumference, and exciting it at the centre, we may obtain the so-called "festoon figures" (Fig. 69), which have been known since the time of Chladni. The theory of such figures is imperfectly, if at all, understood. Employing larger plates, there may be produced simultaneously several different sonorous figures. Sometimes the circumference is divided into a greater number of parts than the central portion. In such a case, several tones, some of which may be in unison, are produced. Fig. 70 shows a complicated subdivision of this character.

If in lieu of sand a very light powder, like lycopodium, be strewn on a vibrating plate, the aspect of the figures produced will be entirely unlike those given by sand. The powder, instead of arranging itself along the nodal

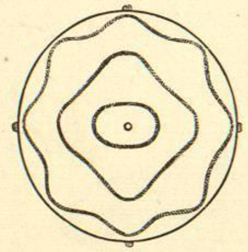


FIG. 69.

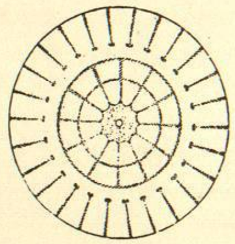


FIG. 70.

lines, as sand does, is collected in little heaps at the points of greatest agitation. Experimeters from the time of Chladni tried to account for the phenomenon, but it was reserved for Faraday to offer an explanation, as simple as it is natural. According to this illustrious physicist, the light powder is held in the centre of the ventral segments, where the motion is greatest, by little whirlwinds of air which are excited by the rapid and violent movements of the plate. The sand, on the contrary, in virtue of its greater density, is able to escape from these miniature cyclones, and hence if the plate be strewn with sand and powder at the same time, the two will be separated as soon as the plate is set in vibration. The sand collects along the nodal lines, and the lycopodium gathers at the points of greatest motion. That Faraday's theory is correct is

proved by making the experiment *in vacuo* (Fig. 71). Here the plate is placed in a bell-glass from which the air has been exhausted, and is set in vibration by rubbing with a resined cloth the wooden rod to which it is attached. Immediately the plate is excited, sand and lycopodium alike are collected along the nodal lines and curves.

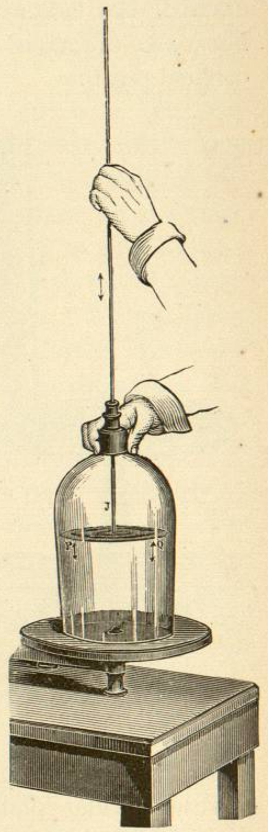


FIG. 71.

Before you (Fig. 72) is a large brass plate mounted on a strong support, and above it is fixed a resonant tube, so adjusted that it can be lengthened or shortened at will. Sprinkling the plate with lycopodium powder, and setting it in vibration, we get the same results as with the plate we have been using. Where the violin-bow is drawn across the edge of the plate is obviously the centre of a ventral segment, and the corresponding radial nodal lines are on either side of this point of maximum vibration. By shifting the bow to the right or the left of this point we evidently cause the nodal lines also to move in a similar manner. This is evidenced by the movements of the little heaps of lycopodium powder, and also by variations in the intensity of the tones emitted by the plate; for if the resonant tube is adjusted, as it now is, so that its note is in unison with that yielded by the plate, an augmentation of sound is produced every time a ventral segment passes under the tube. When, on the other hand, a node passes under the tube, there is a corresponding diminution of sound.

These oscillations and turnings of the ventral segments and nodal lines, and the consequent variations in the intensity of the tone, produced in the manner indicated, are what might have been predicted without making the experiment. But Savart discovered that a similar displacement of the nodal lines may take place when the vibrating plate is left to itself. When, after the plate is excited, the violin-bow is quickly withdrawn, the nodal lines are observed to oscillate on either side of their original position. If now the plate be bowed strongly, and always at the same point, the amplitude of these oscillations may become so great that the nodal lines may be carried to the middle of the segments which separate them in their primitive position. Under such circumstances, an additional stroke of the bow will cause the nodal lines to pass this point and to assume the positions at first occupied by contiguous lines. A vigorous application of the bow, always at the same point, will now enable us to keep up this displacement, and to cause the nodal lines to travel around the entire circumference of the plate. But such a displacement can take place only in circular plates in which the pitch of the note emitted is independent of the position occupied by the nodal lines.

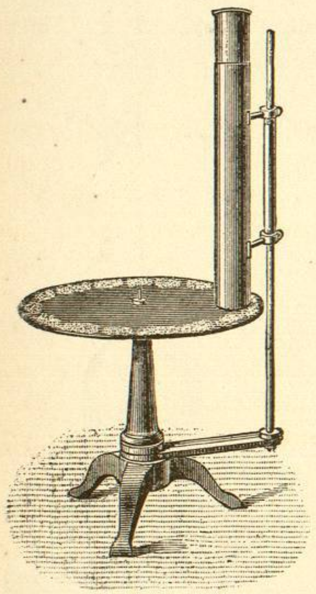


FIG. 72.

Instead of using lycopodium to show the movements we have been studying, we may, like Savart, employ a beam of light. Our lantern is now so adjusted that a beam from it is reflected from the polished surface of the plate, and we thus have an enlarged image of the plate on the screen. Setting the disk in vibration as before, we see the image on the screen transformed into a species of star, the rays

of which correspond to the nodal lines of the plate. If the nodal lines on the plate are made to oscillate or to turn, the rays of the image on the screen oscillate or turn in a similar fashion. By means of a very vigorous use of the bow it is possible to make these rays turn so rapidly that, owing to the persistence of vision, they will coalesce and give a luminous image on the screen like that which is afforded when the disk is at rest.

Savart attributes this curious phenomenon to the lack of homogeneity in the plate employed. No matter how carefully the plate may be wrought, it is nearly, if not quite, impossible to fashion it so that it will be perfectly homogeneous. It will therefore, according to Savart, have two diameters, corresponding respectively to its maximum and its minimum resistance to flexure. If the point of excitation by the bow be properly chosen, the nodal lines will arrange themselves along these diameters, and remain stationary. If, on the contrary, the disk is attacked at some other point, the amount of flexure on either side of the bow, by reason of the difference of elasticity in these two points, will not be the same in both cases. The nodal lines will accordingly oscillate about the point of excitation, or, if the amplitude of oscillation be sufficiently great, they will, as we have witnessed, make an excursion around the entire circumference of the plate.

Although much yet remains to be learned regarding the laws of vibrating plates, Chladni has made us acquainted with those which depend on the thickness and diameter of the plates employed. Before you (Fig. 73) are six brass plates, three of which are circular, and three square. In these plates those of the same size have their thicknesses in the ratio of 1 : 2, while those of the same thickness have diameters which are likewise in the ratio of 1 : 2. Exciting two of the circular plates of the same diameter, one of which is twice as thick as the other, you will observe that two sounds are produced, that due to the thicker plate being an octave higher than the other. Hence Chladni's first law, which says that *for two plates of like form and* *similar subdivision, as disclosed by the figures produced, the numbers of vibrations are directly proportional to the thickness of the plates.*

Exciting another pair of plates, either square or circular, of the same thickness, but having diameters which are as 1:2, we find that the smaller plate yields a note just a double octave above that emitted by the larger plate. Hence the second law, which declares that *for two plates of the same thickness, but of different diameters, the figures produced being the same, the numbers of vibrations vary inversely as the squares of their diameters.*

From these two laws we may deduce a third. *If the thicknesses*, as is here the case with two of the plates, *are*

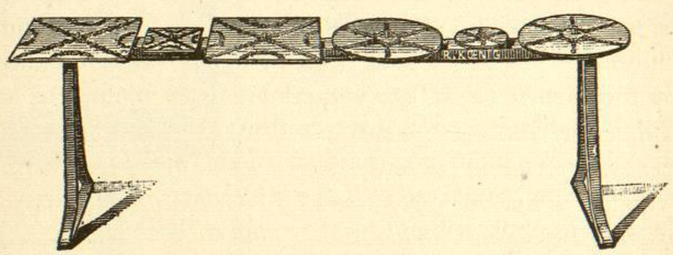


FIG. 73.

proportional to the other dimensions, that is, if the plates are similar solids, the numbers of vibrations are inversely as the homologous sides. Taking a plate, either square or circular, having twice the thickness and twice the diameter of another, the larger plate will emit a note that is an octave below that given by the smaller plate.

The last law holds true equally for solids, liquids, and gases, and must, therefore, be considered as a general expression for the laws of vibratory movement. Savart has shown that for bars of the same material and of similar form, the number of vibrations, as in plates, is inversely as the homologous sides. For spheres of the same substance, or cubes, or cylinders, or other solids of comparable dimensions, the law is equally true. Before you are suspended two spheres of iron, — one six inches, and the

other three inches in diameter. Striking them so as to elicit their fundamental notes, we find that the larger sphere yields a note an octave below that emitted by the smaller.

Mersenne discovered that the number of vibrations executed by drums of similar form, but of different sizes, is inversely as their homologous dimensions.[1] This philosopher, as we shall see in our next lecture, also remarked that the same law obtains for sonorous tubes, both open and stopped. It was reserved for Savart, however, to give an experimental proof of the law. This he did by exciting vibratory motion in masses of air contained in cases and tubes of various forms and sizes.

Causing two cubical boxes, whose linear dimensions are in the ratio of 2:1, to speak, we shall find that the note emitted by the larger box is an octave below that emitted by the smaller one. Employing sonorous cases of spherical, cylindrical, or tetrahedral form, the result would be the same; namely, *that the notes emitted by masses of air in vibration are in all cases inversely as the linear dimensions of the cases in which the air is contained.* We shall reserve the experimental illustration of this law for our next lecture, where it will find an appropriate place.

It is but a step from plates to bells. A disk is to a bell, essentially what a rod is to a tuning-fork. In both disks and bells the mode of subdivision is the same. The number of vibrating segments is always even, and the prime note, in both instances, always corresponds to a division into four segments. As in disks, so in bells, the movements of adjacent segments must at any given time be in opposite directions. Under no other circumstances could the intervening node be formed.

The existence of nodes and ventral segments in bells is beautifully shown by this large glass bell (Fig. 74), around the edge of which are suspended four ivory balls. When the bell is excited by a violin-bow in such a manner that the balls touch the nodes, the motion is very slight.

[1] Harm., lib. xii. Prop. 18.

When, on the other hand, they are near the centre of ventral segments, they are forcibly repelled.

Filling a similar glass bell, *A* (Fig. 75), with water, and exciting it, as before, so as to yield its fundamental tone, the mode of vibration of the bell is disclosed by the condition of the water within. The surface of the liquid shows two nodal lines, *f e* and *g h*, which cut each other in the centre at right angles. Between these nodal lines the water is more or less agitated, as is evidenced by the ripples and crispations that play over its surface. The centres of the ventral segments — where the motion of

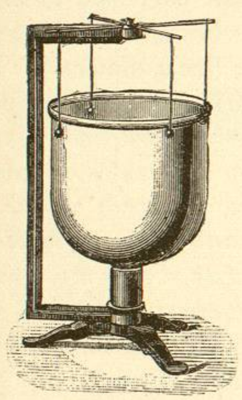


FIG. 74.

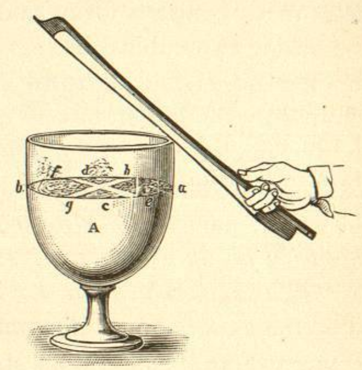


FIG. 75.

the bell, as well as of the water, is at a maximum — are at the points *a, b, c, d*. A few vigorous sweeps of the bow across the edge of the bell would develop vibrations of such amplitude as to shatter it into fragments.

The least number of segments in which a bell can vibrate is, as has been stated, four; and this division always obtains when the bell is yielding its lowest, or ground, tone. The next subdivision would be into six segments, and then into eight, ten, twelve, etc.; the number of segments, as in disks, being always even.

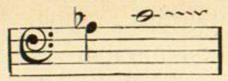
If a bell were perfectly regular and homogeneous throughout, the frequencies of the notes corresponding to 2, 3, 4, 5 meridianal nodal lines would be as the squares of these

numbers; that is, as 2^2, 3^2, 4^2, 5^2. Supposing the prime note of the bell to be C_1, its first three upper partials would be D_2, C_3, $G\sharp_3$. The vibration numbers would thus follow the same law as governs circular plates having similar subdivisions. Such a bell would, like a disk, be characterized by many inharmonic partials, and would not answer the purpose for which bells, especially large ones, are ordinarily employed. Hence the empirical form — a sort of truncated conoid — in which large bells are now always cast.

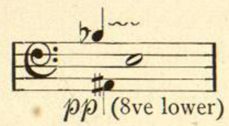
The best form was found only after many ages of study and experiment, and the form aimed at was one that would bring out the fundamental tone and such of the upper partials as would harmonize with the prime. The diameter and height of the bell, the thickness and width of the sound-bow, its weight and size as compared with the rest of the bell, the material used (ordinarily copper and tin, in varying proportions), the relative weight of the clapper, — all these are problems that must be worked out, not theoretically, but experimentally, before the casting of your modern large, harmoniously toned bells can be attempted. Van den Gheyn (1550) and Hemony (1650) are the princes of the art of bell-founding. To them we are indebted for the types and models that are now followed by all bell workers. They have done for bells what Amati and Stradivarius did for violins. They have not only supplied us with models, but they have produced the most perfect work of their kind that the world has yet seen.

According to Hemony, a good bell should have three octaves, two fifths, one major and one minor third. The great bell of the cathedral of Erfurt, celebrated, not only for its size, but also for the fine quality of the metal from which it was cast, has E_1 for its prime, and this is accompanied by the following upper partials: E_2, $G\sharp_2$, B_2, E_3, $G_3\sharp$, B_3, $C_4\sharp$. I give in musical notation the approximate pitches of the compound note of three large bells that are widely celebrated: —

St. Paul's, London. Large Hour-Bell.



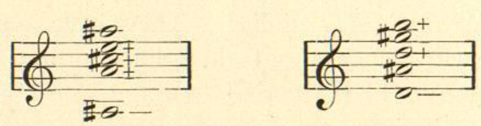
Big Ben of Westminster.



Great Tom of Oxford.

The loudest notes are indicated by minims, the weaker ones by crotchets. The wavy lines following some of the notes are to show that the notes vary in pitch.

Two of the bells of the peal at Terling, examined by Lord Rayleigh, give partials that are more inharmonic than those we have been considering. In musical notation the partials of the compound tones of these two bells would be written as follows: —



The signs + and —, as in the previous instances, signify that the notes after which they occur are to be slightly sharped or flatted.

It is rarely, if ever, that a bell can be cast so as to be perfectly symmetrical and homogeneous throughout. When, therefore, it is set in vibration, it frequently, by reason of its lack of homogeneity, divides itself into segments that emit two distinct sounds differing from each other slightly in pitch. This difference in pitch gives rise

to the beats, or the risings and fallings of sound, that are noticed in most bells, especially when their tones are dying out.

Small bells do not yield such pure tones as larger ones, because of the presence of many inharmonic upper partials. In large bells, as we have seen, such inharmonic partials are weakened or entirely eliminated by the form in which the bells are cast. For this reason small bells, like hand-bells, and even such as are ordinarily used for carillons, are poorly adapted to purposes of music. They are frequently employed, it is true, but the inharmonic partials, which are always prominent in greater or less numbers, render the music produced by them dissonant in the highest degree, and all but unendurable.

The number of vibrations of bells of similar form varies inversely as their homologous dimensions. Thus bells of the same form, but of different sizes, will vary inversely as their diameters. Two bells, whose diameters are as 2 : 1, would consequently yield notes an octave apart, the smaller bell emitting the higher note. It has also been found that the notes emitted by bells vary inversely as the cube roots of their weights. Working in accordance with these two laws, the bell-founder can cast a peal of bells that will approximate to any intervals that may be required. I say approximate to, as it is impossible in this instance, as in so many others, to carry out in practice exactly the indications of theory.

Membranes are closely related to plates in their modes of vibration. The chief difference is that the former are thinner and more flexible than the latter. They are ordinarily of paper, sheet rubber, or gold-beater's skin, and are stretched on a wooden frame with a tension uniformly distributed in all directions. They, like plates, have been carefully studied both theoretically and experimentally. They may be caused to vibrate either by percussion or by sounding near them a note in unison with their proper period of vibration. They exhibit Chladni's figures readily, and the resemblance of these figures to those excited on plates of the same form and size is very marked. The laws which govern the formation of the figures are apparently different in the two cases, and in some respects, indeed, these laws are as yet but imperfectly understood.

The mathematical researches of Poisson, Euler, Kirchhoff, Clebsch, and Mathieu, and the experimental investigations of Savart, Bourget, and Bernard, show that for the order of succession of the nodal lines of membranes, and their successive transformations calculated for the same sound, there is a striking agreement between the results of theory and experiment. The law governing the intervals between the various possible notes of a membrane requires further examination. So far the intervals given by experiment are always greater than those required by theory, and the difference is more pronounced as the membrane is thinner, and as the sounds approach more nearly to the fundamental.

In the following table, taken from the memoir of M. Bourget, are given the theoretical notes corresponding to the simpler nodal lines of circular membranes. In the illustrations given, Fig. 76, the nodal lines are either circles, or diameters including equal angles, or combinations of circles and diameters equally inclined towards each other, according to theory. When the membrane is properly stretched, the figures are perfectly regular, and present exactly the dimensions required by theory. The first figure represents a membrane vibrating as a whole, and yielding its prime. Supposing its fundamental to be C_1, its first upper partial with one diameter will be $G_1\sharp_-$, the ratio of whose vibrations to those of the prime is, as the numbers show, 1.594: 1.000. When the membrane vibrates so as to form two diameters, it emits the note $C_2\sharp$, whose frequency is 2.136 times that of the fundamental; and when it develops three diameters, the note yielded is F_{2-}, with a frequency 2.653 times that of its prime. By inspecting the table, one can tell at a glance the notes and rates of vibrations that appertain to the different figures

	0 Diameter.		1 Diameter.		2 Diameters.		3 Diameters.		4 Diameters.	
	Radii of Nodal Circles.	Corresponding Sounds.	Radii of Nodal Circles.	Corresponding Sounds.	Radii of Nodal Circles.	Corresponding Sounds.	Radii of Nodal Circles.	Corresponding Sounds.	Radii of Nodal Circles.	Corresponding Sounds.
0 CIRCLE		1,000 C_1		1,594 $G_1\sharp$ –		2,136 $C_2\sharp$		2,653 F_2 –		3,156 $G_2\sharp$ –
1 CIRCLE	0,436	2,296 D_2 +	0,546	2,918 G_2 –	0,610	3,501 $A_2\sharp$ –	0,654	4,060 C_3 +	0,686	4,602 D_3 +
2 CIRCLES	0,278 0,638	3,600 $A_2\sharp$ +	0,377 0,690	4,231 $C_3\sharp$	0,442 0,724	4,833 $D_3\sharp$ +	0,490 0,750	5,414 F_3 +	0,528 0,770	5,979 G_3 –
3 CIRCLES	0,204 0,468 0,734	4,905 E_3 –	0,288 0,527 0,764	5,542 $F_3\sharp$ –	0,348 0,569 0,785	6,155 $G_3\sharp$ –	0,393 0,602 0,802	6,748 A_3 +	0,431 0,627 0,816	7,328 $A_3\sharp$ –
4 CIRCLES	0,161 0,370 0,580 0,790	6,211 $G_3\sharp$ –	0,233 0,426 0,618 0,809	6,851 A_3 +	0,287 0,469 0,647 0,824	7,471 B_3 –	0,329 0,503 0,671 0,836	8,074 C_4 +	0,364 0,531 0,691 0,846	8,663 $C_4\sharp$ +

given. The signs + and — have the same signification as in other parts of the lecture.

On examining the preceding table it will be found that the higher upper partials succeed each other very closely, and that the interval separating them is in some cases less than a semitone. Hence we infer that within certain determinate limits a membrane is capable of vibrating in unison with any note whatever. This is especially true of the tympanic membrane of the ear. Here, however,

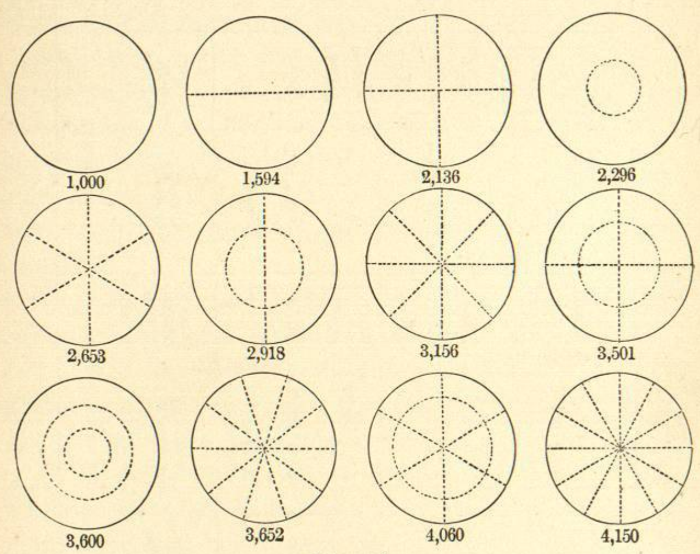


FIG. 76.

owing to the chain of ossicles connected with the tympanum, and the ligaments which bind the ossicles together, the tension of the auricular membrane can be varied within quite wide limits. For this reason the tympanum responds with such readiness to all notes, from the most grave to the most acute.

The same phenomenon is observed in the disks and diaphragms of telephones and phonographs. Such diaphragms, in addition to responding to vibrations of a certain determinate period, depending on the nature and form of the disks, have also a general resonance in vir-

tue of which they are sensitive to any vibratory motion whatever. By reason of this general resonance, which they possess, and their competency to respond to vibrations of different periods, the telephone and phonograph are capable of transmitting and recording all sounds within the limits of ordinary audition.

The audiphone is another illustration of the same fact. As shown in Fig. 77, it is a fan-shaped sheet of hardened india-rubber, the upper part of which is held against the teeth of the upper jaw. Owing to its general resonance, it vibrates in unison with all sounds. The sounds thus collected, as it were, are transmitted by the teeth and

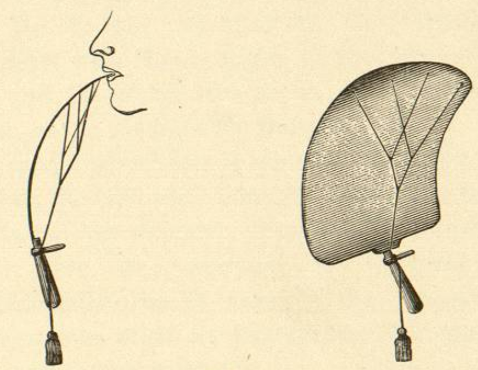


FIG. 77.

bones of the head to the auditory nerve. By this means, deaf persons who suffer from some disease or malformation of the external ear, but whose auditory nerves are intact, are able to hear with comparative ease and satisfaction. Chladni's figures excite our admiration and wonder. But these designs, complicated as some of them are, are excited by comparatively simple vibrations. The vibratory motions induced in the audiphone and in the disks of the telephone and phonograph are infinitely more complex and varied, and calculated, when we reflect on the matter, to excite our sense of wonder far more than anything disclosed by the experiments of Chladni, Savart, or Bourget. And yet further. The equations of the mathematician, and the experiments of the physicist, may tell us something about the laws governing the simpler vibrations of plates and membranes, but no mathematical *tour de force*, however transcendent, no experiment, however ingenious or refined, will ever be competent to unravel the infinitude of motions — changing as they do with the slightest modifications in pitch, intensity, and quality of tone — which characterize that most marvellous and most sensitive recipient of vibratory movement, the tympanic membrane of the human ear.

CHAPTER VI.

SONOROUS TUBES.

IN the two preceding lectures we studied sounds generated by solid bodies. In all the instances considered, the air served simply as a medium for the transmission of the sonorous waves to the ear. To-day we shall devote our attention to the investigation of sounds which have their origin in the vibrations of the air itself, and for which the air, as in the case of solids, serves also as the medium for transmission.

All musical instruments in which a vibrating column of air serves as the sonorous body are known by the general name of wind-instruments. They, like the other instruments we have been studying, are of great antiquity. This is especially true of some of the simpler forms of wind-instruments, such as the syrinx, or pandean pipes, the flute, and the trumpet.

According to Diodorus Siculus, their invention is to be ascribed to some shepherd who had studied the whistling of the wind among the reeds, and who endeavored to reproduce what he found in nature. Lucretius expresses the idea beautifully when he says, —

> "And Zephyr, whistling through the hollow reeds,
> Taught the first swains the hollow reeds to sound;
> Whence woke they soon those tender-trembling tones
> Which the sweet pipe, when by the fingers prest,
> Pours o'er the hills, the vales, and woodlands wild,
> Haunts of lone shepherds and the rural gods."[1]

[1] Et Zephyri, cave per calamum, sibila primum
Agresteis docuere cavas inflare circutas.
Inde minutatim dulceis didicere querelas,