and the experiments of the physicist, may tell us something about the laws governing the simpler vibrations of plates and membranes, but no mathematical *tour de force*, however transcendent, no experiment, however ingenious or refined, will ever be competent to unravel the infinitude of motions — changing as they do with the slightest modifications in pitch, intensity, and quality of tone — which characterize that most marvellous and most sensitive recipient of vibratory movement, the tympanic membrane of the human ear.

CHAPTER VI.

SONOROUS TUBES.

IN the two preceding lectures we studied sounds generated by solid bodies. In all the instances considered, the air served simply as a medium for the transmission of the sonorous waves to the ear. To-day we shall devote our attention to the investigation of sounds which have their origin in the vibrations of the air itself, and for which the air, as in the case of solids, serves also as the medium for transmission.

All musical instruments in which a vibrating column of air serves as the sonorous body are known by the general name of wind-instruments. They, like the other instruments we have been studying, are of great antiquity. This is especially true of some of the simpler forms of wind-instruments, such as the syrinx, or pandean pipes, the flute, and the trumpet.

According to Diodorus Siculus, their invention is to be ascribed to some shepherd who had studied the whistling of the wind among the reeds, and who endeavored to reproduce what he found in nature. Lucretius expresses the idea beautifully when he says, —

"And Zephyr, whistling through the hollow reeds,
Taught the first swains the hollow reeds to sound;
Whence woke they soon those tender-trembling tones
Which the sweet pipe, when by the fingers prest,
Pours o'er the hills, the vales, and woodlands wild,
Haunts of lone shepherds and the rural gods."[1]

---

[1] Et Zephyri, cave per calamum, sibila primum
Agresteis docuere cavas inflare circutas.
Inde minutatim dulceis didicere querelas,

That a sound can be produced from a vibrating column of air independently of the material of which the pipe enclosing the air is made, may be shown by a very simple experiment.

I have here a brass tube about twenty inches long and an inch and a half in diameter. Holding it longitudinally with one hand, and striking one of its open ends with the palm of the other hand, the enclosed air is set in vibration with sufficient force to yield a distinct musical note. If now the hand is quickly withdrawn from the end of the tube, another note is heard; but its pitch is an octave higher than that first emitted. In the former case the air vibrates as it does in a stopped pipe; and in the latter case it obeys the laws governing the vibrations of aërial columns in open pipes. We shall study these laws subsequently. In both instances, it must be remarked, it is the air that vibrates and produces the sound heard, and not the material of the tube which encloses the air.

That such is the case, is easy of demonstration. Striking the tube with my finger, or with a small billet of wood, so as to evoke the prime tone of the metal, we have a note that is much more acute than either of those produced when the enclosed air was in a state of tremor. We thus learn that the air-column within a tube may be caused to vibrate independently of the tube itself, and that the notes emitted by the former are entirely different in pitch from those that may be elicited from the latter.

We may vary the experiment by using pipes of different materials. Here are three different pipes, — one of brass, one of wood, and one of cardboard. Causing them to "speak" successively, you perceive that the pitch of the note in the three cases is identical. If the materials of which the pipes are made had any influence on the number of vibrations, the pitch in these three instances would be different. But the pitch of the three pipes being the same shows that the frequency of the notes generated is independent of the materials of the pipes, and is due solely to the length of the enclosed column of air, which in the instances now under discussion is itself the true sonorous body.

If instead of air the three pipes just used were filled with gases of different densities, the result would no longer be the same. If we were to fill one with air, another with hydrogen, and the third with carbon-dioxide, we should find that there would be a very marked difference in the pitch in the three cases. We saw in our first lecture that the velocity of sound varies for the different gases, and that it is less for carbon-dioxide and greater for hydrogen than it is for air. As pitch varies directly as velocity, and as the velocity of sound in hydrogen is almost four times as great as in air, the note emitted by the pipe filled with this gas would be very nearly two octaves above that produced with the pipe containing air. For a similar reason, the note yielded by the pipe containing carbon-dioxide would be graver than that in which air is the sonorous body.

There are many ways of exciting an air-column so as to make it yield a musical note. A simple and instructive way is by means of a tuning-fork. The column of air in the glass cylinder, *C* (Fig. 78), is thus acted upon by a tuning-fork, *D*, to one of the prongs of which is attached a disk, *A*, of the same diameter as the cylinder. By means of the disk the vibrations of the fork are communicated to all the particles of air at the opening of the tube. By pouring mercury into the tube, the proper sound of the air-column can be made to synchronize with that of the tuning-fork.

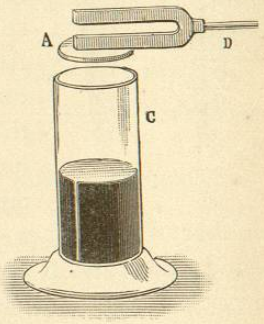

FIG. 78.

Tibia quas fundit, digitis pulsata canentum,
Avia per nemora ac sylvas saltusque reperta,
Per loca pastorum deserta, atque otia dia.

De Rerum Natura, lib. v. 1381 et seq.

See also Ovid, Fab. xv., "Syrinx changed into Reeds," and Virgil, Eclogue ii. 32, 36.

The moment when the two notes are in unison is declared by a remarkable augmentation of sound. We shall study this phenomenon more attentively when we come to investigate the nature and cause of resonance. Suffice to say now that a column of air is always most strongly reinforced when its period is perfectly isochronous with that which throws it into vibration.

Wind-instruments used in music are rendered sonorous by mouthpieces or by reeds. Hence their division into mouth-instruments and reed-instruments.

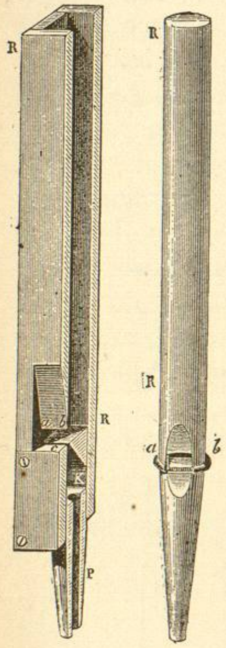

FIG. 79.

Here (Fig. 79) are two organ-pipes, one made of wood and prismatic in form, the other of metal and cylindrical in form. The first is open at the top, and the latter closed. Hence the names used, — open pipes and stopped pipes. The air is admitted through the foot, *P*, into the chamber, *K*, whence it escapes through a slit, *c*. The sharp bevelled edge, *a b*, is called the lip. The space between the slit, *c*, and the lip, *a b*, is called the mouth, or embouchure.

The precise manner in which vibrations in such pipes are executed seems still to be but imperfectly understood. According to the view which generally obtains, when a current of air enters the embouchure a fluttering or hissing noise is produced. This fluttering, like most noises, is made up of a large number of discordant sounds. From the mass of confused sounds the tube *R R* selects one which it strongly reinforces. It can, however, reinforce that note only whose period synchronizes with its own. We have, then, repeated here, but in another form, the experiment of the tuning-fork and the glass cylinder. In the cylinder the air-column was excited by a tuning-fork vibrating in unison with it. In the organ-pipe the vibrations are set up by the current of air which issues intermittently from the embouchure.

According to a more recent theory, advocated especially by M. Cavaillé-Coll, Herr Schneebeli, and Mr. Hermann Smith, the vibrations excited in the aërial column within the pipe are produced by the sheet or blade of air issuing from the slit acting as a reed. Cavaillé-Coll styles this air-blade a free aërial reed ("anche libre aerienne"); Herr Schneebeli calls it a "Luft-lamelle," an aërial lamina; while Mr. Smith denominates it an "aëro-plastic reed," or simply an "air-reed." Novel as it may appear, this view seems to have a solid foundation in fact, and the many and ingenious experiments made in support of the theory are apparently inexplicable on any other assumption. According to Mr. Smith, the air-reed, on issuing from the slit, does not strike the edge of the lip, as the old theory maintains, but passes very near its outer surface. Like a metal reed, whose action we shall study presently, the air-reed oscillates backwards and forwards, and generates in the air-column within the pipe the alternate condensations and rarefactions which are essential to the production of a musical note. Judging by the experiments appealed to in corroboration of it, — time forbids our discussing them here, — it would appear that the new theory is virtually established, and on a basis that is unassailable. As a working hypothesis, I think we are justified in regarding it the more probable of the two theories which now generally prevail.

Organ-pipes like those which we are now using are called, indifferently, mouth-pipes, flute or flue pipes. All parts of the mouthpiece are fixed, and ordinarily the pipe is designed to yield but one note. For this reason they are said to be of constant pitch. In instruments, however, like the flute or flageolet, which act on the same principle as an ordinary organ-pipe, a number of notes may be produced, and hence they are said to be of variable pitch.

The locomotive whistle is but a modified form of the organ-pipe. Inspection of Fig. 80 will show that the former differs from the latter in having a circular instead of a rectilinear embouchure, *a a*, above which is placed the sharp edge, *b b*, of the bell, *T*. The mode of action in both cases is essentially the same.

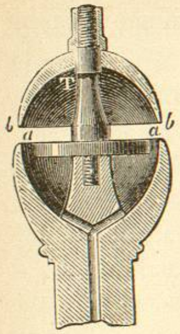

FIG. 80.

Daniel Bernouilli was the first to establish the laws which govern the notes emitted by organ-pipes. For their more elaborate experimental verification we are indebted to Mersenne, Savart, Wertheim, and Koenig.

The first law is that the pitch of the note is inversely as the length of the tube. Placing three similar tubes, *K*, *K′*, *K″*, on the wind-chest, *A B*, of the acoustic organ (Fig. 81), and admitting air into them, they are found to give notes that are separated from each other by an exact octave. The largest tube sounds the note C$_2$, the next C$_3$, and the shortest one C$_4$. By selecting tubes, with diameter of cross section very small compared to the length of tube, whose relative lengths are as the numbers 1, $\frac{4}{5}$, $\frac{2}{3}$, we should, as in the case of vibrating strings, obtain notes constituting the perfect major chord. Supposing the lowest note to be C$_3$, the other two would be respectively E$_3$ and G$_3$.

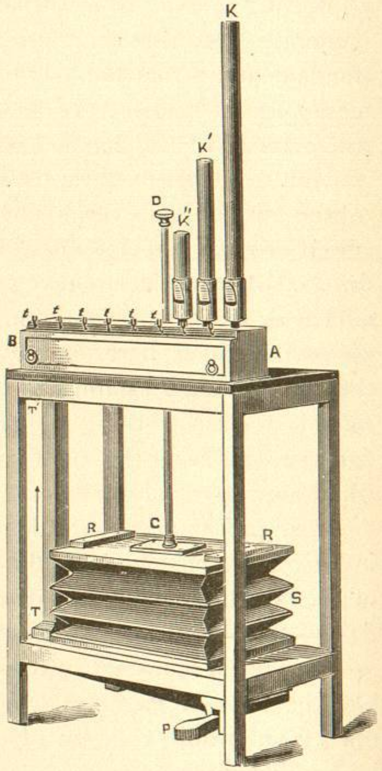

FIG. 81.

Let us now choose eight pipes whose lengths are to each other as the wave-lengths of the different notes of the diatonic scale, beginning with C$_2$. Placing them in the apertures *t*, *t*, *t*, etc., and forcing air into them, we find on touching in succession their corresponding keys that we have all the notes of the gamut between C$_2$ and C$_3$. The result obtained here is identical with that obtained with vibrating strings. As with strings, the number of vibrations varies inversely as the length of the string, so with tubes the number of vibrations varies inversely as the length of the columns of air which they enclose.

But a mass of air in vibration may, like solids, generate several partial tones in addition to its fundamental. Mersenne remarked that one may easily elicit from a harmonic trumpet the tonic, the octave, the twelfth, and the double octave, but no intermediate notes. Sauveur made a similar observation; but it was Daniel Bernouilli who first discovered the law governing the succession of harmonic partials both in open and in stopped pipes.

The experimental illustration of these laws is quite simple. For this purpose we require a long tube of small diameter (Fig. 82), provided at the bottom with a stopcock, *R*. The tube, which is open at the top, is placed in one of the apertures of the wind-chest of the acoustic organ, and the air is allowed to enter through *T T′*, from the bellows, *R R S*. When the stopcock is partially open and the pressure is suitably regulated, a deep, pure tone is produced. The note you now hear is the fundamental, the lowest the tube is capable of yielding. But by admitting more air, and especially by increasing the pressure, — which is effected by bearing down on the rod *D C*, or the pedal *D*, — we obtain a note an octave higher than the one you have just heard. Augmenting the pressure, a still higher note is produced.

FIG. 82.

The musicians present will recognize this as the third harmonic partial, or the fifth of the second octave. By increasing the pressure still more, and turning the stopcock so as to admit a full blast of air, we elicit still higher notes. We now have the second octave above the fundamental, — now the third of the second octave, — and now the fifth. You are familiar with the order of occurrence of these notes; we have adverted to them many times before. They are, in fact, the harmonic partials which succeed each other as the numbers 1, 2, 3, 4, 5, 6.

With the pipe before us, we have readily obtained six partial tones, and might, if it were desirable, elicit several others. The theoretic number of such partial tones has, indeed, no limit. Experimentally it is possible to demonstrate the presence of at least twenty. But to do this, special appliances are required.

The experiments just made enable us to formulate a second law for sonorous tubes; namely, that an open tube is competent to execute vibrations whose relative numbers are to each other as 1 : 2 : 3 : 4 : 5 : 6. The notes thus generated constitute the complete series of harmonic partials. As you will remember, we have the same order of succession of notes for the transverse and longitudinal vibrations of strings, and for the longitudinal vibrations of rods that are free at both ends.

We now replace the tube just used with a stopped pipe of the same length and diameter. Proceeding as before, we educe the fundamental and a series of higher notes. But now the order of succession of the upper partials is different. The first note above the prime is not the octave, as in the open pipe, but the fifth of the second octave. We thus have, as the first note heard above the fundamental, the third instead of the second partial. The next higher note audible is not the fourth partial, as before, but the fifth; and the one following, as is found by experiment, is not the sixth, but the seventh in the order of the harmonic series. Hence the partials in stopped pipes succeed one another in the order of the odd numbers 1, 3, 5, 7, etc., and not as they do in open pipes, where the whole series of partials is found. This fact, as we shall learn later on, will account for the marked difference of quality which distinguishes the two classes of pipes.

We now fix in the wind-chest, side by side, the two tubes with which we have been experimenting, and upon causing them to speak, we observe another fact which distinguishes an open from a closed pipe. Although both pipes are of the same size, the pitch of the notes is not the same. The sound yielded by the open pipe is just an octave higher than that produced by the closed one. This is a general law, — an open pipe yielding its prime tone is always an octave higher than a stopped one of the same length and diameter.

You cannot fail to remark the difference in the quality of sound which characterizes the pipes. That of the open one is full, rich, and brilliant; that of the stopped one is, in comparison, jejune, poor, and dull.

Like solids, vibrating air-columns admit of subdivision, as is evidenced by the formation of upper partials, which we have been studying. They have, consequently, nodes and ventral segments, or points of maximum and minimum motion. The vibrations of the air particles as to their direction of motion follow the same laws as govern the longitudinal vibrations of strings and rods. They make their excursions to and fro parallel to the axis of the tube in which the air is enclosed. Their nodes are, therefore, points of no motion, but of varying density; the centres of their ventral segments are points of maximum motion, but of a density which is constant, being the same as that of the air external to the tube.

In Fig. 83, I, II, III, IV, we have represented the subdivisions of an open pipe when emitting its fundamental and first three upper partials. In I, corresponding to the prime tone of the pipe, there is but one node, *N*, which is at the centre. At both ends of the pipe are centres of ventral segments, *V V*. It is obvious that such must be the case, as the air at these points, being in communication with the external atmosphere, must be always of the same density. The arrows indicate that the paths of movement on opposite sides of a node are always in opposite directions, and the perpendicular lines reveal the position of the nodal planes. The symbolic wave-forms show that when an open tube yields its prime tone, it divides into two semi-ventral segments, making thus one complete ventral segment for the fundamental. When the pipe emits its first upper partial, as the wave-form indicates, it is divided into four semi-ventral, or two complete ventral segments. Similarly, for the second and third upper partials, there are six and eight semi-ventral or three and four complete ventral segments. The nodes increase in number according to the same ratio. There is one node, which is in the middle of the pipe, for the fundamental. For the first upper partial, or second partial simply, we have two nodes, each of which is one fourth the length of the pipe distant from its corresponding extremity. Similarly for the third and fourth partials there are respectively three and four nodes. The pitch of notes in open pipes is, consequently, directly proportional to the number of nodes, or the number of complete ventral segments.

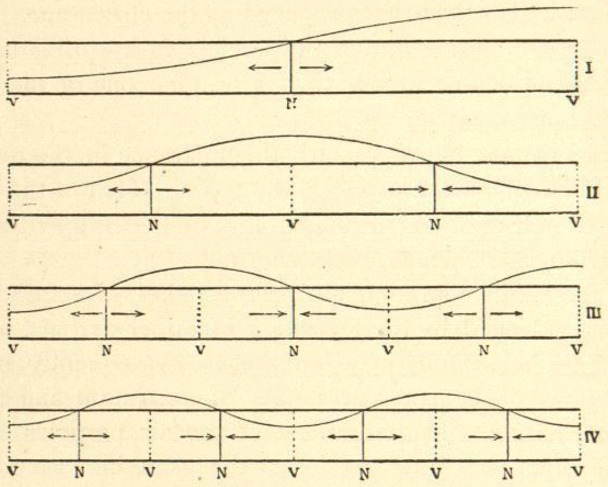

FIG. 83.

In the case of stopped pipes it is different. Instead of yielding notes according to the order of the natural numbers, they emit, as we have seen, notes corresponding only to the odd numbers. Inspection of Fig. 84, I, II, III, IV, will make apparent the reason for this difference.

The open end of a stopped pipe, for the same reason that obtains in an open pipe, must be the middle of a ventral segment. It is here that the direct pulses are originated, and for this reason it must be a point of maximum vibration. At the closed end, on the contrary, there must necessarily be a node, because longitudinal vibrations of the air-particles are here impossible. The simplest way, therefore, in which a stopped pipe can vibrate is that indicated in I, in which the open end answers to the middle of a ventral segment, *V*, and the closed end to a node, *N*. The pipe in this case, as is evident, forms only a semi-ventral segment, and is one half the length of an open pipe yielding the same note. In an open pipe the wave-length corresponding to the prime tone is, as the symbolic curve shows, twice the length of the

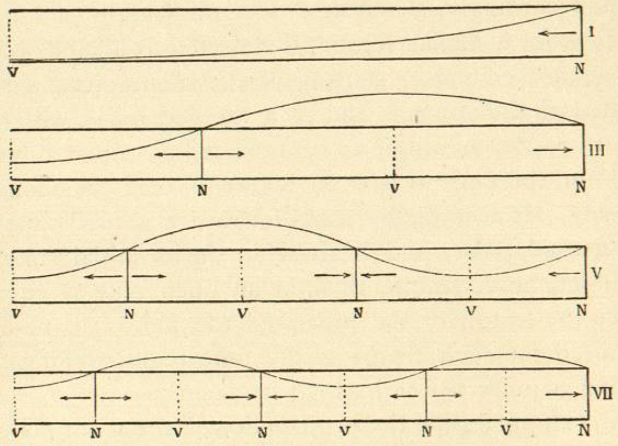

FIG. 84

pipe. In a closed pipe, however, the wave-length is four times the length of the pipe.

A little reflection will show why this is so. A condensed pulse, starting from the mouthpiece of an open pipe, is reflected from the other end as a rarefied pulse. This change from condensation to rarefaction is due to the lesser density of the air on the outside of the tube as compared with that of the condensed pulse within. When a condensation arrives at the open end of the tube, there is a sudden expansion, which gives rise to a rarefaction that is propagated back through the pipe. A condensation, accordingly, is reflected as a rarefaction, and a rarefaction, for a similar reason, is reflected as a condensation. A condensed pulse, starting at the mouthpiece, and reflected at the opposite end as a rarefied pulse, will, on its return to the mouthpiece, be reflected a second time, and will, for the same reason as before, undergo a change of density. It accordingly starts forward a second time as a condensed pulse, and is therefore in its initial state. A complete wave-length, then, in an open pipe is equal to twice the length of the pipe, and the period of vibration required for such a pipe is the time required for propagating a pulse through twice its length.

In a stopped pipe the propagation of the sonorous pulse follows a different law; for if a condensed pulse excited at the embouchure be propagated to the closed end, it will there, owing to the resistance offered, be reflected unchanged. It will accordingly return to its starting-point as a condensed pulse, but on arrival there will be reflected a second time. This time, however, the condensed pulse will be changed into a rarefied one, and for the same reason as a similar change is effected in an open pipe. On reaching the closed end the rarefied pulse will be reflected again, but reflected as a rarefied pulse. Arriving at the mouthpiece a second time, another change of density occurs, and the rarefied pulse once more becomes a condensed pulse. It is now in its initial condition. We have a complete vibration, but only after the pulse has travelled

four times the length of the pipe. In pipes of the same length, therefore, the wave-length of a stopped pipe is twice that of an open one, and the pitch of the former is an octave lower than that of the latter.

The vibrations of the air-columns of pipes, like the vibrations of strings, give rise to stationary undulations. In both instances they are produced by the combination of direct and reflected waves, which are equal and similar to one another. In the production of upper partials the air-column always subdivides itself into a greater or less number of such stationary undulations, separated by a corresponding number of nodal surfaces. At equal distances on opposite sides of the nodal plane the air-particles have equal and opposite velocities. For this reason the air at a node is always subjected to equal and opposite forces, and hence remains unchanged in position.

As the air of a vibrating segment sways to and fro, and as the motions of any two adjacent segments are opposite in direction, it follows that any two consecutive nodes must always be in opposite conditions of condensation and rarefaction. As in the stationary undulations of strings, so also in those of air-columns, the middle points of ventral segments are where the amplitude of motion is greatest. But while the amplitude of motion at these points is greatest, the variations of density, as has been observed, are least. The density of the air at these points is the same as it is at the open ends of pipes, where variations of density are precluded by free communication with the atmosphere outside.

These considerations follow naturally from the demands of mathematical theory. But the conditions which theory demands can all be shown by experiment to have an actual existence.

Savart has taught us a simple method of determining experimentally the position of nodes in sonorous pipes, whether open or closed. A little wooden ring, *S*, to the bottom of which is attached a thin membrane, is suspended by a string inside an open organ-pipe, *T* (Fig. 85), when