William Hamilton, Maxwell, Hertz, and others, but above all by that brilliant young French physicist, Augustin Jean Fresnel. It was he that put the truth of the wave-theory of light beyond further question by his celebrated *experimentum crucis*, in which he obtained total interference of luminous rays both by reflection and refraction.

# CHAPTER VIII.

## BEATS AND BEAT-TONES.

IN our last lecture we dealt with vibrations that are so related to each other that their resultant effect is either resonance or total interference. We found that when two sounds are in unison, and in the same phase, they tend to reinforce each other; and that when the same sounds are in opposite phases, — their intensity being equal, — one cancels the other, and silence is the result. Under these conditions we discovered that the result must always be either augmentation or annihilation of sound, — no other result being possible.

It is, however, comparatively seldom that we deal with two sounds that are exactly in unison. We are more frequently called upon to consider notes whose rates of vibration differ from each other by a greater or less amount. What, then, is the result, when two notes differing more or less from each other in pitch are sounded simultaneously? This question — one that is of special interest to musicians — I shall endeavor to answer in to-day's lecture. What we have learned about resonance and interference has paved the way for our work to-day, — for the discussion, namely, of what we shall, after Koenig, designate as beats and beat-tones.

Before you are the two C forks used in our last lecture. I damp one of them by attaching a small pellet of wax to one of its prongs. On exciting it with the bow, you perceive that it gives a slightly lower note than it did before. The extra load it has to carry retards its motion, and it executes, in consequence, a smaller number of vibrations than previously, and a smaller number, too, than is made by its unencumbered companion.

If now both forks are sounded simultaneously, what will be the result? Something entirely different, apparently, from what was considered in our last lecture, and, yet, as we shall see, something closely related to the phenomena then discussed. You hear peculiar risings and fallings of sound, peculiar throbbing notes, disclosing an augmentation of sound resembling resonance, and a diminution that approaches interference. This, in fact, is what we actually have, — alternate conditions of resonance and of total interference. As, however, the totality of interference lasts but a very small fraction of a second, the sound seems to be continuous and to vary only in intensity.

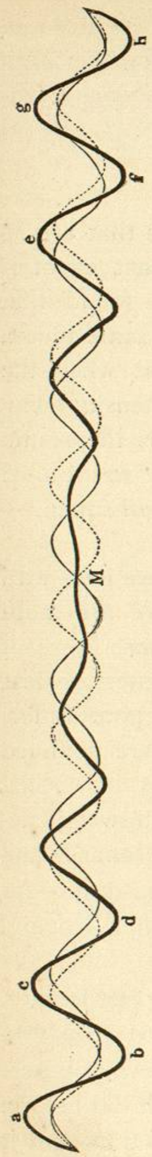

FIG. 140.

That extinction of sound actually occurs, can be demonstrated in various ways. For the present, however, we shall simply consider the phenomena in the light of sinuous curves representing the sounds produced. As before, we shall call the two forks A and B. Suppose now that A, which we shall consider as the loaded fork, makes eight vibrations, while B executes nine. The difference in their frequencies, as a matter of fact, is not so great; but this is immaterial. Viewing them as vibrating in the ratio of 8 : 9, we construct their curves accordingly. Let the light continuous curve (Fig. 140) represent the condensations and rarefactions originated at the fork A, and the dotted curve those proceeding from B. By combining these two curves, as in previous instances, and remembering that the perpendiculars of the resultant curve are always equivalent to the algebraic sum of those of its constituents, we have as a resultant in this case the curve a, b, c, d, e, f,



g, h. In the figure we notice that the two systems of waves commence and terminate at the same points. Hence, at a, b, c, d, e, f, g, h, the crests and troughs will be correspondingly larger. At M, however, crest meets trough, and at this particular point there can be no disturbance.

Translated into the language of sound, these curves signify that when waves of condensation concur, resonance is the result, and that when condensation meets rarefaction, silence ensues. Between the points of maximum resonance and total interference, — that is, between A and M, — there is a gradual diminution of sound; and between the positions of interference and greatest resonance — that is, between M and b — there is a corresponding augmentation. Hence the alternate risings and fallings of sound that are heard when two forks, such as A and B, are sounded together after their unison has been disturbed by so loading one of them as to lower slightly its frequency. Such alterations in the loudness of sounds are called beats, and, as we shall see, are of the utmost importance in acoustics, as well as in music.

When the frequencies of two notes differ from each other by one vibration, there is one alteration of intensity, and, consequently, one beat per second. If two notes differ from each other by two vibrations there will be two risings and sinkings, and, therefore, two beats per second. And, in general, the number of beats per second arising from two notes near unison, sounding at the same time, is equal to the difference of their frequencies.

Let us now apply this knowledge to the determination of the frequency of the loaded fork A. Unencumbered, it executes exactly 512 vibrations per second, as does also its companion B. Loaded, its vibration is something less. Let us see how much. Exciting A and B simultaneously, you hear the same loud distinct beats that were perceived in our previous experiment. Watch in hand, I count the number of beats heard in ten seconds. The number is twenty, and the number of beats for one second is, therefore, two. Subtracting this from 512, we have 510 as the frequency of the fork A as now loaded.

By means of a little wax, a small coin is attached to the fork A. It is thus damped still more. The number of beats audible per second is greater than before. Observation shows that we have thirty-five beats in ten seconds, and, consequently, three and a half in one second. The frequency of the fork A is now reduced to 508.5 vibrations.

Loading the fork A still more, the intervals of reinforcement and diminution succeed each other more rapidly, until finally the beats become so numerous that it is impossible to count them directly. We now become conscious of an unpleasant sensation, which musicians call discord. When two sounds near the middle of the scale give rise to thirty-three beats per second, the discord that ensues is, according to Helmholtz, at a maximum.

But tuning-forks are not our only means of exhibiting the phenomenon of beats. Any two sonorous bodies will, if slightly out of unison, manifest the same alterations in intensity when caused to sound simultaneously.

Let us try these two large open organ-pipes. They are now in unison, each emitting the note $C_2$. By moving downward the slider at the top of one of them, we diminish the length of the vibrating column of air, and at the same time change the pitch of the note emitted. On causing the two pipes to speak, you at once hear, as in the case of the dissonant tuning-forks, loud and very marked beats. If we move the slider upwards the beats succeed each other less frequently, until, finally, when the two pipes sound in unison, they disappear altogether.

We can, however, cause them to break forth again, without touching the slider. It is sufficient to bring the finger near the embouchure of one of the pipes, thus lowering its note, to evoke slow or rapid beats at will. The number of beats, in this case, will depend on how much the embouchure of the pipe is covered. Similarly, by placing the hand on the top of the pipe, and covering



it more or less, we may lower the note, and thereby obtain beats of varying degrees of rapidity.

By means of the pipes furnished with manometric capsules used in our last lecture (Fig. 135), we can observe with the eye the character of these beat-producing tones. To this end, we connect the capsules of the two pipes to the same jet, and ignite the gas that is caused to issue from it. So long as the notes from the pipes are in unison, the flame is quiescent. But no sooner is unison disturbed by moving the slider of one of the pipes, or by putting the finger before the embouchure, than we have beats that cause the flame to dance in time with them. If the beats follow each other quickly, the flame dances with corresponding rapidity. If the beats are slow, as is the case when the two notes are near unison, the flame at once declares the fact.

If now the cubical mirror before the flame be rotated, we have an elongated image of the flame that exhibits most beautifully its intermittent action, and pictures clearly the alternations of resonance and interference. The luminous band seen in the mirror reminds us of the resultant curve given in Fig. 140; the serrated parts of the band correspond to the crests and troughs of the curve, and indicate greater or less coalescence and reinforcement of sound, while the continuous portion of the luminous ribbon, like the middle part, M, of the curve, is certain evidence of total interference.

A very pretty and striking method of observing beats is afforded by means of two singing flames. Before you are two singing flames (Fig. 141) in unison. By raising or lowering a telescopic slider attached to one of the tubes, we can easily change the pitch of the note emitted by the column of air vibrating within the tube. As soon as we thus disturb the unison of the two notes, you hear loud beats that succeed each other with more or less rapidity, — just in proportion as we increase or diminish the interval between the two tones. At the same time you observe a characteristic flickering of the flame. It dances to the beats and keeps perfect time with them.

Beats are very marked in pipe or reed organs tuned according to equal temperament. The so-called tremolo effects given by certain stops of these instruments are due to beats. But bells give rise to beats more readily, perhaps, than other forms of sonorous bodies. This is particularly the case with large bells, and, as we have learned, arises from the impossibility of casting them so that they will be perfectly symmetrical and homogeneous throughout. When ringing, the bell is divided into sections of different sizes, whose periods of vibration differ more or less from one another.

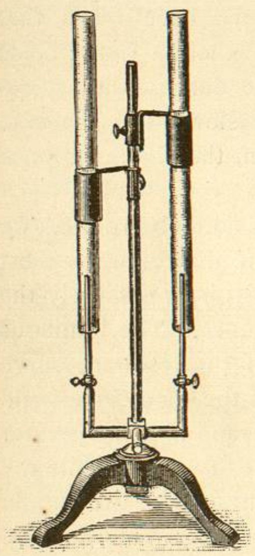

FIG. 141.

On the table are two rare antique Japanese gongs, which either singly or together give forth beats in a most remarkable manner. They are made of bronze, and are quite thin; but the purity and softness of the notes which they emit, and the length of time during which they continue to vibrate after being struck, are quite surprising. The sounding of a number of such bells, properly tuned, in the ancient temples of Japan, must have been productive of effects that were not only pleasing to the ear, but also conducive to solemn religious emotion.

The gongs are placed on small, soft mats, to give mellowness to the tone; and for a similar reason they are struck, not with hard hammers, but with padded sticks. I now strike the large gong, and a delightfully soft and pure note is the result. The beats engendered succeed each other in such a way as to produce a tremolo effect rivalling that afforded by the most perfect musical instrument. On exciting the smaller gong we secure similar results, the only difference being that in this case the pitch is higher. Both gongs, as you observe, are especially rich in upper



partials. By properly striking the gongs, their first upper partials can be made to sound quite as loud as their primes. The primes and first upper partials can now be heard distinctly in all parts of the hall.

When both gongs are struck at the same time, we get a most confused combination of sounds. And the fact that the gongs, when sounding their primes, are slightly out of tune, only intensifies the dissonance when their upper partials are brought out with any degree of force. When, then, both gongs are sounded simultaneously, we have the beats due to each taken separately, and the beats caused by the interferences not only of the primes with each other, but also of the upper partials with each other, and of these partials with their primes. Some of the beats, as you will perceive by listening closely, are very slow, others more rapid, and others again so rapid that they give rise to a rough, rolling noise that is quite painful to the ear. This harshness is observed in chimes of bells when not carefully tuned. It is more prominent in bells than in the gongs we have used, because the tones of the former are more piercing than those of the latter.

Beats furnish us the simplest and the most delicate means of determining when two notes are in unison. Let me illustrate. I take the sonometer and place the bridge as nearly as possible midway between the two supports of the wire. As nearly as I can judge by the eye, the two divisions of the string are equal in length. They should, therefore, give the same note. I excite one section of the string, and as soon as the note produced is extinguished, the other section is agitated. As far as the ear can estimate, there is no difference in the two tones. If we now sound the two divisions of the string together, we at once hear beats that declare the absence of perfect unison. The beats are not very rapid, it is true, because there is very little difference between the frequencies of the two notes. But this difference, slight as it is, manifests itself at once.

By means of beats we are able to distinguish from each other notes that do not differ from each other in frequency by more than one fifth of a vibration in a second. Scheibler's marvellously accurate system of tuning is based on beats entirely. According to his system, there is no attempt made to bring the note of a string, pipe, or reed into unison with a standard of pitch directly. The work is done indirectly, but with a degree of accuracy that is well-nigh absolute. For this purpose a specially constructed set of forks is required, giving notes just four vibrations lower or higher than those which are to be attuned. To tune a piano, for instance, its note of $A_3$ is made to give just four beats per second with a fork that makes exactly that number with a standard $A_3$ fork, whose absolute number of vibrations is known. We are thus certain that the piano-string executes the same number of vibrations as the fork taken as the standard of pitch. By this method any one who can count beats is capable of tuning.

On the table are two sets of forks, — thirteen in each set, — one of which gives the tempered chromatic scale from $C_3$ to $C_4$, according to French pitch, — $A_3 = 435$ vibrations per second, — while the other furnishes the same notes heightened by precisely four vibrations, and generating, consequently, four beats per second.

Allow me to show you how such forks are used. I will take A of the second set of forks, — these are called *auxiliary forks*, — and adjust the string on our sonometer so that it will generate just four beats per second when sounded with the fork chosen. A few moments only are required for the adjustment. When it is once attained, as we know by counting the beats, we are certain that the string is executing exactly 435 vibrations per second, and emitting the note $A_3$ of the standard of pitch of the French Conservatory.

In a similar manner we could, by means of these forks, tune all the notes of an entire octave — from $C_3$ to $C_4$ — of any musical instrument whatever. Musicians, however, are not so exact. They are satisfied to get the pitch of one note right, — generally $A_3$, as above, or $C_4$, — and then



proceed from this one note to tune all the others by ear, by estimation of the fifths. The accuracy of tuning in this manner varies, of course, with the delicacy of the tuner's ear. For this reason no two persons, except by chance, would tune exactly alike. And for a similar reason, no one, who is thus guided solely by his ear, could tune in succession two instruments that would be perfectly in unison.

For perfect tuning, one of Scheibler's tonometers is indispensable. The two sets of forks before you are sometimes called tonometers, because Scheibler's method is used in connection with them. But the tonometers which were devised and used by Scheibler consisted of a series of forks not only extending over a whole octave, as do those on the table, but also giving four beats for every possible note within the octave. Thus, one of his tonometers intended for the octave $A_2$ to $A_3$, German pitch, — that is, from 220 to 440 vibrations per second, — embraced fifty-six tuning-forks. Beginning with $A_2$ of 220 vibrations, each fork in succession of this tonometer was tuned exactly four beats higher than the one preceding.

Koenig makes, on Scheibler's principle, superb tonometers of sixty-seven forks for the octave from $C_3$ to $C_4$. In addition to this, he has, with the expenditure of infinite labor and skill, constructed a like tonometer, as we saw in our second lecture, for the entire compass of musical sounds. By means of this unique instrument one may determine with ease the absolute pitch of every note from $C_{-2}$ to $F_9$.

By means of one of Lissajous' apparatus, as modified by Mercadier and constructed by Duboscq, I am able to give you a most telling optical illustration of the phenomenon of beats. The apparatus consists of two tuning-forks (Fig. 142), one of which is provided with a coil so that it may be kept in vibration electrically. The fork A carries a style on one of its prongs, while one of the prongs of the fork B bears a piece of smoked glass. This latter fork is also furnished with sliding weights, by means of which it may be made to give various intervals with the fork A.

The ends of both forks, with style just touching the smoked glass plate, is adjusted over the condenser of the lantern. The fork B is now set in vibration by passing an electric current through the coil fastened between the two prongs. This causes the style of the fork A to inscribe a straight line on the smoked glass.

If now the fork A is also caused to vibrate, it will tend to make this straight line longer or shorter, according as it moves in the same direction as the fork B, or in an

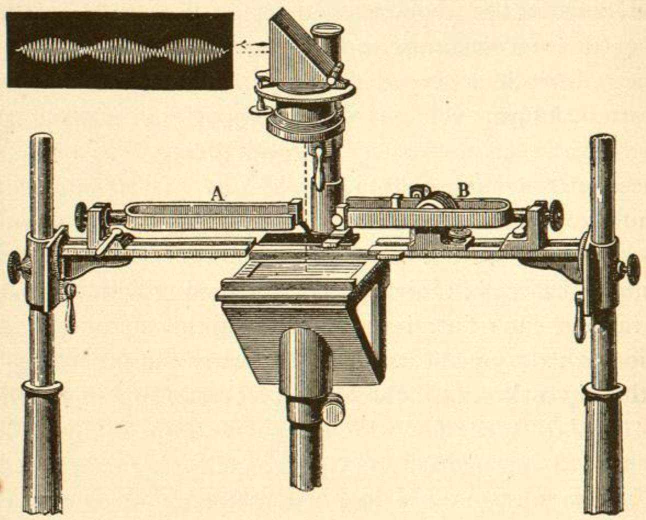


FIG. 142.

opposite direction. When, then, the two forks are in unison, they will reinforce or enfeeble each other according as they are in the same or in opposite phases. This reinforcement or enfeeblement will be indicated by the line traced on the glass, which will be longer or shorter when the two forks are simultaneously acting than when only one is in motion. When the fork A is moved in a line parallel to its axis, the straight line on the glass will change into a beautiful sinuous curve like those shown in the second lecture. The objective and the right-angled prism at the top of the lantern enable me to project on



the screen all the figures that the style of one fork may inscribe on the glass of the other.

I disturb the unison of the two forks by moving the sliding weights on the fork B. When both forks are at rest the result of this disturbance of unison is to cause the line inscribed on the glass plate to alternately lengthen and shorten, as we see by the image on the screen. The number of alterations in any given time will depend on the number of beats per second made by the two forks; and the number of beats, as we have seen, depends on the difference of the frequencies of the forks. If there is only one beat per second, the alterations in the length of the line will occur once every second. If there are two or more beats per second, the lengthening and shortening of the line will take place correspondingly often. Under these circumstances, if the fork A is moved slowly and uniformly to the left, — that is, in a direction parallel to its axis and to the length of the plate, — we observe a sinuous line as before, but one whose indentations have a varying amplitude from a maximum to zero. This variation in the amplitude of the curves shown on the screen exhibits to the eye the difference in the rates of vibration of the two forks, while their beats declare the same thing to the ear.

I now adjust the sliding weights again, and while the two forks are in vibration I move A to the left, as before, and you have the result on the screen as a beautiful undulating curve, which tells more clearly than words the nature of the combined motion of the two forks. The forks used are not tuned to give any particular note, nor are they constructed to give a very loud sound; but if you will listen attentively, you will be able to perceive beats succeeding each other at the rate of about two per second. And if you compare the number of beats with the rhythmic action of the image on the screen, you will find that the beats produced synchronize perfectly with the formation of the spindle-shaped segments of the sinuous curve on the screen.

M. Lissajous has taught us how to vary this experiment so as to obtain the same results in an equally striking and pleasing manner. His method is so beautiful, and its applications are so general and of such importance, that every one interested in acoustics should be familiar with it. We are again indebted to M. Mercadier for devising for us a modified form of Lissajous' original apparatus. Mercadier's apparatus is more convenient than the one Lissajous

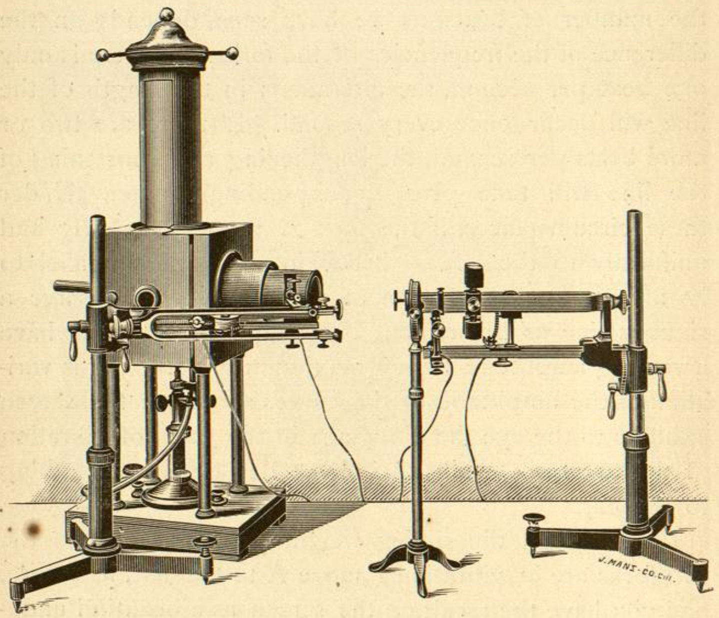

FIG. 143.

used, and enables me to show Lissajous' figures, as they are called, to a large number of persons at the same time.

We use two tuning-forks similar to those used in the preceding experiment. Both are mounted, so as to be kept in vibration at will by an electric current. The only respect in which the forks now used differ from those just employed is that the style and vibrating plate are replaced by polished steel mirrors attached to the ends of the prongs of each fork. One of the forks (Fig. 143) is so



placed that its mirror receives a beam of light coming from the lantern to the left. The light is then reflected from this fork to the mirror on the second fork, and thence reflected to the screen, through a lens, supported on an appropriate stand. One Grenet cell is connected with

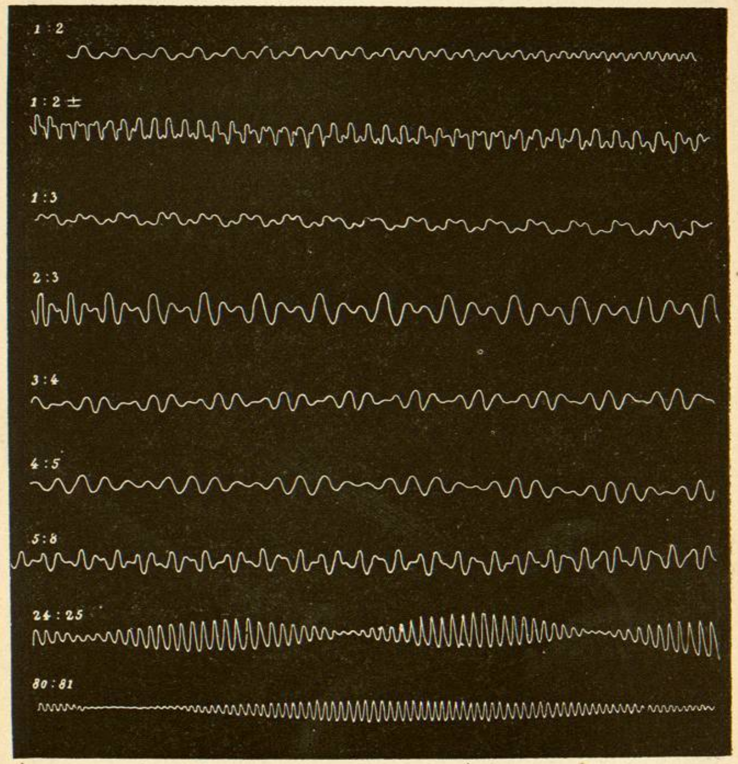


FIG. 144.

each fork, which is thus kept vibrating as long as may be desired. When the two forks are in unison, they tend to reinforce or to weaken each other, according as they are in the same or in opposite phases.

When the forks are so adjusted that they vibrate in the same plane, the image of light seen on the screen can be made to go through all the various changes, and in the same manner, as the inscriptions on the smoked glass in