we can comprehend the nature of the link that binds mind and matter, then, and not till then, may we hope to have some insight into the nature of the phenomena here presented to us, to understand how motion can originate sensation, and how vibrations of different periods can be changed, translated, as it were, into what appeals to our senses, as heat, light, and sound.

CHAPTER X.

MUSICAL INTERVALS AND TEMPERAMENT.

THE sounds with which the acoustician deals range in frequency from sixteen to nearly fifty thousand vibrations per second. Of these, only a comparatively small number are employed in music, and they must always bear to each other certain definite relations of pitch. The ratios of frequencies which characterize such sounds are called *intervals*. Thus, two notes whose frequencies are as 2 : 1 constitute the interval of an octave. Two notes whose vibration-numbers are as 3 : 2 give the interval of a fifth. These intervals are independent of the position that the notes may occupy on the scale. Provided the ratio of their frequencies remains the same, the interval retains the same name, whether the notes are in one part of the scale or in another.

The *gamut*, or *diatonic scale*, embraces a series of eight notes, the first and last of which have the same names, and are separated from one another by the interval of an octave. The notes of the gamut have been designated by the letters —

$$C, \quad D, \quad E, \quad F, \quad G, \quad A, \quad B, \quad C_2.$$

Considering the frequency of C as unity, the frequencies of the notes of the scale, including the *tonic*, or first note, will be proportional to the numbers —

$$1, \quad \tfrac{9}{8}, \quad \tfrac{5}{4}, \quad \tfrac{4}{3}, \quad \tfrac{3}{2}, \quad \tfrac{5}{3}, \quad \tfrac{15}{8}, \quad 2.$$

Dividing each of these notes by that which precedes it, we obtain the intervals between the successive notes of the scale. The intervals of the major scale are as follows: —

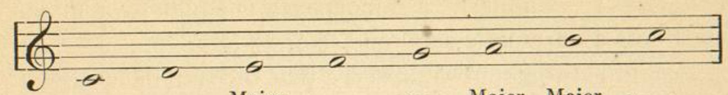
C,	D,	E,	F,	G,	A,	B,	C_2,
$\frac{9}{8}$,	$\frac{10}{9}$,	$\frac{16}{15}$,	$\frac{9}{8}$,	$\frac{10}{9}$,	$\frac{9}{8}$,	$\frac{16}{15}$.	

The consecutive intervals from C to C_2 are called *a second, a major third, a fourth, a fifth, a major sixth, a major seventh,* and *an octave* respectively. C, from which the intervals are reckoned, is called the *tonic*, or *key-note.* Musicians call the fifth note above the key-note the *dominant*, and the fifth note below, the *sub-dominant.* When the key-note is C, the dominant is G, and the sub-dominant is G in the octave below C.[1]

In the foregoing scale, as will be observed, there are only three different intervals, viz., $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$. The first, $\frac{9}{8}$, is called a *major tone;* the second, $\frac{10}{9}$, is named a *minor tone;* and the last is known as a *major* or *diatonic semitone.* The last interval, although called a semitone, is a little more than the half of a major tone. Adding together two semitones, — which is done by multiplying the frequency-ratio, $\frac{16}{15}$, by itself, — we obtain a number which is slightly greater than $\frac{9}{8}$: — $\frac{16}{15} \times \frac{16}{15} = \frac{256}{225}$; $\frac{256}{225} : \frac{9}{8} :: 2048 : 2025$. Two semitones are therefore greater than a major tone in the ratio of $2048 : 2025$. Subtracting a major semitone from a minor tone gives a *minor* or *chromatic semitone,* $\frac{10}{9} \div \frac{16}{15} = \frac{25}{24}$. This interval is the smallest usually employed in music. A less interval, and one of considerable importance in theoretical music, is a *comma.* It is yielded by subtracting a minor from a major tone, $\frac{9}{8} \div \frac{10}{9} = \frac{81}{80}$. Tones, like major and minor tones, that differ from each other by only a comma, are considered in music to have the same value. The same may be said of major and minor semitones. The ratio between these two being less than a comma, — $\frac{16}{15} \div \frac{25}{24} = \frac{128}{125}$, — they are regarded as semitones of equal value.

[1] When necessary, the subdominant is transposed into the octave of the tonic.

The various notes of the diatonic scale are, with respect to their frequencies, related to one another as follows: —



Tonic.	Second.	Major Third.	Fourth.	Fifth.	Major Sixth.	Major Seventh.	Octave.
C_3	D	E	F	G	A	B	C_4
256 :	288 :	320 :	341.3 :	384 :	426.6 :	480 :	512
1 :	$\frac{9}{8}$:	$\frac{5}{4}$:	$\frac{4}{3}$:	$\frac{3}{2}$:	$\frac{5}{3}$:	$\frac{15}{8}$:	2

Using the smallest whole numbers expressing these ratios, we have —

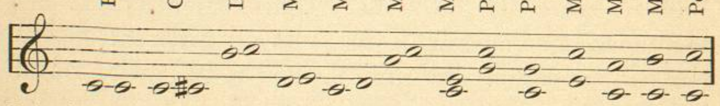
$$24 : 27 : 30 : 32 : 36 : 40 : 45 : 48$$

All the intervals here given, with the exception of the second and the major seventh, are what are known as *consonant* intervals. The second and the seventh form intervals that are called *dissonant.* The ratios corresponding to the former are expressed by small whole numbers, and the more consonant the interval, the smaller the whole number expressing the ratio. Hence, after unison, the most consonant interval is the octave. After the octave come in succession the fifth, the fourth, the major third, and the major sixth.

Dissonant intervals, on the contrary, are characterized by ratios composed of large numbers, and the amount of dissonance to which any two sounds may give rise may, at least in the middle portion of the musical scale, be determined by the ratio of their vibration-frequencies. Thus the interval of a major tone, $\frac{9}{8}$, is, in the lower portions of the musical scale, markedly dissonant. A diatonic or a chromatic semitone, whose vibration ratios are respectively $\frac{16}{15}$ and $\frac{25}{24}$, are, in similar portions of the scale, far more dissonant. The intervals $\frac{2}{1}$, $\frac{3}{2}$, $\frac{4}{3}$, are called respectively the perfect octave, the perfect fifth, and the perfect fourth, to distinguish them from certain diminished or augmented intervals of the same name. They are said to form *perfect consonances,* in contradistinction to the intervals $\frac{5}{4}$, $\frac{5}{3}$, and the minor third, $\frac{6}{5}$, and the minor sixth, $\frac{8}{5}$, which are denominated *imperfect consonances.* The minor third is obtained by subtracting a chromatic semitone from a major third: $\frac{5}{4} \times \frac{24}{25} = \frac{6}{5}$. Similarly, a minor sixth is equal to a major sixth less a chromatic semitone, $\frac{5}{3} \times \frac{24}{25} = \frac{8}{5}$.

From the foregoing we observe that the sum of two intervals is obtained by multiplying, not by adding, their ratios together. Thus a fifth added to a fourth yields an octave, $\frac{3}{2} \times \frac{4}{3} = \frac{12}{6} = 2$. When we wish to subtract one interval from another, we divide the ratio of one by that of the other. Thus, a fifth minus a major third equals a minor third, $\frac{3}{2} \div \frac{5}{4} = \frac{12}{10} = \frac{6}{5}$.

The various intervals thus spoken of are written in musical notation as follows: —



	Perfect unison.	Chromatic semitone.	Diatonic semitone.	Minor tone.	Major tone.	Minor third.	Major third.	Perfect fourth.	Perfect fifth.	Minor sixth.	Major sixth.	Major seventh.	Perfect octave.
Ratio of vibration	$\frac{1}{1}$	$\frac{25}{24}$	$\frac{16}{15}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{2}{1}$
Logarithm of ratio	0	18	28	46	51	79	97	125	176	204	222	273	301

In addition to the intervals given in the above table there are several others used in music, obtained by the inversion of the former; but we have no time to consider them here.

In the preceding diagram, as will be remarked, the intervals are expressed in logarithms[1] as well as by fractions. In point of clearness and intelligibility the logarithmic are much superior to fractional values. By means of logarithms we can tell at a glance the interval between any two sounds whatever, provided we know the numbers of their vibrations. It is immaterial whether they belong to the

[1] The logarithms, it will be noticed, are considered as whole numbers, the usual index and decimal point being omitted.

musical scale or not. Expressed in logarithms, the interval of a comma is 5; the interval between C♯ and D♭, called an enharmonic diesis, is 6; while that of a mean semitone, as employed on tempered instruments, is 25.

To find the sum of two intervals, we add together their logarithmic values. The logarithms of the intervals of a fourth and a fifth are, as given above, 125 and 176 respectively. But $125 + 176 = 301$, the value of an octave. A major third, 97, added to a minor third, 79, equals a fifth, 176. The difference between two intervals is found by subtracting the logarithm of the lesser interval from that of the greater. A fourth, 125, minus a major third, 97, is equivalent to 28, a diatonic semitone. A major tone, 51, less a minor tone, 46, gives a comma, 5.

From what has been said, we learn that from the very large number of different sounds only a small proportion of them can be used for purposes of music; and those that are so used must form a certain fixed determinate series. We cannot slide, or proceed by a continuous transition, from one sound to another, but must advance by degrees; that is, we must ascend or descend by definite steps.[1] In the octave there are seven such steps, of varying size and position. Between these steps of unequal height, the demands of our modern music sometimes require the interposition of other steps, of lesser magnitude. Such a succession of sounds is called a *scale*, from the Latin word *scala*, "ladder," or "stairway." The French use the word *échelle*, which also means a ladder. The German term for scale is still more expressive. It is *tonleiter*, — that is, a "tone-ladder," or a ladder of musical sounds.

A question now naturally suggests itself. Is the diatonic scale, of which we have been speaking, something conventional and empirical, or is it founded on some law of Nature? The majority of musicians, I am inclined to think, would claim for the scale a natural origin. It so

[1] As an ornament, under the name *portamento*, a continuous slide is sometimes permitted in vocal and instrumental music. In instruments with fixed notes, as is obvious, a slide is impossible

well satisfies our ideas of cadence, modulation, and tonality, it comes so natural to sing it, it is so pleasing to the ear, that there is a wide-spread impression that it must rest on natural laws, and is therefore quite independent of all æsthetical principles.

The late M. Fétis, the eminent musical historian, says in reference to this subject: "It is an opinion generally held that the succession of sounds known in the modern music of European nations, and formulated by their major and minor scales, is the result of some fundamental and immutable law, and that diatonic music — *i. e.*, music in which the sounds succeed one another in tones and semitones — is the music of Nature.

"According to the doctrines of many theorists and historians of the art, the sentiment of the necessity of these diatonic relations of sound ought to have preceded every other conception of tonality, and man would have been incapable of imagining a kind of music inconsistent with these relations.

"I do not hesitate to declare that this opinion is absolutely contrary to what history teaches us by facts of the most unquestionable authority.

"We learn by these facts that diatonic music is not the most ancient; on the contrary, we have proof that none of the nations of antiquity adopted it, and that there exist still peoples to whom it is entirely strange. The examples of music of the ancients are sufficient to prove the non-existence of this assumed natural law of diatonic progression. It is not difficult to establish among primitive nations systems of sounds differing altogether from it; and it is possible to trace the progressive transformations by which the modern diatonic scale has been developed, at a comparatively late period, from some of the primordial systems differing from it almost entrely."[1]

The first one to adopt a succession of notes that approached our modern diatonic scale was Pythagoras, the

[1] Quoted from "Histoire Générale de la Musique," par E. F. Fétis, in "The Philosophy of Music," by Mr. William Pole, F.R.S

founder of theoretical music.[1] The diatonic scale now used was introduced by Zarlino, and the first account of it is found in his "Instituzioni Armoniche," published in 1558. According to numerous and varied experiments made by MM. Cornu and Mercadier, the best performers on stringed instruments still follow the Pythagorean scale when playing a melody, and adopt that of Zarlino only when they play pieces in which two or more notes are sounded simultaneously. Accepting their results as true, — and they seem to be well established, — we must conclude that the scale of Pythagoras is more suitable for melody; while the modern diatonic scale — the scale of Zarlino — is preferable for harmony.

All nations that have had any pretensions to anything approaching a musical system have adopted the division of the scale into cycles of octaves. The Chinese divide the octave theoretically into twelve equal parts, corresponding to our semitones. Practically, however, they use only five notes, whose intervals correspond to the black notes of the pianoforte. The same pentatonic division of the octave is also found to a certain extent in the so-called Scotch music.

The Arabs recognize both the octave and the fifth in their system of music. But their system is even more complex than that of the Chinese. It is divided into sixteen or seventeen unequal intervals, and is entirely different from anything that obtains among Western nations.

The Hindoos theoretically divide the octave into twenty-two parts. Their practical scale, however, consists of only seven degrees. In addition to the octave and the fifth, they also employ the interval of the fourth.

[1] The intervals of the Pythagorean scale are as follows: —

C		D		E		F		G		A		B		C_2
	$\frac{9}{8}$		$\frac{9}{8}$		$\frac{256}{243}$		$\frac{9}{8}$		$\frac{9}{8}$		$\frac{9}{8}$		$\frac{256}{243}$	

In this scale, as will be observed, there are only two intervals, — the tone and the semitone, or hemitone as it was called by the Greeks. The intervals of the fourth, the fifth, and the octave are the same as in Zarlino's scale. The major third, sixth, and seventh are increased by a comma, while the semitone is diminished by a like amount.

The music of the Persians, like that of the Hindoos, is characterized by the subdivision of the octave into minute intervals. But while the latter had, at least in theory, twenty-two divisions, the former had twenty-four. Each interval, therefore, of a Persian octave would be equivalent to what we call a quarter tone.

The Persian system of music has a special interest for us, because of its influence on Greek music, from which our own was eventually evolved. The Oriental nations of to-day, like the Greeks of ancient times, had a delicate estimation of sounds that to us is quite astonishing. The Greeks frequently used quarter tones, as do the Arabs and Persians of the present time. It is for this reason that an analysis of the music of Oriental nations has always been such a puzzle to musicians and scientific men. Even now, after all the study that has been bestowed upon the musical systems of the Orient, there are still wanting many important data to enable us to form a correct theory regarding even one of the numerous systems that there prevail.

It is sufficient, however, for our present purpose to know that different musical systems have obtained during the course of the world's history; that even now there are various systems in vogue, whose differences are so great that, with the exception of the octave and the fifth, they have scarcely anything in common. From these facts, and from what precedes, it is, then, evident that the diatonic scale has not that basis in Nature that so many people maintain it has. It is, on the contrary, the outgrowth of long centuries of study and experiment, and the product of the æsthetic sense of the untold number of musicians and composers who have given us our present system of music, and made it what it is to-day.

But although the diatonic scale, in its entirety, is not the scale of Nature — that is, a series of notes dictated by certain physical or physiological laws — that it is so often claimed to be, it would be a mistake to contend that it has no foundation whatever in Nature or on Nature's laws.

There are, at least, parts of the scale that have a natural origin, and seem to arise from the very nature of sound itself. But the parts of the scale that can be shown to rest on an undeniable physical basis are a very small proportion of the scale taken as a whole

The first interval that can be proved to have a natural origin is unquestionably the octave. We have learned that nearly all musical sounds are compound, and that it is possible, even by the unaided ear, to recognize some of the constituents of a given compound note. Among those most readily detected in stringed instruments, such as were used by the Greeks, would be the second and third harmonic partials. But these constitute the octave and the twelfth of the fundamental, and, as has been stated, can be recognized even in the human voice. And for people like the Greeks, who had such acute ears for small intervals of tone, it would be unreasonable to suppose that they were incapable of hearing the partials that can be perceived by even an untrained musical ear

Again, the octave is, from its very nature, only a replicate — a kind of repetition — of the fundamental. So much is this the case that even practised musicians sometimes mistake the second for the first partial. And then, furthermore, we must not forget the natural tendency, with which every one is cognizant, that there is to sing in octaves. Thus, a boy or a woman, in accompanying a melody sung by a bass voice, or played on a bass or barytone instrument, will naturally sing an octave higher. And provided they have an average musical ear, they can do this without any musical education whatever. "When, then, a higher voice," says Helmholtz, "executes a melody an octave higher than a grave voice, we hear again a part of what we heard before; namely, the evenly numbered partial tones of the grave voice, and at the same time we hear nothing that we had not previously heard."

The third partial, or the twelfth above the prime, is, as you know, just a fifth above the second partial. In many instances it can be more readily detected in a compound tone than the octave itself. And so perfectly does the interval of the fifth answer the requirements of the ear that even unpractised singers find it quite natural to take a fifth to a chorus that does not quite suit the pitch of their voice.

What has been said of the octave and the fifth may, in a limited manner, be predicated also of the fourth. But I do not think that we can claim a natural origin for any of the other intervals of the diatonic scale.

The fact that the intervals of the octave, the fifth, and the fourth have a physical basis in the partials of compound tones accounts most probably for the manner of tuning the earliest forms of the Greek lyre. The lyre, as we are informed by Boethius, was, to the time of Orpheus, an instrument of four strings, whose intervals would be represented by the notes $C : F : G : C_2$. Only the order of succession of the notes is indicated by the letters given, as their pitch is unknown.

The remaining intervals of the diatonic scale are more or less arbitrary and the results of numberless experiments to secure such notes as would best answer the purposes of melody and harmony. No one who has examined the subject would for a moment maintain that there is anything in Nature to suggest the intervals of a tone or a semitone. We readily sing the diatonic scale, with its different tones and semitones, as a matter of education; but it is quite certain that no one uninstructed in music would ever naturally sing this scale, however accurate and delicate his ear.

But another question now presents itself. Why was not the octave divided into equal instead of unequal parts? The answer generally given to this question is that it would be difficult for the ear to appreciate uniform divisions, and because, too, of the difficulty, if not impossibility, of the unaided voice to divide the octave into any given number of equal parts. Hence the unequal divisions, some of which were suggested by the harmonic partials that now characterize our major scale.

A succession of single tones, in an order pleasing to the ear, constitutes what is called *melody*. An air or tune sung by a single voice or played on an instrument of any kind, one note at a time, is a melody. As to its structure, music was first developed in the form of melody. Indeed, according to Helmholtz, melody is the essential basis of music. It has been cultivated from the earliest times, and in the musical systems of most of the Oriental nations it is the only form of music yet known.

The simultaneous sounding of two or more tones whose intervals are concordant produces *harmony*, and the combination of two or more notes that are thus harmonious constitutes *a chord*.[1] Two notes constitute a *dyad*, three form a *triad*, and four a *tetrad*. In order that a chord may be consonant, all the intervals composing it must be concords. Three notes, whose rates of vibrations are as $1 : \frac{5}{4} : \frac{3}{2}$, or as $4 : 5 : 6$, — that is, a triad made up of a tonic, a major third, and a fifth, give us the *perfect major chord*. To this chord it is usual to add the octave of the tonic, which gives us four notes that are separated from each other by the intervals $\frac{5}{4}$, $\frac{6}{5}$, $\frac{4}{3}$. By changing the order of these intervals so that they read $\frac{6}{5}$, $\frac{5}{4}$, $\frac{4}{3}$, we obtain the *perfect minor chord*. As you will observe, the only difference between the major and the minor chords is in the order in which the intervals $\frac{5}{4}$ and $\frac{6}{5}$, the major and minor thirds, succeed each other. In the former chord the major third comes first; in the latter it is the minor third that takes precedence. These two chords are at the basis of our modern system of music; and although the difference in ratios of their constituent intervals is very slight, their effect on the ear and the mind is so great that they are

[1] Hauptmann, the great authority on musical theory, draws a very precise distinction between melody and harmony. The former, according to him, conveys the idea of motion, the latter the idea of rest. Melody must go on, otherwise it ceases to be melody. In harmony, however, even though it stand still, the musical idea is complete. There are, indeed, progressions in harmony, but these constitute a succession of distinct ideas, each more or less complete in itself. In melody, on the contrary, it is succession only that forms one idea as a whole.

employed to express entirely different ideas and passions. In musical notation the perfect major and minor chords are written: —

These chords may be inverted in various ways, and caused to go through quite a cycle of changes; but we have not the time to consider them here.

The essential distinction, then, between melody and harmony is seen to be that in the former there is a succession of simple notes, while in the latter we have a chord or a succession of chords. And this, in truth, is the great difference between the music of the ancient nations, who knew little or nothing of harmony, and our modern music; between the systems of music that still prevail in the semi-civilized nations of Asia and that which obtains amongst the more cultured peoples of Europe and America.

We have seen that concords follow each other in the order of smoothness, — from the interval of the octave to that of the minor sixth. But the smoothest intervals are by no means the most gratifying to the ear. This is shown in the frequent use of thirds and sixths in two-part music as compared with the employment of the concords. Thirds and sixths, at least in our modern music, are considered to have a charm and richness that the fifth, the fourth, and the octave possess in only a comparatively inferior degree. Certain æsthetic reasons have been assigned for this preference, but it seems to result rather from habit and education. Time was when both thirds and sixths were used very sparingly by musicians, and when they were considered at best as only imperfectly consonant. Both these intervals were unknown to the Greeks and the Romans, and were introduced into our present system of music in comparatively recent times.

An interval of special interest to acousticians and to theoretical musicians is the harmonic, or sub-minor seventh,

the vibration-frequencies of whose notes are 4:7. But this interval, although in some instances more harmonious than the minor sixth, 5:8, is not used in practical music. No satisfactory explanation seems to have been yet offered why it has not been adopted. The most that can be said against it is that it is strange, and that it has an effect on the ear that is quite different from that of any of the intervals with which we are familiar. It cannot be urged, as is sometimes done, that the interval is dissonant, because, as just stated, it is often less so than an interval that is frequently employed, — the minor sixth. Indeed, the sub-minor seventh is, in some instances at least, quite pleasing and harmonious, and it may in certain cases contribute very materially to the richness and brilliance of certain chords played on instruments that are tuned in pure intonation.

Having spoken at some length of scales, intervals, and chords, we are now prepared for their experimental examination. For this purpose we shall use tuning-forks and sirens, as they are better adapted to our purpose than anything else.

Before you is a set of tuning-forks on resonant cases, giving the diatonic scale. They embrace the octave extending from C_3 to C_4. When they are sounded in succession the musicians present will find that the intervals, although mathematically exact, are slightly different from those to which they are accustomed. Some of the notes appear too flat, others too sharp. The reason for this will be manifest when we come to consider the tempered scale which is now universally used in music.

When the notes constituting the consonant intervals of which we have been speaking are sounded simultaneously, the result is entirely different from that yielded when the same notes are sounded on any of our keyed instruments like the organ or pianoforte. I sound in succession the fifth, the fourth, the major and the minor thirds, and the sixth, and they all give consonances that are marvellously pure and harmonious. And not only do we hear the notes that are emitted by the forks that are agitated by the bow, but also their corresponding beat-tones. They come forth at times so loud and clear that it is difficult to believe that they are not produced by corresponding forks. I now sound C_4 and the seventh harmonic partial of C_2. The frequencies of the notes of these forks are as 4:7. We have, therefore, the tabooed harmonic, or the sub-minor seventh. The result, I think all will confess, is almost as gratifying to the ear as some of the consonant intervals to which you have just been listening. I can even fancy that some of the musicians present would be glad to have this interval introduced into our musical system forthwith. The effect is new, I admit, but I think that not even the most pronounced partisans of our present system of music would declare it to be unpleasant. In the music of the future it may be reckoned as a consonance. Who knows? The major thirds and sixths, and still more the minor thirds and sixths, had to struggle a long time for recognition. But it came at last, and they are now among the most popular intervals in music. And so may it be in a measure with the sub-minor seventh. It has many friends even now, and their number is daily increasing. Music is pre-eminently a progressive art, and it is difficult to foresee what modifications it will admit in the not distant future.

A siren devised by Oppelt enables us to push our investigations still farther. This instrument — an elaborate form of the instrument invented by Seebeck — consists of a large disk of copper (Fig. 174), pierced with 24 concentric circles of holes. Fifteen of the circles yield simple notes, 5 give different intervals of the diatonic scale, and the remaining ones furnish 4 of the more common chords.

The siren is now mounted on a rotator and caused to revolve. When a stream of air is directed by means of a suitable tube against any of the circles of holes, you hear notes exactly like those produced by Seebeck's siren. Blowing against the circle having 12 holes, and then against that having 24 holes, we produce, as you hear, two notes that are separated from each other by the interval of

an octave. There are other circles having 36, 72, 96, 144, and 192 holes. 36 and 72, 72 and 144, 96 and 192, taken in pairs, have the same ratio, viz., 1 : 2, and hence give the same interval as the circles having 12 and 24 holes. The circles that have respectively 12 and 18 apertures — ratio 2 : 3 — yield the interval of a fifth. Similarly, the circles having 12 and 15 holes — ratio 4 : 5 — give the major third. In like manner, we might by suitable combi-

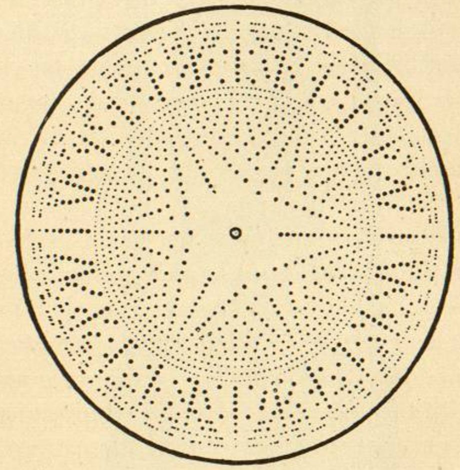


FIG. 174.

nations obtain all the other intervals of which we have been speaking.

But the important fact in this experiment, and the one to which I wish especially to direct your attention, is that these intervals are entirely independent of the pitch of the notes composing them. Whether the disk be rotated rapidly or slowly, the frequency-ratio of any two notes remains the same. Provided, then, that the relative pitch of any two notes remains constant, the interval remains unchanged, whatever the position of the interval in the scale.

From the intervals we have been considering, it is but a step to chords of three or more notes. We shall try here only the perfect major chord, — C, E, G, — the relative