frequencies of whose notes is 4 : 5 : 6. When a current of air is directed against the holes composing this chord, and the disk is caused to revolve, the harmony at once bursts forth pure and clear. Whether the disk move slowly or rapidly, whether the pitch of the notes be high or low,

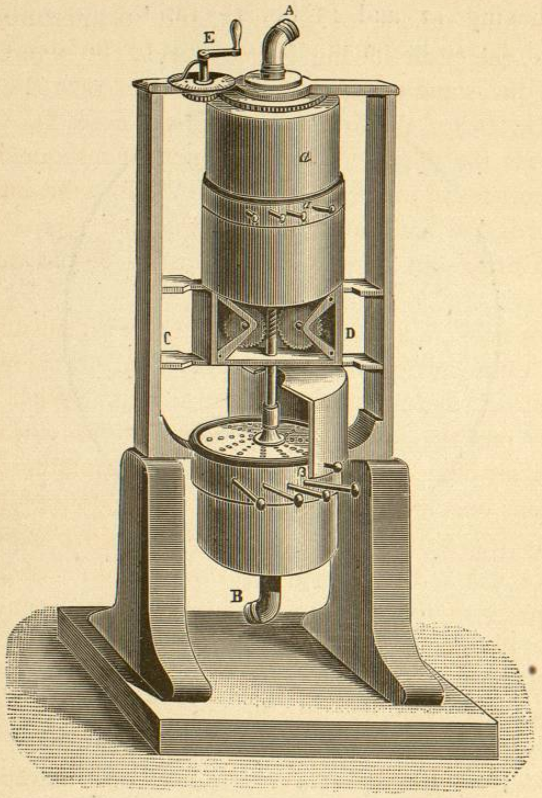

FIG. 175.

the character of the chord, as you perceive, remains unchanged.

But let me make you acquainted with a more elaborate and a more available instrument than the siren of Oppelt. Before you (Fig. 175) is a double siren devised by Helmholtz. It is composed of two of Dove's polyphonic sirens, connected by a common axis. Dove's siren differs

from the siren of Cagniard de la Tour in having two or more circles of holes, — that of Cagniard de la Tour having, as you remember, only one. The Dove sirens used in the instruments before you have each four series of holes, disposed in concentric circles. The lower disk presents four series, of 8, 10, 12, and 18 holes; the upper disk has four others, of 9, 12, 15, and 16 apertures. If, therefore, the circle having 8 holes yields the note C_1, the siren, at the same velocity of rotation, will give the notes C_1, E_1, G_1, D_2 for the lower disk, and the notes D_1, G_1, B_1, and C_2 for the upper. The instrument, consequently, is competent to produce all the more important intervals and chords we have been considering, and serves admirably to bring out all the characteristics of the diatonic scale.

The orifices *A* and *B* are connected with an acoustic bellows by means of an India-rubber tube. When air is urged into *A*, the upper siren alone sounds. When air is admitted into the lower siren, it only becomes vocal. If, however, air be admitted into both orifices simultaneously, both sirens become sonorous. The number of revolutions made by the sirens is recorded by the clock-work *CD*. The keys at *a* and *b* correspond each to a series of orifices in the parts of the air-chambers opposite the openings *A* and *B*. By means of a toothed wheel and pinion at *E*, not only the disk of the upper siren, but also the air-chamber above the disk, can be made to rotate both forwards and backwards. Both the upper and lower sirens are surrounded by brass boxes, divided into halves so as readily to be attached to or removed from the instrument. One half the box is removed from the lower siren, while both halves are seen enclosing the one above. These boxes act as resonators, and their office is to augment the volume of the prime, while the upper partials of the compound tone of the siren are correspondingly damped. The moment the tone of the siren is in unison with that of the box, the note emitted bursts forth with extraordinary purity and power.

When air is simultaneously urged into both wind-chests, with the two circles of twelve apertures open, we have perfect unison. If, now, a series of 8 holes in the lower, and 16 in the upper siren be opened, we obtain the interval of an octave. Opening the series of 9 in the upper and 18 holes in the lower siren, the same interval is given, although the absolute rates of vibration have been increased. But the ratio of the two rates remains the same, being in both cases as 1 : 2. Opening a series of 10 apertures in the lower, and 15 in the upper siren, or of 12 holes in the upper and 18 in the lower, we have, in both instances, the interval of the fifth, because in both cases the ratio of the rates of vibration is as 2 : 3. By opening the series of 9 and 12, or of 12 and 16, — in both of which cases the ratio is 3 : 4, — we obtain the interval of a fourth. Similarly the two series of 8 and 10, and 12 and 15, yield the interval of a major third, expressed by the ratio 4 : 5. In like manner the series 10 and 12, or 15 and 18, give the interval of a minor third, whose frequency-ratio is 5 : 6. When we open the series having 8 and 9 apertures, we obtain the interval of a major tone. The series 9 and 10, for a like reason, give the interval of a minor tone. The series whose orifices number 15 and 16 respectively yield, when sounded at the same time, the interval of a major semitone.

The last three intervals, when the siren is moving at an ordinary velocity, are remarkably dissonant. The reason of this is because of the beats, which are very loud and distinct. We can, however, so increase the velocity of the siren as to cause the beats corresponding to the intervals 8 : 9 and 9 : 10 to coalesce and give rise to pure, clear beat-tones. The harshness of the interval is now far less than it was before. This experiment succeeds particularly well with the interval corresponding to a major tone, 8 : 9. The beat-tone in this case is three octaves below the lowest constituent of the interval, but it is sufficiently loud to be heard throughout the hall.

Helmholtz's siren affords us a simple means of illustrating the phenomena of beats and interference. If we open the two series of twelve orifices each, and urge air through the sirens, we have, as just seen, perfect unison. The sound from one siren reinforces that from the other, and the result is a much greater volume of tone than either one, singly, is competent to produce. This, however, holds true only when the apertures in the sirens have the same motion with reference to the orifices in the wind-chests. But, as we have seen, we are able, by means of the wheel and pinion, *E*, to turn the upper cylinder either in the same direction in which the siren moves, or in the direction opposite. When the cylinder is rotated so that its orifices meet those of the siren, the apertures pass each other more rapidly than when the cylinder is motionless. The pitch of the note of the upper siren is thus rendered higher than the pitch of the note from the lower one, and the result, as declared by the powerful beats, is interference. For every complete revolution of *E* there are produced four beats, for the prime tone of the instrument. If the motion is reversed, the orifices of siren and cylinder pass each other less frequently, and the result is that the pitch of the note emitted by the upper siren is lower than the pitch of the note from the lower siren. Again, we have interference, and beats are heard as before. If one revolution of *E* towards the right give rise to four beats, and heightens the pitch of the upper siren by four vibrations, a single revolution to the left will lower the pitch by the same amount, and the tone of the upper siren will have four vibrations less than the tone of the lower one. It is obvious that we have here another illustration of Doppler's principle, which was discussed *in extenso* in our third lecture.

So far, we have been studying musical intervals acoustically. But we can also study them mechanically and optically. Indeed, paradoxical as it may appear, the most delicate and most accurate means at our disposal for examining musical intervals is the optical method. We shall consider this presently. As an introduction to it, we shall investigate the nature of the vibrations of the compound pendulum devised by Professor Blackburn, of Glasgow, in 1844.

A modified form of such an instrument (Fig. 176) is before you. The bob is a thick disk of lead, into which is fitted a glass funnel filled with fine sand. Instead of a single string, as is used in an ordinary pendulum, we have here an arrangement calculated to give a much more complicated motion. When the pendulum, as shown in the figure, oscillates in a direction at right angles to the cross-piece from which it is suspended, its length is equal to the distance from the lower part of the cross-piece to the centre of the disk. If, however, the pendulum move in a direction parallel to the line joining the two upright pieces, its length will then be measured from the point *r* — a small ring of metal — to the centre of the disk. If moved in either direction, as stated, the motion will be in a straight line, — the direction in one case being perpendicular to what it is in the other. But if the pendulum is started from the point *D*, which is in a line making an angle of forty-five degrees with the line joining the two uprights, we get quite a different result. In this case, the sand from the funnel will trace a curve instead of a straight line, the nature of the curve depending on the relative lengths of the two pendulums. I say *two* pendulums, for that in reality is what we have. The

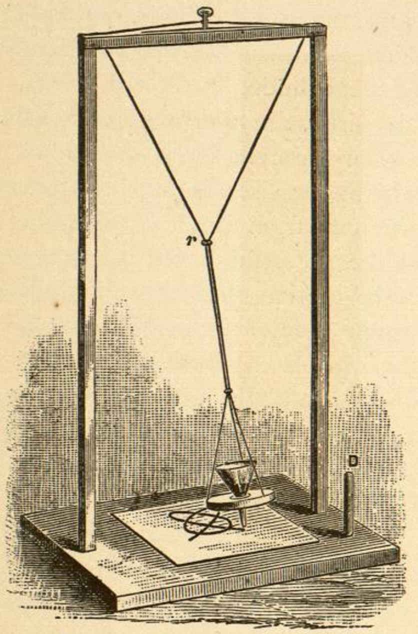

Fig. 176.

point of support for the shorter pendulum is the metal ring *r*, and the point of support of the longer one is the lower part of the cross-bar.

If the shorter curve is one fourth the length of the longer one, the former will execute twice the number of vibrations that the latter will in the same period of time. This is in accordance with the law that the times of the vibrations of any two pendulums vary inversely as the square roots of their lengths. But the bob cannot move in two directions

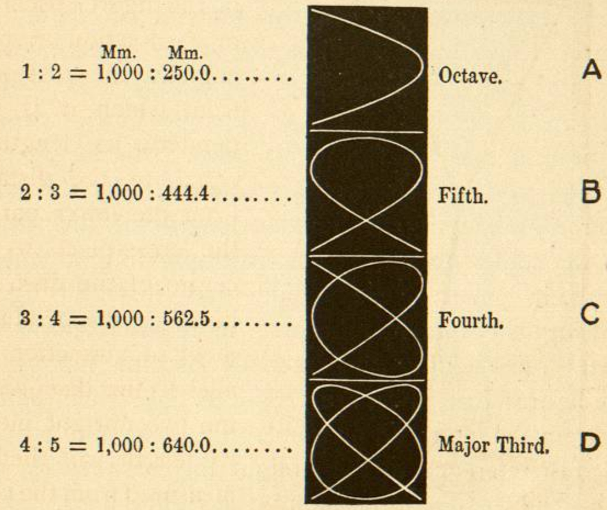

Fig. 177.

at the same time. It will, consequently, move along a path intermediate between the two straight lines just spoken of, and the resultant due to the combination of the two vibrations is a parabola, — *A* (Fig. 177). The rates of vibrations of the two pendulums in the case just considered are as 1 : 2. But this ratio also expresses the interval of the octave. The figure *A* therefore is the curve that corresponds to this interval.

If we change the position of the ring *r* so as to alter the relative lengths of the two pendulums, and start the bob from *D*, as before, we shall obtain an entirely different figure from the one just exhibited. Making the lengths of the two pendulums as 4 : 9, the sand from the funnel will describe figure *B*. But the square roots of 4 and 9 are 2 and 3 respectively. While, therefore, the longer pendulum makes two vibrations, the shorter one executes three. But the ratio 2 : 3 expresses the interval of the fifth, and hence figure *B* may be considered as the visible expression of this interval.

Making the relative lengths of the two pendulums 9 and 16, — the square roots of which are 3 and 4, — we obtain figure *C*, corresponding to the interval of the fourth. Similarly, if we make the lengths of the pendulums as 16 : 25, we shall obtain figure *D*. The square roots of 16 and 25 are respectively 4 and 5. But these ratios express the vibration ratio of the major third. Figure *D*, consequently, corresponds to this interval. In the same manner, by changing the relative lengths of the pendulums, we could obtain figures corresponding to all the intervals in music. We should find that the figures expressing the intervals become more complex as the numbers representing the intervals become larger.

The figures just given are produced only when the bob starts from the point *D*, or from some point similarly situated with reference to the straight line between the two uprights and the one intersecting it at right angles. If the bob be made to start from points other than those mentioned, entirely different figures will be produced.

Mr. Tisley has invented a compound pendulum, which, for the variety, beauty, and delicacy of the figures it is competent to produce, is in every way superior to the one we have been using. Such a one, connected with a vertical lantern, is now before you. It consists of two pendulums, *P P'*, balanced on knife-edges at *A A'*. From the points *c' c* project two brass arms *cp* and *c'p*, which, when the pendulums are at rest, are at right angles to each other. These arms are given perfect freedom of motion in every direction by being connected with the pendulums by ball-and-socket joints at *c* and *c'*. By means of the threads *t*

and *t'*, connected with delicate adjustable springs attached to the arms *cp* and *c'p*, the tracing point at *p* may be readily lowered and raised without in any way affecting the vibrations of the pendulums. Sliding brass plates are attached to the pendulum rods, and are intended to receive the weights, which serve the purpose of bobs. The sum of the weights ordinarily varies from five to twelve pounds. The relative lengths of the two pendulums are altered at will by placing the weights at different heights. *W* is a smaller weight sliding along the pendulum rod, and is counterpoised by the weight *T*. These small weights enable one to adjust the pendulums very accurately, and to change, if need be, their rates of vibration even while in motion.

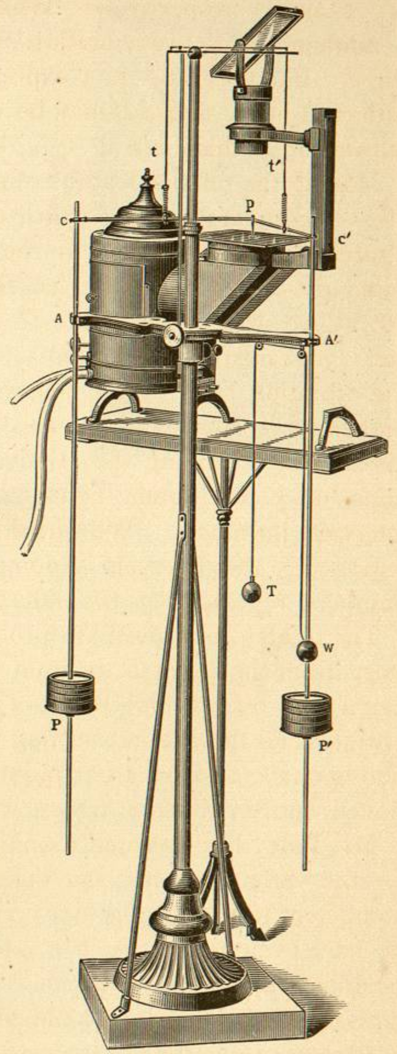

Fig. 178.

On the condenser of the vertical lantern rests a plate of glass blackened by camphor-smoke. The pendulums are so adjusted that one of them vibrates twice while the other executes three vibrations. If, then, they be both made to oscillate simultaneously, they should cause the tracing-point, *p*, to describe a curve corresponding to the musical interval of a fifth. The pendulums are started, and instantly there flashes out on the screen, where all was darkness before, a beautiful bright curve, which becomes more and more complicated. Finally, the tracing-point has returned to its starting-point, and the curve delineating the interval of a fifth is complete. But as the pendulums continue to vibrate there is inscribed on the plate a second figure within the first. Both are identical in all respects except size. The reason of this is due to the gradually decreasing amplitude of oscillation of the pendulums. Thus, by allowing the pendulums to vibrate for some moments, a number of figures, equally beautiful and equally symmetrical, are inscribed on the glass plate, one within the other. We have now on the screen a visible expression (Fig. 179) of the sonorous vibrations composing the interval of a perfect fifth. By sliding the weight, *W*, up or down the rod, we should disturb the perfection of the interval, and introduce corresponding changes in the figure.

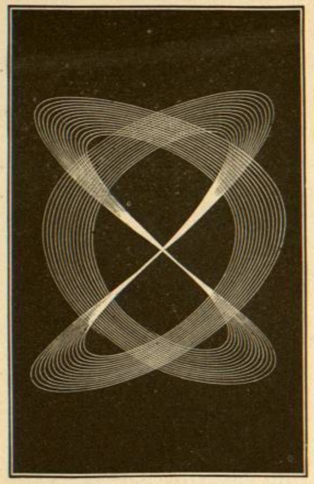

FIG. 179.

Only a moment is required to adjust the pendulums for the interval of a fourth. Substituting a new glass plate for the one now on the lantern, and setting the pendulums going as before, we have designed for us a figure that is even more beautiful and more complex than that corresponding to a fifth. As before, we have a series of curves within curves (Fig. 180), as elegant in form as they are marvellous in regularity. By suitably adjusting the relative lengths of the pendulums, it is manifest that we could secure an infinite number of tracings, corresponding not only to all the intervals used in music, but also answering to all possible rates of vibration.

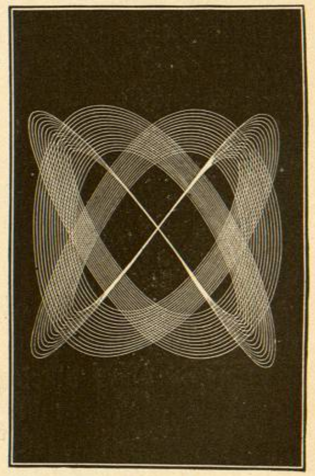

FIG. 180.

In Fig. 181 we have a tracing that is quite different from anything that we have yet seen. A slight change in the relative lengths of the pendulums is all that is required to transform some of the simpler figures we have been studying into others of bewildering intricacy. And yet, notwithstanding the maze-like complexity of these tracings, they are, one and all, as faultlessly symmetrical as they are novel and exquisite.

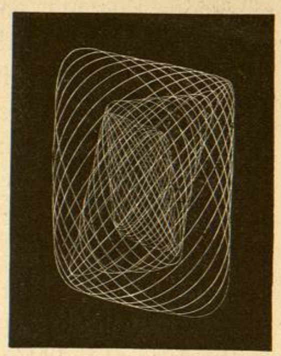

FIG. 181.

In 1827 Wheatstone devised a simple little contrivance for showing the figures corresponding to the various musical intervals, that reproduces admirably all the various curves afforded by the pendulum. To this little piece of apparatus he gave the name of "caleidophone." It is nothing more than an elastic rod of steel (Fig. 182) attached to a firm support. If the rod be cylindrical, and its flexural rigidity for all transverse directions be the same, it will, when set in vibration, move in one plane, like a simple pendulum. But

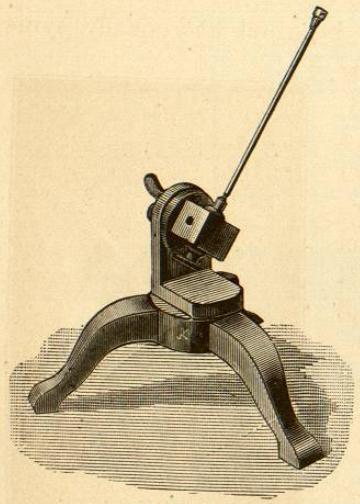

FIG. 182.

if the flexural rigidity be unequal, either through lack of homogeneity of the material of the rod, or on account of its form, there will be a composition of two rectangular vibrations that are, as in the compound pendulum, mutually perpendicular. Thus, if $O\,b\,a$ (Fig. 183) be the cross-section of a prismatic rod, the rod will tend to vibrate more rapidly in the direction Oy than in the direction Ox. If, however, the rod be flexed to some point intermediate between the lines Oy and Ox, and then set free, it will no longer vibrate in a single plane, but will execute a curve varying as the ratio of the sides a and b. If $a:b$ as 1 : 2, the curve due to the compounding of the two rectilinear vibrations will be that corresponding to the interval of the octave. The rod before us is made to give the figure of the octave.

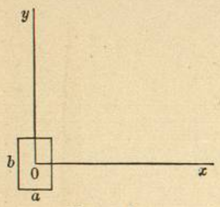

FIG. 183.

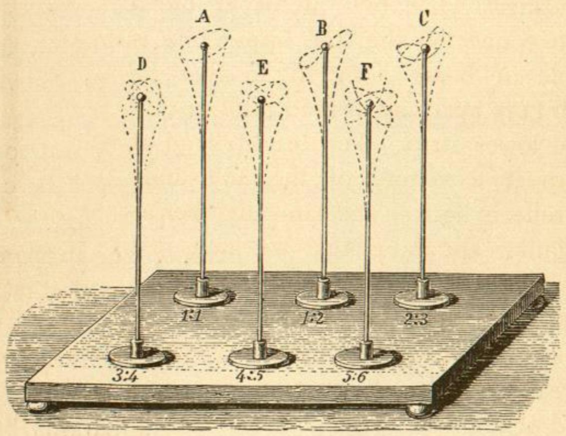

FIG. 184.

At its upper extremity is a small, highly polished mirror, which reflects a beam of light coming from our lantern. On the ceiling is depicted the curve of the figure 8, answering to the curve of the octave.

If, in place of the rod just used, we were to take others,

in which $a:b=2:3$, $3:4$, $4:5$, or $5:6$, we should obtain curves corresponding to the fifth, the fourth, the major, and the minor thirds respectively. On the table is a small stand in which are fixed six rods (Fig. 184) so constructed that they give all the common intervals from unison to the minor third.

The rods, so far employed, are competent to describe curves corresponding only to a single interval each. But Lippich has devised a universal caleidophone (Fig. 185), with which we are able to obtain figures answering to any interval whatever. It consists of a long strip of steel fastened at its lower end to a solid support. To the upper end of the strip is attached a similar strip of steel, the direction of the greater cross section of the latter being perpendicular to that of the former. The two pieces of metal are so connected that the upper one is capable of being adjusted so that its length may bear any desired ratio to that of the lower strip. The bright bead at the upper extremity of the adjustable strip reflects light in the same manner as the similar apparatus that we have just used. It is manifest, from what has been said, that this form of caleidophone, simple as it appears, is capable of yielding as great a variety of curves as the compound pendulum. The results of the one beautifully corroborate those of the other, and both fully respond to all the requirements of theory regarding the composition of the rectangular vibrations of pendulums and elastic rods.

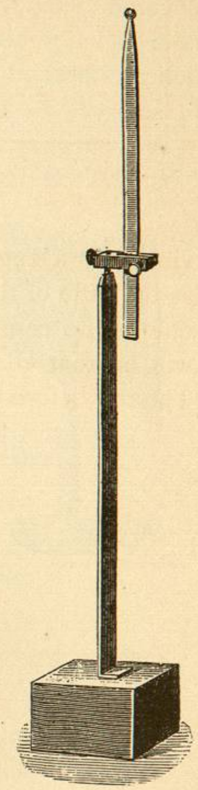

FIG. 185.

But, you may ask, where is the connection between the figures we have been studying and the musical intervals to which they are said to correspond? Neither the pendulums nor the rods emit any sound whatever. The latter may, indeed, in some cases, yield notes, but they are at best very faint. It is important, therefore, to establish a connection that cannot be gainsaid between the various curves given, and the musical intervals that they are said to represent. The optical method of M. Lissajous, discovered in 1857, shows the connection in a most remarkable manner, and at the same time affords the most delicate method of tuning sonorous bodies that is known to science.

We have already had occasion to see something of Lissajous' method, but not precisely in its bearing on

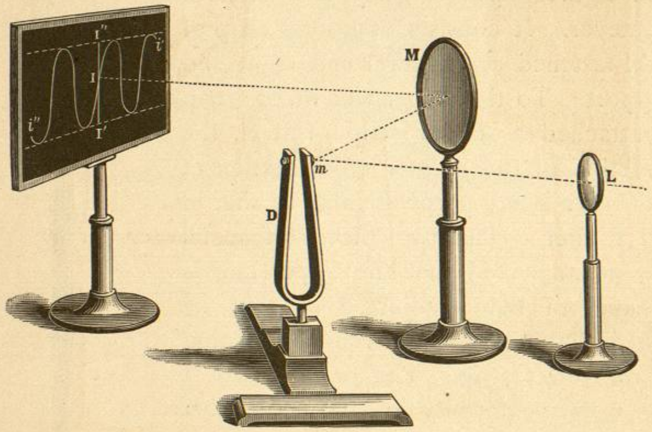

FIG. 186.

musical intervals. This method, which is now so celebrated, and which is now always employed when it is desired to have intervals of tuning-forks, or other instruments, absolutely exact, is, in principle, only a modification of Wheatstone's discovery. In place of rods, Lissajous used tuning-forks, to one of the branches of which are attached small mirrors.

For the sake of illustration, I shall use the simplest form of apparatus. A beam of light from our lantern passes through the lens *L* (Fig. 186), and impinges on the mirror *m* of the upright fork *D*. The light is reflected from the mirror, *m*, of the fork *D* to the mirror *M*, and thence to

the screen. While the fork is quiescent, only a bright spot of light is seen on the canvas. As soon, however, as the fork is set in motion, the spot of light, *I*, becomes a vertical line, *I I'*, parallel to the branches of the fork. If, now, the mirror, *M*, be rotated about its vertical axis, the straight line is transformed into a beautiful sinuous curve, *i' i''*. This change of a luminous point into a straight line, and then into an undulating curve of light, is, as you know, due to the persistence of impressions made on the retina.

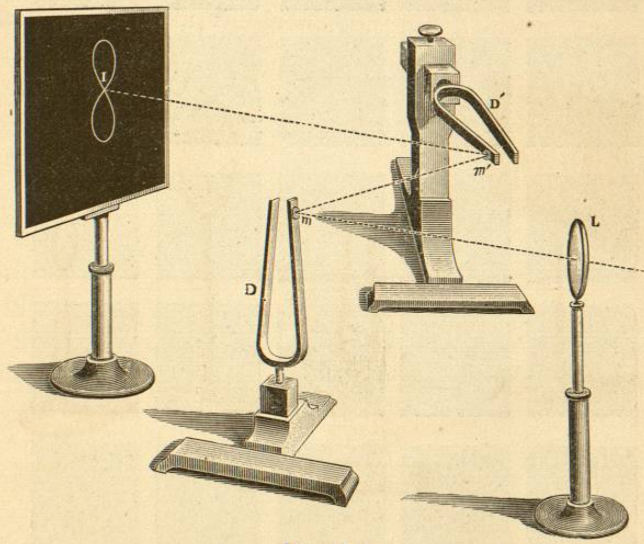

FIG. 187.

Let us now substitute for the mirror *M* a second fork, *D'*, whose plane of vibration is perpendicular to that of the fork *D*. If the fork *D* remain in quiescence, and *D'* be caused to oscillate, the point of light on the screen will describe a horizontal line which is parallel to the branches of the fork. This line is perpendicular, therefore, to that made by the fork *D*. Both forks are excited, and we have now on the screen a curve (Fig. 187), which, we have said, corresponds to the interval of an octave. But how do we know this? Because, aside from the fact that the frequencies of the forks are stamped on their stems, we can, on listening to them, hear that one yields a note exactly an octave higher than that emitted by the other.

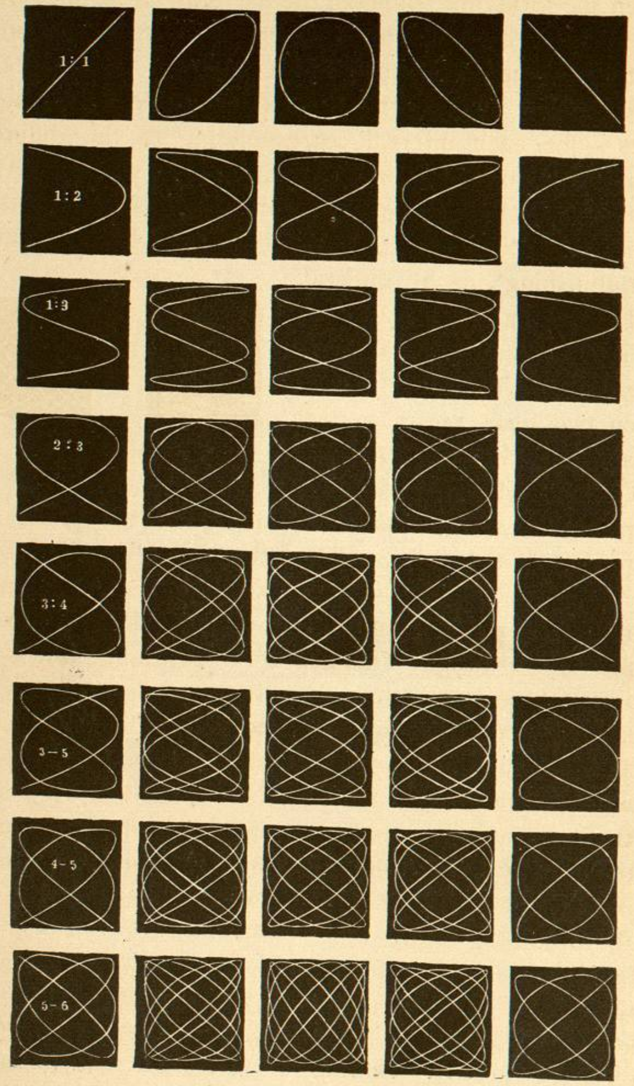

Fig. 188.

By taking forks whose frequencies are as 1 : 1, 1 : 2, 1 : 3, 2 : 3, 3 : 4, 3 : 5, 4 : 5, and 5 : 6, we may obtain all the curves

exhibited in Fig. 188. There can be no doubt about the figures corresponding to the intervals named, because, when the forks are sounded, the ear tells us at once what the intervals are, and these, we find, always correspond to certain characteristic curves. In Fig. 188 there are five curves — for unison, the curves may become straight lines — for each interval corresponding to the different phases in which the forks may happen to vibrate. As a matter of fact, if the intervals are not absolutely exact, there is an indefinite number of forms for the curve distinguishing each interval, and there is a constant change, while the forks are vibrating, of one form into the other. Thus, when two forks are in perfect unison, their characteristic curve is a circle. It may also be an ellipse, or a straight line, depending on the phases of vibrations of the forks. But if the unison be disturbed, even never so slightly, we immediately observe a change, more or less rapid, from a circle into an ellipse, from an ellipse into a straight line, and from a straight line back into a circle. At one time the ellipse, as also the straight line, is inclined to the right; at another, to the left. Each cycle of changes shows all possible forms intermediate between a straight line and a circle. What has been said of the transformation undergone by forks whose unison is disturbed may be iterated regarding the changes that may characterize any of the intervals whose curves are given in the adjoining figure.

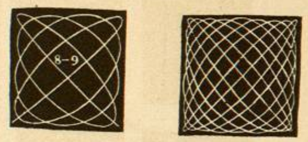

Fig. 189.

In Fig. 189 we have two phases of a more complicated curve, — that corresponding to the interval of a major second, — whose vibration-frequencies, as you remember, are 8 : 9.

By using Mercadier's electric forks, which we have had occasion to employ more than once heretofore, we can, by means of the movable weights on the branches, have the intervals so accurately adjusted that the curves will suffer