no variation whatever. The figure yielded, whatever it may be, or whatever phase it may present, will then remain fixed and invariable as long as the forks are in vibration.

This fact, as is evident, can be used to advantage in what has been aptly termed optical tuning. All that is necessary is to have a standard tuning-fork, executing any given number of vibrations per second. To simplify the work as much as possible, Lissajous invented what is known as an optical comparator, or vibration microscope. An improved form of this instrument was subsequently devised by Helmholtz. It differs from that of Lissajous in being provided with an electro-magnet, so that it can be kept in vibration as long as may be desired. Such a comparator (Fig. 190) is before you. It is composed of an electric fork, attached to a solid support, and a microscope. The objective of the microscope is borne by one of the prongs of the fork, which makes a right angle with the tube. When the fork is set in motion the objects visible in the field of the microscope seem to move in the same direction as does the fork. If now a second tuning-fork, whose prongs are perpendicular to those of the first, be caused to oscillate, a point on the second fork will appear to describe a curve, whose form will depend on the vibration-frequencies of the two forks used. If the intervals of the fork be perfect, some of the forms seen in Fig. 188 will appear, and the form first seen will persist as long as the interval remains undisturbed. If, however, the interval be

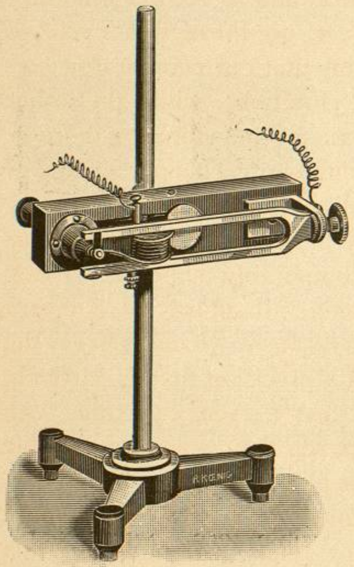

FIG. 190.

perturbed in any way whatever, by a change in the temperature of the forks, for instance, the figure seen is no longer constant. It immediately begins to pass through a cycle of changes, producing some of the various curves in Fig. 188. The longer the time required for effecting a complete cycle of changes, the nearer the intervals of the forks are perfect. The vibration-microscope before us is made to execute exactly 128 vibrations per second. If, now, the figure yielded by this fork, and a second one supposed to be in unison with it, go through a cycle of changes in ten minutes, it means that our comparator executes $10 \times 60 \times 128 = 76800$ vibrations, while the other fork, during the same period, makes one vibration more or one vibration less than this number. The percentage of error in this instance is very slight indeed.

This method of tuning may be applied to any sonorous bodies whatever, and is incomparably superior to any other method we have yet seen. It affords us a means of determining, without any assistance whatever from the ear, any musical interval with a precision that is virtually absolute. By this means a deaf person can tune with almost infinitely greater exactitude than would be possible for the most delicate and most practised musical ear.

Koenig's clock-fork, or tonometer (Fig. 191), is a more elaborate form of comparator than Lissajous' vibration microscope. As an instrument of precision, it is wellnigh perfect. It consists of a large tuning-fork, making sixty-four vibrations per second, which, like Lissajous' comparator, is connected with a microscope. Each prong is provided with a micrometer screw having a heavy head, by means of which the rate of the fork can be adjusted with the utmost precision. Between the prongs is a delicate thermometer for indicating the temperature. The escapement of the clock, with which the fork is connected, is so regulated that the tuning-fork performs the same functions as does the pendulum or balance-wheel in an ordinary clock. The vibratory motion of the fork is rendered continuous by the impulse it receives from the escapement-wheel at each vibration. It was by means of this marvellous piece of mechanism that Koenig determined the frequency of the *Diapason Normal* of the French Conservatory, and proved that its pitch was slightly different from what it was supposed to be. It is this instrument also that he used in determining the frequencies of many

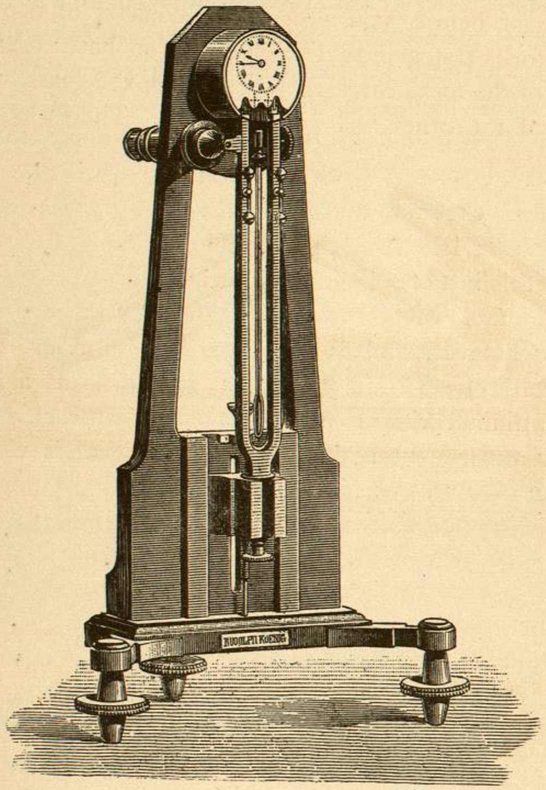

FIG. 191.

of the forks that we have been using in the course of these lectures. Hence their unfailing accuracy, — an accuracy that it would be impossible to secure by any other known means.

M. Lissajous, in connection with M. Desains, has furnished us with another method of obtaining acoustic figures corresponding to any given musical interval. It is

known as the graphical method, and may be viewed as supplementary to the optical method which we have just examined. We have already had occasion, especially in the eighth lecture, to employ the graphical method, so that the principle involved is quite familiar to you. We shall now have recourse to a more delicate piece of apparatus than any we have yet employed when using this method. The instrument before you (Fig. 192) consists of two large tuning-forks fastened to a heavy cast-iron base. A prong of one of the forks carries a piece of smoked glass, while a prong of the other bears a light style. The forks, which

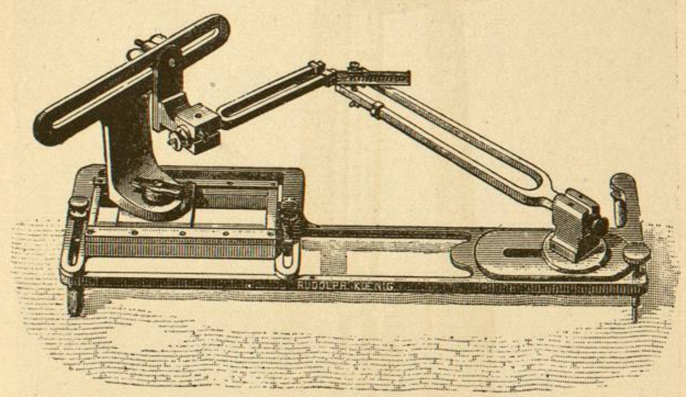

FIG. 192.

are in perfect unison, are now placed at right angles to each other and set in vibration. On moving the fork, to which the style is attached, along a groove, a beautiful trace, corresponding to the interval 1 : 1, — unison, — is given on the smoked glass. Here, as in the optical method, we have the composition of two rectilinear motions, and the result is a curve, Fig. 193, which is characteristic for the interval named. Employing forks whose vibration-frequencies are as 1 : 2, 1 : 2 ±, 5 : 6, and 15 : 16, we obtain the elegant tracings exhibited in the adjoining figure.

All the various methods we have used for elucidating the nature of musical sounds admirably supplement each other, and unequivocally substantiate all the deductions of theory. In observing the intimate connection between simple mathematical ratios and musical consonances, we cannot help calling to mind the saying of Pythagoras, "All is harmony and number." The relations between simple numbers and musical harmony is indeed so marked as to arrest the attention of even the most casual observer. There is something of truth, therefore, in Leibnitz's definition of music, when he says it is "an occult exercise of the mind unconsciously performing arithmetical calculations." [1]

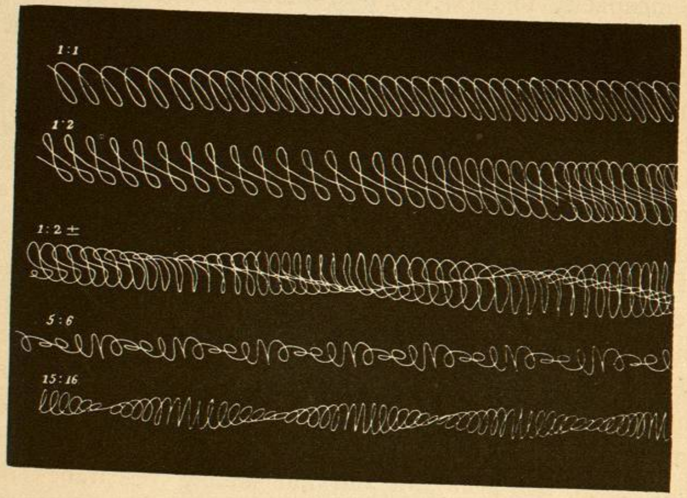

FIG. 193.

We have frequently, in the course of our lectures, used the words *consonance* and *dissonance*, and spoken of intervals as being *consonant* or *dissonant*. It is now time that we should understand the full signification of these terms, and inquire into the causes of consonance and dissonance, and learn why some intervals produce dissonant, and some consonant sensations.

Every one knows, whether he have an ear for music or not, that two or more sounds simultaneously produced

[1] "Musica est exercitium arithmeticæ occultum nescientis se numerare animi."

may in certain instances have a harsh, jarring effect, while in other cases the result of the combination of several notes is pleasing and harmonious. This, however, is only saying that there is such a thing as dissonance as contradistinguished from consonance, that certain musical intervals are rough and grating, while others are smooth and flowing, but it does not offer any explanation of the phenomena observed.

According to the great geometer, Euclid, "Consonance is the blending of a higher with a lower tone. Dissonance is incapacity to mix, when two tones cannot blend, but appear rough to the ear." The illustrious Euler, as the result of profound mathematical investigations, concludes that the mind is pleasurably or unpleasurably affected according as the musical intervals heard are simple or complex. As stated by Helmholtz, "Consonance is a continuous, dissonance an intermittent, sensation of tone." [1] These definitions, however, are little more than statements of a fact. Even the definition of Helmholtz, often as it has been quoted, does not give us the desired information. Indeed, before the investigations of Koenig on the nature of beats, and the researches of Mayer on residual sonorous sensations, a philosophical distinction between consonance and dissonance was an impossibility.[2] But strange as it may seem, the profound and painstaking researches of these two distinguished physicists seem to be entirely

[1] "Consonanz ist eine continuirliche, Dissonanz eine intermittirende Tonempfindung."

[2] As early as the beginning of the last century, Sauveur had outlined the true theory of consonance and dissonance, but it was allowed to fall into oblivion. "Beats," he tells us, "do not please the ear because of the inequality of the sound, and it is quite likely that it is the absence of beats which renders octaves so agreeable. Following out this idea, it is found that the chords in which beats are not heard are precisely the ones which musicians treat as consonances, and that those in which beats are heard are dissonances. When a chord is dissonant in one octave and consonant in another, it is because there are beats in one, and none in the other. Such a chord is deemed an imperfect consonance" ("Histoire de l'Académie Française" for the year 1700, page 143). Most of the acoustical discoveries of Sauveur are to be found in the Memoirs of the French Academy of Sciences.

ignored. They were the first to give us quantitative determinations of the relations between different tones, — all previous determinations being only qualitative, — and the first to put us in possession of the facts necessary to draw the line of demarcation between intervals that are consonant and those that are dissonant. In their works we find the key to the solution of the most vexed questions of musical harmony. And yet, with the exception of a published lecture by Prof. S. P. Thompson,[1] and a few brief notices by Mr. A. J. Ellis,[2] their admirable investigations and the important laws which they disclose are, by English readers at least, virtually unknown.

Koenig's researches, as we have seen, revealed the fact that beats, when sufficiently numerous, may coalesce so as to produce a musical note. Hence the beat-tones, which are commonly known as grave harmonics, differential notes, resultant notes, etc. Professor Mayer finds, by a long series of most arduous observations,[3] extending over the entire musical scale, that the time during which the sensation of sound persists in the ear after the vibrations of air near the tympanic membrane have ceased, varies with the pitch of the note observed. The results of Professor Mayer's experiments are given in the following table: —

N.	V.	B.	D.
C_1	64	16	$\frac{1}{16}$ = .0625 sec.
C_2	128	26	$\frac{1}{26}$ = .0384 "
C_3	256	47	$\frac{1}{47}$ = .0212 "
G_3	384	60	$\frac{1}{60}$ = .0166 "
C_4	512	78	$\frac{1}{78}$ = .0128 "
E_4	640	90	$\frac{1}{90}$ = .0111 "
G_4	768	109	$\frac{1}{109}$ = .0091 "
C_5	1024	135	$\frac{1}{135}$ = .0074 "

[1] The Physical Foundation of Music; being an Exposition of the Acoustical Researches of Dr. Rudolph Koenig of Paris, delivered in the Royal Institution of Great Britain, June 13, 1890.

[2] See Ellis's Helmholtz.

[3] See The American Journal of Science and Arts, October, 1874.

In Column N are given the names of the notes, and in Column V their corresponding frequencies. Column B exhibits the smallest number of beats per second which the note must make with another note in order that the two may constitute the nearest consonant interval. The duration of the beats in fractions of a second are given in Column D. Thus the lowest number of beats that C_1 can give with another note in order that the sensation may be continuous, is 16. The duration of the residual sensation for C_1 is consequently the $\frac{1}{16}$ of a second. But $64 : 64 + 16 = C_1 : E_1$, the interval of a major third. For the next higher octave, we have, according to the table, $128 : 128 \times 26 = C_2 : E\flat_2$. In this instance a minor third is the nearest consonant interval. For the notes C_3 and C_4 the nearest consonant intervals are respectively about $\frac{1}{4}$ and $\frac{1}{2}$ of a semitone less than a minor third. C_5 forms a consonance with a note that is but a single tone higher, while C_6 makes a consonance with a note that is separated from it by an interval which is less than a semitone.

This is certainly contrary to all the generally received opinions of musicians, who consider the intervals of whole tones and semitones as invariably dissonant. They admit, it is true, especially when their attention is called to the fact, that the dissonance of these intervals is less in the higher than in the lower parts of the scale. But they will persist in calling the intervals of whole tones and semitones dissonant, in whatever part of the scale these intervals may happen to be found. Facts, however, are stubborn things, and Professor Mayer has demonstrated that intervals universally regarded by musicians as dissonant are, at least in the higher parts of the scale, quite perfect consonances. Similarly, intervals in the lower parts of the scale that musicians always treat as consonances, Professor Mayer shows are, in reality, dissonances. Thus, the nearest consonant interval for C_1, according to the above table, is a major third. But both in this part of the scale and in that below, musicians make use of a minor third which is demonstrably dissonant.

The conclusions arrived at by Koenig and Mayer establish the fact that whenever two notes, whatever their position in the scale, are separated by an interval sufficiently large to allow the beats to blend into a continuous tone, the result is consonance. When the beats do not blend, there is dissonance. The cause of dissonance, therefore, is beats, which, like a flickering light, give rise to a discontinuous sensation. When the sensation is made continuous by the coalescing of the beats, the result is consonance. These statements may be regarded as two laws, but laws that admit of exceptions. We saw in Lecture Eighth that the same generator may produce, at one and the same time, both beats and beat-tones of the same number of vibrations. We learned also that the same phenomenon is exhibited, especially well, by means of heavy tuning-forks of high pitch.

I sound the two forks C_5 and D_5, which form a major second, and at once, in addition to the notes corresponding to these two forks, we hear a deep beat-tone identical with the note produced by a fork, having a frequency of C_2. I do not think that any of the musicians present would pronounce the effect dissonant; and yet, according to musical theory, the interval of C_5 and D_5 always produces dissonance. In a similar manner are sounded the forks D_5 and E_5, separated from each other by a minor tone. The beat-tone is the same as before, — C_2. In both instances the effect is strange, if you will, but certainly not dissonant, in the sense in which the term is ordinarily understood.

We now take the forks E_6 and F_6, separated from each other by a semitone. When both are sounded together, we hear, in addition to the proper notes of the forks, the deep beat-tone F_2, which breaks forth with astonishing clearness. Again, if we employ the forks B_6 and C_7 — likewise separated by the interval of a semitone — we have produced, when they are sounded simultaneously, a beat-tone, C_3, of singular volume and power. Even in these cases, where the interval between two generators is only

a semitone, the result is smooth and continuous. The sounds of the forks are acute, it is true; but the effect of the combination is neither harsh nor grating on the ear. No appreciable beats are heard in either case, and the interval of a semitone in this region of the scale must be pronounced a consonance, musical theory to the contrary notwithstanding.[1]

Mr. Ellis obtained similar results from two flageolets.[2] When one instrument yielded $F\sharp_6$, and the other G_6, the beat-tone produced would have been G_2, had the interval been pure; but as it was, the beat-tone approximated F_2 more closely than G_2. What is remarkable in this case is that no beats whatever are perceptible, and the beat-tone generated is far below any note that the instrument itself is capable of producing.

All the interesting phenomena which we have just been examining can be beautifully shown by means of a species of harmonium now before you.[3] It was specially designed by Mr. Ellis as an instrument for demonstrating the facts on which musical theory depends. It is tuned by means of a set of forks so as to give intervals that are perfectly pure. It is essentially an experimental instrument, and its range is too limited for the purposes of practical music.

By sounding simultaneously the two notes constituting any of the ordinary musical intervals, we at once hear the corresponding beat-tone burst forth with surprising clearness and strength. Testing some of the notes in the upper part of the scale, which are separated by a tone or a semitone, we obtain a result that is essentially the same as those yielded by tuning-forks and flageolets. Both kinds of intervals yield smooth and distinct beat-tones, and frequently without any perceptible traces of beats. This, then, is an additional illustration of the fact that the inter-

[1] I have repeated the foregoing experiments for the distinguished violinist, Remenyi, and he fully concurs in the views herein expressed regarding the nature of consonance and dissonance.

[2] Ellis's Helmholtz, pp. 153 and 173.

[3] The violin and violoncello also serve the same purpose admirably.