vals of a tone and a semitone, far from being dissonant, may, in the higher regions of the musical scale, be quite consonant.

It must, however, be observed that compound tones, such as those given by reeds, do not always give such pleasing effects as do the simple tones of tuning-forks. The reason of this is that although the primes of compound tones may be consonant, the upper partials may, and often do, beat with their primaries and with each other, and thus give rise to a roughness of tone that is never observed in the case of the same intervals when simple tones are tried.

It is obvious that what has been said of consonance and dissonance, as considered in any of the musical intervals, applies with equal truth to chords composed of three or more notes. Chords are consonant when unaccompanied by beats; they are dissonant when beats are present. Occasionally the beats, especially those resulting from the upper partials of compound tones, are so feeble that they are quenched almost, if not entirely, by the louder continuous tones of the chords. When the beats are so feeble as not to be recognized in the mass of the tone, or so weak as not to produce any disagreeable effect on the ear, they are viewed as being virtually absent, and the chords are regarded as consonant. It is also important to bear in mind that chords, like intervals, may be dissonant in one part of the scale, and consonant in another. It is a mistake, then, for the reasons adduced, to attempt to draw a line of demarcation between consonance and dissonance either in the case of intervals or of chords. Nature has indicated no such boundary lines, and it is futile for musicians—as many of them often do—to pin their faith to theories that can be disproved by the simplest experiments.

In order to exhibit at a glance the comparative dissonance and consonance of the different intervals of the scale from C_3 to C_5, Helmholtz constructed a diagram, of which Fig. 194 is a slightly modified reproduction. The intervals are indicated by distances measured off on the

horizontal line C_3, C_5. Their relative dissonances and consonances are denoted by the perpendiculars from the curve to the horizontal base-line. As will be observed, the curve from C_3 to C_4 is somewhat different from that extending from C_4 to C_5. The curve for octaves above C_5, or below C_3, would show corresponding differences,—the consonances being more numerous in the upper, and less numerous in the lower regions of the musical scale. This curve was calculated for the compound note of a violin. The curves corresponding to the notes yielded by other instruments would, of course, exhibit different outlines, and be modified according to the relative number and intensity of the partials present. Mr. Sedley Taylor, in referring to this curve, picturesquely says: "If we regarded the outline as that of a mountain chain, the discords would

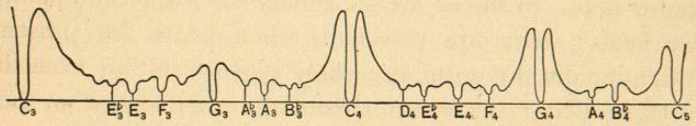

Fig. 194.

be represented by peaks, and the concords by passes. The lowness and narrowness of a particular pass would measure the smoothness and definition of the corresponding musical interval."

So far we have been considering only exact intervals, and only one scale, the natural, or diatonic, scale of C major. But musicians, in order to secure all the variety and effects that characterize our modern music, must employ a large number of scales in both the major and the minor modes. Hence the necessity for additional notes, and hence, also, the origin of the so-called chromatic scale. This scale contains 12 instead of 7—or, counting the octave, 13 instead of 8—notes within the interval of the octave. By the addition of 5 additional semitones, the whole notes are sharpened or flatted by a chromatic semitone; and the octave is thus divided into 13 semitones. These 13 semitones are all that are usually employed within the octave of such keyed instruments as the organ. harmonium, and pianoforte.

But if pure intervals are to be preserved, the number of intervals to the octave should be much greater. The reason of this is, that the sharp of one note is not, as is so often supposed, the same as the flat of the note following. Thus, if C_3 have a frequency of 256 vibrations, the vibration-numbers of $C\sharp_3$ and $D\flat_3$ will be respectively 266.66 and 276.48. Between these two notes, usually considered as the same, there is, as will be observed, a difference of nearly ten vibrations. Again, $E\sharp_3$ and $F\flat_3$ give, respectively, 333.33 and 327.68 vibrations, the former being higher than the latter. The vibration-numbers of $B\sharp_3$ and $C\flat_4$ are 500 and 491.52, in the order named, the sharp of the lower note having a higher pitch than the flat of the upper one. In the octaves above C_3 the differences in the number of vibrations between the sharps and flats of two adjoining notes is proportionately greater.

If, in like manner, we should flat and sharp all the eight notes of the octave, we should obtain twenty-four notes to each octave; and this for one key simply. Similar differences would be observed between the sharps and flats of the notes of the remaining twenty-three major and minor scales.

From what has been said it is obvious that the thirteen tones of the chromatic scale are entirely inadequate to the purposes of music, if pure intervals are to be preserved, and if the performer is to have the power to modulate from one key into another. Mr. Ellis has calculated that theory requires no less than seventy-two keys to the octave, in order that the musician may have complete command over all the keys employed in modern music.

The mechanical difficulties in the way of constructing instruments with such a large number of keys to the octave, not to speak of the difficulties that such an arrangement would entail on the composer and the performer, have given rise to various kinds of compromises, known as systems of *tempering*, or *temperament*.

Temperament is "*the division of the octave into a number of intervals such that the notes which separate them may be suitable in number and arrangement for the purposes of practical harmony.*"

The first system of temperament — known as *unequal*, or *mean tone*, temperament — was introduced by Zarlino and Salinas in the sixteenth century. The object of this form of temperament was to render the more common scales fairly accurate, while the others are ignored. Such a system of tuning limited the player to a part only of the keys now in use. It had, however, the advantage of giving smoother consonances than when all the scales are used, and retained its ground in parts of Europe and Great Britain until only a few years ago. Even now there are organists who prefer it to the systems now in use.

The system of temperament which now prevails almost universally is known as that of *equal temperament.* Its introduction is generally ascribed to Johann Sebastian Bach, in the early part of the last century. There seems, however, to be no doubt that it was known long anteriorly to that period. In his "Harmonie Universelle," Mersenne gives the correct numbers of the ratios for equal temperament, and says of it that "it is the most used and the most convenient, and that all practical musicians admit that the division of the octave into twelve semitones is easier for the player."[1] In his "Harmonicorum," published in 1648, he makes substantially the same statement.[2]

But whoever may have been the inventor of equal temperament, it prevails now, to the almost entire exclusion of other systems. It has the advantage of simplifying the construction of keyed instruments, and of rendering less

[1] His words in reference to equal temperament are, that it is "le plus usité et le plus commode, et que tous les practiciens avouent que la division de l'octave en 12 demitons leur est plus facile pour toucher les instruments" (Livre III. prop. 12).

[2] Speaking of the division of the octave into twelve equal semitones, — thirteen counting the octave of the tonic, — he says, "Quod temperamentum omnium facillimum esse fatebuntur organarii, cum illud ad praxim redegerint" (Liber IX. prop. 19).

difficult the work of composer and performer; but it sacrifices much of the beauty and harmony of effect that would result from just intonation.

In equally tempered instruments, the only interval accurately tuned is the octave. All the other intervals are more or less out of tune. The fifth is somewhat flatted, whereas the thirds and sixths are much sharped. When the notes of the tempered scale are sounded in succession, no deviation from pure intonation is observable, except by trained musical ears. When, however, two or more notes are sounded simultaneously, especially on instruments which, like the organ or the harmonium, give sustained tones, the departure from pure intonation is at once remarked, even by those who have no ear whatever for music. Beats, more or less numerous, are found to accompany all intervals except the octave. Fortunately, however, for the pianoforte, these beats are evanescent, and hence the effect is not so jarring as with the harmonium or the organ. It was indeed its adaptability to the pianoforte that gave the system of equal temperament the long lease of life that it has enjoyed. And it was the simplicity of the system, more than anything else, that tended to develop the pianoforte and make it what it is to-day, — the "voice of the composer," and the most popular of instruments.

But notwithstanding all the advantages claimed for it, equal temperament deprives us of one of the greatest charms of music, — that of the vivid contrast afforded us by the close juxtaposition of consonant and dissonant chords. This is at once seen by comparing the effect of a piece of music played on an instrument tuned in just intonation, with that of the same piece played on an instrument tuned in equal temperament. A still more striking way of showing the marked difference between music as executed according to pure intonation and according to equal temperament, is to have a quartet of accurate voices sing first according to the former system, unaccompanied, and then according to the latter, accompanied by the pianoforte. The contrast between the purity and brilliancy of the concords of pure intonation, and the harshness and dulness of those of equal temperament, is so decisive as to appeal to any one who has even an ordinary ear for music. Hence it would be much better if singers were taught by accompanying them with the violin or 'cello instead of the piano.

It is pure intonation that renders the music played on slide trombones and string quartets of a character so superior to that executed on ordinary keyed instruments. The instruments of the trombone and violin family are not, like the pianoforte and organ, limited to a few notes, but, like the human voice, are competent to produce an indefinite number of tones and intervals that are absolutely pure.

It is because music sung and played in pure intonation is of such excellence that it should receive more attention than is ordinarily given it. There are, it is true, those who think that the duodecimal division of the octave is quite sufficient for all purposes of melody and harmony, and that nothing better can be had, and who accordingly regard all who favor a change as unreasonable innovators. But it must be admitted by all who have examined the subject that our present musical system is far from perfect. No one, I take it, will refuse to encourage pure intonation, where, as in vocal and stringed harmony, it can be secured as readily as intonation that is confessedly faulty and unnatural.[1]

There is no insuperable obstacle to the introduction of pure intonation into all forms of orchestral music. It would of course necessitate some changes in the forms of instruments as now made, but the changes demanded would not by any means be so great or so numerous as is usually imagined. The labors and experiments of Helmholtz, Bosanquet, Ellis, Poole, White, and Colin Brown have demonstrated that pure intonation is possible, even with keyed instruments like the organ and the harmonium.

[1] See Appendix II.

It may therefore be accepted as a fact that we are yet in a state of transition as regards the system of intonation to be employed in music, and that the not far distant future will witness the introduction of many modifications of the tempered system now in vogue.

Mr. Ellis states the case well when he says that "Equal temperament, or what tuners give us for it, — a very different thing generally, — has indeed become a temporary necessity. . . . But the discoveries of Helmholtz have sounded the knell of equal temperament, which must henceforth be regarded as a theoretical mistake and a practical makeshift, — a good servant, dismissed for becoming a bad master, and now merely retaining office until a successor is installed. . . . At any rate, just intonation, even upon a large scale, is immediately possible. And if I long for the time of its adoption in the interests of the listener, still more do I long for it in the interests of the composer. What he has done of late years, with the rough and ready tool of equal temperament, is a glorious presage of what he will do in the future with the delicate instruments which acoustical science puts into his hands. The temporary necessity for equal temperament is passing away. Its defects have been proved to be ineradicable. An intonation possessing none of these defects has been scientifically demonstrated. It is feasible now on the three noblest sources of musical sound, — the quartet of voices, the quartet of bowed instruments, and the quartet of trombones. The issue is in the hands of the composer. Can any one doubt the result?" [1]

I have now only a few brief observations to make, and my task is finished. In what I have said I have confined myself solely to the elucidation of the subject of physical acoustics, and of its simpler relations to the physical basis of musical harmony. The subject of physiological acoustics, which Johann Müller, Helmholtz, Gavarret Preyer, and others have treated so ably, I have touched upon but

[1] "Illustrations of Just and Tempered Intonation," extracted from the Proceedings of the Musical Association of London, 1874–75.

incidentally, and only when it was found necessary to throw light on the physical aspect of the questions under discussion. Only the merest reference has been made to the subject of musical æsthetics. This was foreign to the scope of our work. Although physics and mathematics are intimately connected with music as a science, they have little or nothing to do with it as an art. The æsthetics of music, therefore, is something that must be considered apart from any of the physical and mathematical relations that have been investigated at such length. To show you how important it is not to lose sight of this observation, I shall quote for you a few paragraphs from one of the greatest authorities on musical æsthetics, Dr. Eduard Hanslick, of Vienna. He says, —

"Finally, let it be observed that musical beauty has nothing to do with mathematics. The idea entertained by certain writers, even those who should know better, regarding the part played by mathematics in musical composition, is singularly vague. Not satisfied with the facts that the vibrations of tones, the differences of intervals, consonance and dissonance, may be traced to mathematical relations, they are convinced that the beauty of a piece of music is also grounded on numbers. The study of harmony and counterpoint is for them a kind of *cabala* which teaches composition by calculation. . . .

"Although mathematics furnishes an indispensable key for the investigation of the physical basis of the tonal art, its importance in musical composition must not be overrated. In a tone-poem, be it beautiful or not beautiful, nothing is calculated mathematically. Creations of the fancy are not arithmetical problems. All monochord experiments, sound-figures, proportions of intervals, etc., are out of place here. The department of æsthetics begins where these elementary relations cease. Mathematics prepares only the simple material for intellectual treatment, and remains concealed in the simplest relations; but musical conceptions come to light without its assistance. . . . What converts music into a tone-poem, and raises it out of the category of physical experiments, is something free and spiritual, and, therefore, something incalculable. Mathematical calculation has as much participation in the art work of music as it has in the productions of the other arts, but no more." [1]

In music we have a beautiful illustration of how science and art go together hand in hand, how one aids the other, how science explains art, and how art often forestalls the conclusions of science. The two working together, with a common end in view, have made music what it is to-day, — the solace and the delight of our race. And we have every reason to hope that the music of the future will be even a closer approximation to those "pure strains ethereal" of which the poet sings, and which we are ever wont to associate with that blessed Fatherland, which is filled

> "With acclamation and the sound
> Symphonious of ten thousand harps that tune
> Angelic harmonies."

[1] Vom Musikalich-Schönen, von Dr. Eduard Hanslick, Professor in the University of Vienna.

APPENDIX.