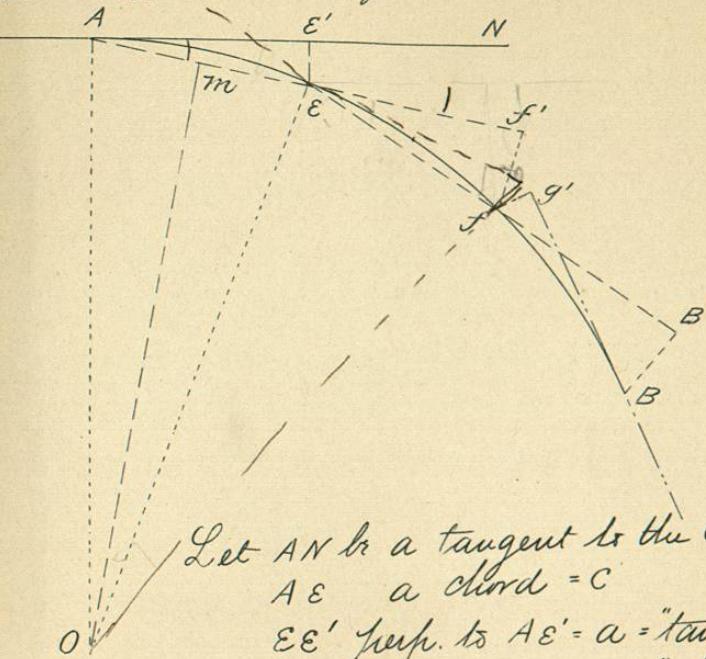


Problem Given D and the stations of P.C and P.I.  
Required, to lay out the curve the method  
of Deflection Distances.

I. When the curve begins and ends at an even station.



Let AN be a tangent to the curve AB

$$AE \text{ a chord} = c$$

$EE'$  perp. to  $AE$  =  $a$  = "tangent deflection"  
 $ff'$  = "chord deflection"

$$AO = EO = R$$

Draw  $Om$  perpendicular to  $AE$ .

Then  $EE' : AE = me : EO$

$$a : c = \frac{c}{2} : R$$

$$a = \frac{c^2}{2R} \quad (26.)$$

$$ff' = 2a \quad Af' = AE \text{ produced.}$$

When  $AE$  is a full station of 100 feet

$$\alpha_{100} = \frac{100^2}{2R}$$

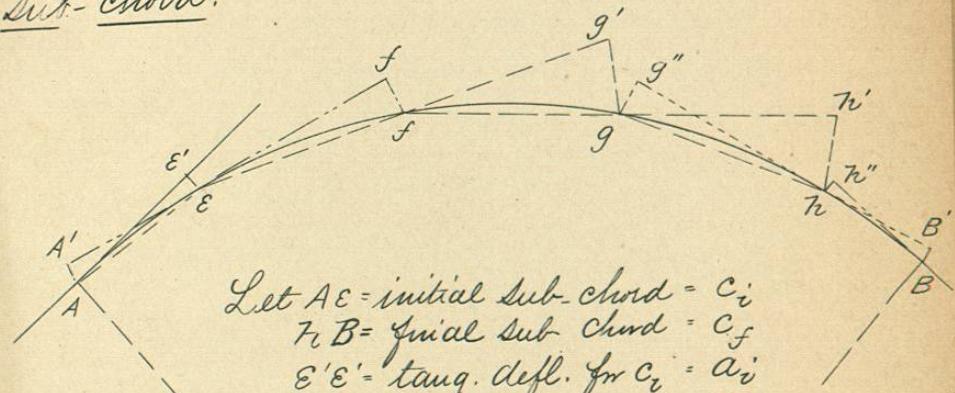
### Fieldwork.

The P.C. and P.I. are assumed to have been set.

- (a.) Calculate  $a_{100}$ .
- (b.) Set point  $E$  distant 100 ft. from  $A$  and distant  $a_{100}$  from  $A'E'$  ( $A'E' < 100$  ft.;  $A'E'E = 90^\circ$ )
- (c.) Produce  $A'E$  to  $f'$  ( $EF' = 100$  ft.) and find  $f$  distant  $2a_{100}$  from  $f'$  ( $EF = 100$  ft.)
- (d.) Proceed similarly until  $B$  is reached (P.I.)
- (e.) At station preceding  $B$  (P.I.), lay off  $fg' = a_{100}$  ( $fg'B = 90^\circ$ )
- (f.)  $g'B$  is tangent to the curve at  $B$  (P.I.)

Problem. Given D and the stations of P.C. and P.I.  
Required to lay out Curve by Defl. Dist.

II. When the curve begins and ends with a sub-chord.



Let  $AE$  = initial sub-chord =  $c_i$   
 $NH$  = final sub chord =  $c_f$   
 $E'E'$  = tang. defl. for  $c_i$  =  $a_i$   
 $n''n$  = " " " "  $c_f$  =  $a_f$

$$\text{by (26.) } a_i = \frac{c_i^2}{2R}; \quad a_f = \frac{c_f^2}{2R}; \quad a_{100} = \frac{100^2}{2R}.$$

$$\begin{aligned} a_i : a_{100} &= c_i^2 : 100^2 & a_i = a_{100} \frac{c_i^2}{100^2} \\ a_f : a_{100} &= c_f^2 : 100^2 & a_f = a_{100} \frac{c_f^2}{100^2} \end{aligned} \quad (27)$$

In general it is better to use (27) than  $a_i = \frac{c_i^2}{2R}$ .

Example Given P.T. 20 + 42 6° Curve R  
P.C. 16 + 25

Required all data necessary to lay out curve by "Deflection Distances." Calculate without tables. Results to 100 ft.

$$\text{Radius } 1^{\circ} \text{ Curve} = \frac{5730}{6}$$

$$\begin{array}{r} 1910) 10000. (\text{S. 235+} \\ \underline{955} \\ 450 \\ 382 \\ \hline 680 \\ 573 \\ \hline 1070 \\ \underline{955} \end{array}$$

$$\alpha_{100} = \frac{100^2}{2 \times 955} = 5.24$$

$$2 \alpha_{100} = 10.47$$

$$\alpha_{75} = .75^2 \times 5.24$$

$$= 2.95$$

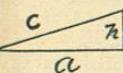
$$\alpha_{42} = .42^2 \times 5.24$$

$$= 0.92$$

Searles' Tables give  $\alpha_{100} = 5.234$  (mean value.)

The distance AE' is slightly shorter than AE. It is generally sufficient to take the point E' by inspection simply. If desired for this or any other purposes, a simple approximate solution of right triangles is as follows.

Problem. Given, the hypotenuse (or base) and altitude. Required, the difference between base and hypotenuse. or in the figure C-a.



$$\begin{aligned} c^2 - a^2 &= h^2 \\ (c-a)(c+a) &= h^2 \\ c-a &= \frac{h^2}{c+a} = \frac{h^2}{2c} \text{ (approx.)} \\ c-a &= \frac{h^2}{2a} \text{ (approx.)} \end{aligned} \quad (28.)$$

Wherever h is small in comparison with a or c the approximation is good for ordinary purposes.

Example.  $C = 100$     $n = 10$

$$C - a = \frac{100}{200} = 0.50$$

$$a = 99.50$$

The precise formula gives 99.499

Fieldwork      for Case II p. 54.

- (a.) Calculate  $a_{100} - a_i - a_f$ . Remember that tangent deflections are as the squares of the chords  $a_{100}$  may be found generally in Table IV Searles as "tangent offset"
- (b.) Find the point  $E$  distant  $a_i$  from  $A'E'$  and distant  $c_i$  from  $A$  ( $A'E'E = 90^\circ$ )
- (c.) Erect auxiliary tangent at  $E$  (lay off  $AA' = a_i$ )
- (d.) From  $A'E$  produced, find point  $f$  ( $ff' = a_{100}$ ;  $Ef = 100$ ;  $Eff = 90^\circ$ )
- (e.) From  $Ef$  produced find point  $g$  ( $gg' = 2a_{100}$ ;  $fg' = fg = 100$ )
- (f.) Similarly for each full station use  $2a_{100}$  etc.
- (g.) At last even station in curve to erect an auxiliary tangent (lay off  $gg'' = a_{100}$ ;  $gg''h = 90^\circ$ ).
- (h.) From  $g''h$  produced find  $B$  ( $B'B = a_f$  etc.)
- (i.) Find tangent at  $B$  ( $hh'' = a_f$ ;  $hh''B = 90^\circ$ ).

The values of  $a_{100} - a_i - a_f$  should be calculated to the nearest  $\frac{1}{100}$  ft.

Caution. The tangent deflections vary as the squares of the chords, not directly as the chords

Curves may be laid out by this method without a transit, by the use of plumb line or "flag" in sighting in points, and with fair degree of accuracy.

For calculating  $a_{100} - a_i - a_f$  it is sufficient in most cases to use the approx. value  $R_x = \frac{5730}{D_x}$ . A curve may be thus laid out without the use of transit or tables.

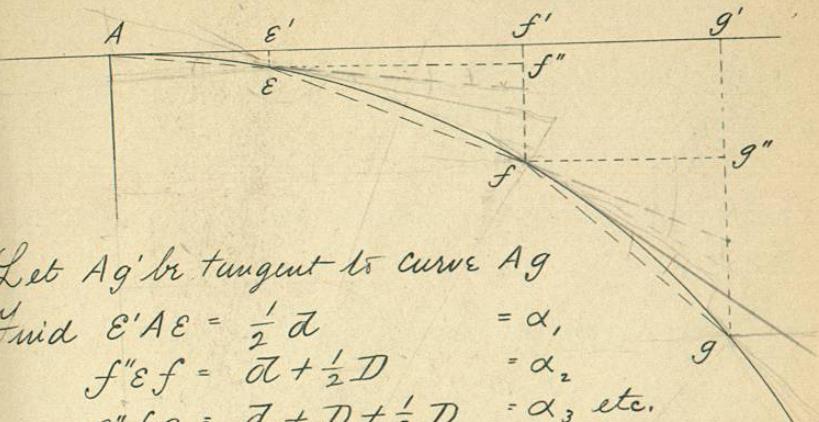
For many approximate purposes it is well and useful to remember that the "chord deflection" for  $1^\circ$  curve is 1.75 ft. nearly, and for other degrees in direct proportion. A head chainman may thus put himself nearly in line without the aid of the transitman.

The method of "Deflection Distances" is not well adapted for common use, but will often be of value in emergencies.

Problem Given  $D$  and stations of P.C. and P.T.  
Required to lay out the curve by "Deflection Distances" when the first sub. chord is small.

Caution. It will not be satisfactory in this case to produce the curve from this short chord.

Problem Given  $D$  and stations of P.C. and P.I.  
Required to lay out the curve by the method of  
Offsets from the Tangent.



Let  $Ag'$  be tangent to curve  $Ag$

$$\text{Find } E'A\bar{E} = \frac{1}{2} d = \alpha,$$

$$f''\bar{E}f = \bar{d} + \frac{1}{2} D = \alpha_2$$

$$g''\bar{f}g = \bar{d} + D + \frac{1}{2} D = \alpha_3 \text{ etc.}$$

$$AE' = C_i \cos \alpha,$$

$$EF'' = 100 \cos \alpha_2$$

$$FG'' = 100 \cos \alpha_3$$

$$EE' = C_i \sin \alpha,$$

$$FF'' = 100 \sin \alpha_2$$

$$GG'' = 100 \sin \alpha_3$$

$$FF' = EE' + FF''$$

$$GG' = FF' + GG'' \text{ etc}$$

When  $A\bar{E} = 100$ , then  $\frac{1}{2} d$  becomes  $\frac{1}{2} D$ .

### Fieldwork.

(a) Calculate  $AE'$ ,  $E'F'$ ,  $F'G'$ ,  
 $EE'$ ,  $FF'$ ,  $GG'$

(b) Set  $E'$ ,  $F'$ ,  $G'$

(c) Set  $E$  by distance  $A\bar{E}$  ( $C_i$ ) and  $EE'$

(d.) Set  $f$  " "  $E\bar{F}$  (100) and  $FF'$

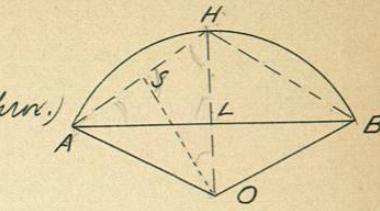
(e.) Set  $g$  " "  $E\bar{G}$  (100) and  $GG'$ .

For the computations indicated above, use always natural sines and cosines.

When  $C = 100$  ft. or less, an approximate formula will generally suffice.

Problem Given  $R$  and  $c$

Required  $M$  (approx.)



$$HL = M: AH = \frac{AH}{2} : R$$

$$M = \frac{AH^2}{2R} \quad \text{Where } AB \text{ is small compared with } R$$

$$AH = \frac{C}{2} \text{ (approx.)}$$

$$M = \frac{C^2}{8R} \text{ (approx.)} \quad (32)$$

Example Given  $C = 100$   $D = 9^\circ$

Required  $M$ .

$$R_9 = \frac{573^\circ}{9} = \frac{636.7}{5093.6} \quad (1.963 = M)$$

Precise value

$$M = 1.965$$

$$\begin{array}{r} 10000. \\ 50936 \\ \hline 490640 \\ 458424 \\ \hline 322160 \\ 305616 \\ \hline 16544 \end{array}$$

Table XII Seales gives middle ordinates for curving rails of certain lengths.

Problem Given  $R$  and  $c$

Required ordinate at any given point  $Q$ .

Approximate Method

I. Measure  $LQ = q$

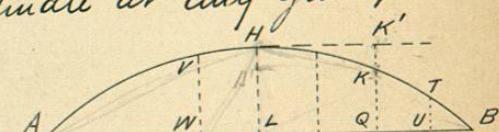
$$HL = M = \frac{\left(\frac{C}{2}\right)^2}{2R}$$

$$KK' = \frac{HK^2}{2R}$$

$$KK': M = HK^2 : \left(\frac{C}{2}\right)^2$$

$$KK' = \frac{q^2}{\left(\frac{C}{2}\right)^2} M \text{ (approx.) since } HK = q \text{ (approx.)} \quad (33)$$

$$KQ = M - KK'$$



$$HK^2 = 2R \times KK'$$

$$KK' = \frac{HK^2}{2R}$$

When  $\frac{q}{\frac{c}{2}} = \frac{1}{2}$  as in figure  $KK' = \frac{M}{4}$  and  $KQ = \frac{3}{4}MK$  (approx.)

when  $\frac{q}{\frac{c}{2}} = \frac{1}{4}$   $VIV = \frac{15}{16}MK$  (approx.)

"  $\frac{q}{\frac{c}{2}} = \frac{3}{4}$   $TU = \frac{7}{16}MK$  (approx.)

It may be shown that the curve thus found is accurately a parabola, but for short distances this practically coincides with a circle.

### Approximate Method

#### II. Measure LQ and QB

$$M = \frac{\left(\frac{c}{2}\right)^2}{2R} \quad KK' = \frac{q^2}{2R} \text{ (approx.)}$$

$$KQ = \frac{\left(\frac{c}{2}\right)^2 - q^2}{2R} = \frac{\left(\frac{c}{2} + q\right)\left(\frac{c}{2} - q\right)}{2R} \text{ (approx.)}$$

$$KQ = \frac{AQ \times QB}{2R} \text{ (approx.)} \quad (34.)$$

Sometimes one, sometimes the other of these methods will be preferable

Example Given  $C = 100$   $D = 9^\circ$

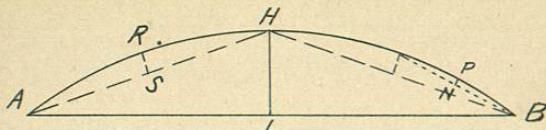
$M = 1.965$  from Tables

Required ordinate at point 30 ft. distant from center toward end of chord.

I.	II.
$30 \text{ ft.} = \frac{30}{50} \times \frac{C}{2}$	$AQ = 80$
$KK' = \frac{q}{25} \times 1.965$	$BQ = 20$
$25) \underline{17.685}$	$R_q = 5730. \quad 1273.4) \underline{1600} (1.207$
$M = \frac{1.965}{1.207}$	$R_q = 636.7 \quad \underline{1273.4}$
ordinate = 1.258	$2R_q = 1273.4 \quad \underline{32660}$
	$\underline{25468} \quad \underline{71920}$
	$\underline{63670} \quad \underline{8200}$

Precise result for data abm = 1.260

Problem Given  $R$  and  $c$   
Required a series of points on the curve.



$$MC = HL = \frac{c^2}{8R} \text{ (approx.)}$$

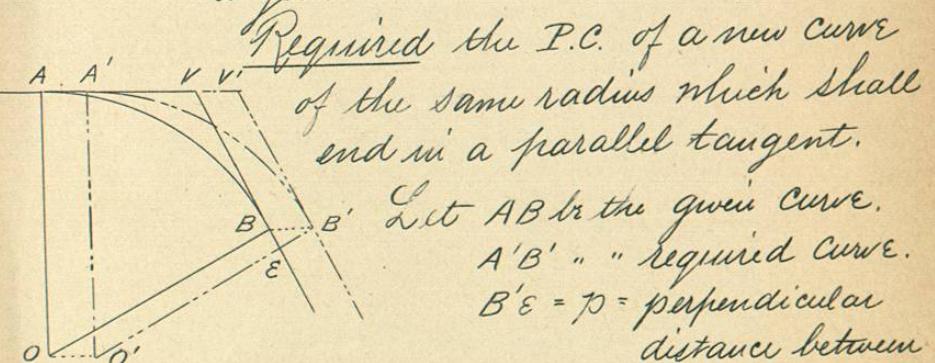
$$RS = \frac{AH^2}{8R} \text{ (approx.)} \quad AH = \frac{c}{2} \text{ (approx.)}$$

$$RS = \frac{\frac{c^2}{4}}{8R} = \frac{MC}{4} \text{ (approx.)}$$

$$PN = \frac{RS}{4} \text{ (approx.) etc as far as desirable.}$$

This method is useful for many general purposes, bending rails among others.

Problem Given a simple curve joining two tangents



Required the P.C. of a new curve of the same radius which shall end in a parallel tangent.

Let  $AB$  be the given curve.

$A'B'$  .. " required curve.

$B'E = p$  = perpendicular distance between tangents.

Join  $BB'$

Then  $AA' = OO' = BB'$  also  $B'B'E = V'VB = I$

$BB' \sin I = p \quad BB' = AA' = \frac{p}{\sin I} \quad (35.)$

When the proposed tangent is outside the original tangent, the distance  $AA'$  is to be added to the station of the P.C. When inside is to be subtracted.