

Parabolic Curves.

Instead of circular arcs to join two tangents, parabolic arcs have been proposed and used, in order to do away with the sudden changes in direction which occur when a circular curve leaves or joins a tangent. Parabolic curves have failed to meet with favor for railroad curves for several reasons.

1. Parabolic curves are less readily laid out by instrument than are circular curves.

2. It is not easy to compute at any point the radius of curvature for a parabolic curve; this may be necessary, either for curving rails or for giving the elevation to the outer rail.

3. The use of the "Spiral" or other "Easement" or "Transition" curves, secures the desired result in a more satisfactory way.

There are however many cases (in Landscape Gardening or Elsewhere) where a parabolic curve may be useful either because it is more graceful or because without instruments it is more easily laid out, or for some other reason.

It is seldom that parabolic curves would be laid out by instrument.

Properties of the Parabola.

§ 131 - 132 - 133 - 134 Runkle.

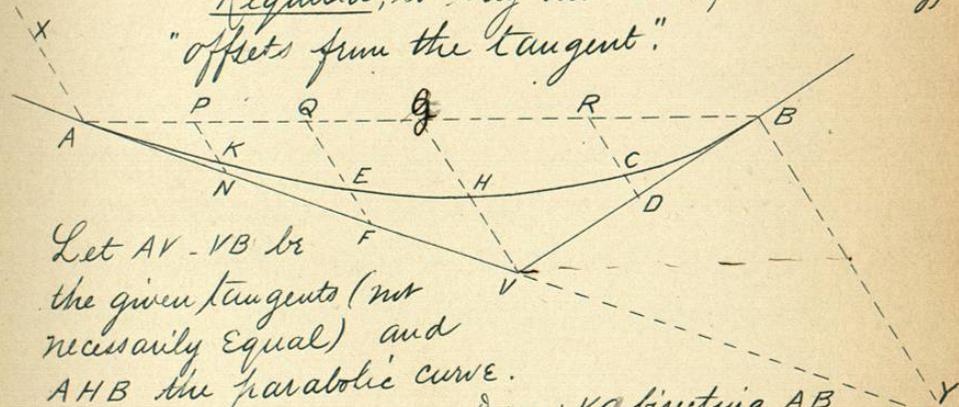
(a.) The locus of the middle points of a system of parallel chords of a parabola is a straight line parallel to the axis of the parabola (i.e. a diameter).

- (b.) The locus of the intersection of pairs of tangents is in the diameter
- (c.) The tangent to the parabola at the vertex of the diameter is parallel to the chord bisected by this diameter.
- (d.) Diameters are parallel to the axis
- (e.) The equation of the parabola, the coordinates measured upon the diameter and the tangent at the end of the diameter is

$$y'^2 = \frac{4p}{\sin^2 \theta} x' \quad \text{or}$$

$$y^2 = 4p'x \quad (121.)$$

Problem. Given, two tangents to a parabola also the position of P.C. and P.T.
 Required, to lay out the parabola by "offsets from the tangent".



Let AV - VB be the given tangents (not necessarily equal) and AHB the parabolic curve.
 Join the chord AB. Draw VG bisecting AB
 Draw AX - BY parallel to VG. Produce AV to Y.
 Then VG is a diameter of the parabola
 AX parallel to VG is also a diameter.
 The Equation of the parabola referred to AX and AY as axes is $y^2 = 4p'x$.

Hence $AV^2 : AY^2 = HV : BY$
 $AV^2 : (2AV)^2 = HV : 2GV$
 $1 : 4 = HV : 2GV$
 $HV = \frac{GV}{2}$

(122.)

Next bisect VB at D

Draw CD parallel to AX

Then $BD^2 : BV^2 = CD : HV$
 $CD = \frac{HV}{4}$

Similarly make AN = NF = FV

Then $KN = \frac{HV}{9}$
 $EF = \frac{4}{9} HV$

In a similar way, as many points as are needed may be found.

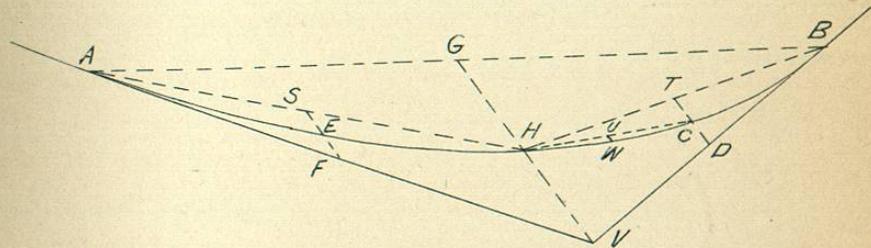
Fieldwork.

- (a.) Find G bisecting AB.
- (b.) Find H bisecting GV.
- (c.) Find points P-Q and N-F dividing AG-AV proportionately; also R and D dividing GB and BV proportionately.
(Use simple ratios when possible as $\frac{1}{2}$, $\frac{1}{3}$ etc.)
- (d.) Lay off on PN, the calculated distance KN (in figure $KN = \frac{HV}{9}$); on QF lay off EF; and on RD lay off CD (in figure $CD = \frac{HV}{4}$).

For many purposes or in many cases it will give results sufficiently close, to proceed without establishing P-Q-R, the direction of NK-EF-CD being given approximately by eye. When the

angle AVG is small (as in the figure) it will generally be necessary to find $P - Q - R$.
 When the angle AVH is large (greater than 60°) and the distances $NK - EF - CD$ are not large, it will often be unnecessary. No fixed rule can be given as to when approximate methods shall be used. Experience educates the judgment so that each case is settled upon its merits.

Problem Given, two tangents to a parabola, also the positions of P.C. and P.T.
Required, to lay out the parabola by "middle ordinates"



The ordinates are taken from the middle of the chord, and parallel to GV in all cases.

Fieldwork.

- (a.) Establish H as in last problem.
- (b.) Lay off $SE = \frac{1}{4} HV$; also $TC = \frac{1}{4} HV$
- (c.) Lay off $UW = \frac{1}{4} TC$ and continue thus until a sufficient number of points is obtained.

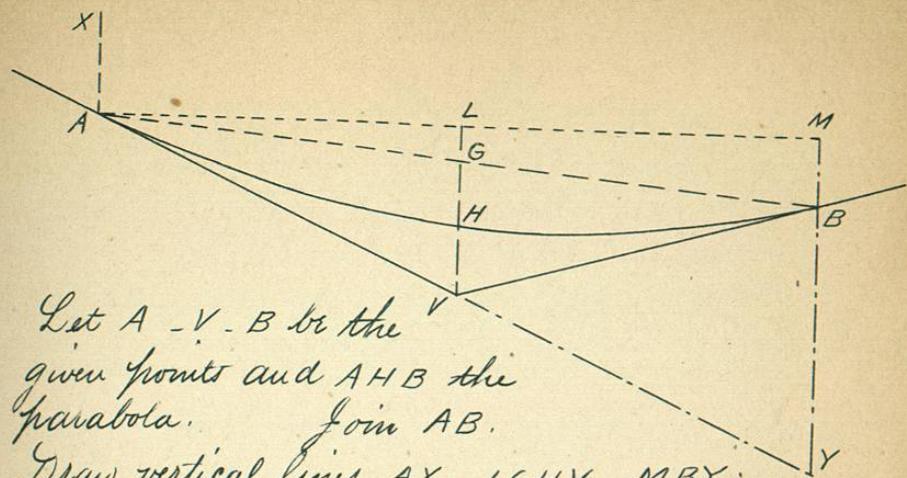
The length of curve can be conveniently found only by measurement in the ground.

Vertical Curves.

It is convenient and customary to fix the grade line upon the profile as a succession of straight lines; also to mark the elevation above datum plane, of each point where a change of grade occurs; also to mark the rates of grade in feet per station of 100 feet. At each change of grade an angle is formed. To avoid a sudden change of direction it is customary to introduce a vertical curve at every such point, if the angle is large enough to warrant it. The curve commonly used for this purpose is the parabola. A circle and a parabola would substantially coincide where used for vertical curves. The parabola effects the transition rather better theoretically than the circle, but its selection for the purpose is due principally to its greater simplicity of application. It is generally laid to extend an equal number of stations on each side of the vertex.

Problem. Given, the elevations at the vertex and at one station (100') each side of vertex.

Required, the elevations of the vertical curve opposite the vertex.



Let $A - V - B$ be the
given points and AHB the
parabola. Join AB .

Draw vertical lines $AX - LGHV - MBY$
and horizontal line ALM .

Produce AV to Y .

In the case of a vertical curve, the horizontal
projections of AV and VB are equal, and here
each equals 100 feet = $AL = ML$

Therefore $AG = GB$ and $AV = VY$

VG is a diameter of the parabola.

AX is also a diameter.

$$VH = \frac{VG}{2}$$

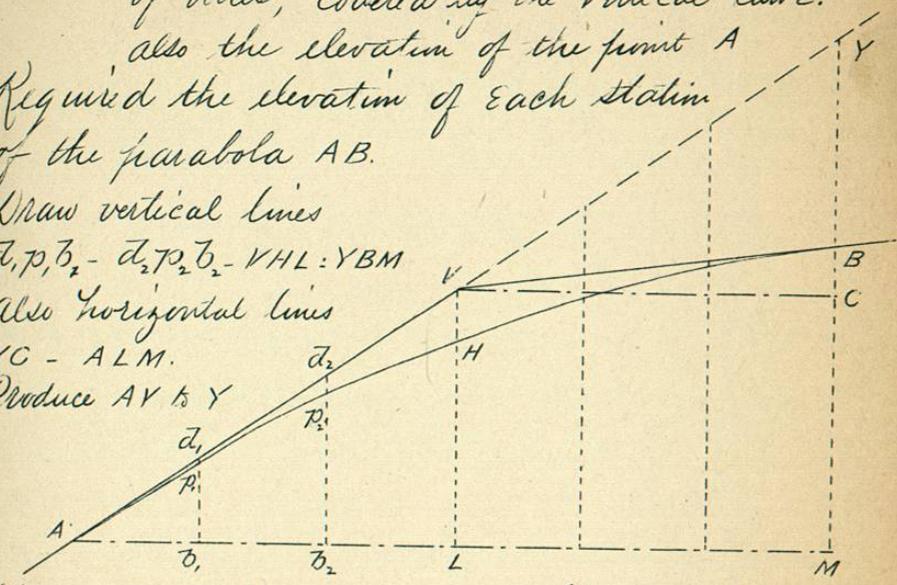
$$\text{Elev. } H = \frac{1}{2} \left(\frac{\text{Elev. } A + \text{Elev. } B}{2} + \text{Elev. } V \right) \quad (123.)$$

This affords a simple and quick method of
finding H when the vertical curve extends
only one station each side of vertex, which is
the most common case. When the vertical
curve extends more than one station each
side of the vertex, another method is preferable,
which is also applicable to the above case,
and is in some respects preferable for that
also.

Problem Given, the rate of grade g of AV
 and " " " " g' of VB,
 the number of stations n , on each side
 of vertex, covered by the vertical curve.
 also the elevation of the point A

Required the elevation of each station
 of the parabola AB.

Draw vertical lines
 $\bar{a}_1, p_1, \bar{b}_1 - \bar{a}_2, p_2, \bar{b}_2 - VHL: YBM$
 also horizontal lines
 $VC - ALM.$
 Produce AV to Y



Let $a_1 =$ offset \bar{a}_1, p_1 , at the first station from A.
 $a_2 =$ " \bar{a}_2, p_2 " " second " " A.
 $a_3 =$ etc

Then $a_2 = 2^2 a_1 = 4a_1$
 $a_3 = 3^2 a_1 = 9a_1$
 $a_{2n} = (2n)^2 a_1 = 4n^2 a_1 = YB$

$YB = YC - BC$
 $4n^2 a_1 = ng - ng'$ or $a_1 = \frac{g-g'}{4n}$ (124.)

Due regard must be given to the signs of g and g' in this formula, whether + or -.

From the Elevation at A we may now find the required Elevations since we have given g and we also have a_1

$a_2 = 4a_1$
 $a_3 = 9a_1$, etc.

A method better and more convenient for use is given below

$$\begin{aligned} d_1 b_1 &= g & p_1 b_1 &= g - a_1 \\ d_2 b_2 &= 2g & p_2 b_2 &= 2g - a_2 = 2g - 4a_1 \\ d_3 b_3 &= 3g & p_3 b_3 &= 3g - a_3 = 3g - 9a_1 \\ d_4 b_4 &= 4g & p_4 b_4 &= 4g - a_4 = 4g - 16a_1 \text{ etc.} \end{aligned}$$

Again

$$\begin{aligned} p_1 b_1 &= g - a_1 & &= g - a_1 \\ p_2 b_2 - p_1 b_1 &= 2g - 4a_1 - (g - a_1) = g - 3a_1 \\ p_3 b_3 - p_2 b_2 &= 3g - 9a_1 - (2g - 4a_1) = g - 5a_1 \\ p_4 b_4 - p_3 b_3 &= 4g - 16a_1 - (3g - 9a_1) = g - 7a_1 \text{ etc.} \end{aligned}$$

On a straight grade, the elevation of any station is found from the preceding, by adding a constant g .

On a vertical curve the elevation of each station is found from the preceding by adding, in a similar way, not a constant, but a varying increment, being for the

$$\begin{array}{l} 1^{\text{st}} \text{ station from } A = g - a_1 \\ 2^{\text{nd}} \text{ " " } A = g - 3a_1 \\ 3^{\text{rd}} \text{ " " } A = g - 5a_1 \end{array} \left. \begin{array}{l} \text{changing by successive} \\ \text{differences of } 2a_1, \\ \text{in each case.} \end{array} \right\}$$

The labor involved is not materially greater, in many cases, for a vertical curve than for a straight grade. This method has the additional advantage that a correct final result at the end of the vertical curve makes a "check" upon all intermediate results.

$\begin{array}{r} 135.00 \\ 117.50 \\ \hline 13.50 \end{array}$	$\begin{array}{r} 137.50 \\ 135.00 \\ \hline 2.50 \end{array}$
---	--

Example.

Given, Grades as follows: —

Sta.	Elev.	Rate
5	117.50	+3.50
10	135.00	+0.50
15	137.50	+0.50

Then $a_1 = \frac{g-g'}{4n} = \frac{3.00}{12} = 0.25$

Sta.	Elev.		
5	117.50		
	+ 3.50		= g
6	121.00		
	+ 3.50	3.50	= g
7	124.50	- 0.25	= a ₁
	+ 3.25	3.25	= g - a ₁
8	127.75	- 0.50	= - 2a ₁
	+ 2.75	2.75	= g - 3a ₁
9	130.50	- 0.50	= - 2a ₁
	+ 2.25	2.25	= g - 5a ₁ , etc
10	132.75	- 0.50	
	+ 1.75	1.75	
11	134.50	- 0.50	
	+ 1.25	1.25	
12	135.75	- 0.50	
	+ 0.75	0.75	
13	136.50		End of Curve
	+ 0.50		= g'
14	137.00		
	+ 0.50		
15	137.50		

3.25
2.5
2.00

The elevation for Sta. 15 thus obtained agrees with the elevation shown in the data, All the intermediate elevations are therefore "checked"

Problem Given $g - g' - a_1$
Required n .

From (124.) $n = \frac{g-g'}{4a_1}$ (125.)

For practical considerations a_1 should not exceed 0.25

or $n \geq \frac{g-g'}{4 \times 0.25}$ or $n \geq g-g'$ (126.)

