

Another method which has been used for calculating irregular cross sections is to plot them on cross section paper and get the area by planimeter. In very irregular cross sections this method would prove economical as compared with direct computation by ordinary methods, but by the use of suitable tables or diagrams (to be explained later) it is probable that in almost every case equal speed and equal precision could be obtained; for this reason the use of the planimeter is not recommended.

Having found the values of A for each cross section, S is found in each case by the formula above given

$$S = \frac{A_0 + A_1}{2} \cdot \frac{T}{27} \text{ (in cu. yds.)}$$

It is found that this formula is only approximately correct. Its simplicity and substantial accuracy in the majority of cases render it so valuable that it has become the formula in most common use. It gives results in general larger than the true solidity.

Prismoidal Formula.

"A prismoid is a solid having for its two ends, any dissimilar parallel plane figures of the same number of sides, and all the sides of the solid plane figures also"

Any Prismoid may be resolved into Prisms pyramids and wedges having as a common altitude the perpendicular distance between the two parallel end planes.

Let A_0 and A_1 = areas of End planes

M = area of middle section parallel to the end planes.

l = length of prismoid, or perpendicular distance between end planes

S = solidity of the prismoid.

Then it may be shown that

$$S = (A_0 + 4M + A_1) \frac{l}{6}$$

Let B = area of lower face or base of a Prism, wedge or pyramid.

b = area of upper face.

m = middle area parallel to upper and lower faces.

a = altitude of Prism, wedge or pyramid.

S = solidity "

Then the area of the upper face b in terms of lower base B will be for

Prism $b = B$ Wedge $b = 0$ Pyramid $b = 0$

and the middle area m will be for

Prism $m = B$ Wedge $m = \frac{B}{2}$ Pyramid $m = \frac{B}{4}$

The Solidity S will be for

$$\text{Prism } S = aB = \frac{a}{6} \cdot 6B = \frac{a}{6}(B + 4B + B) = \frac{a}{6}(B + 4m + b)$$

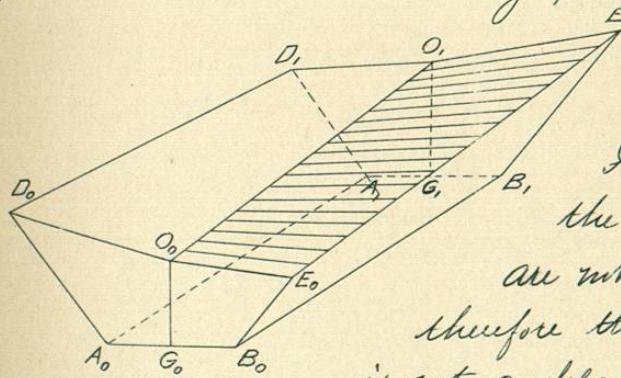
$$\text{Wedge } S = \frac{aB}{2} = \frac{a}{6} \cdot 3B = \frac{a}{6}(B + \frac{4B}{2} + 0) = \frac{a}{6}(B + 4m + b)$$

$$\text{Pyramid } S = \frac{aB}{3} = \frac{a}{6} \cdot 2B = \frac{a}{6}(B + \frac{4B}{4} + 0) = \frac{a}{6}(B + 4m + b)$$

Since a prismoid is composed of prisms, wedges and pyramids, the same expression may apply to the prismoid, and this may be put in the general form $S = (A_0 + 4m + A_1) \frac{v}{6}$ (147.)

using the notation of the preceding page.

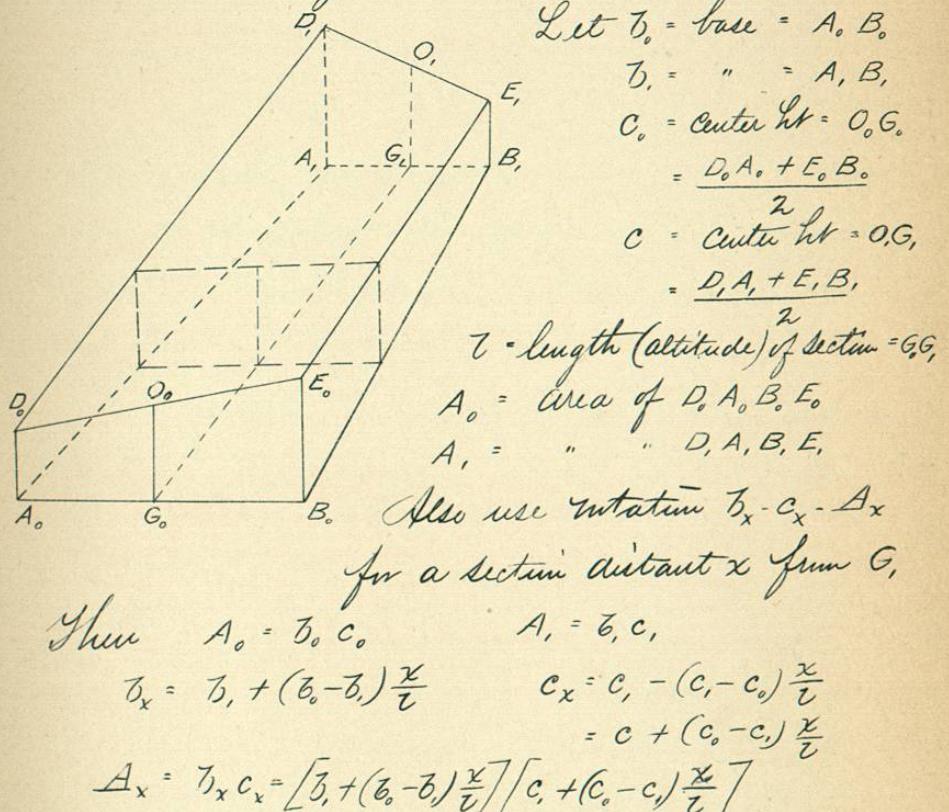
A regular section of earthwork having for its surface, a plane face, is a prismoid. Most sections of earthwork have not this surface plane, and are not strictly prismoids.



In this figure the lines E_0O_0 and E_1O_1 are not parallel and therefore the surface $O_0O_1E_0E_1$ is not a plane. The most common assumption as to this surface, is that the lines O_0O_1 and E_0E_1 are right lines, and that the surface $O_0O_1E_0E_1$ is a warped surface, generated by a right line moving as a generatrix always parallel to the plane $O_0G_0B_0E_0$ and upon the lines O_0O_1 and E_0E_1 as directrices as indicated in the figure. The surface thus.

generated is a warped surface called a "hyperbolic paraboloid". It can be shown that the "prismoidal formula" applies also to this solid which is not however properly a prismoid.

In the following figure which has perpendicular sides $D_0 A_0 A, D_0 - E_0 B_0 B, E$, and the lines $D_0 E_0$ and D, E , right lines,



$$\text{Then } A_0 = B_0 c_0 \quad A_1 = B_1 c,$$

$$B_x = B_0 + (B_0 - B_1) \frac{x}{l} \quad c_x = c_0 - (c_0 - c_1) \frac{x}{l} \\ = c_0 + (c_1 - c_0) \frac{x}{l}$$

$$A_x = B_x c_x = \left[B_0 + (B_0 - B_1) \frac{x}{l} \right] \left[c_0 + (c_1 - c_0) \frac{x}{l} \right]$$

$$\text{Solidity } S = \int_0^l \left[B_0 + (B_0 - B_1) \frac{x}{l} \right] \left[c_0 + (c_1 - c_0) \frac{x}{l} \right] dx \\ = B_0 c_0 l + \left[B_0 (c_1 - c_0) + c_0 (B_0 - B_1) \right] \frac{l^2}{2} + \frac{(B_0 - B_1)(c_0 - c_1) l^3}{3} \\ = \frac{l}{6} \left\{ 6 B_0 c_0 + 3 B_0 c_1 + 3 B_1 c_0 + 2 B_1 c_1 \right. \\ \left. - 3 B_0 c_1 - 2 B_1 c_0 - 2 B_0 c_1 \right\} \\ = \frac{l}{6} (2 B_0 c_0 + 2 B_0 c_1 + B_1 c_0 + B_1 c_1) \quad (148)$$

Apply the "Prismoidal Formula" to the same section.
The base and center height of the middle section are:

$$B_m = \frac{B_o + b}{2}$$

$$C_m = \frac{c_o + c}{2}$$

$$A_o =$$

$$B_o C_o \quad A. = B. c.$$

$$M = \frac{B_o + b}{2} \cdot \frac{c_o + c}{2} = \text{area of middle section}$$

$$S' = (A_o + 4M + A.) \frac{c}{6}$$

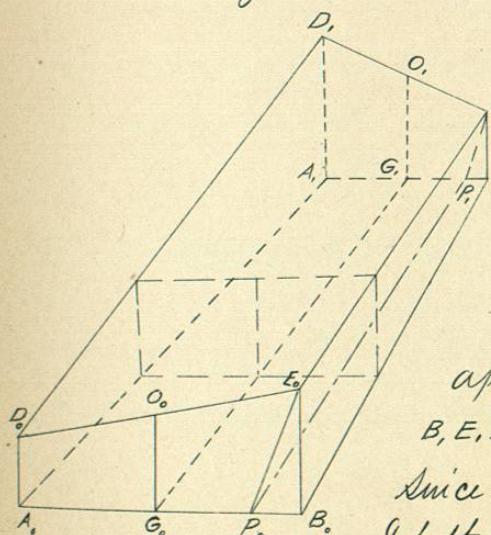
$$= (B_o C_o + B_o C_o + B_o c_o + B_o c_o + B_o c_o)$$

$$= \frac{c}{6} (2B_o C_o + 2B_o c_o + B_o c_o + B_o c_o) \quad (149.)$$

This is the same as formula (148.) found
above to be correct for the warped surface.

Therefore the "Prismoidal Formula" applies to
the section shown on page 146.

The sections of Earthwork commonly used in
railroad work are bounded not by perpendicular
sides, but by inclined planes.



In the figure, suppose
a plane to be passed
through the line E_o E,
cutting A_o B_o at P_o
and A, B, at P,

The prismoidal formula
applies to the solid E_o P_o B_o
B, E, P, cut out by this plane
since the solid is a true prismoid.
If the prismoidal formula applies
to the entire solid and also to the part cut out,
it must apply to the remaining solid D_o A_o P_o E_o P_o A_o D.

and this represents in form one side of a regular three level section of Earthwork, in which D_A is the center height, and E_P the slope.

If the prismatical formula applies to the section upon one side of the center, it applies also to the other side, and so to the entire section.

The "Prismatical Formula" is of wide application since it applies to prisms, wedges, pyramids, and to solids bounded by warped surfaces generated as described, it follows that it applies to any solid bounded by two parallel plane faces and defined by the surfaces generated by a right line moving upon the perimeters of these faces as directrices. It may also be stated here without demonstration, that it also applies to the parts of all solids generated by the revolution of a curve section, as well as to the complete solids, for instance the sphere,

The prismatical formula is generally accepted as correct for the computation of Earthwork and similar solids. The failure to use it results generally from the additional labor necessary for its use.

In three level sections of Earthwork, a result correct by the prismatical formula may be secured, and the work simplified, by calculating the quantities first by the incorrect method of

"End areas", and then applying a correction which we may call "The Prismoidal Correction"

Let S_E' = Solidity by "End Areas"

S_p = " " " Prismoidal Formula

Then $C = S_E' - S_p$ = Prismoidal Correction

In the figure p. 147

$$S_p = \text{by formula (48.)} = \frac{\pi}{6} (2b_1 c_1 + 2b_0 c_0 + b_1 c_0 + b_0 c_1)$$

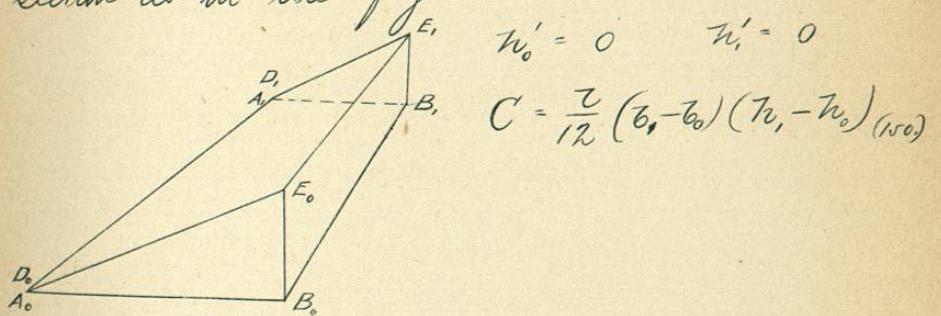
$$S_E' = \frac{\pi}{2} (b_1 c_1 + b_0 c_0) = \frac{\pi}{6} (3b_1 c_1 + 3b_0 c_0)$$

$$\begin{aligned} C &= S_E' - S_p = \frac{\pi}{6} (b_1 c_1 + b_0 c_0 - b_1 c_0 - b_0 c_1) \\ &= \frac{\pi}{6} (b_1 - b_0)(c_1 - c_0) \end{aligned}$$

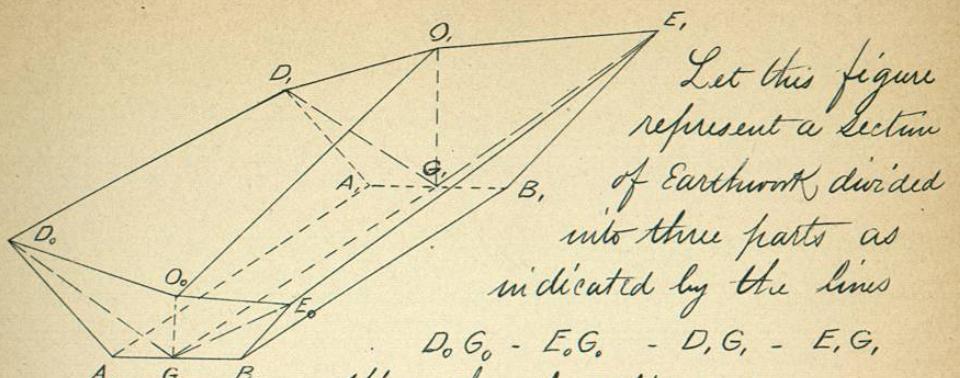
$$\begin{array}{ll} \text{Let } D_0 A_0 = h_0' & D_1 A_1 = h_1' \\ E_0 B_0 = h_0 & E_1 B_1 = h_1 \end{array}$$

$$\begin{aligned} \text{Then } C &= \frac{\pi}{6} (b_1 - b_0) \left(\frac{h_0 + h_1'}{2} - \frac{h_0 + h_0'}{2} \right) \\ &= \frac{\pi}{12} (b_1 - b_0) (h_1 + h_1' - h_0 - h_0') \end{aligned}$$

When the solid assumes a triangular cross-section as in the figure.



If any solid be divided into a number of solids each of triangular cross section, the above correction may be applied to each such triangular solid, and the sum of the corrections will be the correction for the entire solid.



Let this figure represent a section of earthwork divided into three parts as indicated by the lines

$$D_0G_0 - E_0G_0 - D_0E_0$$

Then for the solid $O_0D_0G_0E_0E, G, D, O,$

$$C = \frac{\tau}{12} [(c - c_o)(\bar{d}_v - \bar{d}_{v_o}) + (c - c_o)(\bar{d}_r - \bar{d}_{r_o})] \\ = \frac{\tau}{12} (c - c_o)(\bar{d}_v + \bar{d}_r - \bar{d}_{v_o} - \bar{d}_{r_o})$$

Let $D_v = \bar{d}_v + \bar{d}_r$, and $D_o = \bar{d}_{v_o} + \bar{d}_{r_o}$

$$C = \frac{\tau}{12} (c - c_o)(D_v - D_o)$$

For the solid $G_0B_0E_0E, B, G,$

$$C = \frac{\tau}{12} (b - b_o)(n_r - n_{r_o}) = \frac{\tau}{12} (0)(n_r - n_{r_o}) = 0$$

Similarly for the solid $A_0G_0D_0D, G, A,$ $C = 0$

Hence for the entire solid $A_0B_0E_0O_0D_0D, O, E, B, A,$

$$C = \frac{\tau}{12} (c - c_o)(D_v - D_o) \quad (151.)$$

When $\tau = 100$

$$C = \frac{100}{12 \times 27} (c - c_o)(D_v - D_o) \\ = \frac{1}{3.24} (c - c_o)(D_v - D_o) \text{ in cu. yds} \quad (152.)$$

Since $C = S_E - S_P$

$$S_P = S_E - C$$

When $(c - c_o)(D_v - D_o)$ is positive, the correction C is to be subtracted from $S_E.$ (153.).

When $(c - c_o)(D_v - D_o)$ is negative, the arithmetical value of C is to be added.

The latter case occurs infrequently in practice

In general, for sections of Earthwork the pyramidal correction as given above, applies only when the width of base is the same at both ends of the section. There are certain special cases however which often occur, and which allow of the convenient use of this formula for pyramidal correction. Referring to the figure on page 136, and the corresponding notes on page 133, the correction can be correctly applied in the case of the excavation from Sta 2+64 to 2+76 as follows:

Compute S_E and then apply C, using at Sta 2+64 $D_0 = 23.2$ and at Station 2+76 $D_0 = 11.9 = d_m$, or the distance out on one side only. This may readily be demonstrated to be true if the correction to the right of the center be taken using formula (151.) and the correction to the left using formula (150.) and the two corrections (right and left) be added.

Formula (150.) can also be used to find the correction for the triangular pyramids (for excavation Sta 2+76 to 2+91 and embankment 2+64 to 2+76). Each end of the pyramid being considered to have a triangular section. A much simpler way to find the correction for a pyramid is this.

$$C = S_E - S_P = \frac{1}{3} S_E \text{ as may readily be shown.}$$