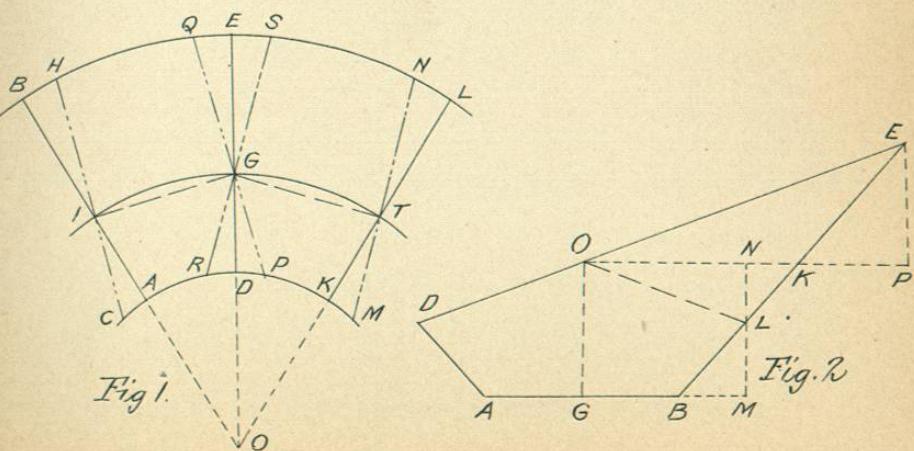


requirements of modern railroad practice.

Some engineers whose opinions are entitled to careful consideration object to the use of the prismoidal formula or prismoidal correction in any form, some as an unnecessary refinement, and some on the ground that certain practical considerations render the results near the truth when the method of averaging end areas is used without applying the prismoidal correction. Probably the greater part of the best engineering practice favors the use of the prismoidal correction.

#### Correction for Curvature.

In the case of a curve, the ends of a section of earthwork are not parallel, but are in each case normal to the curve. In calculating the solidity of a section of earthwork we have hitherto assumed the ends parallel, and for curves this would be perpendicular to the chord of the curve between the two stations.



Then as shown in Fig. 1 (where  $IG$  and  $GT$  are center line chords) the solidity (as above) of the sections  $IG$  and  $GT$  will be too great by the wedge shaped mass  $RGP$ , and too small by  $QGS$ . When the cross sections on each side of the center are equal, these masses balance each other. When the cross section on one side differs much in area from that on the other, the correction necessary may be considerable.

In Fig. 2. use  $c h_a h_r d_r d_s$  as before  
Let  $D$  = degree of curve.

Make  $BL = AD$  and join  $OL$

Then  $ODAG$  balances  $OLBG$  and there remains an unbalanced area  $OLE$

Draw  $OKP$  parallel to  $AB$ .

By the "Theorem of Pappus"  
(see Lanza App. Mech.)

"If a plane, lying wholly on the same side of a straight line in its own plane, revolves about that line, and thereby generates a solid of revolution, the volume of the solid thus generated is equal to the product of the revolving area, and of the path described by the center of gravity of the plane area during the revolution."

The correction for curvature, or the solidity developed by this triangle OLE (Fig. 2) revolving about OG as an axis, will be its area  $\times$  this distance described by its center of gravity. The distance out (horizontal) to the center of gravity from the axis (center line) will be  $\frac{2}{3}$  of the mean of the distances out to E and to L or  $= \frac{2}{3} \cdot \frac{d_r + d_v}{2}$  and the distance described will be  $\frac{2}{3} \cdot \frac{d_r + d_v}{2} \cdot QGS$

$$\text{The area } OLE = OK \times \frac{NL + PE}{2}$$

$$= \left( \frac{\bar{E}}{2} + sc \right) \frac{h_r - h_v}{2}$$

Therefore the correction for curvature

$$C = \left( \frac{\bar{E}}{2} + sc \right) \frac{h_r - h_v}{2} \cdot \frac{d_r + d_v}{3} \text{ angle QGS}$$

When IG and GT are each an even station or 100 ft in length QGS = D and

$$C = \left( \frac{\bar{E}}{2} + sc \right) \frac{h_r - h_v}{2} \cdot \frac{d_r + d_v}{3} \text{ angle D}$$

$$\text{arc } 1^\circ = .01745$$

$$C = \left( \frac{\bar{E}}{2} + sc \right) \frac{h_r - h_v}{2} \cdot \frac{d_r + d_v}{3} \cdot .01745D$$

$$= \left( \frac{\bar{E}}{2} - sc \right) (h_r - h_v) (d_r + d_v) \times .00291D \quad (\text{cu. ft.})$$

$$= \left( \frac{\bar{E}}{2} - sc \right) (h_r - h_v) (d_r + d_v) \times .00011D \quad (\text{cu. yds.})$$

When IG or GT or both are less than 100 ft.

$$\text{Let } IG = T_o$$

$$GT = T_i$$

$$\text{Then } QGE = \frac{T_o}{100} \cdot \frac{D}{2} \text{ and } SGE = \frac{T_i}{100} \cdot \frac{D}{2}$$

$$QGS = \frac{T_o + T_i}{200} D$$

$$C = \left( \frac{\bar{E}}{2} + sc \right) (h_r - h_v) (d_r + d_v) \frac{T_o + T_i}{200} \times .00011D \quad (\text{cu. yds.})$$

(158.)

The correction  $C$  is to be added when the greater area is on the outside of the curve, and subtracted when the greater area is on the inside of the curve.  
 When the center height is 0 as in Fig 3,  
 we may consider this a regular section in  
 which  $c = 0$   $h_r = 0$  and  $\bar{d}_r = \frac{b}{2}$ ; then  

$$C = \left( \frac{b}{2} \cdot h_r \left( \bar{d}_r + \frac{b}{2} \right) \frac{L_0 + L_r}{200} \times .00011 D \right) \text{ cu. yds.}$$
  

$$(059)$$

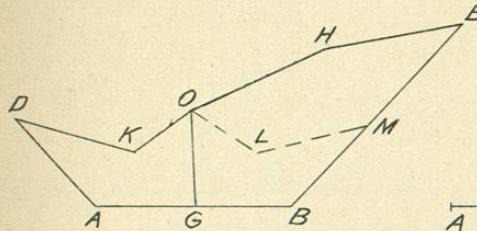


Fig. 4

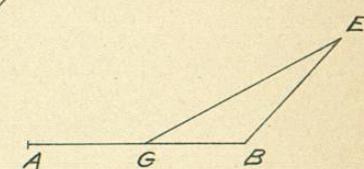
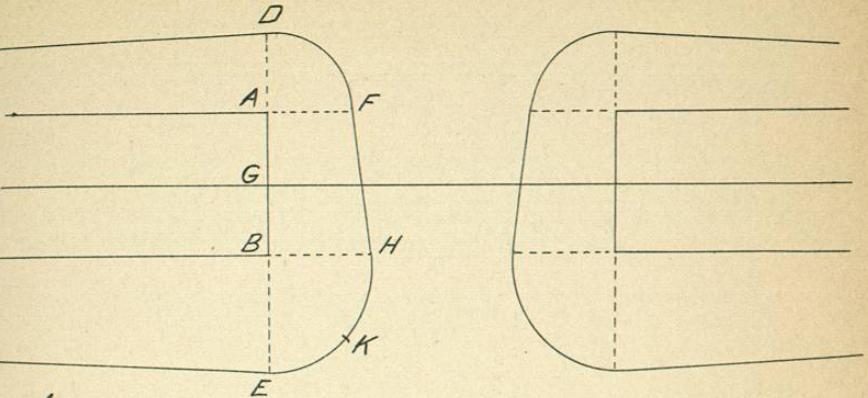


Fig. 3

In the case of irregular sections as shown in Fig. 4, the area and distance to center of gravity of OHEML may be found by any method available, and the corrections figured accordingly. The correction for curvature is, in present railroad practice, more frequently neglected than used. Nevertheless, its amount is sufficient in many cases to fully warrant its use.

Opening in Embankment.



When an opening is left in an embankment  
there remains outside the regular sections, the  
mass DEKHF.

This must be calculated in 3 pieces,

$$ADF - BEKH - ABHF$$

$$\text{Let } b = \text{base} = AB$$

$d_r$ ,  $d_l$  = distances out right and left

$$\begin{aligned} p_r &= BH \\ p_l &= AF \end{aligned} \quad \text{taken parallel to center line}$$

$$\begin{aligned} s_r &= \text{heights at } B \\ s_l &= \text{heights at } A \end{aligned}$$

$$s_1 = \text{solidity } ADF; \quad s_2 = BEKH; \quad s_3 = ABHF$$

Then (approximately) following the "Theorem  
of Pappus"

$s_1$  = mean of triangles AD and AF  $\times$  distance  
described by center of gravity.

$$\frac{\text{Area } AD + \text{Area } AF}{2} = \frac{\frac{f_r}{2}(d_r - \frac{b}{2}) + \frac{f_l}{2}p_l}{2} p_r$$

$$= \frac{f_r}{2} \cdot \frac{d_r + p_l - \frac{b}{2}}{2} = \text{mean area}$$

Distance described by center of gravity is found thus  
length  $AD = \bar{d}_r - \frac{6}{2}$  and  $AF = p_r$

$$\frac{\text{length } AD + \text{length } AF}{2} = \frac{\bar{d}_r + p_r - \frac{6}{2}}{2} = \text{mean length.}$$

$$\text{distance out to center of gravity} = \frac{1}{3} \cdot \frac{\bar{d}_r + p_r - \frac{6}{2}}{2}$$

$$\text{distance described by center of gravity} = \frac{\bar{d}_r + p_r - \frac{6}{2}}{6} \cdot \frac{\pi}{2}$$

$$S_1 = \frac{f_r}{2} \cdot \frac{\bar{d}_r + p_r - \frac{6}{2}}{2} \cdot \frac{\bar{d}_r + p_r - \frac{6}{2}}{6} \cdot \frac{\pi}{2}$$

$$= f_r \left( \bar{d}_r + p_r - \frac{6}{2} \right)^2 \frac{\pi}{48}$$

$$= \frac{f_r}{15} \left( \bar{d}_r + p_r - \frac{6}{2} \right)^2 \text{ nearly}$$

(160.)

$$S_2 = \frac{f_r}{15} \left( \bar{d}_r + p_r - \frac{6}{2} \right)^2 \text{ nearly.} \quad (161.)$$

The work of deriving these formulas is approximate throughout, but the total quantities involved are in general not large, and the error resulting would be unimportant.

There seems to be no method of accurately computing this solidity, which is adapted to general railroad practice.

$$S_3 = \frac{\text{area } AF + \text{area } BH}{2} \times AB$$

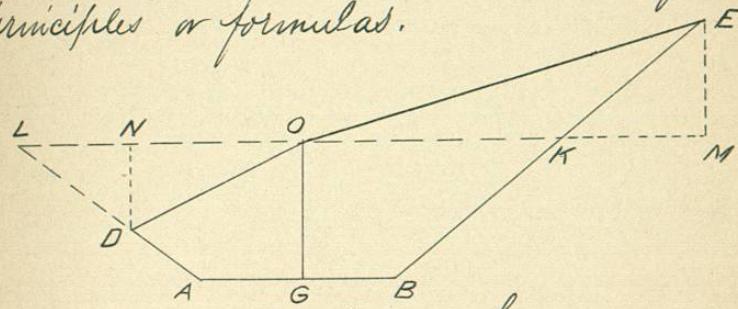
$$= \frac{f_r p_r + f_r p_r}{4} \cdot b$$

(162.)

### Earthwork Tables.

The calculation of quantities can be much facilitated by the use of suitably arranged "Earthwork Tables".

For regular "Three Level Sections" very convenient tables can be calculated upon the following principles or formulas.



Use notation as before for

$$c - h_r - h_v - d_r - d_v - s - \bar{v} - A - S.$$

$$\begin{aligned} \text{Then } A &= ABKL + OKE - ODL \\ &= c(\bar{v} + sc) + \frac{\bar{v} K X E M}{2} - \frac{\bar{v} L X N D}{2} \\ &= c(\bar{v} + sc) + \frac{\bar{v} K}{2} (EM - ND) \\ &= c(\bar{v} + sc) + \frac{\bar{v} + sc}{2} (h_v - c - c + h_r) \end{aligned}$$

$$A = c(\bar{v} + sc) + \frac{\bar{v} + sc}{2} (h_v + h_r - 2c) \quad (63)$$

For a prism of base  $A$  and  $v = 50$

$$S = 50 A \text{ (cu.ft.)} = \frac{50}{27} A \text{ (cu.yds)}$$

$$S = \frac{50}{27} c(\bar{v} + sc) + \frac{25}{27} (\bar{v} + sc)(h_v + h_r - 2c) \quad (64)$$

cu.yds.

For cross sections of a given base and slope, that is given  $\bar{v}$  and  $s$  constant, we may calculate for successive values of  $c$  - and

tabulate values of  $I_r$  and  $C$  as follows:-

	$I_r$	$C$
$c$	$\frac{50}{27} c(\bar{b} + sc)$	$\frac{25}{27} \left( \frac{\bar{b}}{2} + sc \right)$

$I_r$  represents the solidity for the level section.  $C$  is for use as a correction.

The formula then adapts itself to this table for any desired values of  $c - h_v - h_r$

$$S = I_r + C(h_v + h_r - 2c) \quad (164)$$

Having found for successive stations  $S_o$  and  $S'$ , (each for a prism  $t = 50$ ), then for the full section by "end areas",

$$S'_{100} = S_o + S' \quad \text{for}$$

$$S'_{100} = \frac{A_o + A'}{2} \cdot \frac{100}{27} = \frac{50 A_o}{27} + \frac{50 A'}{27}$$

$$S'_{100} = S_o + S' \quad (165)$$

When  $t$  is less than 100

$$S_t = (S_o + S') \frac{t}{100} \quad (166)$$

In level sections  $h_v = h_r = c$

$$h_v + h_r - 2c = 0 \quad \text{and the formula}$$

$$S' = I_r + C(h_v + h_r - 2c) \quad \text{becomes}$$

$$S' = I_r \quad \text{for level sections} \quad (167)$$

and the quantities for any given values of  $c$  can be directly taken from column  $I_r$  without any correction from column  $C$ .

In preliminary estimates or wherever center heights only are used, these tables are rapidly used.

Tables accompany these notes, calculated for

$$1 - B = 20 \quad S = 1\frac{1}{2} \text{ to } 1$$

$$2 - B = 14 \quad S = 1\frac{1}{2} \text{ to } 1$$

An example will illustrate their use.

$$B = 14 \quad S = 1\frac{1}{2} \text{ to } 1$$

Notes	Sta 1	$\frac{13.0}{-4.0}$	- 3.7	$\frac{12.4}{3.6}$
0		$\frac{10.6}{-2.4}$	- 2.5	$\frac{10.3}{-2.2}$

#### Calculations

$$\begin{array}{rcl} 3.7 & I_r = 134.0 & C = 11.6 \\ & + 2.3 & \frac{0.2}{2.32} \\ & \hline S_r = 136.3 & + 0.2 \end{array} \quad n_r + h_r = 7.6$$

$$\begin{array}{rcl} 2.5 & I_r = 82.2 & C = 10.0 \\ & - 4.0 & \frac{0.4}{4.00} \\ & \hline S_o = 78.2 & - 0.4 \end{array} \quad n_o + h_o = 4.6$$

$$S_{100} = S_r + S_o = 214.5$$

Tables of the sort described have been published as follows:—

"The Civil Engineer's Excavation and Embankment Tables" by Clarence Pullen and Charles C. Chandler, published by the "J. M. W. Jones Stationery and Printing Co" Chicago.

Tables are calculated for  $B = 12 - 14 - 16 - 18 - 20$

$$S = \frac{1}{4} - \frac{1}{2} - 1 - 1\frac{1}{2}$$

Accompanying these is also a "Table of Prismoidal Correction" calculated by the formula

$$C = \frac{1}{3.24} (C_o - C_r)(D_o - D_r)$$