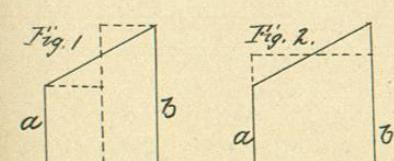
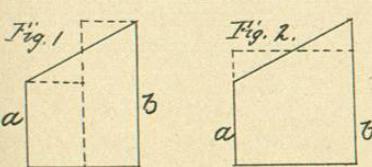


4. Between any two points where the diagram is intersected by any horizontal line, excavation equals embankment.

5. The area cut off by any horizontal line is the measure of the "haul" between the two points cut by that line.

It may be necessary to explain the latter point at somewhat greater length.

Any quantity (such as dimension weight or volume) is often represented graphically by a line; in a similar way, the product of two quantities (such as volume into distance or as foot pounds) may be represented or measured by an area. In the case of a figure other than a rectangle, the value or product measured by this area may be found by cutting up the area by lines, and these lines may be vertical lines representing volumes or horizontal lines representing distance. The result will be the same in either case. An example will illustrate.



In the two figures let  
 $\underline{a}$  and  $\underline{b}$  represent lbs.  
 $\underline{c}$  " " feet.  
and the area of the trapezoid represent a certain number of foot-pounds. The trapezoid may be resolved into rectangles

by the use of a vertical line as shown in Fig 1,  
or by a horizontal line as in Fig. 2.

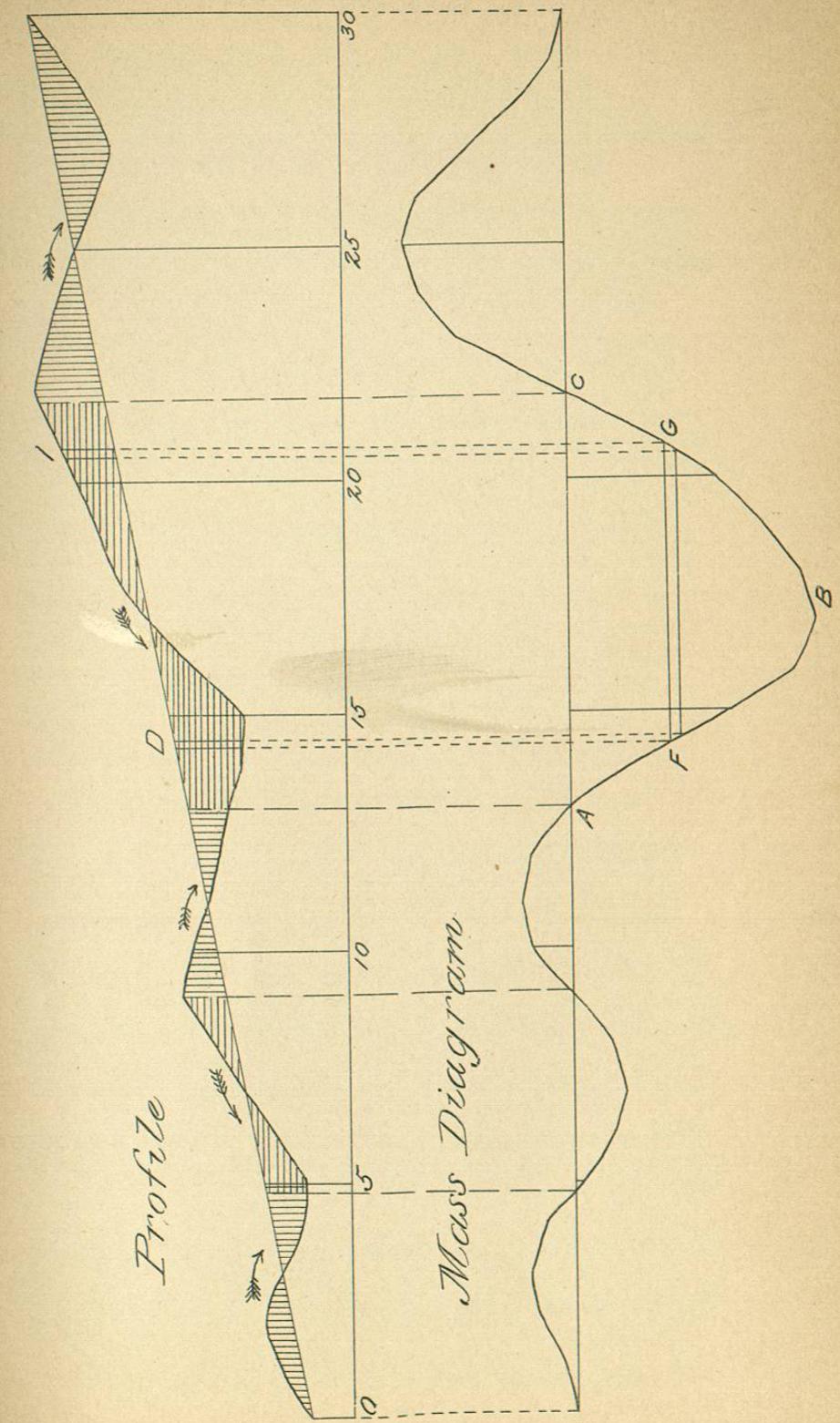
$$\text{In Fig 1, the area is } a \times \frac{c}{2} + b \times \frac{c}{2}$$

$$\text{" Fig 2, " " " } \frac{a+b}{2} \times c$$

the result being the same in both cases.

In an entirely similar way, the area ABC (p. 206) represents the "haul" of Earthwork (in cu. yds. moved 1 ft.) between A and C, and this area may be calculated by dividing it by a series of vertical lines representing solidities, as shown in the Fig. p. 201. That this area represents the haul between A and C may appear as follows. Take any elementary solidity  $dS$  at D (p. 206). Project this down upon the diagram at F and draw the horizontal lines FG. Between the points F and G (or between D and I) therefore, excavation equals embankment, and the mass  $dS$  must be hauled a distance FG, and the amount of "haul" on  $dS$  will be  $dS \times FG$ . Similarly with any other elementary  $dS$ .

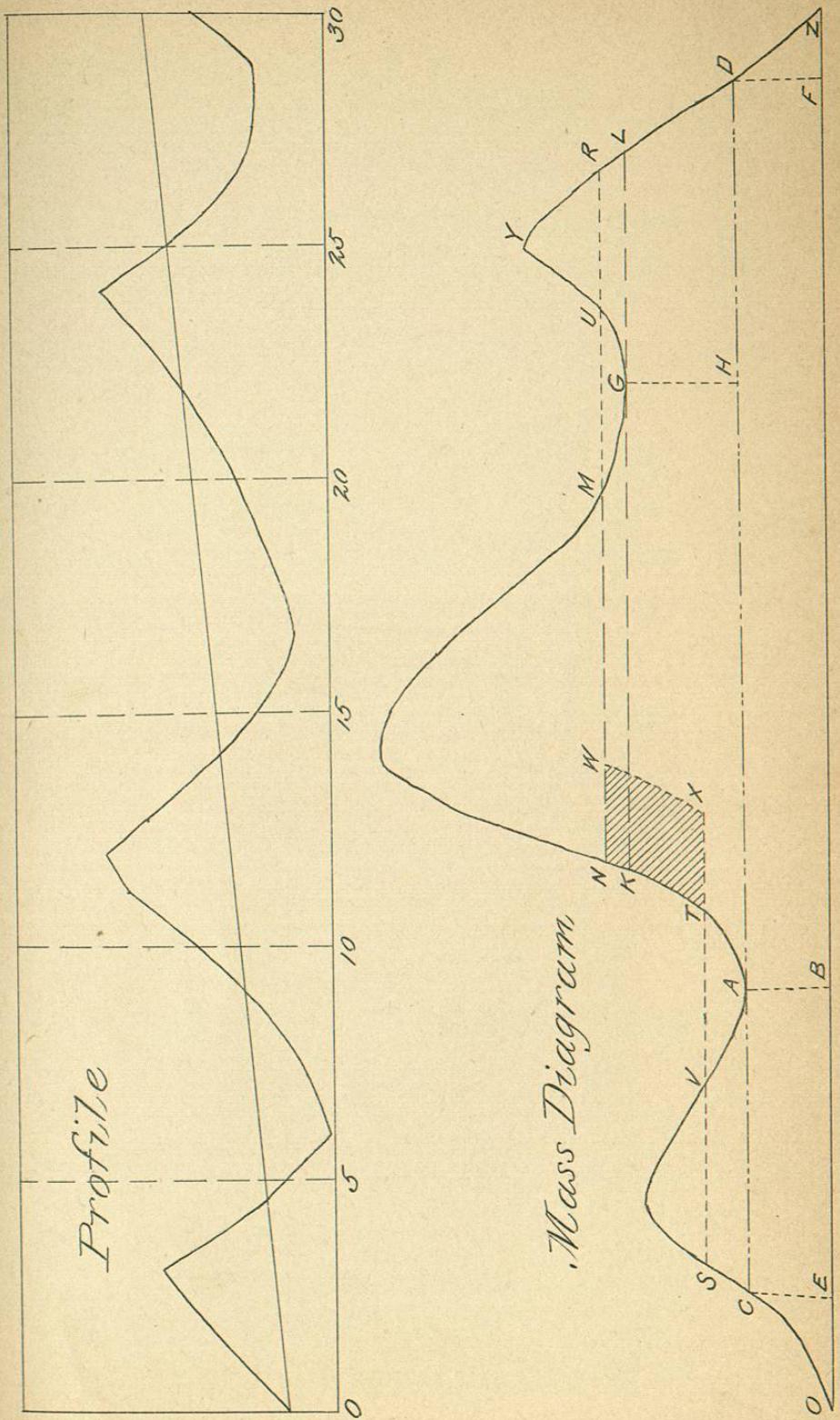
The total "haul" between A and C will be measured by the area ABC. This area is most conveniently measured by the trapezoids formed by the vertical lines representing solidities. The average length of haul will be



this area divided by the total solidity (represented in this case on p. 201 by the longest vertical line 2084).

The construction of the "Mass Diagram" as a series of trapezoids, involves the assumption that the center of gravity of a section of earthwork lies at its mid-section, which is only approximately correct. If the line joining the ends of the vertical lines be made a curved line, the assumption becomes more closely accurate, and if the area be calculated by "Simpson's Rule" or by Planimeter, results closely accurate will be reached.

It will be further noticed that in the "diagram", hill sections represent haul forward; and valley sections, haul backward (see p. 206). The mass diagram may therefore be used to indicate the methods by which the work shall be performed; whether excavation at any point shall be hauled forward or backward; and more particularly, to show the point where backward haul shall cease, and forward haul begin, as in the figure p. 206, which shows a very simple case, the cuts and fills being evenly balanced, and no haul over



over 900 feet. In the figure p. 208, the excavation from Sta. 0 to 14 is very much in excess of Embankment, and vice versa from Sta. 14 to 30. The mass diagram indicates a haul of nearly 3000 ft. for a large mass of Earthwork as shown by the ordinate AB. It will not be economical to haul the material 3000 ft.; it is better to "waste" some of the material near Sta. 0, and to "borrow" some near Sta. 30 if this be possible as is commonly the case. If we draw the line CD, the cut and fill between C and D will still be equal, and the volume of cut measured by CE can be wasted, and the equal volume of fill measured by DF can be borrowed, to advantage. It can be seen that there is still a haul of nearly 2000 ft. (from A to D) on the large mass of Earthwork measured by GH. It is probable that it will not pay to haul the mass GH, or any part of it, as far as AD. We must find the limit beyond which it is unprofitable to haul material, rather than borrow and waste. Let  $c$  = cost of 1 cu. yd. Excavation or Embankment.  
 $n$  = cost of haul on 1 cu. yd. 100 ft.  
 $n$  = length of haul in "Stations" of 100 ft. each.

Then when 1 cu. yd. of excavation is wasted, and  
1 cu. yd. of embankment is borrowed  
the cost =  $\frac{2c}{n}$

When 1 cu. yd. of excavation is hauled  
into embankment the cost =  $c + nh$

The limit of profitable haul is reached  
when  $2c = c + nh$  or

$$\text{when } n = \frac{c}{h}$$

Example. When excavation or embankment is  
18 cents per cu. yd. and haul is  $1\frac{1}{2}$  ct.

$$n = \frac{18}{1.5} = 12 \text{ stations.}$$

When  $c = 16$  and  $h = 2$   $n = 8$  stations.

In the former case 1200 ft. haul, we  
should draw its mass diagram (p. 208), the  
line KGL. KG is less than 1200 ft.

The line should not be lower than G, for  
in this case, the haul would be as great as  
KL or more than 1200 ft.

In the latter case (800 ft. haul) the line  
would be carried up to a point where  $NM = 800$  ft.  
The masses between N and A also C-O can be  
better wasted than hauled, and the masses be-  
tween M-G also L-Z can be better borrowed  
than hauled (always provided that there are  
suitable places at hand for borrowing and wasting).

Next, produce NM to R. The number of yards  
borrowed will be the same  $R-Z$  or  $M-G + L-Z$ .  
That arrangement of work which gives the smallest

"haul" (product of cu.yds  $\times$  distance hauled), is the best arrangement. The "haul" in one case is measured by GLRYU and in the other by MGV + URY. If MGV is less than GLRU, then it is cheaper to borrow (a) R-Z rather than (b) M-G and L-Z.

In a similar way material N-T and S-O can be wasted more economically than N-A and O-O. The most economical position for the line MR is when  $MV = UR$ . For ST when  $SV = VT$ . Any change from these positions of MR and ST will show an increase of area representing "haul".

The case is often not as simple as that here given. Very often the material borrowed or wasted has to be hauled beyond the limit of "free haul". The limit beyond which it is unprofitable to haul will vary according to the length of haul on the borrowed or wasted material; the limit will in general be increased by the length of haul on the borrowed or wasted material. The haul on wasted or borrowed material as NT may be shown graphically by  $NTXW$  - where  $NW = TX$  shows the length of haul and  $NTXW$  the "haul" (mass  $\times$  distance).

The mass diagram can be used also for finding the limit of "free haul" on the profile, and various applications will suggest themselves to those who become familiar with its use and the principles of its construction. Certainly one of its most important uses is in allowing "haul" and "borrow and waste" to be studied by a diagram giving a comprehensive view of the whole situation. There are few if any other available methods of accomplishing this result.