

Railroad Curves and Earthwork.

The operations of "locating" a railroad, as commonly practised in this country, are three in number:—

I. Reconnoissance.
II. Preliminary Survey.
III. Location Survey.

I. Reconnoissance.

The reconnoissance is a rapid survey, or rather a critical examination of country, without the use of the ordinary instruments of surveying. Certain instruments are however used; the Aneroid Barometer, for instance. It is very commonly the case that the termini of the railroad are fixed, and often intermediate points also. It is desirable that no unnecessary restrictions as to intermediate points should be imposed on the engineer to prevent his selecting what he finds to be the best line, and for this reason it is advisable that the reconnoissance should, where possible, precede the drawing of the Charter. The first step in reconnoissance should be to procure the best available maps of the country; a study of these will generally furnish to the engineer a guide as to the routes or section of country that should be examined. If maps of the Geological Survey are at hand, with contour maps and other topography carefully shown, the reconnoissance can be largely determined upon these maps

Lines clearly impracticable will be thrown out, the maximum grade closely determined, and the field examinations reduced to a minimum. No route should be accepted finally from any such map, but a careful examination should be made over the routes indicated on the contour maps. The examination, in general, should cover the general section of country, rather than be confined to a single line between the termini. A straight line and a straight grade from one terminus to the other is desirable, but is seldom possible and is in general far from possible. If a single line only is examined, and this is found to be nearly straight throughout, and with satisfactory grades, it may be thought unnecessary to carry the examination further. It will frequently, however, be found advantageous to deviate considerably from a straight line in order to secure satisfactory grades. In many cases it will be necessary to wind about more or less through the country in order to secure the best line. It can be expected when a high hill or a mountain lies directly between the points that a line around the hill, and somewhat remote from a direct line, will prove more favorable than any other. Unless a reasonably direct line is found to satisfactory the examination should embrace all the section of intervening country, and all feasible lines should be examined.

There are two features of topography that are likely to prove of especial interest in reconnoissance, ridge lines and valley lines.

A ridge line along the whole of its course is higher than the ground immediately adjacent to it on each side. That is, the ground slopes downward from it to both sides. It is also called a water shed line.

A valley line, to the contrary, is lower than the ground immediately adjacent to it on each side. The ground slopes upward from it to both sides. Valley lines may be called water course lines.

A pass is a place on a ridge line lower than any neighboring points on the same ridge. Very important points to be determined in reconnoissance are the passes where the ridge lines are to be crossed; also the points where the valleys are to be crossed; and careful attention should be given to these points. In crossing a valley through which a large stream flows, it may be of great importance to find a good bridge crossing. In some cases where there are serious difficulties in crossing a ridge, a tunnel may be necessary. Where such structures, either bridges or tunnels are to be built, favorable points for their construction should be selected and the rest of the line be compelled to conform.

Distances may be determined with sufficient accuracy in many cases from the map where a good one exists. Where this method is impossible or seems undesirable, the distance may be determined in one of several different ways. When the trip is made by wagon, it is customary to use an Odometer, an instrument which measures and records the number of revolutions of the wheel to which it is attached, and thus the distance travelled by the wagon. There are different forms of odometer. In its most common form, it depends upon a hanging weight or pendulum, which is supposed to hold its position, hanging vertical, while the wheel turns. The instrument is attached to the wheel between the spokes and as near to the hub as practicable. At low speeds it registers accurately; as the speed is increased, a point is reached where the centrifugal force neutralizes or overcomes the force of gravity upon the pendulum, and the instrument fails to register accurately, or perhaps at high speeds to register at all. If this form of odometer is used, a clear understanding should be had of the conditions under which it fails to correctly register. A theoretical discussion would readily establish the point at which the centrifugal force will balance the force of gravity. The wheel striking against stones in a rough road will create disturbances in the action of the pendulum, so that the odometer will fail to register accurately at speeds less than that determined upon the above assumption.

Another form of Odometer is manufactured which is connected both with the wheel and the axle, and measures positively the difference in the relative motion between the wheel and axle, and which ought to be reliable for registering accurately. Many Engineers prefer to count the revolutions of the wheel themselves, tying a rag to the wheel to make a conspicuous mark for counting.

When the trip is made on foot, pacing will give satisfactory results. An instrument called the Pedometer registers the results of pacing. As ordinarily constructed, the graduations read to quarter miles and it is possible to estimate to one tenth that distance. Pedometers are made which register paces. In principle the pedometer depends upon the fact that with each step, a certain shock or jar is produced as the heel strikes the ground, and each shock causes the instrument to register. Those registering miles are adjustable to the length of pace of the wearer.

If the trip is made on horseback, it is found possible to get good results with a steady-gaited horse, by determining his rate of travel and figuring distance by the time consumed in travelling. Excellent results are said to have been secured in this way.

It is customary for engineers not to use a a compass in reconnoissance, although this is sometimes done in order to more accurately trace upon the map, the line traversed. A pocket level will be found useful. The skillful use of pocket instruments will almost certainly be found useful, in fact of great value to the Engineer of reconnoissance.

It may in cases occur that no maps of any value are in existence or procurable. It may be necessary in such a case, to make a rapid instrumental survey, the measurements being taken either by pacing, chain or stadia measurements. This is however unusual.

The preliminary survey is based upon the results of the reconnoissance, and the location upon the results of the preliminary survey. The reconnoissance thus forms the foundation upon which the location is made. Any failure to find a suitable line and the best line constitutes a defect which no amount of faithfulness in later work will rectify. The most serious errors of location are liable to be due to imperfect reconnoissance; an inefficient Engineer of reconnoissance should be avoided at all hazards. In the case of a new railroad, it would in general be proper that the Chief Engineer should in person conduct this survey. In the case of the extension of existing lines, this might be impracticable or inadvisable, but an assistant of Known responsibility, ability and experience should in this case be selected to attend to the work. 8.

References

Reconnoissance.

Searles' Field Eng. p. 1

Wellington Ec. Theory Railway Location p. 831.

Beahan Ass'n Eng. Societies 1888

or R.R. Gazette 1888 p. 366 June 8.

McElroy " " 1888 p. 445 July 6

Jameson R.R. & Eng. Journal 1887 and 1888.

Aneroid Barometer Use of

Searles' Field Eng. p. 4 to 7

Godwin Eng. Fld BK p. 165

Van Nostrand Science Series No. 35.

Aneroid Barometer Tables.

Searles' Field Eng. Tables XV and XVI

Henck's Field Book " VIII and IX

Tables XV and XVI of Searles' are applicable to the formula $D = 60384.3(\log h_0 - \log h_1)\left(1 + \frac{t_0 + t_1 - 64°}{900°}\right)$

where D = difference in Elevation between two stations 0 and 1

h_0 = ht. of barom. Sta. 0 h_1 = ht. of barom. Sta. 1

t_0 = temp Fahr. Sta. 0 t_1 = temp Fahr. Sta. 1

For use with Table XV the formula becomes

$$D = 60384.3(\log h_0 - 1 - \overline{\log h_1} - 1)\left(1 + \frac{t_0 + t_1 - 64°}{900°}\right)$$

Table XV gives values of $60384.3(\log h - 1)$

Table XVI " " " $\frac{t_0 + t_1 - 64°}{900}$

Preliminary Survey.

The Preliminary Survey is based upon the results of the Reconnoissance. It is a survey made with the ordinary instruments of surveying. Its purpose is to fix and mark upon the ground a first trial line approximating as closely to the proper final line, as the difficulty of the country and the experience of the Engineer will allow; further than this, to collect data such that this survey shall serve as a basis upon which the final Location may intelligently be made. In order to approximate closely in the trial line, it is essential that the maximum grade should be determined or estimated as correctly as possible, and the line fixed with due regard thereto.

It will be of value to devote some attention here to an explanation about grades and "Maximum Grades"

Grades.

The ideal line in railroad location is a straight and level line. This is seldom if ever realized. When the two termini are at different elevations, a line straight and of uniform grade becomes the ideal. It is commonly impossible to secure a line of uniform grade between termini. In operating a railroad, an engine division will be about 100 miles, sometimes less, often more. In locating any 100 miles of railroad, it is almost certain than a uniform grade cannot be maintained. More commonly there will be a succession of hills, part of the line up grade, part down grade. Sometimes there will be a continuous up grade but not at a uniform rate. With a uniform grade, a locomotive engine will be constantly exerting its maximum pull or doing its maximum work in hauling the longest train it is capable of hauling; there will be no power wasted in hauling a light train over low or level grades upon which a heavier train could be hauled. Where the grades are not uniform, but are rising or falling, or rising irregularly, it will be found that the topography on some particular 5 or 10 miles is of such a character that the grade here must be steeper than is really necessary anywhere else on the line; or there may be two or three stretches of grade where about the same rate of grade is necessary, steeper than elsewhere required. The steep grade thus found necessary at some special point or points on the line of railroad is called the "Maximum Grade" or "Ruling Grade" or "Limiting Grade", it being the grade that limits the weight of train that an engine can haul over the whole division. It should be the effort to make the rate of "Maximum Grade" as low as possible, because the lower the rate of

"Maximum Grade", the heavier the train a given locomotive can haul, and because it costs not very much more to haul a heavy train than a light one. The maximum grade determined by the Reconnoissance should be used as the basis for the Preliminary Survey. How will this affect the line? Whenever a hill is encountered, if the maximum grade be steep, it may be possible to carry the line straight, and over the hill; if the maximum grade be low, it may be necessary to deflect the line and carry it around the hill. When the maximum grade has been once properly determined, if any saving can be accomplished by using it rather than a grade less steep, the maximum grade should be used. The saving made will in general be one or more of three kinds; a. Amount or quantity of excavation or embankment; b. distance; c. curvature.

In some cases, a satisfactory grade, a low grade for a maximum, can be maintained throughout a division of 100 miles in length with the exception of 2 or 3 miles at one point only. So great is the value of a low maximum grade that all kinds of expedients will be sought for to pass the difficulty without increasing the rate of maximum grade, which we know will apply to the whole division.

Sometimes by increasing the length of line, we are able to reach a given Elevation with a lower rate of grade. Sometimes heavy and expensive cuts and fills may serve the purpose. Sometimes all such devices fail, and there still remains an increase of grade necessary at this one point, but at this point only. In such case it is now customary to adopt the higher rate of grade for these 2 or 3 miles and operate them by using an Extra or additional Engine. In this case, the "ruling grade" for the division of 100 miles is properly the "Maximum grade" prevailing over the division generally, the higher grade for a few miles only being known as an "Auxiliary Grade" or more commonly a "Pusher Grade". The train which is hauled over the Engine division is helped over the auxiliary or pusher grade by the use of an additional Engine called a "Pusher". When the use of a short "Pusher Grade" will allow the use of a low "Maximum Grade", there is evident economy in its use. (Reference Wellington Econ. Theory Ry Loc. Chap XVI)

The Preliminary Survey follows the general line marked out by the Reconnoissance, but this rapid Examination of country may not have fully determined which of two or more lines is the best, the advantages may be so nearly balanced. In this two or more Preliminary Surveys must be made for comparison. When the Reconnoissance has fully determined the general route, certain details are still left for the Preliminary Survey to determine. It may be necessary to run two lines, one on each side of a small stream, and possibly a line crossing it several times. The Reconnoissance would often fail to settle little points like this. It is desirable that the Preliminary Survey should closely approximate to the final line, but it is not important that it should fully coincide anywhere.

An important purpose of the Preliminary is to provide a map which shall show enough of the topography of the country, so that the Location proper may be projected upon this map. Working from the line of survey as a base line, measurements should be taken sufficient to show streams and various natural objects, as well as the contours of the surface.

The Preliminary serves several purposes:

1st. To fix accurately the Maximum Grade for use in Location.

2nd To determine which of several lines is best.

3rd To provide a map as a basis upon which the Location can properly be made.

4th To make a close estimate of the cost of the work.

5th To secure in certain cases legal rights by filing plans.

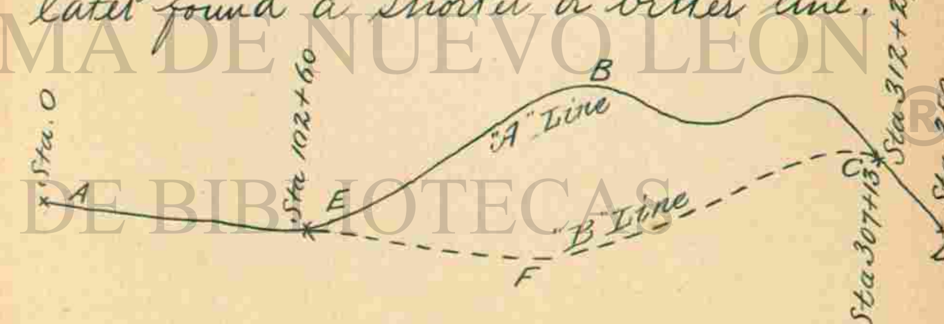
It should be understood that the Preliminary Survey is in general simply a means to an end and rapidity and economy are desirable. It is an instrumental survey. Measurements of distance are taken usually with the chain, although a tape is sometimes used. Angles are taken generally with a transit; some advocate the use of a compass. The line is ordinarily run as a broken line with angles. With a compass no backsight need be taken, and in passing small obstacles, a compass will save time on this account. A transit line can be carried past an obstacle readily by a zig-zag line. Common practice among engineers favors the use of the transit, rather than the compass. Stakes are set at every "station" 100 feet apart, and the stakes are marked on the face, the first 0, the next 1, and so to the end of the line. A stake set 1025 feet from the beginning would be marked 10+25.

Levels are taken on the ground at the side of the stakes, and as much oftener as there is any change in the inclination of the ground. All the surface heights are platted on a profile; and the grade line adjusted.

The line should be run from a governing point towards country allowing a choice of location; that is from a pass or from an important bridge crossing towards country offering no great difficulties. There is an advantage in running from a summit down hill, subject however to the above considerations. In running from a summit down at a prescribed rate of grade, an experienced Engineer will carry the line so that, at the end of a days work, the levels will show the line to be about where it ought to be. For this purpose, the levels must be worked up and the profile platted to date at the close of each day. Any slight change of line found necessary can then be made early ~~in~~ the next morning. A method sometimes adopted in working down from a summit, is for the locating Engineer to plat his grade line on the profile, daily in advance, and then during the day, plat a point on his profile, whenever he can conveniently get one from his leveler, and thus find whether his line is too high or too low.

Occasionally the result of two or three days work will yield a line extremely unsatisfactory, enough so that the work of these two or three days will be abandoned. The party "backs up" and takes a fresh start from some convenient point. In such case the custom is not to tear out several pages of note book, but instead to simply draw a line across the page and mark the page "Abandoned". At some future time the abandoned notes may convey useful information to the effect that this line was attempted and found unavailable. In general, all notes worth taking are worth saving.

Sometimes after a line has been run through a Section of Country, there is later found a shorter or better line.



In the figure used for illustration, the first line "A" Line, is represented by AEBCD, upon which the stations are marked continuously from A to D, 350 stations. The new line "B" Line starts from E, Sta 102+60, and the stationing is held continuous from O to where it connects with the "A" Line at C. The point C is Sta 312+27 of the "A" Line, and is also Sta 307+13 of the "B" Line. It is not customary to restake the line from C to D in accordance with "B" Line stationing. Instead of this, a note is made in the note books as follows:

Sta. 312+27 "A" Line = 307+13 "B" Line.

Some engineers make the note in the following form:-

Sta. 307 to 313 = 86 ft.

The first form is preferable, being more direct and less liable to cause confusion.

All notes should be kept clearly and nicely in a note book, never on small pieces of paper. The date, and the names of members of members of the party should be entered each day in the upper left hand corner of the page. An office copy should be made as soon as opportunity offers, both for safety and convenience. The original notes should always be preserved; they would be admissable as evidence in a court of law where a copy would be rejected. When two or more separate or alternate lines are run, they may be designated Line "A", Line "B", Line "C" or "A" Line, "B" Line, "C" Line.

The Preliminary Line is occasionally run with curves connecting the straight stretches, generally for the reason that a map of such a line is available for filing and certain legal rights result from such a filing.

Organization of Party on Preliminary Survey.

The organization of party may be as follows:-

1. Locating Engineer.

Transit Party:

2. Transitman.
3. Head Chainman.
4. Stakeman.
5. Rear Chainman.
6. Back Flag
7. Axemen (one or more)

Level Party:

8. Leveler
9. Rodman (sometimes 2)

Topography Party:

10. Topographer
11. Assistant

12. Cook
13. Teamster

1. The Locating Engineer is the chief of party, and is responsible for the business management of the camp and party, as well as for the conduct of the Survey. He determines where the line shall run, keeping ahead of the

transit, and establishing points as foresights or turning points for the transitman. In open country, the extra axeman can assist by holding the flag at turning points, and thus allowing the locating engineer to push on and pick out other point in advance. The locating engineer keeps a special note book or memorandum book; in it he notes on the ground the quality of material, rock, earth, or whatever it may be; takes notes to determine the lengths and positions of bridges, culverts, and other structures; shows the localities of timber, building stones, borrow pits and other materials valuable for the execution of the work; in fact makes notes of all matters not properly attended to by the transit, leveling, or topography party. The rapid and faithful prosecution of the work depend upon the locating engineer, and the party ought to derive inspiration from the energy and vigor of their chief, who should be the leader in the work. In open and easy country, the locating engineer may instil life into the party by himself taking the place of the head chainman occasionally. In country of some difficulty, his time will be far better employed in prospecting for the best line.

2. The Transitman does the transit work, ranges in the line from the instrument, measures the angles, and keeps the notes of the

transit survey. The following is a good form for the left hand page of the note book

Station	Point	Deflection	Observed Bearing	Calculated Bearing.
			N. 3°30'E	N. 3°38'E
7	o+24	33°02'R		
6				
5			N. 29°30'W	N. 29°24'W
4	o	12°09'L		
3				
2				
1			N. 17°15'W	N. 17°15'W
0	o			

Notes of topography and remarks are entered on the right hand page, which for convenience is divided into small squares by blue lines, with a red line running up and down through the middle.

The stations run from bottom to top of page. The bearing is taken at each setting and recorded just above the corresponding point in the note book, or opposite a part of the line, rather than opposite the point. Ordinarily, the transitman takes the bearings of all fences and roads crossed by the line, finds the stations from the rear chainman, and records them in their proper place and direction on the right hand page of the note book. Section lines of the United States land surveys should be observed in the same way. The transitman is next in authority to the locating engineer.

and directs the work when the latter is not immediately present. The transitman, while moving from point to point, setting up, and ranging line, limits the speed of the entire party and should waste no time.

3. The Head Chainman carries a "flag" and the forward end of the chain which should be held level and firm with one hand, while the flag is moved into line with the other. He should always put himself nearly in line before receiving a signal from the transitman; plumbing may be done with the flag. When the point is found, the stakeman will set the stake. When a suitable place for a turning point is reached, a signal should be given the transitman to that effect. A nail should be set in top of the stake at all turning points. A proper understanding should be had with the transitman as to signals.

Signals from the transitman.

A horizontal movement of the hand indicates that the rod should be moved as directed.

A swinging movement of the hand, plumb the rod as indicated.

A movement of both hands or waving the handkerchief freely above the head means "all right"

At long distances, a handkerchief can be seen to advantage, or when snow is on the ground something black is better.

Signals from the Head Chainman.

Setting the bottom of flag on the ground and waving the top, means give the line;
Raising the flag and holding it horizontal with both hands, above the head, give line for a turning point.
The "all right" signal is the same as from the transitman.

In all measurements less than 100 feet (or a full chain), the head chainman holds the end of the chain, leaving the reading of the measurement to the rear chainman.

The head chainman regulates the speed of the party during the time that the instrument is in place, and should keep alive all the time. The rear chainman will keep up as a matter of necessity

4. The stakeman carries, marks and drives the stakes at the points indicated by the head chainman. The stakes should be driven with the flat side towards the instrument, and marked in the front with the number of the station. Intermediate stakes should be marked with the number of the last station + the additional distance, in feet and tenths, as 10+67.4. The stationing is not inter-

rupted and taken up anew at each turning point, but is continuous from beginning to end of the survey. At each turning point a plug should be driven nearly flush with the ground, and a witness stake driven in an inclined position at a distance of about 15 inches from the plug and at the side towards which the advance line deflects, and marked W, and under it the station of the plug.

5. The rear chainman holds the rear end of the chain over the stake last set, but does not hold against the stake to loosen it. He calls "chain" each time when the new stake is reached, being careful not to overstep the distance. He should stand beside the line, not on it, when measuring, and take pains not to obstruct the view of the transitman. He checks, and is responsible for, the correct numbering of stakes, and for all distances less than 100 feet, as the head chainman always holds the end of the chain. The stations where the line crosses fences, roads, and streams should be set down in a small note book, and reported to the transitman at the earliest convenient opportunity. The rear chainman is responsible for the chain.

6. The back flag holds the flag as a back sight at the point last occupied by the transit. The only signals necessary for him to understand from the transitman are "plumb the flag" and "all right". The flag should always be in position, and the transitman should not be delayed an instant. The back flag should be ready to come up the instant he receives the "all right" signal from the transitman. The duties are simple, but frequently are not well performed.

7. The axeman cuts and clears through forest or brush. A good axeman should be able to keep the line well, so as to cut nothing unnecessary. In open country, he prepares the stakes ready for the stakeman or assists the locating engineer as fore flag.

8. The leveler handles the level and generally keeps the notes, which may have the following form for the left hand page.

Station	+S	H I	−S	Elevation

The right hand page is for remarks and descriptions of turning points and bench marks. It is desirable that turning points should, where possible, be described, and that all bench marks should be used as turning points. Readings on turning points should be recorded to hundredths or to thousandths of a foot, dependent upon the judgment of the Chief Engineer. Surface readings should be made to the nearest tenth, and Elevations set down to nearest tenth only. A self reading rod has advantages over a target rod for short sights. A target rod is possibly better for long sights and for turning points. The "Philadelphia Rod" is both a target rod and a self reading rod, and is thus well adapted for railroad use. Bench marks should be taken at distances of from 1000 to 1500 feet depending upon the country. All bench marks, as soon as calculated should be entered together on a special page near the end of the book. The leveler should test his level frequently to see that it is in adjustment. The leveler and rodman should together bring the notes to date every Evening and plat the profile to correspond.

The profile of the preliminary line should show:
1. Surface line (in black).
2. Grade line (in red).
3. Grade Elevations at each change in grade (in red).
4. Rate of grade, rise per 100 (in red).
5. Station and deflection at each angle in the line (in black).
6. Notes of roads, ditches, streams, bridges, etc (in black).

9. The rodman carries the rod and holds it vertically upon the ground at each station and at such intermediate points as mark any important change of slope of the ground. The surface of streams and ponds should be taken when met, and at frequent intervals where possible, if they continue near the line. Levels should also be taken of high water marks wherever traces of these are visible. The rodman carries a small note book in which he enters the rod readings at all turning points. In country which is open, but not level, the transit party is liable to outrun the level party. In such cases greater speed will be secured by the use of two rodmen.

10. The topographer is, or should be, one of the most valuable members of the party. In times past, it has not always been found necessary to have a topographer, or if employed, his duty has been to sketch in the general

features necessary to make an attractive map and represent hills and buildings sufficiently well with reference to the line, to show, in, a general way, the reason for the location adopted. Sometimes the Chief of the party has for this purpose, taken the topography. At present the best practice favors the taking of accurate data by the topography party.

The topographer (with one or two assistants) should take the station and bearing (or angle) of every fence or street line crossed by the survey; also take measurements and bearings for platting all fences and buildings near enough to influence the position of the Location; also sketch as well as may be, fences, buildings, and other topographical features of interest which are too remote to require exact measurement; and finally, establish the position of contour lines, streams and ponds, within limits such that the Location may be well and fully determined on the contoured map.

The work of taking contours is accomplished by the use of hand level and tape (pacing may in many cases be sufficient). The elevation at the center line at each station is found from the leveler. Contours are generally taken at vertical intervals of 5 feet. The contour line say to the right of the station is found by reading upon a light hand marked rod, the difference in elevation between the center line at the stake, and the required contour. A cloth tape held by the assistant topographer will serve the purpose of a rod. The next contour to the right can be readily found owing to the fact that the topographer's eye is nearly 5 feet above the ground, making leveling easy. An allowance should be made for the difference between 5 feet and the height of the eye. Sections both right and left should be taken as often as necessary, the distances to each contour measured, and the lines sketched in on the ground, between the points thus determined. Books of convenient size, are made and divided into small cross sections to facilitate sketching. Cross section blocks or pads will be considered equally good by some engineers. The distance to which contour lines should be taken depends on the character of the country. The object should be to take contours as far from the line as is necessary in order to furnish contours requisite for determining the position of the located line.

Instead of a hand level, some engineers use a clinometer and take and record side slopes.

Topography can be taken rapidly and well by use of stadia or of plane table. This is seldom done, as most engineers are not sufficiently familiar with their use.

Some engineers advocate making a general topographical survey of the route by Stadia, instead of the survey above described. In this case, no staking out by "Stations" would be done. All points occupied by the transit should be marked by plugs, which can be used to aid in marking the "location" on the ground after it is determined on the contour map. This method has been used a number of times and is claimed to give economical and satisfactory results. It is probable that this method will have constantly increasing use in the future, and will prove the best method in a large share of cases.

3. Location Survey.

The location survey is the final fitting of the line to the ground. In location, curves are used to connect the straight lines or "tangents", and the alignment is laid out complete, ready for construction.

The party is much the same as in the preliminary, and the duties substantially the same. More work devolves upon the transitman on account of the curves, and more skill is useful in the head chainman in putting himself in position on curves. He can readily range himself on tangent. The form of notes will be shown later. The profile is the same, except that is shows for alignment notes the P.C. and P.T. of curves, and also the degree and central angle, and whether to the right or left.

It is well to frequently connect location stakes with preliminary stakes, when convenient, as a check on the work.

In making the location survey, two distinct methods are in use among Engineers.

1st Method.

Use preliminary survey and preliminary profile as guides in reading the country, and locate the line upon the ground. Experience in such work will enable an engineer to get very satisfactory results in this way in nearly all cases.

The best engineers, in locating in this way, as a rule, lay the tangents first, and connect the curves afterwards. It will appear how this is done later.

2nd method.

Use preliminary line, preliminary profile, and especially the contour lines on the preliminary map, make a paper location, and run this in on the ground. Some go so far as to give their locating engineer a complete set of notes to run by. This is going too far. Whether it is best to go farther than to fix in the map the location of tangents and specify the degree of curve is a question. A conservative method is to do no more than this, and in some cases, leave the degree of curve even an open question. The 2nd method is gaining in favor; but the 1st method is even now much used.

It is well accepted among engineers that no reversed curves should be used; 200 feet of tangent at least should intervene. Neither should any curve be very short, say less than 300 feet in length.

A most difficult matter is the laying of a long tangent, so that it shall be straight. Lack of perfect adjustment and construction of instrument will cause a "swing" in the tangent. The best way is to run for a distant foresight. Another way is to have the transit as well adjusted as possible, and even then change ends every time in reversing so that any error will not accumulate. It will be noticed that the preliminary is run in without curves, because more economical in time. Sometimes curves are run in here, either because the line can be run closer to its proposed position, or sometimes in order to allow of filing plans with the U.S. or separate States.

In location a single tangent often takes the place of a broken line in the preliminary, and it becomes important to determine the direction of the tangent with reference to some part of the broken line. This is readily done by finding the coordinates of any given point with reference to that part of the broken line, assumed temporarily as as a meridian. The course of each line is calculated, and the coordinates of any point thus found. It simplifies the calculation to use some part of the preliminary as an assumed meridian, rather than to use the actual bearings of the lines. The co-ordinates of two points on the proposed tangent, allow the direction of the tangent to be determined with reference to any part of the preliminary. Where the angles are small, an approximation sufficiently close will be secured by assuming in all cases that the cosine of the angle is 1.000000 and that the sines are directly proportional to the angles themselves. In addition to this, take the distances at the nearest even foot, and the calculation becomes much simplified.

The located line, or "Location," as it is often called, is staked out ordinarily by center stakes which mark a succession of straight lines, connected by curves to which the straight lines are tangent. The straight lines are by general usage called "Tangents."

The curves most generally in use are circular curves, although parabolic and other curves are sometimes used. Circular curves may be classed as Simple, Compound, Reversed, or Spiral.

A Simple Curve is a circular arc, extending from one tangent to the next. The point where the curve leaves the first tangent is called the "P.C.," meaning the point of curvature, and the point where the curve joins the second tangent is called the "P.T.," meaning the point of tangency. The P.C. and P.T. are often called the tangent points. If the tangents be produced, they will meet in a point of intersection called the "Vertex," V. The distance from the vertex to the P.C. or P.T. is called the "Tangent Distance," T. The distance from the vertex to the curve (measured towards the center) is called the External Distance, E. The line joining the middle of the Chord, C, with the middle of the curve subtended by this chord, is called the Middle Ordinate, M. The radius of the curve is called the Radius, R. The angle of deflection between the tangents is

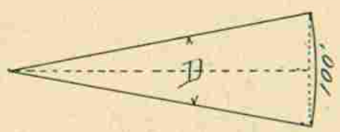
called the Intersection Angle, I.
The angle at the center subtended by a chord of 100 feet is called the Degree of Curve, D. A chord of less than 100 feet is called a Sub-Chord, c, its central angle a Sub-Angle, d.

The measurements on a curve are made (a) from P.C. by a sub-chord (sometimes a full chord of 100 ft.) to the next even station, then (b) by chords of 100 feet each between even stations, and finally (c) from the last station on the curve, by a sub-chord (sometimes a full chord of 100 ft.) to P.T. The total distance from P.C. to P.T. measured in this way, is the Length of Curve, L.

The degree of curve is here defined as the angle subtended by a chord of 100 feet. Some engineers define the degree of curve as the angle subtended by an arc of 100 feet. Either assumption involves the use of approximate methods either in calculations or measurements, if the convenient and customary methods are followed. It is believed that on the merits of the question, it is best to consider the degree of curve as the angle at the center subtended by the chord of 100 feet, as here defined. In addition to this, "Henck's Field Book," published in 1854 and for 25 years from that time the field book most widely used, has so defined it, and the almost universal practice in this country is in harmony with this definition.

Outside of the United States, a curve is generally designated by its Radius R. In the United States for railroad purposes, a curve is generally designated by its Degree D.

Problem. Given R. Required D.



$$R \sin \tfrac{1}{2} D = \frac{100}{2}$$

$$\sin \tfrac{1}{2} D = \frac{50}{R} \qquad (1.)$$

Problem. Given D. Required R

$$R = \frac{50}{\sin \frac{1}{2} D} \qquad (2.)$$

Example. Given $D = 1°$

$$R_1 = \frac{50}{\sin \frac{1}{2} D}$$

50 log 1.698970
0°30' log sin. 7.940842

$R_1 =$ 5729.6 log 3.758128

Problem. Given R_1 the radius of 1° Curve (or D_1)

Required R_x the radius of any given curve of degree $= D_x$.

$$R_1 = \frac{50}{\sin \frac{1}{2} D_1} \qquad R_x = \frac{50}{\sin \frac{1}{2} D_x}$$

$$\frac{R_x}{R_1} = \frac{\sin \frac{1}{2} D_x}{\sin \frac{1}{2} D_1} \qquad R_x = R_1 \frac{\sin \frac{1}{2} D_1}{\sin \frac{1}{2} D_x} \qquad (3.)$$

In the case of small angles, the angles are proportional to the sines (approximately.)

$$R_x = R_1 \frac{\frac{1}{2}D_1}{\frac{1}{2}D_x} = R_x = R_1 \frac{D_1}{D_x} \quad \left\{ \begin{array}{l} \text{but } R_1 = 5730 \\ \text{to nearest foot} \end{array} \right\}$$

$$R_x = \frac{5730}{D_x} \text{ (approx.)} \qquad (4.)$$

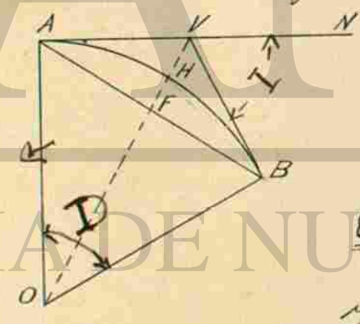
Example

$R_{10} = 573.7$ by (3.) or by Searles Table IV.
$= 573.0$ by (4.) (approx.)

Values of R and D are readily convertible. Table IV Searles, serves this purpose, giving accurate results or values. Approximate values can be found without tables by (4.) The radius of a 1° curve = 5730. should be remembered. In problems later, where either R or D is given, both will, in general, be assumed to be given

---

Problem Given I and R or D.
Required T.



$AOB = NVB = I.$
$AO = OB = R.$
$AV = VB = T$
$T = R \tan \frac{1}{2} I \qquad (5.)$

Example. Given $D = 9; I = 60°48'$
Required T_9

Table IV Searles R_9 log $= 2.804327$
$30°24'$ log tan $= 9.768414$
$T_9 = 373.9$ log 2.572741

Approximate Method.

$T_1 = R_1 \tan \frac{1}{2} I \; ; \; T_x = R_x \tan \frac{1}{2} I$

$\frac{T_x}{T_1} = \frac{R_x}{R_1} = \frac{D_1}{D_x} = \frac{1}{D_x}$ (approx.)

$T_x = \frac{T_1}{D_x}$ (approx) $\qquad (6.)$

Table VI Searles gives values of T_1 for various values of I. ($I = \Delta$ of Searles)

Table V Searles gives a correction to be added after dividing by R_x

Example. as before Given $D = 9$; $I = 60°48'$

Required T_9.

Table VI Searles	T_1	$60°40'$	$= 3352.6$
		$8'$	$= 9.0$
	T_1	$60°48'$	$= 3361.6$
	T_9		$= 373.5$ (approx)
Table V Searles correction			$.4$
(interpolation by inspection simply)	T_9		$= 373.9$ (Exact) the same as before.

Problem Given I and R or D

Required E.

Using previous figure

$$VH = E = R \text{ exsec } \tfrac{1}{2} I \qquad (7.)$$

Table XXIX Searles gives natural Exsec.

Table XXVI " " logarithmic Exsec.

Approximate Method.

By method used for (6.)

$$E_x = \frac{E_1}{D_x} \text{ (approx)} \qquad (8.)$$

Table VI Searles gives values for E_1

Table V Searles gives correction to be used if desired.

Problem Given I and R or D

Required M

$$FH = M = R \text{ vers } \tfrac{1}{2} I \qquad (9.)$$

Table XXIX Searles gives natural vers.
Table XXVI " " logarithmic vers.
Table VIII " " certain middle ordinates.

Problem. Given I and R or D
Required chord AB - C

$$C = 2R \sin \tfrac{1}{2} I \qquad (10.)$$

Table VII Searles gives values for certain long chords.

Transposing, we find additional formulae as follows:—

from (5.) $R = T \cot \frac{1}{2} I$ (11.)

(7.) $R = \dfrac{E}{\operatorname{exsec} \frac{1}{2} I}$ (12.)

(9.) $R = \dfrac{M}{\operatorname{vers} \frac{1}{2} I}$ (13.)

(10.) $R = \dfrac{C}{2 \sin \frac{1}{2} I}$ (14.)

(4.) $D_x = \dfrac{5730}{R_x}$ (approx.) (15.)

(6.) $D_x = \dfrac{T_1}{T_x}$ (approx.) (16.)

(8.) $D_x = \dfrac{E_1}{E_x}$ (approx.) (17.)

Problem Given sub-angle d and R or D
Required sub-chord c

$$c = 2R \sin \tfrac{1}{2} d \qquad (18.)$$

Approximate Method.

$$100 = 2R \sin \tfrac{1}{2} D$$

$$\frac{c}{100} = \frac{\sin \frac{1}{2} d}{\sin \frac{1}{2} D} = \frac{d}{D} \text{ (approx.)} \qquad (19.)$$

The precise formula is seldom if ever used.

Problem. Given sub-chord c and R or D.
Required sub-angle d.

$$d = \frac{c\,D}{100} \qquad (20.)$$

$$\frac{d}{2} = \frac{c}{100}\,\frac{D}{2} \qquad (21.)$$

A modification of this formula is as follows:

$$\frac{d}{2} = \frac{c\,D}{200}$$

for $D = 1$

$$\frac{d}{2} = c\,\frac{60'}{200} = c \times 0.3'$$

For any value D_x

$$\frac{d}{2} = c \times 0.3' \times D_x \text{ (value in minutes)} \qquad (22)$$

This gives a very simple and rapid method of finding the value of $\frac{d}{2}$ in minutes, and the formula should be remembered.

Example. Given sub-chord = 63.7 $D = 6°30'$
Required sub-angle

I. by (20.)
63.7
6.5 = D
3185
3822
414.05
4.14
60'
$d = 4°08'$
$\frac{d}{2} = 2°04'$

II. by (21.)
63.7
3.25 = $\frac{D}{2}$
3185
1274
1911
207.025
2.07
60'
$\frac{d}{2} = 2°04'$

III. by (22.)
63.7
0.3
19.11
6.5
9555
11466
124.215 minutes
$\frac{d}{2} = 2°04'$

Method III seems preferable to I or II.

<u>Problem</u> <u>Given</u> I and D.
<u>Required</u> I_1.

(a.) When the P.C. is at an <u>even</u> <u>station</u>, D will be contained in I a certain number of times $\underline{n}$, and there will remain a sub-angle $\underline{d}$ subtended by its chord $\underline{c}$.

$$\frac{I}{D} = n + \frac{d}{D} = n + \frac{c}{100} \text{ (approx.)}$$

$$100\frac{I}{D} = \quad 100n + c = I_1 \text{ (approx)}$$

(b.) When the P.C. is at a sub-station the same reasoning holds, and

$$I_1 = 100\frac{I}{D} \text{ (approx.)} \qquad (23.)$$

Transposing

$$I = \frac{I_1 D}{100} \text{ (approx.)} \qquad (24.)$$

$$D = \frac{100\, I}{I_1} \qquad (25.)$$

These formulas (23)(24)(25), though approximate are the formulas in common use.

<u>Example</u> <u>Given</u> 7° curve $I = 39°37'$
<u>Required</u> I_1

$I = 39°37'$
$D = 7\,\underline{)\,39.6167°}$
$5.6595+$
$I_1 = 566.0$

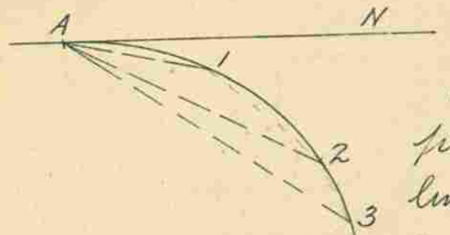
<u>Example</u> <u>Given</u> D and I_1.
<u>Required</u> I.

Given 8° curve
also P.I. = 93+70.1
P.C = 86+49.3

$I_1 = 7\ 20.8$
$D = 8$
57.664
$60'$
39.840
$I = 57°40'$

Method of Deflection Angles.

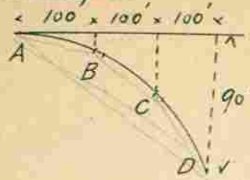
If at any point on an existing curve, a tangent to the curve be assumed, the angles from the tangent to any given points on the curve may be measured, and the angles thus found may be called Total Deflections to those points (as NA1, NA2, NA3)



In laying out successive points upon a straight line (as on a "Tangent"), Each point is generally fixed by (a) measurement from the preceding point, and (b) line; the line on a Tangent being the same for all point.

Similarly in laying out a curve, successive points may be fixed by (a) measurement from the preceding point and (b) line; the line in this case, for the curve, being that found by using the total deflection calculated for that point. In the figure preceding, the chord distance A1 and the total deflection NA1 fix point 1; the chord distance 1-2 and total deflection NA2 fix point 2; and 2-3 and NA3 fix 3. A curve can be conveniently laid out by this method, if the proper total deflections can be readily computed.

<u>Problem</u> <u>Given</u> a parabolic curve



<u>Required</u> - total deflections to B - C - D and chord lengths AB - BC - CD

Give results to nearest minute or nearest $\frac{1}{10}$ foot.

<u>Simple Curves.</u>

In the case of "simple curves" the "total deflections" can be readily computed, and the method of "deflection angles" is well adapted to laying them out.

<u>Problem.</u> To find the Total Deflections for a simple curve having <u>given</u> the Degree.

I. When the curve begins and ends at even stations.

The distance from Station to Station is 100 ft. The deflection angles are required.

An acute angle between a tangent and a chord is equal to one half the central angle subtended by that chord.

$A1 = 100 \quad VA1 = \frac{1}{2} D.$

The acute angle between two chords having their vertices in the circumference, is equal to one half the arc included between those chords.

1 - 2 = 100 and 1 A 2 = $\frac{1}{2}D$. Similarly,
2 - 3 = 100 and 2 A 3 = $\frac{1}{2}D$.
3 - B = 100 and 3 A B = $\frac{1}{2}D$.

This angle $\frac{1}{2}D$ is called by Henck and Searles the Deflection Angle, and will be so called here. Shunk and Trautwine call it the "Tangential Angle". The weight of Engineering Opinion appears to be largely in favor of the "Deflection Angle".

The "Total Deflections" will be as follows:—

V A 1 = $\frac{1}{2}D$
V A 2 = V A 1 + $\frac{1}{2}D$
V A 3 = V A 2 + $\frac{1}{2}D$.
V A B will be found by successive increments of $\frac{1}{2}D$
V A B = V B A = $\frac{1}{2}I$. This furnishes a "check" on the computation.

II. When the curve begins and ends with a Sub-chord.

V A 1 = $\frac{1}{2}d_1$
V A 2 = V A 1 + $\frac{1}{2}D$
V A 3 = V A 2 + $\frac{1}{2}D$
V A B is found by adding $\frac{1}{2}d_2$ to previous "deflection angle".
V A B = V B A = $\frac{1}{2}I$. This furnishes "check".

The total deflections should be calculated by successive increments; the final "check" upon $\frac{1}{2}I$ then checks all the intermediate total deflections.

(k.) Move transit to P.I. (B.)
(l.) Turn vernier back to 0 and beyond 0 to $\frac{1}{2}I$.
(m.) Sight on A.
(n.) Turn vernier to 0.
(o.) Sight towards V (or reverse and sight towards P.) and see the line checks on V or P.

It should be observed that 3 "checks" on the work are obtained.

1. The calculation of the total deflections is checked if total deflection to B = $\frac{1}{2}I$.
2. The chaining is checked if the final sub-chord measured on the ground = calculated distance.
3. The transit work is checked if the total deflection to B brings the line accurately on B.

Caution.

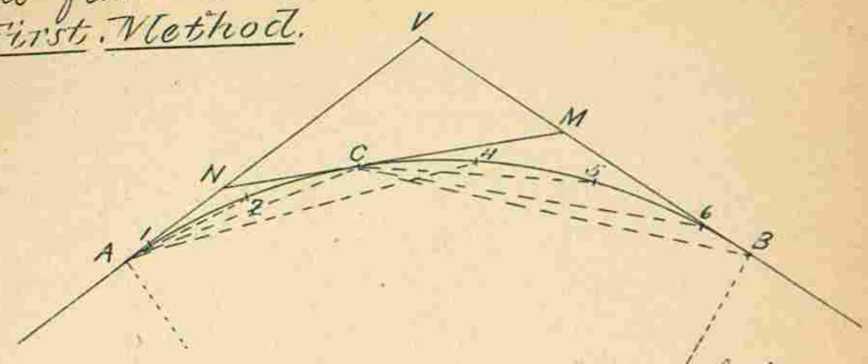
If a curve of nearly 180° = I is to be laid out from A, it is evident that it would be difficult or impossible to set the last point accurately as the "intersection" would be bad. It is undesirable to use a total deflection greater than 30°

It may be impossible to see the entire curve from the P.C. at A.

It will therefore, frequently happen that from one cause or another, the entire curve cannot be laid out from the P.C. at A.

Fieldwork when the entire curve cannot be laid out from the P.C.

First Method.



(a.) Lay out curve as far as C as before.
(b.) Set transit point at some convenient point as C (even station preferably).
(c.) Move transit to C.
(d.) Turn vernier back to 0 and beyond 0 to measure the angle VAC.
(e.) Sight on A.
(f.) Turn vernier to 0. See that transit line is on auxiliary tangent NCM (VAC = NCA being measured by $\frac{1}{2}$ arc AC)
(g.) Set off new deflection angle ($\frac{1}{2}d$ or $\frac{1}{2}D$).
(h.) Set point 4 and proceed as in ordinary cases.

Second Method.

(a.) Set point C as before and move transit to C.
(b.) Set vernier at 0
(c.) Sight on A.
(d.) Set off the proper "total deflection" for the point 4 = VA4.
(e.) Reverse transit and set point 4; NCA + MC4 VA4, Each measured by $\frac{1}{2}$ arc AC4.

(F.) Set off and use the proper "total deflections" for the remaining points.

The second method is in some respects more simple, as the notes and calculations, and also setting off angles are the same as if no additional setting were made. By the first method the deflection angles to be laid off will in general be even minutes, often degrees or half degrees, and are thus easier to lay off. It is a matter of personal choice which of the two methods shall be used.

Fieldwork of Finding P.C. and P.T.

In running in the line, it is common and considered advisable to establish "V", determine the station of "V", and measure the angle I. Having given I only, an infinite number of curves could be used. It is, therefore, necessary to assume additional data to determine what curve to use. It is common to proceed as follows:—

(a.) Assume either (1.) D at once (directly)
(2.) E and calculate D.
(3.) T and calculate D.

It is often difficult to determine off-hand what degree of curve will well fit the ground. Frequently the value of E_x can be readily determined on the ground.

The determination of D from E_x is readily made, using the approx formula $D_x = \frac{E'_1}{E_x}$. Similarly we may be limited to a given (or ascertainable) value of T_x, and from this readily find $D_x = \frac{T'_1}{T_x}$.
The value of D_x adopted will in general, be taken to the nearest $\frac{1}{2}°$ (perhaps only to nearest degree) rather than at the exact value found as above. (Some engineers use 1°40' = 100' and 3°20' = 200' etc. rather than 1°30' or 3°30' etc.)

(b.) From the data finally adopted I is <u>calculated anew</u>.

(c.) The instrument still being at V, the P.T. is set by laying off T.

(d.) The station of P.C. is calculated and P.C. set.

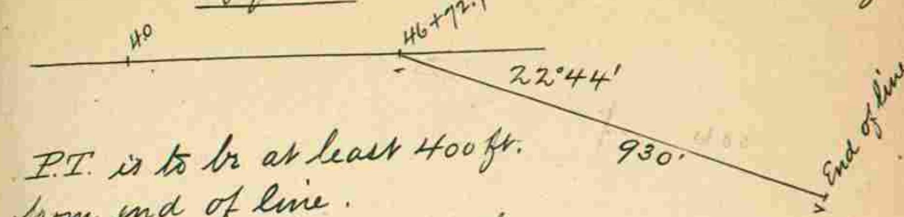
(e.) The length of curve L is calculated and station of P.T. thus determined. (not by adding T to station of V.)

Total deflections should be all calculated, and entered in note book.

Whether D, E, or T shall be assumed, depends upon the special requirements in each case. Curves are often run out from P.C. without finding or using V, but the best engineering usage seems to be in favor of setting V, whenever this is at all practicable, and from this finding the P.C. and P.T.

T = 22°-40' T = 1148.4 930
400
(530

__Example__ __Given__ a line as shown in sketch
__Required__ a curve to connect the tangents



P.T. is to be at least 400 ft. from end of line.
Use smallest degree or half degree consistent with this.
Find degree of curve and stations of P.C. and P.T.

Table VI Searles 22°40' T. = 1148.4
4' 3.4
22°44 1151.8
1060
918

930
400
(530
2.2 –

use 2°30' curve.

T. = 1151.8 (2.5
100 460.7
151
150
180

Table V correction 0
460.7 = T.

V = 46 + 72.7
T' 4 + 60.7
P.C 42 + 12.0
I 9 + 09.3
P.T. 51 + 21.3

22°44'
2.5) 22°7333
909.3 = I

$L = \frac{100 I}{D}$

$I = \frac{LD}{100}$

$LD = 100 I$

$L = \frac{100 I}{D}$

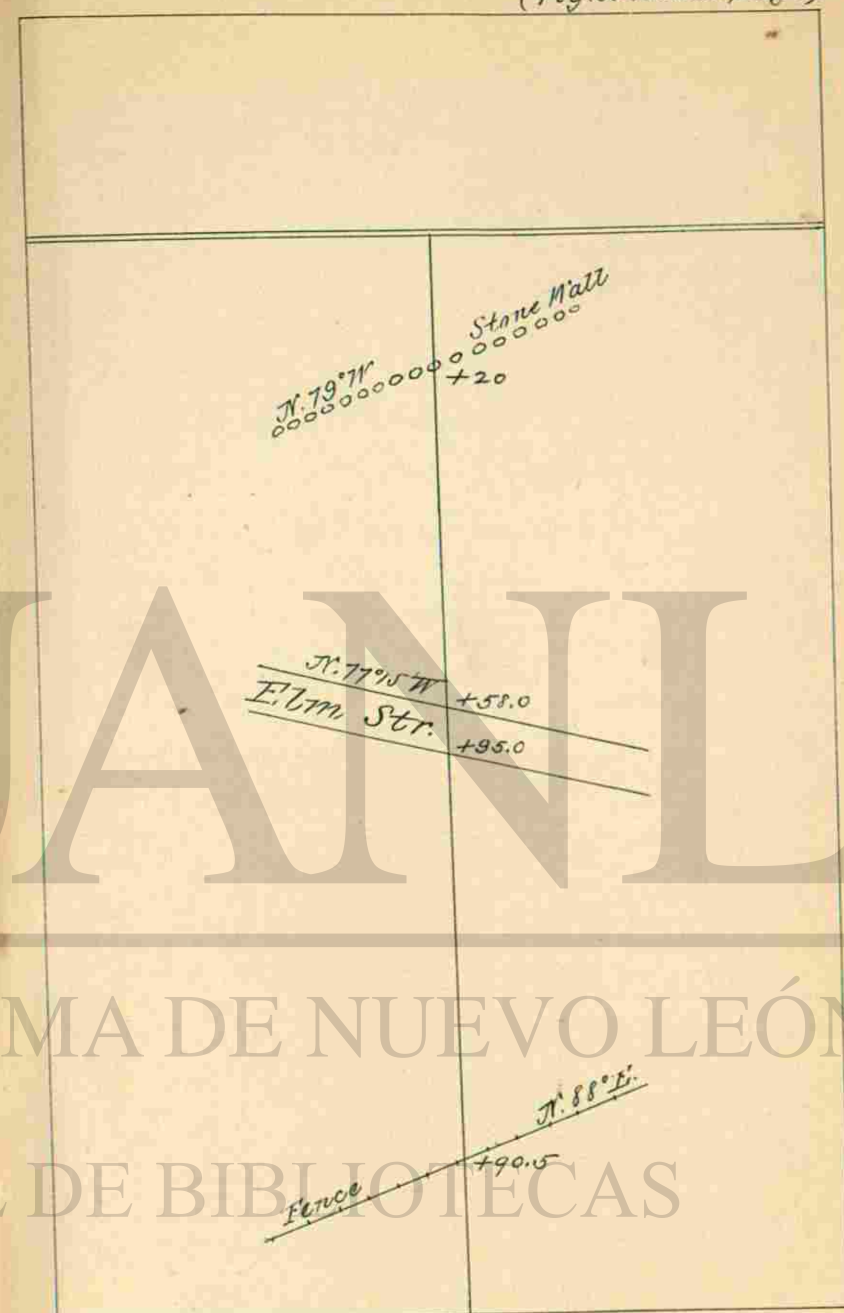
Form of Transit Book (Left hand page)

(Date)
(Names of Party)

Station	Points	Descrip. of Curve	Total Deflect.	Observed Course
114				
113				
112				
111				
110				N.46°00'E.
109	0+90.0 P.T.		11°15'	
108		R=1146.3	9°00'	
107		L=450.0	6°30'	
106	0+68.0 V	T=228.0	4°00'	
		I=22°30'		
105	0+40.0 P.C.	5° Right	1°30'	
104				
103				
102				
101				
100				
99				N.23°15'E.
98				

V is not a point on the curve. Nevertheless it is customary to record the Station found by chaining along the tangent.

(right hand page)



The Station to which the + refers will be clear from the left hand page.

Metric Curves.

In Railroad Location under the "Metric System" a chain of 100 meters is too long, and a chain of 10 meters is too short. Some engineers have used the 30 meter chain, some the 25 meter chain, but lately the 20 meter chain has been generally adopted as the most satisfactory. A "Station" is 10 meters. Ordinarily every second station only is set, and marked Sta 0, Sta 2, Sta 4 etc. On curves, chords of 20 meters are used. Usage among engineers varies as to what is meant by the degree of curve under the metric system. There are two distinct systems used as shown below.

I. The degree of curve is the angle at the center subtended by a chord of 1 chain of 20 meters.

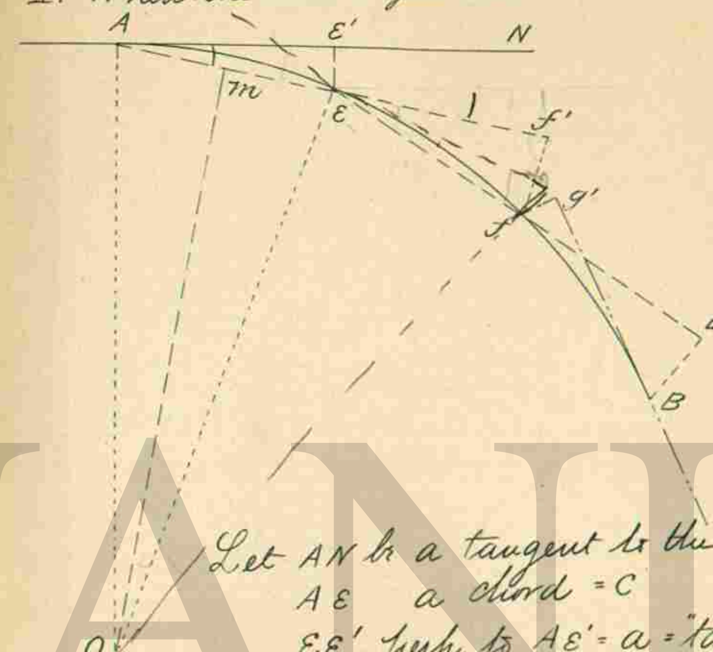
II. The degree of curve is the deflection angle for a chord of 1 chain of 20 meters (or one half the angle at the center).

II. Or very closely. the degree of curve is the angle at the center subtended by a chord of 10 meters (equal to 1 station length)

For several reasons, the latter system is here favored. Tables upon this basis have been calculated, giving certain data for metric curves. Such tables are to be found in Henck's Field Book, but not in Searles'.

Problem Given D and the Stations of P.C and P.T.
Required, to lay out the curve the method of **Deflection Distances.**

I. When the curve begins and Ends at an even Station.



Let AN be a tangent to the curve AB

AE a chord $= C$

EE' perp. to $AE' = a =$ "tangent deflection"

ff' $=$ "chord deflection"

$AO = EO$ $= R$

Draw Om perpendicular to AE.

Then $EE' : AE = mE : EO$

$$a : c = \frac{c}{2} : R$$

$$a = \frac{c^2}{2R} \qquad (26.)$$

$ff' = 2a$ $Af' = AE$ produced.

When AE is a full station of 100 feet

$$a_{100} = \frac{100^2}{2R}$$

Fieldwork.

The P.C. and P.I. are assumed to have been set.

(a.) Calculate a_{100}.

(b.) Set point $\mathcal{E}$ distant 100 ft. from A and distant a_{100} from $A\mathcal{E}'$ ($A\mathcal{E}' < 100$ ft.; $A\mathcal{E}'\mathcal{E} = 90°$)

(c.) Produce $A\mathcal{E}$ to f' ($\mathcal{E}f' = 100$ ft.) and find f distant $2\,a_{100}$ from f' ($\mathcal{E}f = 100$ ft.)

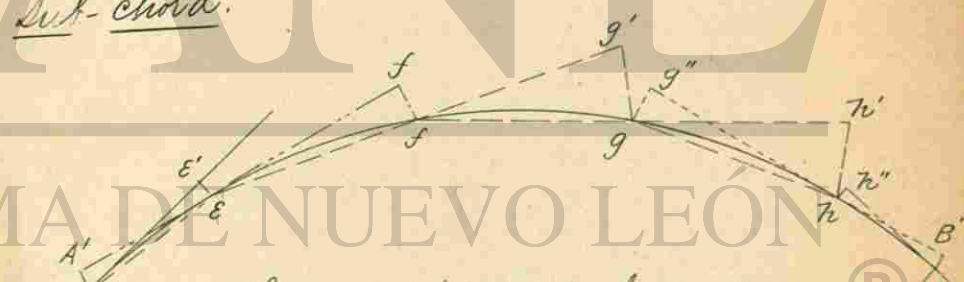
(d.) Proceed similarly until B is reached (P.T.)

(e.) At station preceding B (P.T.), lay off $fg' = a_{100}$ ($fg'B = 90°$)

(f.) $g'B$ is tangent to the curve at B (P.T.)

Problem. Given D and the Stations of P.C. and P.T.
Required to lay out Curve by Defl. Dist.

II. When the curve begins and end with a Sub-chord.



Let $A\mathcal{E}$ = initial sub-chord = c_i

hB = final sub-chord = c_f

$\mathcal{E}'\mathcal{E}$ = tang. defl. for c_i = a_i

$h''h$ = " " " c_f = a_f

by (26.) $a_i = \frac{c_i^2}{2R}$; $a_f = \frac{c_f^2}{2R}$; $a_{100} = \frac{100^2}{2R}$.

$$a_i : a_{100} = c_i^2 : 100^2 \qquad a_i = a_{100}\frac{c_i^2}{100^2}$$
$$a_f : a_{100} = c_f^2 : 100^2 \qquad a_f = a_{100}\frac{c_f^2}{100^2} \quad (27.)$$

In general it is better to use (27) than $a_i = \frac{c_i^2}{2R}$.

Example Given P.T. 20+42 6° Curve R
P.C. 16+25

Required all data necessary to lay out Curve by "Deflection Distances." Calculate without Tables. Results to $\frac{1}{100}$ ft.

Radius 1° Curve = $\frac{5730.}{955.}$ (6
6°

$$a_{100} = \frac{100^2}{2 \times 955.}$$

1910) 10000. (5.235+
955
450
382
680
573
1070
955

$= 5.24$

$2\,a_{100} = 10.47$

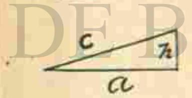
$a_{75} = .75^2 \times 5.24$
$= 2.95$

$a_{42} = .42^2 \times 5.24$
$= 0.92$

Searles' Tables give $a_{100} = 5.234$ (precise value.)

The distance AE' is slightly shorter than AE. It is generally sufficient to take the point E' by inspection simply. If desired for this or any other purposes, a simple approximate solution of right triangles is as follows.

Problem. Given, the hypotheneuse (or base) and altitude. Required, the difference between base and hypothenuse. or in the figure $c-a$.



$$c^2 - a^2 = h^2$$
$$(c-a)(c+a) = h^2$$
$$c-a = \frac{h^2}{c+a} = \frac{h^2}{2c}\ \text{(approx.)}$$
$$c-a = \frac{h^2}{2a}\ \text{(approx.)} \qquad (28.)$$

Wherever h is small in comparison with a or c the approximation is good for ordinary purposes.

<u>Example.</u> $c = 100 \quad h = 10$

$$c - a = \frac{100}{200} = 0.50$$

$$a = 99.50$$

The precise formula gives 99.499

<u>Fieldwork</u> <u>for Case II p. 54.</u>

(a.) Calculate $a_{100} - a_i - a_f$. Remember that tangent deflections are as the <u>squares</u> of the chords

a_{100} may be found generally in Table IV Searles as "tangent offset"

(b.) Find the point E distant a_i from AE' and distant c_i from A $(AE'E = 90°)$

(c.) Erect auxiliary tangent at E (lay off $AA' = a_i$)

(d.) From $A'E$ produced, find point f $(ff' = a_{100}$; $Ef = 100$; $Eff' = 90°)$

(e.) From Ef produced find point g $(gg' = 2a_{100}$; $fg' = fg = 100)$

(f.) Similarly for each full station use $2a_{100}$ etc.

(g.) At last even station on curve h erect an auxiliary tangent (lay off $gg'' = a_{100}$; $gg''h = 90°$).

(h.) From $g''h$ produced find B ($B'B = a_f$ etc.)

(i.) Find tangent at B $(hh'' = a_f$; $hh''B = 90°)$.

The values of $a_{100} - a_i - a_f$ should be calculated to the nearest $\frac{1}{100}$ ft.

<u>Caution</u>. The tangent deflections vary as the <u>squares</u> of the chords, not directly as the chords

Curves may be laid out by this method without a transit, by the use of plumb line or "flag" in sighting in points, and with <u>fair</u> degree of accuracy.

For calculating $a_{100} - a_i - a_f$ it is sufficient in most cases to use the approx. value $R_x = \frac{5730}{D_x}$. A curve may be thus laid out without the use of transit or tables.

For many approximate purposes it is well and useful to remember that the "chord deflection" for 1° curve is 1.75 ft. nearly, and for other degrees in direct proportion. A head chainman may thus put himself nearly in line without the aid of the transitman.

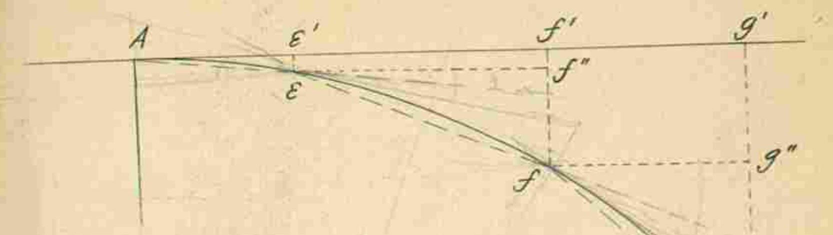
The method of "Deflection Distances" is not well adapted for common use, but will often be of value in emergencies.

Problem Given D and Stations of P.C. and P.T. Required to lay out the curve by "Deflection Distances" when the first sub-chord is small.

Caution. It will not be satisfactory in this case to produce the curve from this short chord.

Problem Given D and Stations of P.C. and P.I.
Required to lay out the curve by the method of

Offsets from the Tangent.



Let Ag' be tangent to curve Ag

Find
$$\varepsilon' A \varepsilon = \tfrac{1}{2} d \qquad = \alpha_1$$
$$f'' \varepsilon f = d + \tfrac{1}{2} D \qquad = \alpha_2$$
$$g'' f g = d + D + \tfrac{1}{2} D \qquad = \alpha_3 \text{ etc.}$$

$$A\varepsilon' = c_1 \cos \alpha_1 \qquad \varepsilon\varepsilon' = c_1 \sin \alpha_1$$
$$\varepsilon f'' = 100 \cos \alpha_2 \qquad ff'' = 100 \sin \alpha_2$$
$$fg'' = 100 \cos \alpha_3 \qquad gg'' = 100 \sin \alpha_3$$
$$ff' = \varepsilon\varepsilon' + ff''$$
$$gg' = ff' + gg'' \text{ etc}$$

When $A\varepsilon = 100$, then $\frac{1}{2} d$ becomes $\frac{1}{2} D$.

Fieldwork.

(a.) Calculate $A\varepsilon' - \varepsilon f' - fg'$,
$\varepsilon\varepsilon' - ff' - gg'$

(b) Set $\varepsilon' - f' - g'$

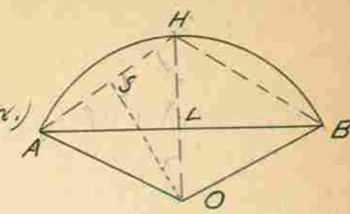
(c.) Set ε by distance $A\varepsilon\ (c_1)$ and $\varepsilon\varepsilon'$

(d.) Set f " " $\varepsilon f\ (100)$ and ff'

(e.) Set g " " $fg\ (100)$ and gg'.

For the computations indicated above, use always natural sines and cosines.

When $C = 100$ ft. or less, an approximate formula will generally suffice.



Problem Given R and c

Required M (approx.)

$HL = M : AH = \frac{AH}{2} : R$

$M = \frac{AH^2}{2R}$ Where AB is small compared with R

$AH = \frac{c}{2}$ (approx.)

$M = \frac{c^2}{8R}$ (approx.) (32.)

Example Given $c = 100$ $D = 9°$

Required M.

$R_9 = \frac{5730}{9} = 636.7$

$\times 8$

$5093.6\,)\,10000.\,(1.963 = M$

50936
490640
458424
322160
305616
16544

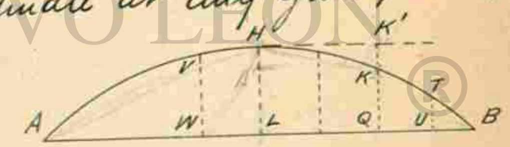
Precise value
$M = 1.965$

Table XII Searles gives middle ordinates for curving rails of certain lengths.

Problem Given R and c

Required ordinate at any given point Q.

Approximate Method



I. Measure $LQ = q$

$HL = M = \frac{\left(\frac{c}{2}\right)^2}{2R}$

$KK' = \frac{HK^2}{2R}$ $KK' : M = HK^2 : \left(\frac{c}{2}\right)^2$

$KK' = \frac{q^2}{\left(\frac{c}{2}\right)^2} M$ (approx.) since $HK = q$ (approx.) (33.)

$KQ = M - KK'$

$HK^2 = 2R \times KK'$

$KK' = \frac{HK^2}{2R}$

When $\frac{q}{\frac{c}{2}} = \frac{1}{2}$ as in figure $KK' = \frac{M}{4}$ and $KQ = \frac{3}{4}M$ (app.)

when $\frac{q}{\frac{c}{2}} = \frac{1}{4}$ $\qquad VW = \frac{15}{16}M$ (app.)

" $\frac{q}{\frac{c}{2}} = \frac{3}{4}$ $\qquad TU = \frac{7}{16}M$ (app.)

It may be shown that the curve thus found is accurately a parabola, but for short distances this practically coincides with a circle.

Approximate Method

II. Measure LQ and QB

$$M = \frac{\left(\frac{c}{2}\right)^2}{2R} \qquad KK' = \frac{q^2}{2R} \text{ (approx)}$$

$$KQ = \frac{\left(\frac{c}{2}\right)^2 - q^2}{2R} = \frac{\left(\frac{c}{2}+q\right)\left(\frac{c}{2}-q\right)}{2R} \text{ (approx.)}$$

$$KQ = \frac{AQ \times QB}{2R} \text{ (approx.)} \qquad (34.)$$

Sometimes one, sometimes the other of these methods will be preferable

Example Given $c = 100$ $\quad D = 9°$

$M = 1.965$ from Tables

Required ordinate at point 30 ft. distant from center toward end of chord.

I. $30 \text{ ft.} = \frac{30}{50} \times \frac{c}{2}$

$KK' = \frac{9}{25} \times 1.965$

1.965×9

$25)\ 17.685$

$.70740$

$M = 1.965$

ordinate $= 1.258$

II. $R_1 = 5730.$

$R_9 = 636.7$

$2R_9 = 1273.4$

$AQ = 80$

$BQ = 20$

$1273.4)\ 1600\ (1.257$

1273.4

32660

25468

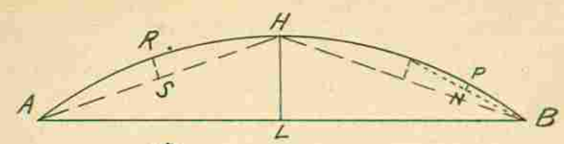
71920

63670

8250

Precise result for data above $= 1.260$

Problem Given R and c
Required a series of points on the curve.



$$M = HL = \frac{C^2}{8R} \text{ (approx.)}$$

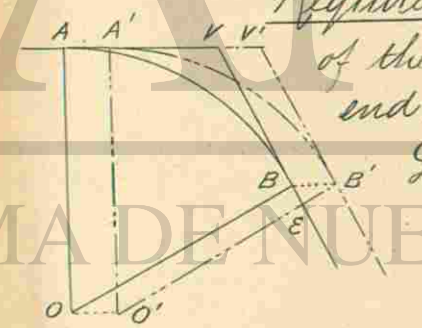
$$RS = \frac{AH^2}{8R} \text{ (approx.)} \qquad AH = \frac{c}{2} \text{ (approx.)}$$

$$RS = \frac{\frac{c^2}{4}}{8R} = \frac{M}{4} \text{ (approx.)}$$

$$PN = \frac{RS}{4} \text{ (approx.)}$$ Etc as far as desirable.

This method is useful for many general purposes, bending rails among others.

Problem. Given a simple curve joining two tangents
Required the P.C. of a new curve of the same radius which shall end in a parallel tangent.



Let AB be the given curve.
$A'B'$ " " required curve.
$B'E = p$ = perpendicular distance between tangents.

Join BB'

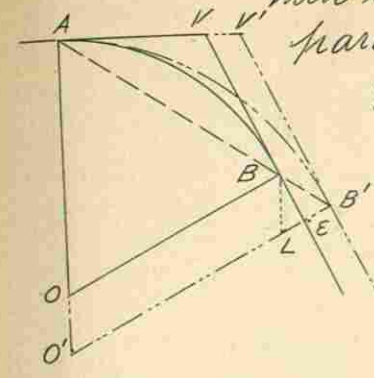
Then $AA' = OO' = BB'$ also $B'BE = V'VB = I$

$BB' \sin I = p \qquad BB' = AA' = \dfrac{p}{\sin I}$ (35.)

When the proposed tangent is outside the original tangent, the distance AA' is to be added to the station of the P.C. When inside is to be subtracted.

__Problem.__ __Given__ a simple curve joining two tangents

__Required__ the radius of a new curve which with the same P.C. shall end in a parallel tangent.



Let AB be the given curve of Radius $R = AO$

$B'E = p$ = perpendicular distance

AB' the required curve, rad $= R'$

Draw chords AB, AB'; also BB' also BL parallel to AO'.

Then $BL = OO' = R' - R = B'L = BL$

$BLB' = AO'B' = I$.

$BL \text{ vers } BLB' = B'E$

$R - R' \text{ vers } I = p \;; \quad R - R_, = \dfrac{p}{\text{vers } I}$ (36.)

When the proposed tangent is __outside__ the original tangent (as in the figure), the above formula applies, and $R' > R$.

When the proposed tangent is __inside__ the original tangent, the formula becomes

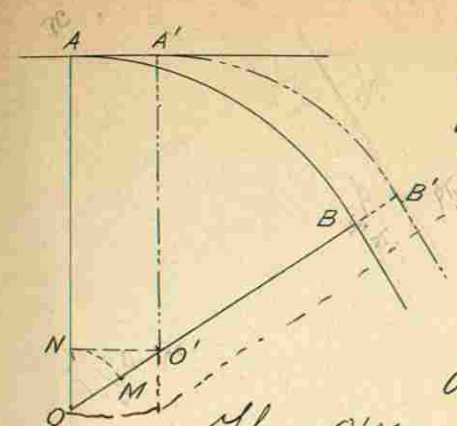
$$R - R_, = \frac{p}{\text{vers } I} \qquad (37.)$$

and $R' < R$

Since $VAB = V'AB'$, AB and $A'B'$ are in the same straight line.

Problem Given a simple curve joining two tangents.

Required, the radius and P.C. of a new curve to end in a parallel tangent with the new P.T. directly opposite the old P.T.



Let AB be the given curve of radius = R.
A'B' the required curve of radius R'
BB' = p
Draw perpendicular O'N and arc NM

Then $O'M = B'M - B'O'$
$= B'M - BM = BB'$
$O'M = p$

$ON \text{ exsec } NOO' = O'M$

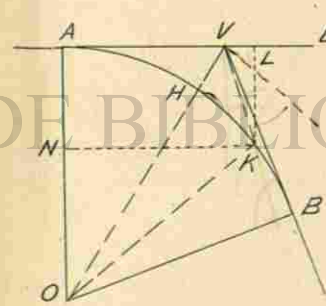
$R-R' \text{ exsec } I = p \;;\; R-R' = \dfrac{p}{\text{exsec } I}$ (38.)

$AA' = O'N = ON \tan NOO'$

$AA' = (R-R') \tan I$ (39.)

When the new tangent is <u>outside</u> the original tangent (as in the figure) $R > R'$ and AA' is added to the Station of the P.C.
When the new tangent is <u>inside</u> the original tangent $R < R'$ and $R'-R = \dfrac{p}{\text{exsec } I}$.
The distance AA' is then subtracted from the Station of P.C.

<u>Problem.</u> To find the simple curve that shall join two given tangents and pass through a given point.



With the transit at V, the given point K can often be best fixed by angle BVK and distance VK. If the point K be fixed by other measurements, these generally can readily be reduced to the angle BVK and distance VK.

(÷T − β)

Problem Given the two tangents intersecting at V, the angle I, and the point K fixed by angle $BVK = \beta$ and distance $VK = b$

Required, the radius R of curve to join the two tangents and pass through K.

In the triangle VOK we have given $VK = b$ and $OVK = \frac{180 - I}{2} - \beta$

Further $VO = \frac{R}{\cos \frac{1}{2} I}$ $\qquad OK = R.$

$$VO : OK = \sin VKO : \sin OVK$$

$$\frac{R}{\cos \frac{1}{2} I} : R = \sin VKO : \cos\left(\tfrac{1}{2} I + \beta\right)$$

$$\sin VKO = \frac{\cos\left(\frac{1}{2} I + \beta\right)}{\cos \frac{1}{2} I} \qquad (40.)$$

From data thus found the triangle VOK may be solved for R.

In solving this triangle, the angle VOK is often very small. A slight error in the value of this small angle may occasion a large error in the value of R. In this case the following **Second Method** of finding R after VOK has been found.

Find $AOK = \frac{1}{2} I + VOK$ also $DVK = I + \beta$

Then $R \text{ vers } AOK = LK$

$= b \sin DVK$

$$R = \frac{b \sin DVK}{\text{vers } AOK} \qquad (41.)$$

Problem Given R – I – β (BVK)

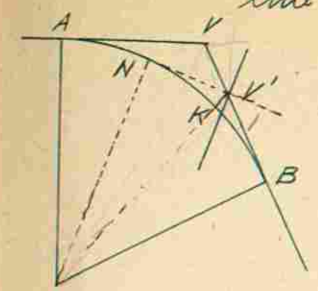
Required b (VK)

In the triangle VOK $\quad OK = R$; $OVK = \frac{R}{\cos \frac{1}{2} I}$

$OVK = 90 - (\frac{1}{2} I + \beta)$. Solve triangle for b.

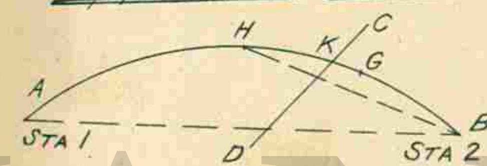
Also find VOK and station of K if desired.

Problem. To find the point where a straight line intersects a curve between Stations.



Find where the straight line VK cuts VB at V'.
Measure KV'B
Use V' as an auxiliary vertex,
Find I' from V'B.
Solve by preceding problem

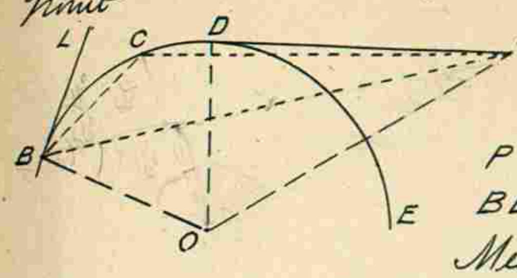
Approximate Method



Set the middle point H by method of ordinates
If the arc HB is sensibly a straight line, find the intersection of HB and CD.
Otherwise set the point G by method of ordinates, and get intersection of HG and CD.
Additional points on the arc may be set, if necessary, and the process continued until the required limit of accuracy can be secured.
The points H and G can be set without the use of a transit with sufficient accuracy for many purposes, a plumb line or flag being used in "sighting in".

Problem Given a curve and a point outside the curve

Required a tangent to the curve from that point



Let BDE be the given curve.
P the point outside the curve.
BL a tangent at B.
Measure LBP, also BP.

In the triangle BPO we have given PBO - BP - BO
Solve the triangle for BOP and OP

$$\text{Then } \cos DOP = \frac{OD}{OP} = \frac{R}{OP}$$

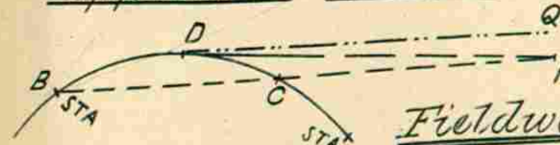
BOD = BOP − DOP.

From BOD find Station of D from known point B.

It should be noted that if log OP is found, this can be used again without looking out the number for OP. Other similar cases will occur elsewhere in calculation.

When for any reason it is difficult or inconvenient to measure BP directly, the angles CBP - BCP and the distance BC may be measured and BP calculated.

Approximate Method.



Fieldwork.

(a) From the Station (B) nearest to the required point D find by approximate method where BP cuts the curve at C. (If C be nearest Sta. produce PC to B).

(B.) Assume a point D so that BD is slightly greater than CD, and with transit at P.C. set the point D (transit point.) truly on the curve.
(c.) Move the transit to D and lay off tangent to the curve at D; this will very nearly strike P.
(d.) If the tangent strikes away from P at Q, measure QDP, and move the point D (ahead or back as the case may be) a distance c due to an angle at the center d = QDP. The tangent from this new point ought to strike P almost exactly.

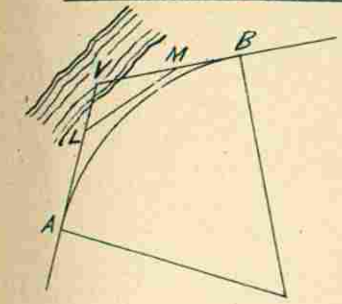
In a large number of cases, the point D will be found on the first attempt, sufficiently close for the required purpose.

If a tangent between two curves is required, similar methods by approximation will be found available.

Obstacles.

When any obstacle occurs upon a "tangent", the ordinary methods of surveying for passing such obstacle, will be used.

When V is inaccessible.



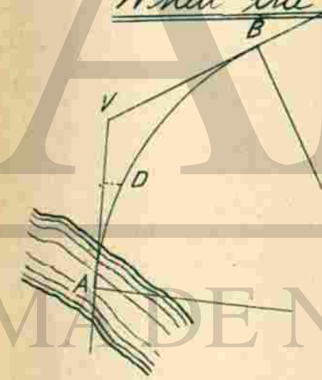
Measure VLM - VML - LM.

$I = VLM + VML$

LV and VM are readily calculated and AL and MB determined.

In some cases the best way is to assume the position of P.C. and run out the curve as a trial line, and finally find the position of P.C. correctly by the method of formula (35.)

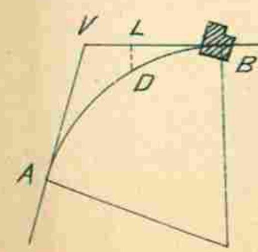
When the P.C. is inaccessible.



Establish some point D (an even station is preferable) by method of "offsets from Tangent" or otherwise.

Move transit to B (P.T.) and run out curve starting from D and checking on tangent VB.

When the P.T. is inaccessible.



With instrument still at V, set some convenient point D, move transit to P.C. and run in curve to D, and then pass the obstacle at B as any obstacle on a tangent would be passed.

When obstacles occur on the curve to prevent running it in, no general rules can well be given. Sometimes re-setting the transit on the curve will serve. Sometimes where one or two points only are invisible from the transit, these can be set by "deflection distances", and the curve continued by "deflection angles", without re-setting the transit. Sometimes "offsets from the tangent" can be used to advantage. Sometimes points can be set by "ordinates" from chords. Sometimes the method shown on page assuming an auxiliary V is the only one possible.

It should be borne in mind that it is seldom necessary that the even stations should be set. If there can be set any points whose stations are known and which are not too far apart, this is generally sufficient.

Finally, for passing obstacles and for solving many problems which occasionally occur, it is necessary to understand the various methods of laying out curves, and to be familiar with the mathematics of curves, and, in addition, to exercise a reasonable amount of ingenuity in the application of the knowledge possessed.

Compound Curves.

When one curve joins another, the two curves having a common tangent at the point of junction, and lying upon the same side of the common tangent, the two curves form a Compound Curve.

When two such curves lie upon opposite sides of the common tangent, the two curves then form a Reversed Curve.

In a compound curve, the point at the common tangent where the two curves join, is called P.C.C. meaning the "point of compound curvature".

In a reversed curve, the point where the curves join is called the P.R.C. meaning the "point of reversed curvature."

Fieldwork.

Laying out a compound curve or a reversed curve.

(a.) Set up transit at P.C.

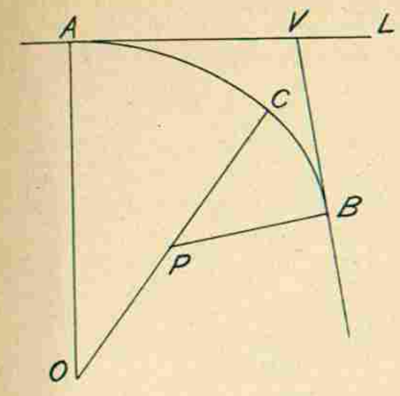
(b.) Run in simple curve to P.C.C. or P.R.C.

(c.) Move transit to P.C.C. or P.R.C.

(d.) Set line of sight on common tangent with vernier at 0

(e.) Run out second curve as a simple curve.

Data Used in Compound Curve Formulas.



In the curve of longer radius

$OA = R_l$

$AOC = I_l$

$AV = T_l$

In the curve of shorter radius.

$PB = R_s$

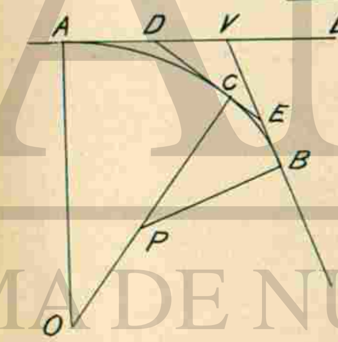
$BPC = I_s$

$VB = T_s$

$LVB = I$

Problem. Given $R_l - R_s - I_l - I_s$

Required $I - T_l - T_s$



Draw the common tangent DCE

Then $I = I_l + I_s$

$AD = CD = R_l \tan \frac{1}{2} I_l$ } or use Searles Table VI and

$EB = CE = R_s \tan \frac{1}{2} I_s$ } corr. Table V.

In the triangle DVE we have

$DE = R_l \tan \frac{1}{2} I_l + R_s \tan \frac{1}{2} I_s$

$VDE = I_l$

$VED = I_s$

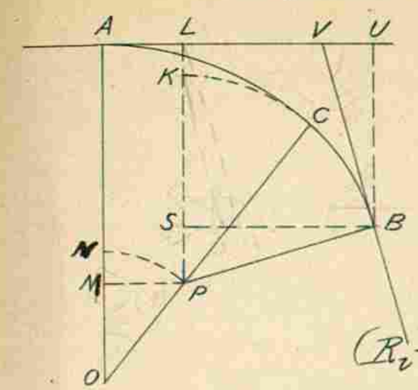
$DVE = 180 - I.$

Solve for VD and VE

$AV = AD + VD = T_l$

$VB = BE + VE = T_s$

Problem Given $T_s - R_s - R_l - I$.
Required $T_l - I_l - I_s$.



Draw arcs NP and KC.
" perpendiculars MP. LP. SB. UB.

Then $AM = LP$

$AN = R_s = KP$

$NM = LK = LS - KS$

$OP \text{ vers } NOP = VB \sin VBS - PB \text{ vers } KPB$

$(R_l - R_s) \text{ vers } I_l = T_s \sin I - R_s \text{ vers } I.$

$$\text{vers } I_l = \frac{T_s \sin I - R_s \text{ vers } I}{R_l - R_s} \qquad (42.)$$

$$I_s = I - I_l$$

$AV = MP + SB - UV$

$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_s \cos I. \qquad (43.)$$

Problem. Given $T_s - R_s - I_s - I$.
Required $T_l - R_l - I_l$.

$I_l = I - I_s$

$$R_l - R_s = \frac{T_s \sin I - R_s \text{ vers } I}{\text{vers } I_l} \qquad (44.)$$

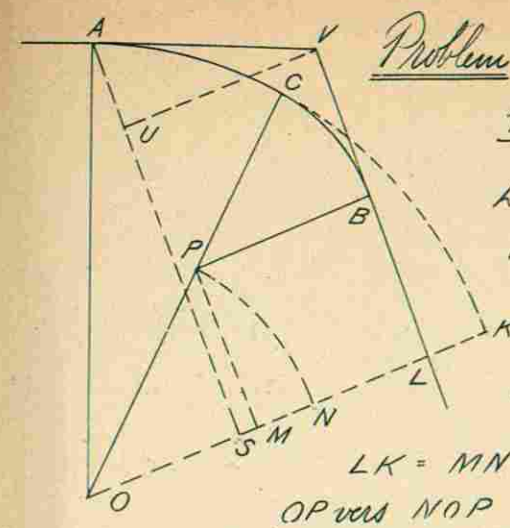
$$T_l = (R_l - R_s) \sin I_l + R_s \sin I - T_s \cos I \qquad (45.)$$

Problem. Given $T_l - T_s - R_s - I$.
Required $R_l - I_l - I_s$.

Show that

$$\tan \tfrac{1}{2} I_l = \frac{T_s \sin I - R_s \text{ vers } I}{T_l + T_s \cos I - R_s \sin I} \qquad (46.)$$

$$R_l - R_s = \frac{T_l + T_s \cos I - R_s \sin I}{\sin I_l} \qquad (47.)$$



Problem Given $T_l - R_l - R_s - I$.
Required $T_s - I_l - I_s$.

Draw arcs $NP - KC$

" perpendiculars $OK - AS . PM - VU$

Then $LM = BP = KN$

$MN = LM - LN = KN - LN = KL$

$$LK = MN = KS - LS$$

$$OP \text{ vers } NOP = AO \text{ vers } AOK - AV \sin VAS$$

$$(R_l - R_s) \text{ vers } I_s = R_l \text{ vers } I - T_l \sin I$$

$$\text{vers } I_s = \frac{R_l \text{ vers } I - T_l \sin I}{R_l - R_s} \qquad (48.)$$

$$I_l = I - I_s$$

$$VB = AS - PM - AU$$

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I \qquad (49.)$$

Problem Given $T_l - R_l - I_l - I$.
Required $T_s - R_s - I_s$.

$$I_s = I - I_l$$

$$R_l - R_s = \frac{R_l \text{ vers } I - T_l \sin I}{\text{vers } I_s} \qquad (50.)$$

$$T_s = R_l \sin I - (R_l - R_s) \sin I_s - T_l \cos I \qquad (51.)$$

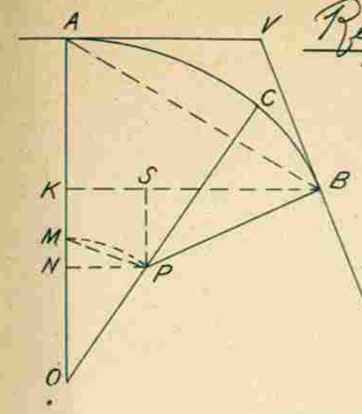
Problem Given $T_l - T_s - R_l - I$.
Required $R_s - I_l - I_s$.

Show that
$$\tan \tfrac{1}{2} I_l = \frac{R_l \text{ vers } I - T_l \sin I}{R_l \sin I - T_l \cos I - T_s} \qquad (52.)$$

$$R_l - R_s = \frac{R_l \sin I - T_l \cos I - T_s}{\sin I_s} \qquad (53.)$$

__Problem.__ __Given__ in the figure $AB - VAB - VBA - R_s$

__Required__ $R_2 - I_2 - I_s - I.$



Draw arc NP

" perpendiculars $KB - MP - SP.$

$$I = VAB + VBA$$

$$\begin{aligned} NM &= AK + KM - AN \\ &= AB \sin VAB + PB \cos SPB - AN \\ &= AB \sin VAB + R_s \cos I - R_s \\ &= AB \sin VAB - R_s \operatorname{vers} I. \end{aligned}$$

$$\begin{aligned} MP &= KB - SB \\ &= AB \cos VAB - PB \sin SPB \\ &= AB \cos VAB - R_s \sin I \end{aligned}$$

$$\tan NPM = \tan \tfrac{1}{2} I_2 = \frac{NM}{MP} \tag{54}$$

$$I_s = I - I_2$$

$$OP = R_2 - R_s = \frac{MP}{\sin I_2} \tag{55}$$

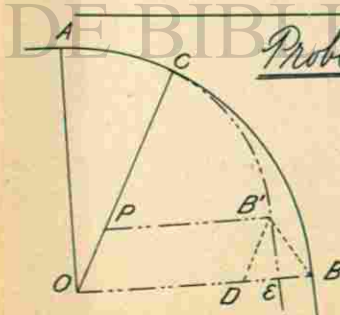
__Problem__ __Given__ in the figure $AB - VAB - VBA - R_2$

__Required__ $R_s - I_2 - I_s - I.$

Find I and show that

$$\tan \tfrac{1}{2} I_s = \frac{R_2 \operatorname{vers} I - AB \sin VBA}{R_2 \sin I - AB \cos VBA} \tag{56}$$

$$R_2 - R_s = \frac{R_2 \sin I - AB \cos VBA}{\sin I_s} \tag{57}$$



__Problem__ __Given__ a simple curve ending in a given tangent. A second curve of given radius is to leave this and end in a parallel tangent.

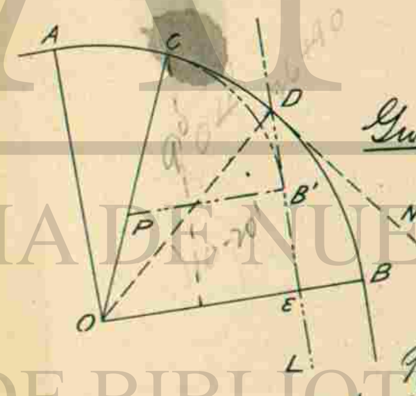
__Required__ the P.C.C.

Let AB be the given curve of radius R_2.
C be the P.C.C.
CB' the second curve of radius R_5.
$BE = p =$ distance between tangents

Then $\text{vers } COB = \dfrac{p}{R_2 - R_5}$ (56.)

It may sometimes be more convenient or quicker to run in a simple curve first and change to a compound curve by the method of this problem, rather than to run in the compound curve at first. When it is impossible or inconvenient to run in the curve as far as B (P.T.) the point of intersection D between the curve and the tangent may be found, the angle LDN measured and BE calculated

$BE = DO \text{ vers } DOB$
$p = R_2 \text{ vers } LDN$ (57.)



Example

Given Notes of Curve — 5° curve R.
22 + 20 P.C.
Proposed tangent intersects curve at 26 + 90.
Angle between tangent and curve = 10°20'.
Required Station of P.C.C. to join proposed tangent.

$p = R_5 \text{ vers } 10°20'$

R_5 log 3.059290
10°20' vers 8.210028
p log 1.269318

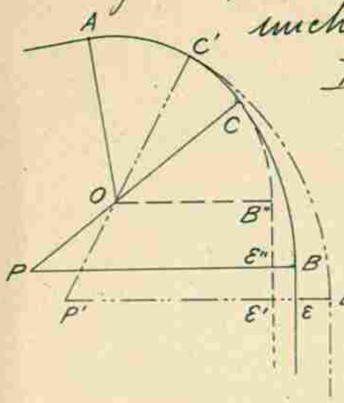
$\text{vers } COB = \dfrac{p}{R_5 - R_7}$

$R_5 = 1146.28$ — p log 1.269318
$R_7 = 819.02$
327.26 → log 2.514893
19°24' vers 8.754425
10°20'
9°04' = $\dfrac{9.0667}{181.3}$ (5°

26 + 90
1 + 81.3
25 + 08.7 P.C.C.

__Problem__ __Given__ a compound curve ending in a tangent

__Required__, to change the P.C.C. so as to end in a given parallel tangent, the radii remaining unchanged.



I. When the __new__ __tangent__ lies __outside__ the __old__ tangent, and the __curve__ __ends__ with curve of __larger__ __radius__.

Let ABC be the given compound curve.

$AC'B'$ the required curve.

Produce arc AC to B''

Draw OB'' parallel to PB and $B''E'$ perpendicular to P'

Let $B'E = p =$ perpendicular distance between tangents.

Then $\quad B'E = B'E' - BE''$

$$B'E = (R_l - R_s)\,\text{vers}\,C'OB' - (R_l - R_s)\,\text{vers}\,COB''$$

$$p = (R_l - R_s)\,\text{vers}\,I_l' - (R_l - R_s)\,\text{vers}\,I_l$$

$$\text{vers}\,I_l' = \text{vers}\,I_l + \frac{p}{R_l - R_s} \qquad (58.)$$

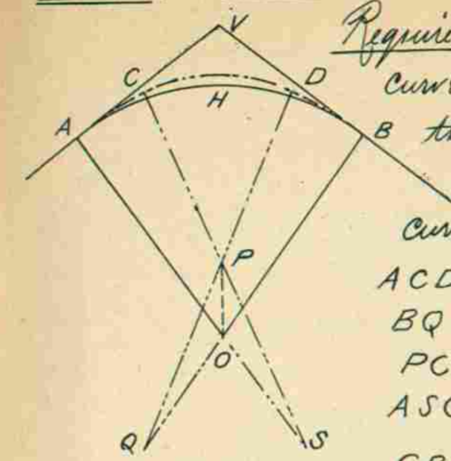
$$AOC = I - I_l$$

II. When the __new__ __tangent__ lies __inside__ the __old__ tangent and the curve __ends__ with the curve of __larger__ __radius__. $\quad \text{vers}\,I_l' = \text{vers}\,I_l - \frac{p}{R_l - R_s} \qquad (59.)$

III. When the __new__ __tangent__ lies __outside__ the __old__ tangent, and the __curve__ __ends__ with curve of __smaller__ __radius__. Show that $\text{vers}\,I_s' = \text{vers}\,I_s - \frac{p}{R_l - R_s} \qquad (60.)$

IV. When the __new__ __tangent__ lies inside the __old__ tangent and the __curve__ __ends__ with curve of __smaller__ __radius__. Show that $\text{vers}\,I_s' = \text{vers}\,I_s + \frac{p}{R_l - R_s} \qquad (61.)$

Problem Given a simple curve joining two tangents. Required to substitute a symmetrical curve with flattened ends, using the same P.C. and P.T.



Let AHB be the simple curve of radius R_c
$ACDB$ the required curve in which

$BQ = AS = R_t$
$PC = R_s$
$ASC = BQD = I_t$
$CPD = I_s$

Then $I = I_s + 2I_t$

In the triangle POQ $\qquad PQ = R_t - R_s$
$\qquad QO = R_t - R_c$

$PQO = I_t$
$POQ = 180 - \frac{1}{2}I$
$OPQ = \frac{1}{2}I_s$

There are Given $I - R_c$ Required $R_t - R_s - I_t - I_s$.
We may assume any two of the latter (except I_t and I_s) and readily calculate the others.

I. Assume R_t and I_t

$I_s = I - 2I_t$

$PQ : QO = \sin POQ : \sin OPQ$

$R_t - R_s : R_t - R_c = \sin \frac{1}{2}I : \sin \frac{1}{2}I_s$

$$R_t - R_s = \frac{(R_t - R_c)\sin\frac{1}{2}I}{\sin\frac{1}{2}I_s} \qquad (62.)$$

II. Assume R_t and R_s

$$\sin\frac{1}{2}I_s = \frac{(R_t - R_c)\sin\frac{1}{2}I}{R_t - R_s} \qquad (63.)$$

III. Assume R_s and I_s

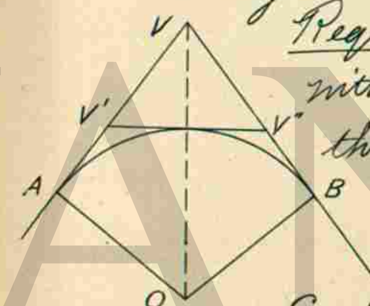
$$I_2 = \tfrac{1}{2}(I - I_s)$$

$$\frac{PQ+QO}{PQ-QO} = \frac{\tan\frac{1}{2}(POQ+OPQ)}{\tan\frac{1}{2}(POQ-OPQ)}$$

$$\frac{R_2-R_s+R_2-R_c}{R_2-R_s-R_2+R_c} = \frac{\tan\frac{1}{2}(180-\frac{1}{2}I+\frac{1}{2}I_s)}{\tan\frac{1}{2}(180-\frac{1}{2}I+\frac{1}{2}I_s)}$$

$$2R_2-R_c-R_s = \frac{(R_c-R_s)\cot\frac{1}{4}(I-I_s)}{\cot\frac{1}{4}(I+I_s)} \qquad (64)$$

Problem. Given a simple curve joining two tangents.



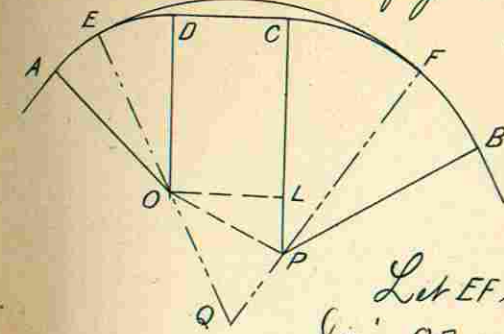
Required to substitute a curve with flattened ends to pass through the same middle point.

Let AB be the given simple curve and H the middle point.

Erect an auxiliary tangent V'HV" at H. The auxiliary intersection angles at V' and V" are readily calculated; also V'H and V"H. Sufficient additional data can be assumed, and the problem solved as a problem in compound curves.

It is not necessary that the curves on the two sides of H should be symmetrical.

<u>Problem</u>. <u>Given</u> two curves with connecting tangent.
<u>Required</u> to substitute a simple curve of given radius, to connect the two.



Let $DC = t$ = the given tangent, connecting the two curves AD and CB, of radii R_s and R_l respectively.

Let EF be required curve of radius R_c.

Join OP and draw perpendicular OL.

Then $\tan LOP = \dfrac{LP}{OL} = \dfrac{R_l - R_s}{t}$

$$OP = \frac{t}{\cos LOP}$$

In the triangle OPQ we have given

$OP = \dfrac{t}{\cos LOP}$; $OQ = R_c - R_s$; $QP = R_c - R_l$

Solve this triangle for OQP - QOP - OPQ

Then $\quad CPF = 180° - (OPQ + OPL)$

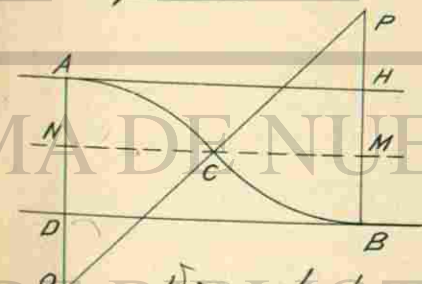
$\qquad\quad EOD = 90° - (QOP + LOP)$

Reversed Curves.

It is very undesirable that reversed curves should be used on main lines or where trains are to be run at any considerable speed. The marked change in direction is objectionable, and an especial difficulty is that there is no opportunity to elevate the outer rail at the P.R.C. The use of reversed curves on lines of railroad is therefore very generally condemned by Engineers. For yards and stations, reversed curves may often be used to advantage, also on street railways and perhaps for other purposes.

Problem. Given the perpendicular distance between parallel tangents, and the common radius of the reversed curve.

Required the central angle of each curve.



Let AH and BD be the parallel tangents.
ACB the reversed curve.
HB = p = perpendicular distance between tangents.

Draw perpendicular NM

Let AOC = BPC = I_r

Then $\text{vers } AOC = \dfrac{AN}{AO} = \dfrac{BM}{PB} = \dfrac{\frac{1}{2}HB}{AO} = \dfrac{\frac{p}{2}}{R} = \text{vers } I_r$ (65.)

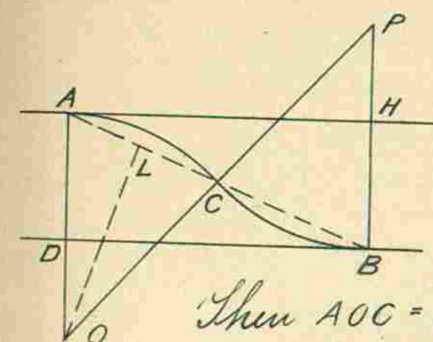
Problem. Given $p - I_r$

Required R

$$R = \frac{\frac{p}{2}}{\text{vers } I_r} \qquad (66.)$$

<u>Problem</u> Given the perpendicular distance between parallel tangents, and chord distance between P.C. and P.T.

<u>Required</u> the common radius of reversed curve to connect the parallel tangents.



Let AH and BD be the parallel tangents.

AB the reversed curve.

$BH = p$

$AB = d$.

Connect AC and CB

Then $AOC = BPC$ and $ACO = PCB$

$\therefore ACB$ is a straight line.

$AO : AL = AB : HB$

$R : \frac{d}{4} = d : p$

$$R = \frac{d^2}{4p} \qquad (67.)$$

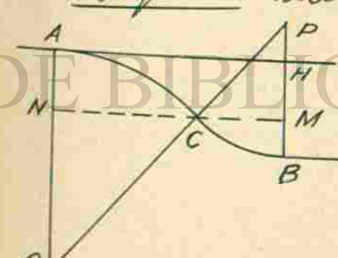
<u>Problem</u> <u>Given</u> R and p

<u>Required</u> d

$$d = \sqrt{4Rp} = 2\sqrt{Rp} \qquad (68.)$$

<u>Problem</u> Given the perpendicular distance between two parallel tangents and the central angle and radius of first curve of reversed curve.

<u>Required</u> the radius of second curve



Let AH and BD be parallel tangents

ACB = reversed curve

$HB = p$

$AO = R_1$

$AOC = CPB = I_r$

$PB = R_2$

$HB = AN + MB$ Draw perpendicular NCM

$= AO \text{ vers } AOC + BP \text{ vers } BPC$

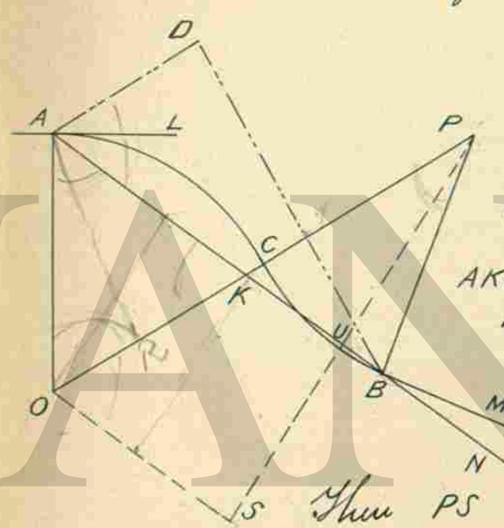
$p = R_1 \text{ vers } I_r + R_2 \text{ vers } I_r$ $\qquad R_1 + R_2 = \dfrac{p}{\text{vers } I_r} \qquad (69.)$

Problem Given $R_1 - R_2 - p$
Required I_r

$$\operatorname{vers} I_r = \frac{p}{R_1 + R_2} \qquad (70.)$$

Problem Given, two points upon tangents not parallel, the length of line joining the two points, and the angles made by this line with each tangent.

Required, the common radius of a reversed curve to connect the two tangents at the given points



Let A and B be the given points.
$AL - BM$ = given tangents.
ACB = required curve.
$LAB = A$ and $MBN = B$
$AB = l$
$AKO = PKB = K$.
Draw PS perpendicular and OS parallel to AB
Also BD perpendicular and AD parallel to OP

Then $PS = PU + SU$

$$OP \sin POS = PB \cos BPU + AO \sin OAB$$

$$2R \sin K = R \cos B + R \cos A$$

$$\sin K = \frac{\cos A + \cos B}{2}$$

$$AOK = O = 180° - OKA - OAK$$
$$= 180° - K - (90° - A) = 90° + A - K.$$

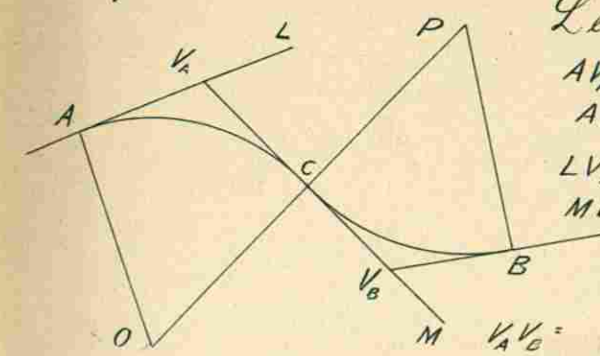
$$BPK = P = 180° - BKP - PBK =$$
$$= 180° - K - (90° - B) = 90° + B - K.$$

$$BD = AB \sin DAB = AO \sin AOK + BP \sin KPB$$
$$l \sin K = R \sin O + R \sin P$$

$$R = \frac{l \sin K}{\sin O + \sin P} \qquad (72.)$$

<u>Problem.</u> <u>Given</u> the length of the common tangent and the angles of intersection with the separated tangents.

<u>Required</u> the common radius of a reversed curve to join the two separated tangents.



Let $V_A V_B$ = common tangent.

$AV_A - BV_B$ = separated tangents

ACB = required curve

$LV_AC = I_A$

$MV_BB = I_B$

$V_AV_B = t$

$$V_AV_B = V_AC + V_BC$$

$$t = R \tan \tfrac{1}{2} I_A + R \tan \tfrac{1}{2} I_B$$

$$R = \frac{t}{\tan \frac{1}{2} I_A + \tan \frac{1}{2} I_B} \qquad (73.)$$

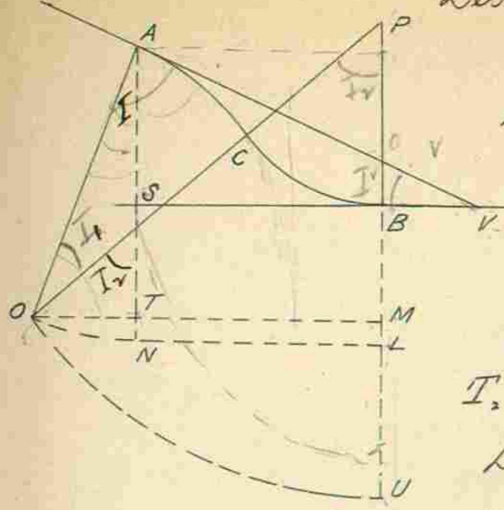
An approximate method is as follows:—

Find I_{A1} for a 1° curve; also I_{B1} (Table VI Searles)

Then $D_x = \dfrac{I_{A1} + I_{B1}}{V_A V_B}$

<u>Problem</u> <u>Given</u> a P.C. upon one of two tangents not parallel, also the tangent distance to the second tangent, also the angle of intersection, also the unequal radii of a reversed curve to connect the tangents.

<u>Required</u>, the central angles of the simple curves, also the tangent distance from V to P.T.



Let $AV = T_1$ = given tang. dist.
A = given P.C.
V = vertex
$AVS = I$
$AO = R_1$ } given radii
$PB = R_2$ } given radii
VS = Second tangent.
ACB = required curve.
$AOC = I_1$ } required angles
$BPC = I_2$ } required angles
$T_2 = BV$ = required tang. dist.
Draw arcs OU – ON
" perpendiculars AN – OM – NL

Then $AO = BU = R_1$

$LU = BU - BL = AN - SN = AS$

$AS = LU = MU - ML$

$AV \sin AVS = PO \text{ vers } BPC - AO \text{ vers } AON$

$T_1 \sin I = (R_1 + R_2) \text{ vers } I_2 - R_1 \text{ vers } I$

$$\text{vers } I_2 = \frac{R_1 \text{ vers } I + T_1 \sin I}{R_1 + R_2} \qquad (74.)$$

$$I_1 = I_2 - I$$

$$T_2 = T_1 \cos I + R_1 \sin I - (R_1 + R_2) \sin I_2 \qquad (75.)$$

<u>Problem</u> <u>Given</u> BV instead of AV, and other data as above

<u>Required</u> I, I_2 etc.

Turnouts.

A turnout is a track leading from a main or other track

Turnouts may be for several purposes.

I. Branch Track (for line used as a Branch Road for general traffic)

II. Siding. (for passing trains at stations, storing cars, loading or unloading, and various purposes)

III. Spur Track (for purposes other than general traffic, as to a quarry or warehouse.)

IV. Cross Over (for passing from one track to another, generally parallel.)

The essential parts of a turnout, are

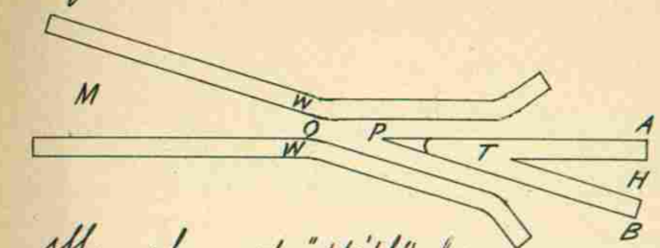
1. The Switch. 2. The Frog. 3 The Guard Rail

1. Some device is necessary to cause a train to turn from the main track; this is called the "Switch".

2. Again, it is necessary that one rail of the turnout track should cross one rail of the main track; and some device is necessary to allow the flange of the wheel to pass this crossing; this device is called a "Frog".

3. Finally, if the flange of the wheel were allowed to bear against the point of the frog, there is danger that the wheel might be turned to the wrong side of the frog point; therefore a Guard Rail is set opposite to the frog and prevents the flange from bearing against the frog point.

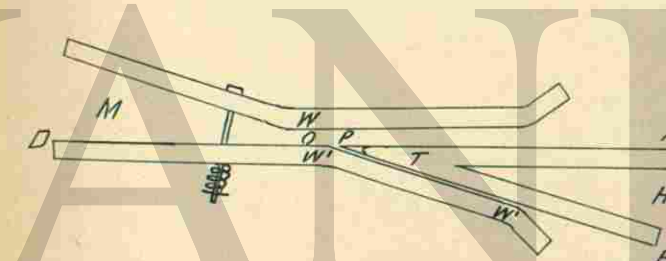
Frogs are of various forms and makes, but are mostly of this general shape, and the parts are named as follows:



M = mouth.
O = throat.
WW = wings.
P = point.
T = tongue.
H = heel.

This shows the "Stiff" frog.

The "Spring" frog is often used where the traffic on the main line is large, and on the turnout small. In the Spring frog W'W' is movable.



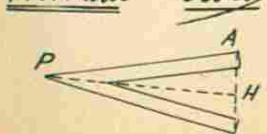
AD represents the main line. and W'W' is pushed aside by the wheels of a train passing over the turnout.

The "*frog angle*" is the angle between the sides of the tongue of the frog = APB.

Frogs are made of certain standard proportions and are classified by their number.

The "*number*" n of a frog is found by dividing the length of the tongue by the width of the heel, that is, $n = \frac{PH}{AB}$

Problem *Given* n. *Required*, Frog Angle F.



$$\tan \tfrac{1}{2} F = \frac{\frac{1}{2} AB}{PH} = \frac{AB}{2PH}$$

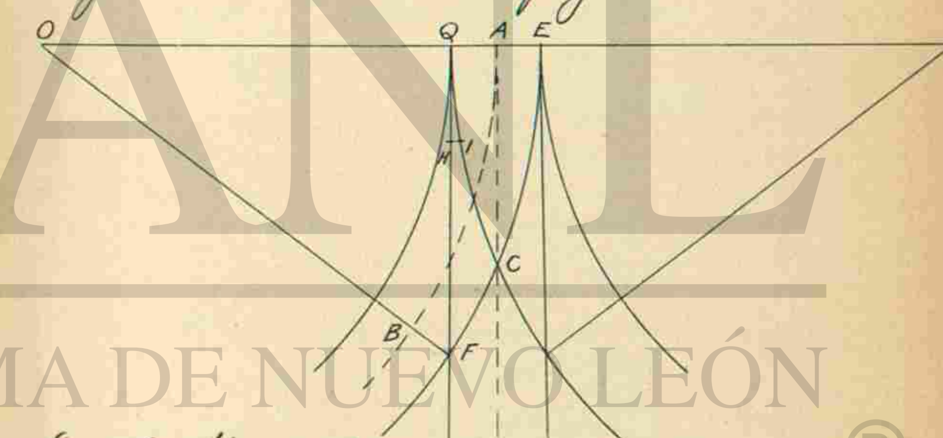
$$\cot \tfrac{1}{2} F = 2n \qquad (76.)$$

A form of switch in common use is the "<u>Stub Switch</u>", which is formed by two rails, one on each side of the track, called the "<u>Switch Rails</u>". One end of the rail for a short distance (often about 5 feet) is securely spiked to the ties, the rest of the rail being free to slide on the ties, so that it may meet the fixed rails of either main track or turnout, as desired. These fixed rails are held by a casting or piece of metal called the "<u>Head Block</u>", and upon which the switch rail slides. A "<u>switch rod</u>" connects the ends of the switch rails with the "<u>switch stand</u>". One end of the rail being spiked down, when the free end is drawn over by the switch rod, the rail is sprung into a curve which may with slight error be considered a circular curve, tangent to the main line (if this be straight.) The distance through which the free end of the rail is drawn or "thrown" by the switch rod is called the "<u>Throw</u>" of the Switch. The free end of the rail is called the "<u>Toe</u>" and the P.C. of the curve, the "<u>Heel</u>" of the switch.

Knowing the "throw" t and the length l of the switch rail, we can deduce the radius R, or degree of curve D, and continuing this curve to the point of frog, we can readily deduce the angle between the rails or the "Frog Angle" necessary.

It is more customary, however, having given the throw (5" - 5½" - 6" are used in different roads) to assume either (1.) the radius (or degree) of turnout curve and from this find F (or n) and l; or (2.) the number n (or angle F) of frog, and from this find R and then l.

When there are two turnouts at the same point, one on each side of the main line, three frogs are necessary, the middle one being called the "crotch frog".



In the figure the

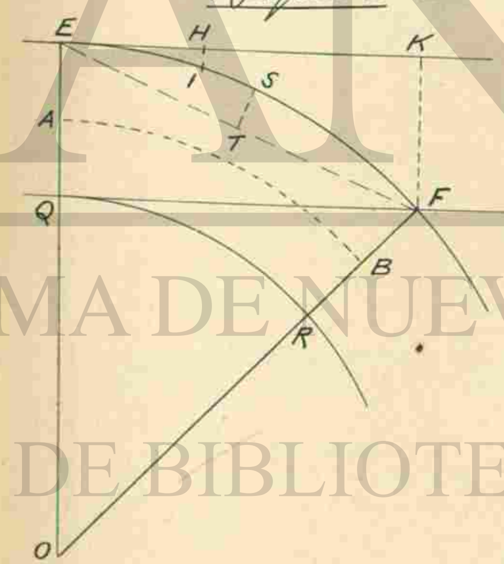
Heel of Switch is at	E or Q	Length of Switch Rail	= QH
Toe " "	H	Throw of Switch	= HI
Head Block	H and I	Lead	= HF
Crotch Frog	C	Frog is at	F
Center of turnout curve	AB		

It is necessary that there should be two numbers of frog, one for the ordinary turnout frog, and another for the "crotch frog." It is advisable that only two be used, and that all turnout curves be arranged to use one or the other of these two frogs. (For double turnouts with point switches a third number may be necessary for a crotch frog.) In this case we should assume n (or F) as the value for one of the two "standard frogs"

On main line it is now customary to use a "split switch" or "point switch", the description and discussion of which will follow the discussion of the stub switch.

<u>Problem</u> <u>Given</u> gauge of track $= g$, frog angle $= F$, and throw of switch $= t$.

<u>Required</u> R, l and $QF = E$.



Let EF, QR be the rails of the turnout

Draw perpendiculars KF, also HI at toe of switch

$EOF = F$.

$\text{vers } EOF = \frac{EQ}{FO}$

$$\text{vers } F = \frac{g}{R + \frac{g}{2}} \qquad (77.)$$

$$R + \frac{g}{2} = \frac{g}{\text{vers } F} \qquad (78.)$$

$QF = OF \sin EOF$

$$E = \left(R + \frac{g}{2}\right) \sin F \qquad (79.)$$

by (26.) $t = \frac{l^2}{2R}$ (approx)

$$l = \sqrt{2Rt} \text{ (approx.)} \qquad (80.)$$

<u>Problem</u> <u>Given</u> $g - t - n$.
<u>Required</u> $R - E - l$.

In the figure preceding, connect EF
Then $EFQ = FEK = \frac{1}{2}F$.

$QF = EQ \cot EFQ$

$E = g \cot \frac{1}{2}F$. (81.)

$E = 2ng$ (82.)

$FQ^2 = FO^2 - QO^2$

$E^2 = (R+\frac{g}{2})^2 - (R-\frac{g}{2})^2$

$E^2 = (R+\frac{g}{2}+R-\frac{g}{2})(R+\frac{g}{2}-R+\frac{g}{2})$

$E^2 = 2R \times g$

$R = \frac{E^2}{2g} = \frac{4g^2n^2}{2g}$

$R = 2n^2g$ (83.)

$l^2 = E^2\frac{t}{g}$

$= 4n^2g^2\frac{t}{g} = 4n^2gt$

$l = 2n\sqrt{gt}$ (84.)

<u>Problem</u> <u>Given</u> g
<u>Required</u> the middle ordinate ST

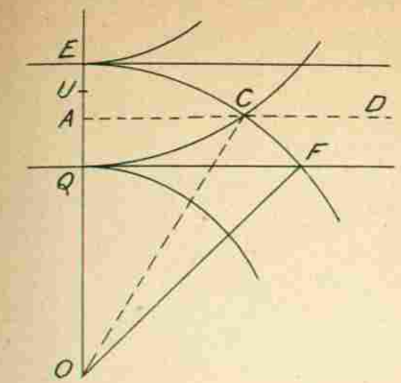
$m = \frac{c^2}{8R}$

EQ is middle ordinate for chord $2FQ$
ST " " " " " EF
$EF = FQ$ (approx.)
$ST : FK = EF^2 : (2FQ)^2$

$ST = \frac{FK}{4}$

$ST = \frac{g}{4}$ (85.)

This is true evidently whatever the degree of curve.



Problem Given g - R

Required, the angle of crotch frog = C

The frog angle at $C = 2AOC$

$$\text{vers } AOC = \frac{EA}{OC}$$

$$\text{vers } \tfrac{1}{2} C = \frac{\tfrac{1}{2} g}{R + \frac{g}{2}} \qquad (86.)$$

Problem Given the number of crossing frog = n_f

Required the number of crotch frog = n_c

$AO = R = 2 n_f^2 g$

If we consider AD to represent a rail and n_q the frog proper for the crossing of QC and AD

Then $UO = 2 n_q^2 \frac{g}{2}$

But the angle between EC and QC = twice the angle between QC and AD

Then $UO = 2(2n_c)^2 \frac{g}{2}$ (approx.)

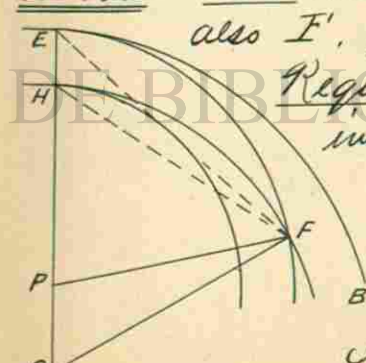
$R = 2(2n_c)^2 \frac{g}{2}$ (approx.)

$2n_f^2 g = 2(2n_c)^2 \frac{g}{2}$ (approx.)

$n_f^2 = (2n_c)^2$ (approx.)

$n_c = 0.7071\, n_f$ (approx.) (87.)

Problem Given. main track of radius R_m; also F', g and n



Required radius R_t of a turnout inside of main track; also F.

Let EB be the outer rail of main track.

EF the outer rail of turnout.

Join EF

Let $EOF = O$; $PFO = F$.

In the triangle EOF $\quad EO = R_m + \frac{g}{2}$

$FO = R_m - \frac{g}{2}$

$EFO - FEO = EFO - EFP = F'$

$EFO + FEO = 180° - O$

Then $\tan\frac{1}{2}(EFO+FEO) : \tan\frac{1}{2}(EFO-FEO) = EO+FO : EO-FO$

$\cot\frac{1}{2}O : \tan\frac{1}{2}F' = 2R_m : g$

$\tan\frac{1}{2}O : \cot\frac{1}{2}F' = g : 2R_m$

$\tan\frac{1}{2}O = \frac{g}{2R_m}\cot\frac{1}{2}F' = \frac{g\,2n}{2R_m}$

$$\tan\frac{1}{2}O = \frac{gn}{R_m} \qquad (88.)$$

Similarly $FPH = F + O$

Join HF and in triangle HPF, $\tan\frac{1}{2}(F+O) = \frac{gn}{R_t}$

$$R_t = \frac{gn}{\tan\frac{1}{2}(F+O)} \qquad (89.)$$

chord $HF \cdot E' = 2\left(R_m - \frac{g}{2}\right)\sin\frac{1}{2}O$ (90.)

Approximate Formula

Let R . D = Radius and Degree of a turnout curve from a straight line to correspond to the given value of F or n.

R_m - D_m = Radius and Degree of main track.

R_t - D_t = Radius and Degree of turnout curve.

Then from (88.) (89.) $R_m = \frac{ng}{\tan\frac{1}{2}O}$; $R_t = \frac{ng}{\tan\frac{1}{2}(O+F)}$

also (83.) $R = 2n^2g = ng \times 2n = \frac{ng}{\tan\frac{1}{2}F}$

$\sin\frac{1}{2}D_m = \frac{50}{R_m} = \frac{50\tan\frac{1}{2}O}{ng}$

$\sin\frac{1}{2}D_t = \frac{50}{R_t} = \frac{50\tan\frac{1}{2}(O+F)}{ng}$

$\sin\frac{1}{2}D = \frac{50}{R} = \frac{50\tan\frac{1}{2}F}{ng}$

$$\sin\tfrac{1}{2}D_m : \sin\tfrac{1}{2}D_t : \sin\tfrac{1}{2}D = \tan\tfrac{1}{2}O : \tan\tfrac{1}{2}(O+F) : \tan\tfrac{1}{2}F$$

$$D_m : D_t : D = O : O+F : F \text{ (approx.)}$$

$$D_m + D : D_t = O+F : O+F \text{ (approx.)}$$

$$D_t = D_m + D \text{ (approx.)} \qquad (91.)$$

Again (90.) $HF = E_1 = 2(R_m - \frac{g}{2}) \sin\frac{1}{2}O$

(82.) $E_1 = 2ng$ (turnout from straight track)

88. $= 2R_m \tan\frac{1}{2}O$

But $\frac{g}{2}$ is small compared with R_m and may be neglected. and for small angles $\sin\frac{1}{2}O = \tan\frac{1}{2}O$ (approx.)

$$HF = E_1 = 2R_m \tan\tfrac{1}{2}O \text{ (approx.)}$$

$$E_1 = 2gn \text{ (approx.)} \qquad (92.)$$

The above formula and (91.) while approximate are the formulas in general use.

It is difficult in practical track work to secure results more precise than would be obtained by the use of these approximate formulas.

<u>Example.</u> <u>Given</u> a 3° curve on main line and a No 9 frog

<u>Required</u> the degree of turnout curve to the inside of the curve

Table XI Searles' shows for a No 9 frog, the

degree of curve = 7°31'04", this is ordinarily

taken = 7°30' = D

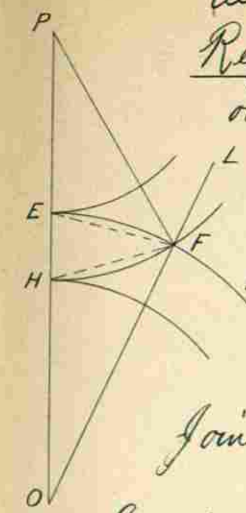
degree of main line = 3°00' = D_m

degree of turnout = 10°30' = $D_t = D + D_m$

By precise formula 10°34' = D_t

Problem Given main track of radius R_m also F - g - n.

Required radius R_t of a turnout curve outside of main track.



I. When the center of turnout curve lies outside of main track

Let EB be the outer rail of main track.

and HF the outer rail of turnout.

Join HF. Let $HOF = O$

Then $PFL = F$

In the triangle HOF $\quad FO = R_m + \frac{g}{2}$

$HO = R_m - \frac{g}{2}$

Also $FHO + HFO = 180° - O$

$FHO - HFO = 180° - PHF - (180° - PHF - F)$

$= F$

Then $\tan\frac{1}{2}(FHO + HFO) : \tan\frac{1}{2}(FHO - HFO) = FO + HO : FO - HO$

$\cot\frac{1}{2}O : \tan\frac{1}{2}F = 2R_m : g$

$$\tan\frac{1}{2}O = \frac{gn}{R_m} \qquad (93.)$$

Similarly $OPF = F - O$

Join EF and in triangle EPF, $\tan\frac{1}{2}(F - O) = \frac{gn}{R_t}$

$$R_t = \frac{gn}{\tan\frac{1}{2}(F - O)} \qquad (94.)$$

$$\text{Chord } EF = F' = 2\left(R_m + \frac{g}{2}\right)\sin\frac{1}{2}O \qquad (95.)$$

Approximate Formulas.

$$D_t = D - D_m \quad \text{(approx.)} \qquad (96.)$$

$$F' = 2ng \quad \text{(approx.)}$$

II. When the center of turnout curve lies on the inside of main track.

By a process entirely similar it may be shown that $\tan \frac{1}{2} O = \dfrac{gn}{R_m}$ (97.)

$$R_t = \frac{gn}{\tan \frac{1}{2}(O - F)} \qquad (98.)$$

$$F = 2\left(R_m + \frac{g}{2}\right) \sin \frac{1}{2} O \qquad (99.)$$

Approximate Formulas.

$D_t = D_m - D$ (approx.) (100.)

$F = 2ng$ (approx.)

Split Switch.

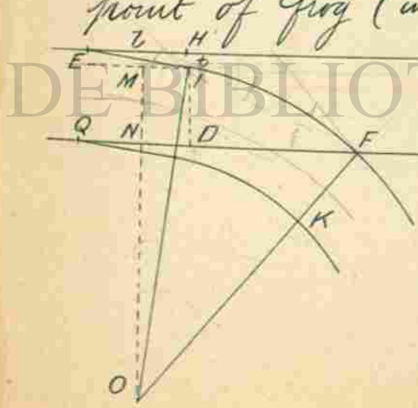
In turnouts from main line, it is customary now to use the split switch. In this switch the outer rail of the main line and the inner rail of the turnout track are continuous. The switch rails are steel rails, each planed down at one end to a wedge point, so that it may be caused to lie close against the track rail and so turn the wheel in the direction intended. An angle is made between the main track and the switch rail. The fixed end of the switch rail is placed at a point corresponding to the "head-block" of the stub switch, and the distance between rails at this point is generally made the same as the "throw" of the stub switch (from 5" to 6"). The switch rail is often made 15 feet in length. The "switch angle" is determined by the length of switch rail and the distance between rails (gauge to gauge) and which we may call t.

With the split switch a common practice is to calculate the turnout for a stub switch; the fixed end of the split switch is then placed at the point where the movable end of a stub switch would be placed, and the point of the switch wherever this will bring it. This gives results approximately correct, and sufficiently good to satisfy the requirements of many roads.

Some prefer a more exact solution.

__Problem.__ __Given__, in a turnout, the gauge of track g, length of switch rail l, the distance between rails t, and frog angle F.

__Required__ the radius of turnout track and distance from movable end of rail to point of frog (or the "lead".)



Let EIF and QK be the rails of the turnout.

Draw MI parallel and OM perpendicular to QF

Let s = switch angle.

$t = HI$

$l = EI$

Then $\sin s = \frac{t}{l}$

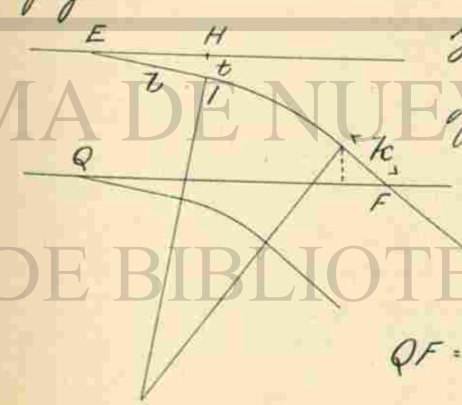
$$MN = MO - NO$$
$$g - t = \left(R + \tfrac{g}{2}\right)\cos S - \left(R + \tfrac{g}{2}\right)\cos F$$
$$R + \frac{g}{2} = \frac{g - t}{\cos S - \cos F} \qquad (101.)$$

$$QF = QD + DF$$
$$= l \cos S + \frac{ID}{\tan IFD}$$
$$QF = \underset{(\text{approx})}{l} + \frac{g - t}{\tan \frac{1}{2}(F + S)} \qquad (102.)$$

Some prefer even greater precision and give weight to the fact that the frog is straight, not curved.

Problem. Given, in a turnout, the gauge of track g; length of switch rail l; distance between rails t; length of frog, end to point K; and the frog angle F.

Required, the radius of turnout curve, and the distance from movable end of rail to point of frog.



Then following the methods of the preceding problem

$$\sin S = \frac{t}{l}$$

$$R + \frac{g}{2} = \frac{g - t - K \sin F}{\cos S - \cos F} \qquad (103.)$$

$$QF = l + \frac{g - t - K \overset{\sin}{\cancel{\cos}} F}{\tan \frac{1}{2}(F + S)} \ (\text{approx.}) \qquad (104.)$$

$+ K \cos F$

Two parallel straight tracks may be conveniently connected by a turnout in four different ways.

I. By a reversed curve, the two curves having equal radii.

II. By a reversed curve, the two curves having unequal radii, and with P.R.C. at point of frog F.

III. By (a.) a simple curve to F, (b.) tangent, and return by (c.) curve of radius equal to the first.

IV. By (a.) a simple curve to F, (b.) tangent, and return by (c.) simple curve of radius unequal to first.

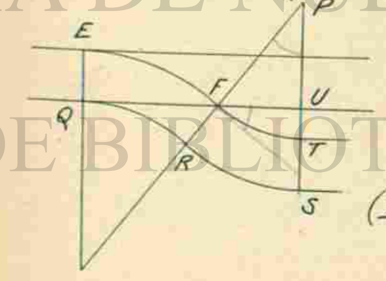
I. <u>Problem</u>. Given the perpendicular distance between two parallel tangents $= p$; also the common radius $= R$.

<u>Required</u> I_r.

Formula (65.) $\text{vers}\, I_r = \dfrac{\frac{1}{2}p}{R}$

II. <u>Problem</u>, Given, the radius of the first curve $= R$, also F and p

<u>Required</u>, the radius of the second curve R_2 to connect the parallel tangents.



If P.R.C. be taken at F
Then $I_r = F$.

$UT = US - TS$

$PT \,\text{vers}\, TPF = US - TS$

$(R - \frac{g}{2})\,\text{vers}\, F = p - g$

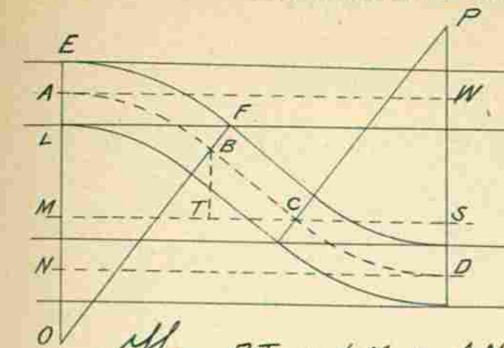
$$R - \frac{g}{2} = \frac{p - g}{\text{vers}\, F} \qquad (106.)$$

Problem Given as above R, $-F-$ p $-n$
Required to show that

$$R_2 - \frac{p}{2} = (p-g)\, 2n^2 \qquad (107.)$$

III. Problem Given $R . F . p$

Required the length l of tangent between the two curves of Equal radii.



Let $AW - ND$ be the center lines of the parallel tracks and $ABCD$ the turnout curve. Draw the perpendiculars $LB - MCS - BT$.

$$\text{Then } BT = LM = AN - AL - MN$$
$$CB \sin BCT = AN - AO \operatorname{vers} AOF - PD \operatorname{vers} DPC.$$
$$l \sin F = p - R \operatorname{vers} F - R \operatorname{vers} F$$
$$\therefore l = \frac{p - 2R \operatorname{vers} F}{\sin F} \qquad (108.)$$

Roughly Approximate Formula.

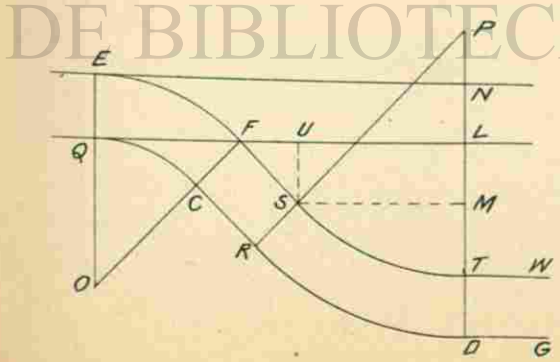
$$R \operatorname{vers} F = g \quad (\text{approx})$$
$$\sin F = 2 \tan \tfrac{1}{2} F = \frac{2}{\cot \frac{1}{2} F} = \frac{1}{n} \quad (\text{approx.})$$

from (108.) $l = (p - 2g)\, n$ (roughly) (109.)

The value thus found is hardly accurate enough for direct use in laying out track, but for "checking" and perhaps for other purposes will be found useful.

IV. Problem Given, $R_1 - g - p - l - F$.

Required, R_2.



Let EN and QL TW and DG be the rails of the parallel tangents and $EFST$ and $QCRD$ the rails of the turnout.

Draw the perpendiculars $US - SM$.

Then $SU = LM = NT - NL - MT$

$$FS \sin UFS = NT - NL - PS \operatorname{vers} SPM$$

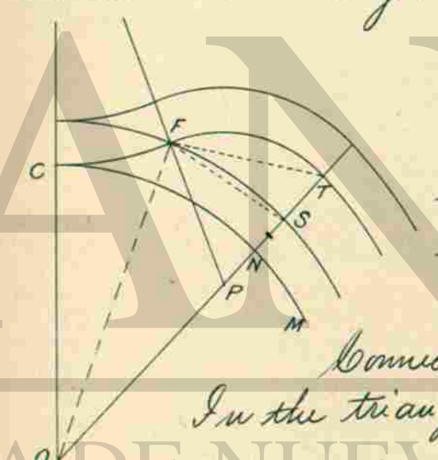
$$l \sin F = p - g - \left(R_2 - \tfrac{g}{2}\right) \operatorname{vers} F$$

$$R_2 - \frac{g}{2} = \frac{p - g - l \sin F}{\operatorname{vers} F} \qquad (110.)$$

Problem Given the radial distance p between a given curved main track and a parallel siding, also frog angle F (or number n) and gauge of track g.

Required the radius of second curve to connect point of frog with siding.

I. When the siding is outside the main track.



Let CM be the inner rail of the given main line.

CFT inner rail of turnout

R_m = radius of main line (center)

R_t = radius of turnout (center)

$p = TN$ = radial distance

Connect FT. FO Let $FOT = O$

In the triangle FTO, $FO = R_m + \frac{g}{2}$

$$TO = R_m - \frac{g}{2} + p$$

also $OFT + OTF = 180° - O$

$$OFT - OTF = OFT - PFT = F$$

Then $\tan \frac{1}{2}(OFT + OTF) : \tan \frac{1}{2}(OFT - OTF) = TO + FO : TO - FO$

$$\cot \tfrac{1}{2} O : \tan \tfrac{1}{2} F = 2R_m + p : p - g$$

$$\tan \tfrac{1}{2} O = \frac{p-g}{2R_m + p} \cot \tfrac{1}{2} F = \frac{p-g}{R_m + \frac{p}{2}} \cdot \frac{\cot \frac{1}{2} F}{2}$$

$$\tan \tfrac{1}{2} O = \frac{p-g}{R_m + \frac{p}{2}} n \qquad (111.)$$

Similarly $FPT = F + O$. Join FS (112.)

In the triangle PFS, $\tan \frac{1}{2}(F+O) = \frac{p-g}{R_t - \frac{p}{2}} n$; $R_t - \frac{p}{2} = \frac{(p-g)n}{\tan \frac{1}{2}(F+O)}$

Length of curve $I = \frac{100(F+O)}{D_t}$ (113.)

<u>Approximate Method</u>

It may readily be shown to be approximately true that if the entire turnout from a straight track be calculated, and the degree of each curve found, then by adding or subtracting the degree of curve of main track, the resulting degree of curve will be the degree required. The distances CF - FT will be the same as in the turnout from straight track. The demonstration will follow in principle closely that given in reaching (91.)

<u>Example.</u>

Turnout from straight line with No. 9 frog - $p = 15$.

$R - \frac{p}{2} = (p-g)\,2n^2 = (15.0 - 4.7)\,2 \times 81 = 1668.6$

$R = 1676.1$; $D = 3°25'$; $F' = 6°22'$ (Table XI)

$I = \dfrac{100 \times 6°22'}{3°25'} = 186.3$

Turnout from curve <u>outside</u> the main track on 4° curve.

<u>Precise Method</u>

$\tan \frac{1}{2} O = \dfrac{p-g}{R_m - \frac{p}{2}}\, n = \dfrac{10.3}{1440.2} \times 9 = \dfrac{92.7}{1440.2}$

92.7	log	1.967080
1440.2	log	3.158422
	tan	8.808658

$\frac{1}{2} O = 3°41'$

$\frac{1}{2} F' = 3°11'$

$\frac{1}{2}(F'+O) = 6°52'$

$R_t - \frac{p}{2} = \dfrac{(p-g)\,n}{\tan \frac{1}{2}(F'+O)} = \dfrac{92.7}{\tan 6°52'}$

	tan	9.080710
92.7	log	1.967080
	log	2.886370

769.8
7.5

$R_t = 777.3$

$D_t = 7°23'$

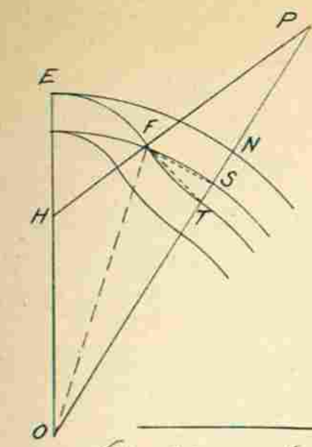
$I_1 = \dfrac{100(F'+O)}{D_t} = \dfrac{100 \times 13°44'}{7°23'} = 186.4$

<u>Turnout from Curve</u> - <u>Approximate Method</u>

$D_t = D + D_m$
$= 3°25' + 4° = 7°25'$ (7°23' precise method).

$I_1 = 186.3$ as with straight track. (186.4 precise method).

II. When the siding is <u>inside</u> the main track.



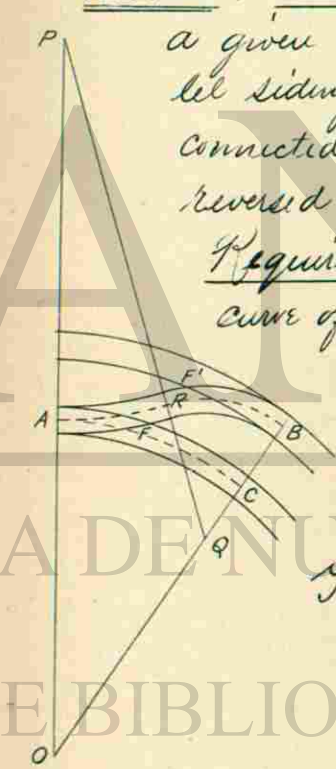
In a similar fashion it may be shown, using this figure, that

$$\tan \tfrac{1}{2} O = \frac{p - g}{R_m - \frac{p}{2}}\, n \qquad (114.)$$

$$R_t - \frac{p}{2} = \frac{(p-g)\,n}{\tan \frac{1}{2}(F - O)} \qquad (115.)$$

$$L = \frac{100\,(F - O)}{D_t} \qquad (116.)$$

<u>Problem</u>. <u>Given</u>, the radial distance between a given curved main track and a parallel siding. The two tracks are to be connected by a crossover which is a reversed curve of given unequal radii. <u>Required</u>, the central angle of each curve of the reversed curve.



Let ARB be center line of turnout.
AC center line of main track.

$AO = R_m$

$AP = R_1$

$RQ = R_2$

Then in the triangle POQ

$PO = R_m + R_1$

$PQ = R_1 + R_2$

$OQ = OC + CB - BQ$

$\quad = R_m + p - R_2$

Solve for OPQ - PQO - POQ, then RQB

In practice this problem might take the following form <u>Given</u> R_m - p - g <u>Assume</u> n (or F) and n' (or F'). From these calculate R_1 and R_2 (or use Table XI Searles) Then solve as above.

Approximate Method.

Where p is very small compared with R_m, the degree of curve used will frequently be found by the formulas (approx.) $D_{t_1} = D - D_m$

and $D_{t_2} = D + D_m$

The length of each part may be found for a cross over between parallel straight tracks and the same length used in the case of the cross over between curves.

The process is similar in every way to that shown by example in the previous problem.

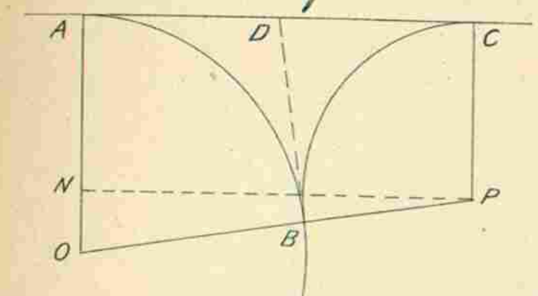
A similar method of treatment will be applicable in all turnouts from curves where the distance between tracks is not too great.

"Y" Tracks.

In many cases where a branch track leaves a main track, there is laid an additional track connecting with the branch, but leading from the main line in a direction opposite from that of the branch track. Such a track is called a "Y" track and the combination of connecting tracks is called a "Y." One convenient use of a "Y" is to turn an entire train at once without uncoupling.

<u>Problem</u> <u>Given</u>, a straight main track, also the P.C. and radius of a simple curve turnout

<u>Required</u>, the distance from P.C. of turnout to P.C. of "Y" track, also the central angles of turnout and of "Y" track to the point of junction.



Let AC be the given straight main track.
AB the turnout
CB the "Y" track.
Draw perpendicular NP
also Common tangent DB

Let $AC = l$; $AOB = I_t$; $CPB = I_y = 180° - I_t$.

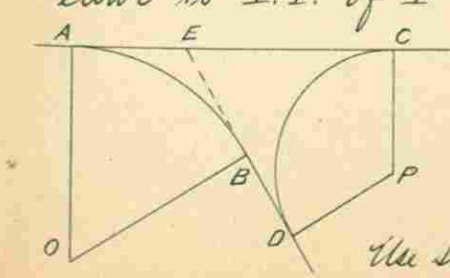
Then $\cos AOB = \dfrac{ON}{OP}$

$$\cos I_t = \frac{R_t - R_y}{R_t + R_y} \qquad (117.)$$

$$l = (R_t + R_y) \sin I_t \qquad (118.)$$

<u>Problem</u> <u>Given</u>, a straight main track, also the P.C., radius and central angle of a simple curve turnout connecting with a second tangent; also the radius of a "Y" track.

<u>Required</u>, the distance from P.C. of turnout to P.C. of "Y" track, and from P.T. of turnout curve to P.T. of "Y" track.



Let AC be the given main track; ABD the turnout; CD the "Y" track.
Let $AO = R_t$; $CP = R_y$
$AC = l$; $BD = m$
Use similar notation for T_t; T_y; I_t; I_y.

Produce BD to E.

Then $AC = AE + EC$

$$= AO \tan \tfrac{1}{2} AOB + CB \tan \tfrac{1}{2} CPD$$

$$l = R_t \tan \tfrac{1}{2} I_t + R_y \cot \tfrac{1}{2} I_t$$

$$l = T_t + T_y \qquad (119.)$$

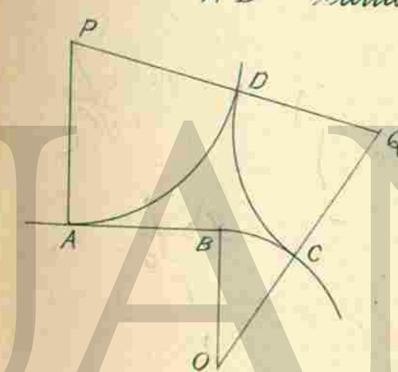
$$BD = ED - EB$$

$$= T_y - T_t \qquad (120.)$$

<u>Problem</u>. In the accompanying sketch where

ABC = main track.

AD = turnout



<u>Given</u>, $AB = l$

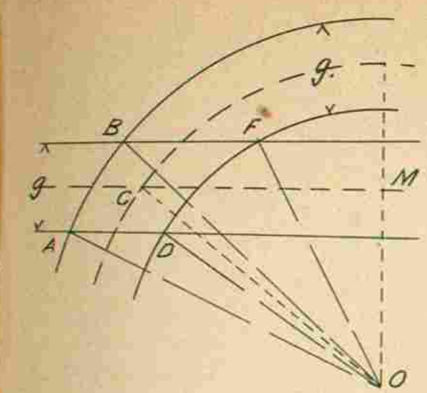
$OB = R_m$

$AP = R_t$

$DQ = R_y$

<u>Required</u>, the points D and C.

Crossings.



Problem. Given a curve crossing a tangent, R - g, and angle C between tangent and curve.

Required frog angles at A - B - F - D

Draw lines AO - BO - FO - DO also MO perpendicular to CM

Then $MO = R \cos C$

$$\cos A = \frac{MO - \frac{g}{2}}{R + \frac{g}{2}}$$

$$\cos D = \frac{MO - \frac{g}{2}}{R - \frac{g}{2}}$$

$$\cos B = \frac{MO + \frac{g}{2}}{R + \frac{g}{2}}$$

$$\cos F = \frac{MO + \frac{g}{2}}{R - \frac{g}{2}}$$

Parabolic Curves.

Instead of circular arcs to join two tangents, parabolic arcs have been proposed and used, in order to do away with the sudden changes in direction which occur where a circular curve leaves or joins a tangent. Parabolic curves have failed to meet with favor for railroad curves for several reasons.

1. Parabolic curves are less readily laid out by instrument than are circular curves.
2. It is not easy to compute at any point the radius of curvature for a parabolic curve; this may be necessary either for curving rails or for giving the elevation to the outer rail.
3. The use of the "Spiral" or other "Easement" or "Transition" curves, secures the desired result in a more satisfactory way.

There are however many cases (in Landscape Gardening or Elsewhere) where a parabolic curve may be useful either because it is more graceful or because without instruments it is more easily laid out, or for some other reason.

It is seldom that parabolic curves would be laid out by instrument.

Properties of the Parabola.

§ 131 - 132 - 133 - 134 Rankle.

(a.) The locus of the middle points of a system of parallel chords of a parabola is a straight line parallel to the axis of the parabola (i.e. a diameter).

(b.) The locus of the intersection of pairs of tangents is in the diameter

(c.) The tangent to the parabola at the vertex of the diameter is parallel to the chord bisected by this diameter.

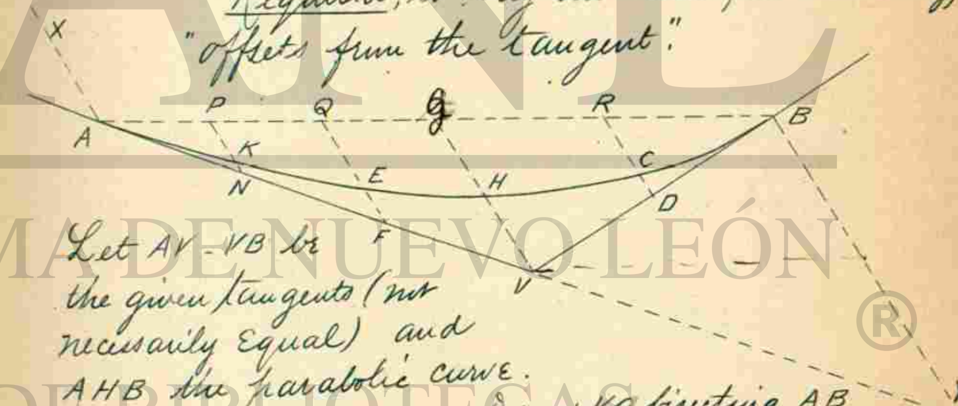
(d.) Diameters are parallel to the axis

(e.) The equation of the parabola, the coordinates measured upon the diameter and the tangent at the end of the diameter is

$$y'^2 = \frac{4p}{\sin^2\theta}x' \qquad \text{or}$$

$$y^2 = 4p'x \qquad (121.)$$

<u>Problem.</u> Given, two tangents to a parabola also the position of P.C. and P.T.
<u>Required</u>, to lay out the parabola by "offsets from the tangent".



Let AV - VB be the given tangents (not necessarily equal) and AHB the parabolic curve.
Join the chord AB. Draw VG bisecting AB
Draw AX - BY parallel to VG. Produce AV to Y.
Then VG is a diameter of the parabola
AX parallel to VG is also a diameter.
The Equation of the parabola referred to AX and AY as axes is $y^2 = 4p'x$.

Hence $AV^2 : AY^2 = HV : BY$

$$AV^2 : (2AV)^2 = HV : 2GV$$
$$1 : 4 = HV : 2GV$$
$$HV = \frac{GV}{2} \qquad (122.)$$

Next bisect VB at D

Draw CD parallel to AX

Then $BD^2 : BV^2 = CD : HV$

$$CD = \frac{HV}{4}$$

Similarly make $AN = NF = FV$

Then $KN = \frac{HV}{9}$

$$EF = \frac{4}{9} HV$$

In a similar way, as many points as are needed may be found.

Fieldwork.

(a.) Find G bisecting AB.

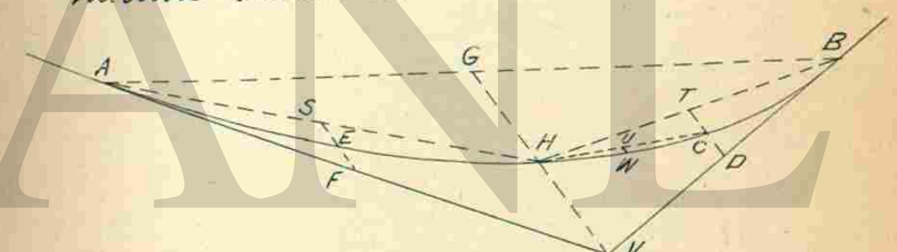
(b.) Find H bisecting GV.

(c.) Find points P - Q and N - F dividing AG - AV proportionately; also R and D dividing GB and BV proportionately.
(Use simple ratios when possible as $\frac{1}{2}$, $\frac{1}{3}$ etc.)

(d.) Lay off on PN, the calculated distance KN (in figure $KN = \frac{HV}{9}$); on QF lay off EF; and on RD lay off CD (in figure $CD = \frac{HV}{4}$).

For many purposes or in many cases it will give results sufficiently close, to proceed without establishing P - Q - R, the direction of NK - EF - CD being given approximately by eye. When the angle AVG is small (as in the figure) it will generally be necessary to find P - Q - R. When the angle AVH is large (greater than 60°) and the distances NK - EF - CD are not large, it will often be unnecessary. No fixed rule can be given as to when approximate methods shall be used. Experience educates the judgment so that each case is settled upon its merits.

<u>Problem</u> <u>Given</u>, two tangents to a parabola, also the positions of P.C. and P.T.
<u>Required</u>, to lay out the parabola by "middle ordinates"



The ordinates are taken from the middle of the chord, and parallel to GV in all cases.

<u>Fieldwork</u>.

(a.) Establish H as in last problem.
(b.) Lay off SE = $\frac{1}{4}$ HV; also TC = $\frac{1}{4}$ HV
(c.) Lay off UW = $\frac{1}{4}$ TC and continue thus until a sufficient number of points is obtained.

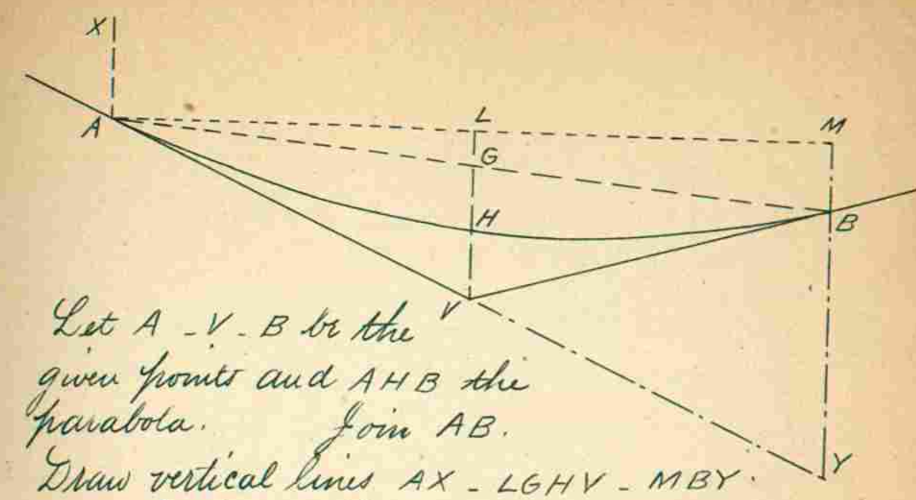
The length of curve can be conveniently found only by measurement on the ground.

Vertical Curves.

It is convenient and customary to fix the grade line upon the profile as a succession of straight lines; also to mark the elevation above datum plane, of each point where a change of grade occurs; also to mark the rates of grade in feet per station of 100 feet. At each change of grade an angle is formed. To avoid a sudden change of direction it is customary to introduce a vertical curve at every such point, if the angle is large enough to warrant it. The curve commonly used for this purpose is the parabola. A circle and a parabola would substantially coincide where used for vertical curves. The parabola effects the transition rather better theoretically than the circle, but its selection for the purpose is due principally to its greater simplicity of application. It is generally laid to extend an equal number of stations on each side of the vertex.

Problem. Given, the elevations at the vertex and at one station (100') each side of vertex.

Required, the elevations of the vertical curve opposite the vertex.



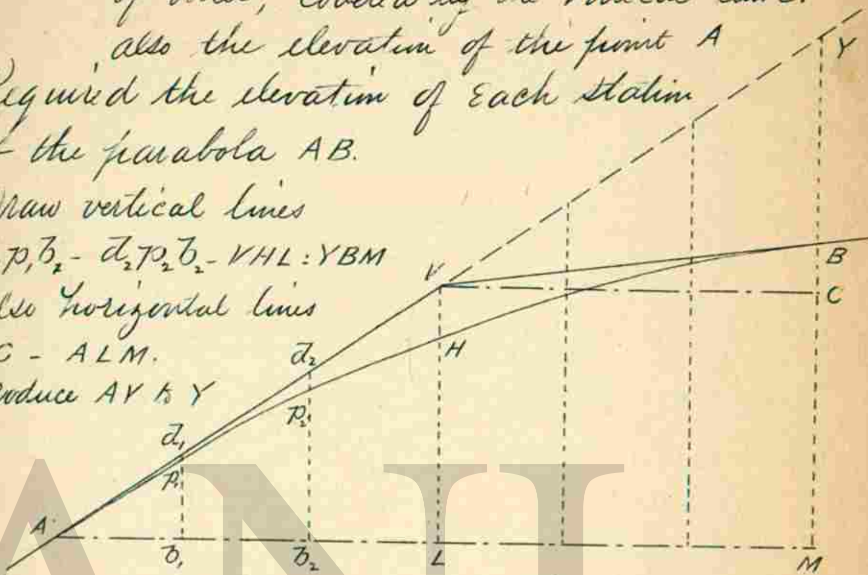
Let A - V - B be the given points and AHB the parabola. Join AB.
Draw vertical lines AX - LGHV - MBY and horizontal line ALM.
Produce AV to Y.
In the case of a vertical curve, the horizontal projections of AV and VB are equal, and here each equals 100 feet = AL = ML
Therefore AG = GB and AV = VY
VG is a diameter of the parabola.
AX is also a diameter.

$$VH = \frac{VG}{2}$$

$$\text{Elev. } H = \frac{1}{2}\left(\frac{\text{Elev. } A + \text{Elev. } B}{2} + \text{Elev. } V\right) \qquad (123.)$$

This affords a simple and quick method of finding H when the vertical curve extends only one station each side of vertex, which is the most common case. When the vertical curve extends more than one station each side of the vertex, another method is preferable, which is also applicable to the above case, and is in some respects preferable for that also.

Problem Given, the rate of grade g of AV
and " " " " g' of VB,
the number of stations n, on each side of vertex, covered by the vertical curve.
also the elevation of the point A
Required the elevation of each station of the parabola AB.



Draw vertical lines
$d_1p_1b_1 - d_2p_2b_2 - VHL : YBM$
Also horizontal lines
$VC - ALM$.
Produce AV to Y

Let a_1 = offset d_1p_1 at the first station from A.
a_2 = " d_2p_2 " " second " " A.
a_3 = etc

Then $a_2 = 2^2 a_1 = 4a_1$
$a_3 = 3^2 a_1 = 9a_1$
$a_{2n} = (2n)^2 a_1 = 4n^2 a_1 = YB$
$YB = YC - BC$
$4n^2 a_1 = ng - ng'$ or $a_1 = \frac{g-g'}{4n}$ (124.)

Due regard must be given to the signs of g and g' in this formula, whether + or −.
From the elevation at A we may now find the required elevations since we have given g and we also have a_1

$a_2 = 4a_1$
$a_3 = 9a_1$ etc.

A method better and more convenient for use is given below

$$d_1b_1 = g \qquad p_1b_1 = g - a_1$$
$$d_2b_2 = 2g \qquad p_2b_2 = 2g - a_2 = 2g - 4a_1$$
$$d_3b_3 = 3g \qquad p_3b_3 = 3g - a_3 = 3g - 9a_1$$
$$d_4b_4 = 4g \qquad p_4b_4 = 4g - a_4 = 4g - 16a_1 \quad \text{Etc.}$$

Again

$$p_1b_1 = g - a_1 \qquad = g - a_1$$
$$p_2b_2 - p_1b_1 = 2g - 4a_1 - (g - a_1) = g - 3a_1$$
$$p_3b_3 - p_2b_2 = 3g - 9a_1 - (2g - 4a_1) = g - 5a_1$$
$$p_4b_4 - p_3b_3 = 4g - 16a_1 - (3g - 9a_1) = g - 7a_1 \quad \text{Etc.}$$

On a straight grade, the elevation of any station is found from the preceding, by adding a constant g.

On a vertical curve the elevation of each station is found from the preceding by adding, in a similar way, not a constant but a varying increment, being for the

$$\left.\begin{array}{lll} 1^{st}\ \text{Station from} & A = & g - a_1 \\ 2^{nd}\ \ \text{"} \quad \text{"} & A = & g - 3a_1 \\ 3^{rd}\ \ \text{"} \quad \text{"} & A = & g - 5a_1 \end{array}\right\} \text{changing by successive differences of } 2a_1 \text{ in each case.}$$

The labor involved is not materially greater, in many cases, for a vertical curve than for a straight grade. This method has the additional advantage that a correct final result at the end of the vertical curve makes a "check" upon all intermediate results.

135.00
17.50
13.50

137.50
135.00
2.50

Example.

Given. grades as follows:—

Sta.	Elev.	Rate
5	117.50	+3.50
10	135.00	+0.50
15	137.50	+0.50

Then $a_1 = \frac{g-g'}{4n} = \frac{3.00}{12} = 0.25$

Sta.	Elev.		
5	117.50		
	+3.50		$= g$
6	121.00		
	+3.50	3.50	$= g$
7	124.50	−0.25	$= a_1$
	+3.25	3.25	$= g - a_1$
8	127.75	−0.50	$-2a_1$
	+2.75	2.75	$g - 3a_1$
9	130.50	−0.50	$-2a_1$
	+2.25	2.25	$g - 5a_1$ Etc
10	132.75	−0.50	
	+1.75	1.75	
11	134.50	−0.50	
	+1.25	1.25	
12	135.75	−0.50	
	+0.75	0.75	
13	136.50		End of curve
	+0.50		$= g'$
14	137.00		
	+0.50		
15	137.50		

The elevation for Sta. 15 thus obtained agrees with the elevation shown in the data, All the intermediate elevations are therefore "checked"

Problem Given $g - g' - a_1$

Required n.

From (124.) $n = \frac{g-g'}{4a_1}$ (125.)

For practical considerations a_1 should not exceed 0.25

or $n \geq \frac{g-g'}{4 \times 0.25}$ or $n \geq g - g'$ (126.)

Spiral Easement Curve.

Upon tangent track ought properly to be level across; upon circular curves, the outer rail should be elevated in accordance with the formula $E = \frac{gv^2}{32.2R}$ In passing directly from tangent to circular curve, there is a point (at P.C.) where two requirements conflict; the track cannot be level across and at the same time have the outer rail elevated. It has been the custom to elevate the outer rail on the tangent for perhaps 100 feet back from the P.C. This is unsatisfactory. It is becoming somewhat common to introduce a curve of varying radius, in order to pass gradually from the tangent to the circular curve. The transition will be most satisfactorily accomplished when the elevation E increases uniformly with the distance l from the P.E. (Point of Easement) where the Spiral Easement curve leaves the tangent; so that $\frac{E}{l}$ is a constant, or

$$\frac{gv^2}{32.2\,Rl} = A \text{ (a constant) or } Rl = \frac{gv^2}{32.2\,A}$$

g - v - A are all constants; we may then put the equation in the form $Rl = C$ (127.)

To further investigate the qualities of this curve, let α = angle of inclination of curve to tangent at any point.

Then $R\,d\alpha = dl$

$$d\alpha = \frac{dl}{R} = \frac{l\,dl}{C}$$

$$\alpha = \frac{l^2}{2C} \qquad (128.)$$

Again $dx = dl \sin\alpha$ and $dy = dl \cos\alpha$

Since all values of α within the limits of the Spiral Easement curve must be small, we may with slight error assume for simplicity that

$\sin\alpha = \alpha$ and $\cos\alpha = 1$

then $dx = \alpha\, dl$ and $dy = dl$ (approx.)

whence $y = l$

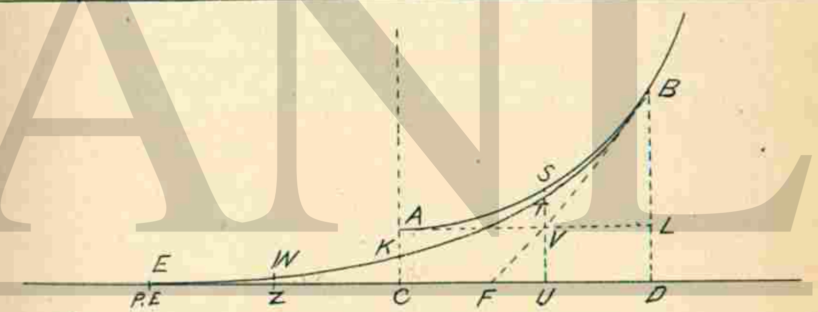
from (127.) $Ry = C$ (129.)

(128.) $\alpha = \dfrac{y^2}{2C}$ (130.)

$dx = \alpha\, dl = \alpha\, dy = \dfrac{y^2}{2C}\, dy$

whence integrating $x = \dfrac{y^3}{6C}$ (approx.) (131.)

This Equation shows the curve to be approximately a "cubic parabola".



A useful and simple application of the cubic parabola to railroad work is shown below.

Fieldwork.

(a.) Lay off tangent ED.

(b.) Offset from any point C on tangent and run in circular curve AB (offset by any distance AC desired).

(c.) Find the distance AL such that BL = 3 AC

(d.) Make EC = AL

(e.) Mark K so that AK = KC.

(f.) Fix any desirable point W by the formula
$x = x_B\left(\frac{y}{y_B}\right)^3$.

(g.) Fix any desirable point T by the formula
$ST = x_B\left(\frac{y_B - y}{y_B}\right)^3$

It will in general be sufficiently accurate to take AB = AL and SB = VL

It remains to be proved with the fieldwork indicated above that:—

I. The curve passing through the points E - W - K - B as described, is in fact a cubic parabola

II. The curves, circular and parabolic, have a common tangent at B.

III. The curves circular and parabolic, have the same radius at B

IV. The offset ST is correctly calculated.

I. In the figure, let AB = the circular curve
ED = the tangent
EKB = the cubic parabola
AL = a tangent to AB at A
BF = " " " EKB at B
BD = 4 AC
EC = CD
AC = LD

Then from the equation of the cubic parabola

BD = 8 KC
4AC = 8 KC
AC = 2 KC (132.)
BD = 4AC
BL = 3 AC (133.)

The position of W follows directly from the equation of the cubic parabola.

II. The tangent to the cubic parabola at any point is found thus:—

$$\tan\alpha = \frac{dx}{dy} = \frac{\frac{y^2 dy}{2c}}{dy} = \frac{y^2}{2C}$$

For the point B $\quad \tan\alpha_b = \dfrac{y_b^2}{2C}$

but $\tan\alpha_b = \dfrac{BD}{FD} = \dfrac{x_b}{FD}$ or

$$FD = \frac{x_b}{\tan\alpha_b} = \frac{\frac{y_b^3}{6C}}{\frac{y_b^2}{2C}} = \frac{y_b}{3} = \frac{2}{3}CD$$

By similar triangles $VL = \frac{3}{4}FD = \frac{1}{2}CD$

Therefore for the cubic parabola $AV = VL$

and approximately the tangents $AV = VB$

The two tangents to the circle at A and B must be Equal. Hence when BL is small compared with AL the tangent to the cubic parabola approximately coincides with that of the circle.

In the case of a very sharp curve, or where BL is large, the approximation may not be sufficiently close.

III. From (129) and (131) $R_\varepsilon = \dfrac{C}{y_b} = \dfrac{\frac{y_b^3}{6x_b}}{y_b} = \dfrac{y_b^2}{6x_b}$

$$R_c = \frac{(\text{Chord } AB)^2}{2BL} = \frac{AL^2}{2BL} \text{ (approx.)}$$

$$= \left(\frac{y_b}{2}\right)^2 \div 2\times\frac{3}{4}x_b$$

$$= \frac{y_b^2}{6x_b}$$

Therefore R_c and R_ε are approximately Equal.

IV. Required in the figure, the offset ST.

$$US = AC + VS$$

$$= \frac{x_3}{4} + BL\left(\frac{AT}{AL}\right)^2 \text{(approx.)}$$

$$= \frac{1}{4}x_3 + \frac{3}{4}x_3\left(\frac{y - \frac{y_3}{2}}{\frac{y_3}{2}}\right)^2$$

$$= \frac{1}{4}x_3 + \frac{3}{4}x_3\left(\frac{4y^2}{y_3^2} - \frac{4y}{y_3} + 1\right)$$

$$= x_3 + x_3\left(\frac{3y^2}{y_3^2} - \frac{3y}{y_3}\right)$$

$$UT = x_3 \frac{y^3}{y_3^3}$$

$$ST = US - UT = x_3\left(1 - \frac{3y}{y_3} + \frac{3y^2}{y_3^2} - \frac{y^3}{y_3^3}\right)$$

$$= x_3\left(1 - \frac{y}{y_3}\right)^3$$

$$= x_3\left(\frac{y_3 - y}{y_3}\right)^3 = x_3 \frac{VL^3}{ED^3}$$

If we make $VL = EZ$ then $ST = WZ$.

The Spiral Easement Curve may be readily laid out by <u>Deflection Angles</u>, if preferred.

<u>Fieldwork</u>:

(a.) Lay off the tangent ED

(b.) Offset from any point C on tangent, and set point A from which to run in circular curve AB (offset by any distance desired).

(c.) Find distance AL such that $BL = 3\,AC$

(d.) Assuming the arc $AB = AL$, find the central angle (for the arc AB) = AOB (e.) Take deflection angle DEB $= \frac{AOB}{3} = i_b$

(f.) For other deflection angles use the formula

$i = i_b \frac{y^2}{y_b^2}$

(g.) With ⊼ at A run out circular curve from B.

It remains to be proved that with this fieldwork

A. The formula $i = i_b \frac{y^2}{y_b^2}$ is correct.

B. The deflection angle $DEB = \frac{AOB}{3}$

A. Let i = deflection angle from P.T. to any point xy.

Since $x = \frac{y^3}{6C}$, $\tan i = \frac{x}{y} = \frac{y^2}{6C}$

For small angles, tangents are proportional to the angles. Here i is always small

Therefore $i : i' = y^2 : y'^2$ (approx) (132.)

B. In curve AB

$BL = \frac{AB^2}{2R}$ (AB here being the chord)

$BD = \frac{4}{3} \cdot \frac{AB^2}{2R} = \frac{2AB^2}{3R}$

$ED = 2AB$ (approx.)

$\tan i_b = \frac{BD}{ED} = \frac{AB}{3R}$

$\tan \frac{1}{2} AOB = \frac{BL}{AL} = \frac{AB^2}{AL \times 2R} = \frac{AB}{2R}$ (approx.)

Tangents of small angles are proportional to the angles. Hence

$i_b : \frac{AOB}{2} = 2 : 3$

$i_b = \frac{AOB}{3}$ (133.)

For all Easement curves likely to be used, the approximations here given will lead to no errors of consequence. The greatest lack of precision will be in the position of the point B, and since it is undesirable that any error, however small, should be carried on, if it be possible to avoid it, there is an advantage in running in the circular curve with the transit at A, and with a back-sight parallel to the tangent ED, rather than with the transit at B.

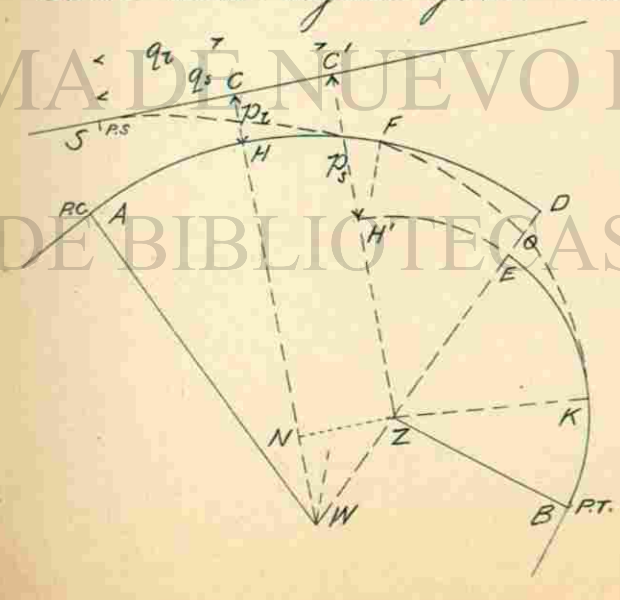
If preferred, the length of line AL or AB may be assumed and AC calculated. This method will simplify the calculation of angles if AL or AB be taken in round numbers.

Compound Curves.

In the case of Compound curves, it is proper and desirable that Spirals or Easement curves should be introduced between the two curves forming the compound curve. It is apparent without formal demonstration that where one curve is offset from another by a given distance, p we may readily calculate the length of cubic parabola to connect a tangent and a curve whose degree is the difference between the degrees of the curves under consideration, and having the same offset p from tangent to curve. The offsets calculated may then be laid off from either circular curve, as is most convenient. The middle point of the cubic parabola will be on the offset measured by p and midway between the two circular curves. The principle involved is the same as that discussed in connection with turnouts from curves, where the approximation was pointed out. pp. 93-94-102.

Problem. Given, the degrees of two parts of a compound curve.

Required, to connect the two curves by a Searles' spiral; and to find the offset, and also the points where the spiral leaves one curve and again joins the other.



Let the two circular curves be AD with center W. HEB " " Z. Let the required spiral be FK This spiral must be selected so that at F it will have the proper curvature to join the circle W, and at K the proper curvature to join circle Z. If the spiral be continued back to the P.S. at S, we may show the tangent at S by SCC'. The points F and K must be regular points on the spiral.

Draw perpendiculars WC and ZC' and ZN and extend circle Z to H' parallel to SCC'.

The notation $q_z - p_z - q_s - p_s$ then is clear.

We require offset $O = DE$.

Having selected a spiral which shall properly fit at F and K, we may find for this spiral $q_z - p_z - q_s - p_s$

$$\text{The } \tan CWD = \frac{ZN}{NW} = \frac{q_s - q_z}{R_z + p_z - (R_s + p_s)} \qquad (134.)$$

$$O = DE = DW - ZE - ZW$$

$$= R_z - R_s - \frac{q_s - q_z}{\sin CWD} \qquad (135.)$$

The point F may be found from D since angle $FWD = CWD - s_F$

The point K may be found by similar process. The point D is assumed on the ground and is the point selected for compounding the curves.

Example.

Given, a compound curve 2° and 5°.
Required, to select a Spiral (Searles'); to find offset between 2° and 5° curves; also points where the Spiral connects with 2° and 5° curves.
From Table VI Searles' Spiral
and " IV " Field Engineering
Select a Spiral and compile the following Table

Degree	n - c	q	p	R	R + p
2°	3 - 33	48.997	0.236	2864.93	2865.17
5°	9 - 33	146.846	3.855	1146.28	1150.14
	Difference	97.849		1718.65	1715.03

97.849 log 1.990556
1715.03 log 3.234272
3°15'55" tan 8.756284 sin = 8.755579
97.849 log = 1.990556
1717.82 log 3.234977
$R_2 - R_5$ = 1718.65
∴ offset = 0.83

Find point F

$S_2 = 1°$
3°15'55"
2°c) 2°15'55" = 2°.159167' = 2.2653°
1
113.26 = FD

Find point K

S_5 = 7°30'
3°15'55"
5°c) 4°14'05" = 4°.14087' = 4.2348°
84.70 = EK

Length Spiral n c
6 - 33 = 198.00 → 197.96 = FD + EK
check.

Computation of Earthwork.

The first step in the process is staking out, or "Setting Slope Stakes", as it is commonly called.
There are two important parts of the work of setting slope stakes
1. Setting the stakes.
2. Keeping the notes.
The data for setting the stakes are:—
A. The ground with center stakes set at every Station (sometimes oftener).
B. A record of bench marks and of elevations and rates of grades established.
C. The base and side slopes of the cross-section for each class of material.
In practice, notes of alignment, a full profile and various convenient data are commonly given in addition to the above.

1. Setting the Stakes. The work consists of
(a.) Marking upon the back of the center stakes the "Cut" or "Fill" in feet and tenths, as C 2.3 or F 4.7
(b.) Setting side stakes or slope stakes at each side of the center line at the point where the side slope intersects the surface of the ground and marking upon the inner side of the stake the "cut" or "fill" at that point.

The process of finding the cut or fill at the center stake is as follows.

Given, for any station the height of instrument $= h_i$ and the elevation of grade $= h_g$.

Then the required rod reading for grade

$$= r_g = h_i - h_g \qquad (136.)$$

It is not necessary to figure h_g for each station. Let $h_{g_0} = h_g$ at Sta 0

$h_{g_1} = h_g$ " " 1

$h_{g_2} = h_g$ " " 2 etc.

Also use similar notation for r_g.

Let g = rate of grade (rise per station)

Then $h_{g_1} = h_{g_0} + g$

$h_{g_2} = h_{g_1} + g$

$h_{g_3} = h_{g_2} + g$. etc

$$r_{g_0} = h_i - h_{g_0}$$

$$r_{g_1} = h_i - h_{g_1}$$

$$= h_i - (h_{g_0} + g) = h_i - h_{g_0} - g$$

$$r_{g_1} = r_{g_0} - g \qquad (137.)$$

Similarly $r_{g_2} = r_{g_1} - g$ etc

It will be necessary or certainly desirable to figure h_g and r_g anew for each new h_i.

It is well to figure h_g and r_g (as a check) for the last station before each turning point.

Example.

$h_i = 106.25$

Sta 0. Grade Elevation 100.00 rate +1.00

5. " " 105.00 " +0.50

10 " " 107.50

$= r_{g_0} = 106.25 - 100.00 = 6.25 \qquad 6.25$

$r_{g_1} = \qquad 6.25 - 1.00 = 5.25$

$r_{g_2} = \qquad 5.25 - 1.00 = 4.25$

$r_{g_3} = \qquad 4.25 - 1.00 = 3.25$

$r_{g_4} = \qquad 3.25 - 1.00 = 2.25$

$r_{g_5} =$ change in rate $\qquad 2.25 - 1.00 = 1.25$

$r_{g_6} = \qquad 1.25 - 0.50 = 0.75$

$r_{g_7} = \qquad 0.75 - 0.50 = 0.25$ ✓

It is found necessary to take a T.P. here and we therefore find $h_{g_7} = h_{g_5} + 2g$

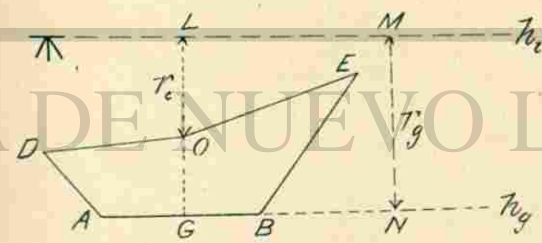
$= 105.00 + 1.00 = 106.00$

$r_{g_7} = h_i - h_{g_7} = 106.25 - 106.00 = 0.25$ ✓

Therefore all intermediate values $r_{g_1} - r_{g_2}$ etc. are "checked".

We now have found r_g.

Next, holding the rod upon the surface of the ground at the center stake we obtain the rod reading $r_c = LO$.



The cut or fill c

$= OG = MN - LO$

$= r_g - r_c$ (138)

In the figure given the value of r_g and c are positive; a positive value of $\underline{c}$ indicates a "cut", a negative value of $\underline{c}$ indicates a "fill". It can be shown that in the two cases of "fill"

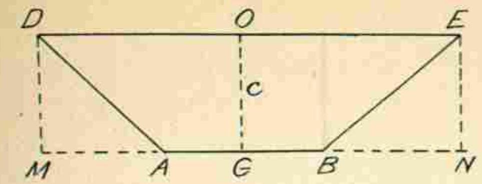
(a.) when h_i is greater than h_g and

(b.) when h_i is less than h_g

the formula given will hold good by paying due attention to the sign of r_g, whether + or −.

2. Setting the stake for the side slope.

(a.) When the surface is level.



Let AB = base of section = b

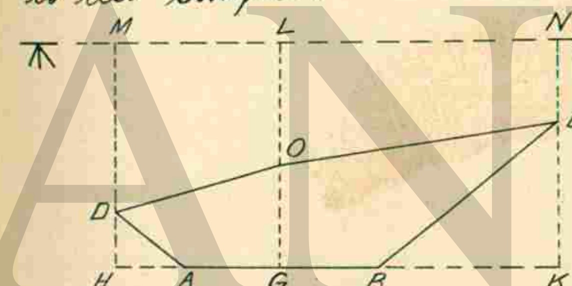
OG = center ht = c

$\frac{BN}{EN} = \frac{AM}{DM}$ = side slope = s

$OD = OE$ = distance out = d center to slope stake

Then
$$d = GB + BN$$
$$= \tfrac{1}{2}b + s \times DM = \tfrac{1}{2}b + s \times EN$$
$$= \tfrac{1}{2}b + sc \qquad (139.)$$

(b.) When the surface is not level, the process is less simple.



Let $b = AB$ = base

$c = OG$ = center height (or cut).

s = slope

$h_r = EK$ = side height right.

$h_l = DH$ = " " left.

$d_r = GK$ = distance out right.

$d_l = GH$ = " " left.

Then
$$\left.\begin{aligned} d_r &= \tfrac{1}{2}b + sh_r \\ d_l &= \tfrac{1}{2}b + sh_l \end{aligned}\right\} \qquad (140.)$$

But h_r and h_l are not known. It is evident from the figure that $h_r > c$ and $h_l < c$ in the case indicated, and therefore $d_r > \frac{1}{2}b + sc$

$d_l < \frac{1}{2}b + sc$

It would be possible in many cases to take measurements such that the rate of slope of the lines OE and OD would be known, and the positions of E and D determined by calculation from such data. But speed and results finally correct are the essentials in this work and these are best secured by finding h_l and h_r and the corresponding d_l and d_r upon the ground by a series of approximations as described below.

Having determined c, make an estimate at once as to the value of h at the point where the slope will intersect the surface, and calculate $d = \frac{1}{2}b + sh$ to correspond.

Measure out this distance, set the rod at the point thus found, take the rod reading, and if the cut or fill thus found from the rod reading yields a value of d equal to that actually measured out, the point is correct.

Otherwise make a new and closer approximation from the better data just obtained, and repeat the process until a point is reached where the cut or fill found from the rod reading yields a distance out equal to that taken on the ground. Then set the stake and mark the cut or fill upon the inner side as previously stated.

This process may impress some as unscientific, and at first trial as slow, but with a little practice, it is surprising how rapidly, almost by instinct, the proper point is reached, often within the required limits of precision at the first trial, while more than two trials will seldom be necessary except in difficult country. The instrumental work is the same in principle as at the center stake.

Let r_r = NE = rod reading at slope stake right

then $KN - NE$ or $r_g - r_r = h_r$

where r_g is the same for center, right and left of section.

It requires a certain amount of field practice to appreciate the process here outlined, but which in practice is very simple.

In some cases it may be necessary to make one or more resettings of the level in order to reach the side stakes from the center stake. In this case of course a new r_g must be calculated from the new h_i. This introduces no new principle, but makes the work slower. A "slope-board" or "level-board" may be used to advantage in many cases. In certain sections of country this might be considered almost indispensable. It consists simply of a long straight edge of wood (say 15 ft. long) with a level mounted in the upper side. It is used with any self-reading rod. A rod quickly hand marked will serve the purpose well. Having given the cut or fill at the center or at any point in the section, the leveling for the side stakes and any additional points, can readily and with sufficient accuracy, be done by this "level board," and the necessity for taking new turning points and resetting the level avoided.

II. Keeping the Notes.

The form of note book used for keeping the notes of slope stakes and of center cuts and fills, often called "cross-section" notes, is shown in the following two pages.

The left hand column for stations should read from bottom to top.

The column of surface elevations is not obtained directly from the levels, but results from adding to the grade elevation at any station, the cut or fill at that station, paying due attention to the signs. This column of surface elevations need not be entered up in the field, but may be filled in as office work.

The column of grade elevations consists of the grade elevations as figured for each station.

The figures marked + are cuts in feet and tenths, and those marked − are fills; the figures above the cuts and fills are the distances out from the center, and the position in the notes, whether right or left of the center, corresponds to that in the ground.

The columns in the right hand page are used for entering, when computed, the "quantities" or number of cubic yards in the section of Earthwork.

(Left Hand Page)

Station	Surface Elev.	Grade Elev.	Cross Section					
5	97.1	105.00	$\frac{18.4}{-7.6}$		-7.9			$\frac{19.4}{-8.3}$
+69.7 P.T.	94.4	104.70	$\frac{22.1}{-10.1}$		-10.3			$\frac{23.0}{-10.7}$
4	96.9	104.00	$\frac{19.3}{-8.2}$		-7.1			$\frac{17.0}{-6.7}$
+27.2 P.C.	98.0	103.27	$\frac{16.6}{-6.4}$		-5.3			$\frac{12.4}{-3.6}$
3	98.1	103.00	$\frac{16.0}{-6.0}$		-4.9			$\frac{10.9}{-2.6}$
+91	100.6	102.91	$\frac{13.3}{-4.2}$		-2.3			$\frac{10.0}{0.0}$
+76	102.8	102.76	$\frac{10.3}{-2.2}$		0.0			$\frac{11.9}{+1.9}$
+64	103.7	102.64	$\frac{10.0}{0.0}$		$+1.1$			$\frac{13.2}{+3.2}$
+50	106.4	102.50	$\frac{13.4}{+3.4}$		$+3.9$			$\frac{17.1}{+7.1}$
2	115.1	102.00	$\frac{16.7}{+6.7}$		$+13.1$			$\frac{26.7}{+16.7}$
1	117.7	101.00	$\frac{22.7}{+12.7}$	$\frac{10.0}{+17.2}$	$+16.7$	$\frac{10.0}{+13.1}$		$\frac{22.2}{+12.2}$
0	109.2	100.00	$\frac{18.0}{+8.0}$	$\frac{9.0}{+10.1}$	$+9.2$	$\frac{8.5}{+7.8}$	$\frac{18.4}{+14.7}$	$\frac{24.6}{+14.6}$

(Right Hand Page)

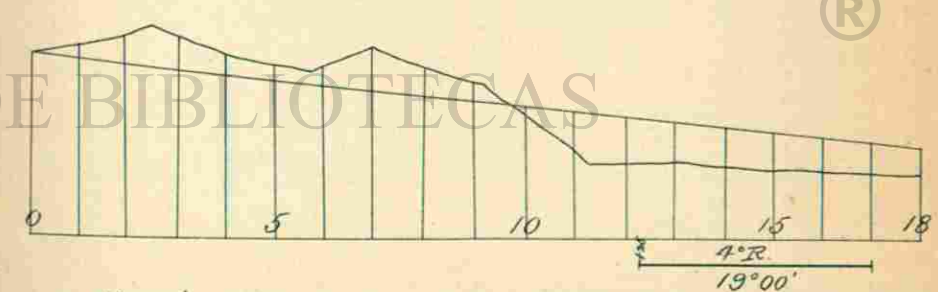
Excavation L. Rock	S. Rock	Earth	Embankment	General Notes.

The column "General Notes" is used for entering extra measurements (of ditches etc.) not included in the regular cross section notes; also notes of material "hauled"; classification of material and various other matters naturally classed under the head of "Remarks."

When the surface is irregular between the center and side stakes, additional rod readings and distances out are taken, and the results entered as shown for station 0 on p. 134, the section itself being as shown below in the sketch

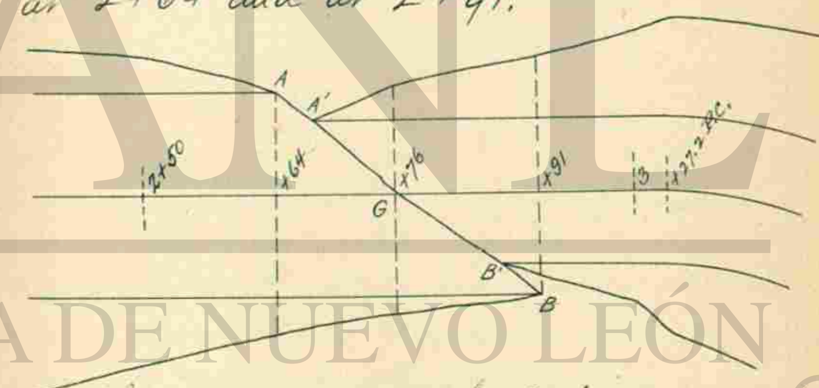
Station 0

Cross sections are taken at every full station, at every P.C. or P.T. of curve, wherever grade cuts the surface, and in addition, at every break in the surface. In the figure below, showing a profile, sections should be taken at the following stations.



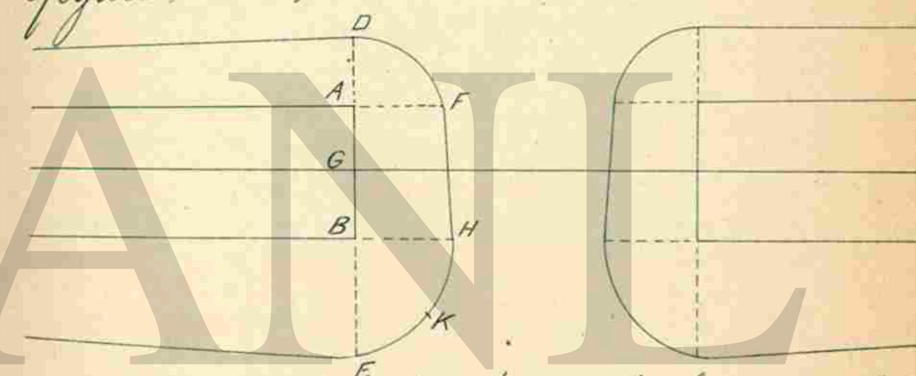
at Stations 0 - 1 - 2 - 2+52 - 3 - 4 - 5 - 5+80 - 6 - 7 - 8 - 9 - 9+11 9+62 - 10 - 11 - 11+20 - 12 - 12+25 P.C. - 13 - 14 - 15 - 16 - 17 P.T. 18.

It is not necessary to actually drive stakes in all cases where a cross section is taken and recorded, but in every case where they will aid materially in construction, stakes should be set. It is best to err on the safe side, or liberal side. In passing from cut to fill, it is customary to take full cross sections, not only at the point where the grade line cuts the surface at the center line of survey, but also where the grade cuts the surface at the outside of the base, both right and left, as in the figure below, which illustrates the notes on p. 134. Full cross sections are taken not only at Stations 2+76, but also at 2+64 and at 2+91.



Stakes are set at the center G and at the points A and B where the outside line of the base of Excavation cuts the surface. It is not customary to set stakes or record the notes for the points A' and B' where the outside line of the base of Embankment cuts the surface. The stakes at A - G and B are a sufficient guide for construction, and the solidities or "Quantities" would in general be affected only slightly by the additional notes if they were made. When the line AGB crosses the center line nearly at right angles it would not be necessary to take more than one section so far as the notes are concerned. It is well however to set the stakes A and B exactly in their proper position.

Wherever an opening is to be left in an embankment for a bridge or for any other structure, stakes should be set as in the figure below:



at A and B (at the side of the base and top of the slopes AF and BH) stakes should be set marked "BANK TO GRADE"; and at F and H (at the foot of the slopes) stakes should be set marked "TOE OF SLOPE"; where the bank is high an additional stake K at foot of slope may be set as an aid to construction. The stakes at D and E should also be set as ordinary slope stakes.

The "level notes" proper, or the record of heights of instrument, bench marks, turning points, etc. used in setting slope stakes, are usually kept separate from the cross section notes. They should be kept either in the back of the cross section book or in a level book carried for that purpose. Keeping these or any notes on a slip of paper is bad practice.

Computation of Earthwork.

Earthwork can be most readily computed when the surface is level across the section, but this is seldom the case, and for purposes of final computation it is not often attempted to take measurements upon that basis.

In general, in railroad work, the ground is sufficiently regular to allow of "Three Level Sections" being taken, one level (Elevation) at the center and one at each slope stake as shown by these notes $\frac{11.3}{+2.6}$ $+4.2$ $\frac{12.8}{+5.5}$

The term "Three Level Section" is usually applied only to regular sections where the widths of base on each side are the same.

In regular three level sections the calculation of quantities can be made quite simple.

For the final estimation of quantities, it is best to use three level sections as far as possible.

In many cases where "Three Level Sections" are not sufficient, it may be possible to use "Five Level Sections", consisting of a level at the center, one at each side where the base meets the slope, and one at each side slope stake as shown by the following notes. (Base 20 slope 1 to 1)

$\frac{22.7}{+12.7}$ $\frac{10.0}{+17.2}$ $+16.7$ $\frac{10.0}{+13.1}$ $\frac{22.2}{+12.2}$

The term "five level section" is usually applied only to regular sections where the base and the side slopes are the same on each side of the center.

Where the ground is very rough, levels have to be taken wherever the ground requires, and the calculations must be made to suit the requirements of each special case, although certain systematic methods are generally applicable.

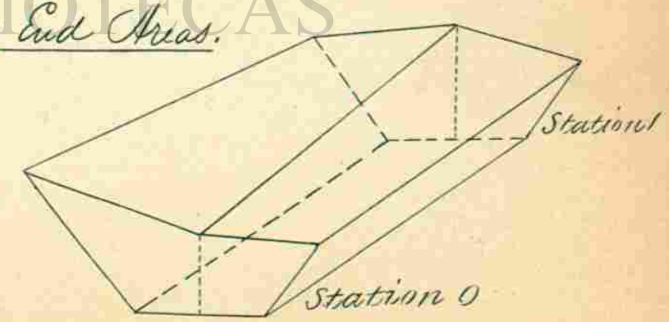
Methods of Calculating Earthwork.

In calculating the solidities or "quantities" of Earthwork, the principal methods used are as follows:—

1. Averaging End Areas
2. Prismoidal Formula
3. Middle Areas
4. Equivalent Level Sections
5. Mean Proportionals
6. Henck's

1. Averaging End Areas.

This is the simplest method



Let A_0 = area of cross section at Station 0.
A_1 = " " " " " " 1.
l = length of section Sta 0 to Sta 1.
S = Solidity of section of Earthwork (Sta 0 to 1)

Then $S = \frac{A_0 + A_1}{2} \, l$ (in cubic feet) (141.)

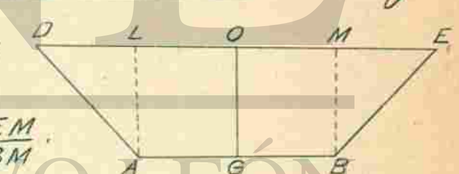
$= \frac{A_0 + A_1}{2} \cdot \frac{l}{27}$ (in cu. yds.) (142.)

As (141.) is capable of expression $S = A_0 \frac{l}{2} + A_1 \frac{l}{2}$ it is practically based on the assumption that the solidity consists of two prisms one of base A_0 and one of base A_1 and each of a length or altitude of $\frac{l}{2}$.

To use this method, we must find the area A of each cross section; the cross section may be:—

1. Level 2. Three Level 3. Five Level 4. Irregular.

1. Level Cross Section.

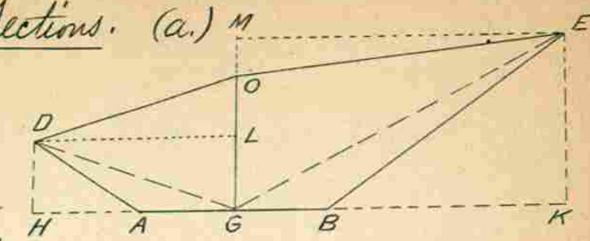


Let b = base = AB
s = side slope = $\frac{DL}{AL} = \frac{EM}{BM}$
c = center ht = OG
A = area of cross section

Then $DL = EM = sc$

$A = AB \times OG + DL \times AL$
$= bc + sc^2$
$= c(b + sc)$ (143.)

2. Three Level Sections. (a.)



Let b = base = AB
s = side slope
c = center ht.
h_r, h_l = side hts EK, DH
d_r, d_l = distances out ME, DL
A = area of cross section

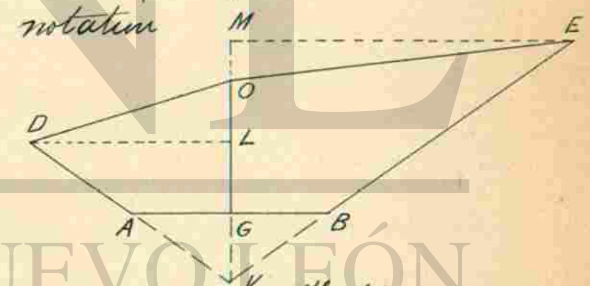
Then $A = \frac{1}{2} OG \times DL + \frac{1}{2} OG \times ME + \frac{1}{2} GB \times EK + \frac{1}{2} AG \times DH$

$$= \frac{1}{2} c(d_l + d_r) + \frac{1}{2}\frac{b}{2}(h_r + h_l)$$

$$= \frac{c(d_l + d_r) + \frac{b}{2}(h_l + h_r)}{2} \qquad (144.)$$

(b.) Another solution apparently less simple, is nevertheless more rapid for calculating a series of cross sections, and is as follows.

Using the same notation



$$\frac{GB}{GV} = s$$

$$GV = \frac{GB}{s} = \frac{b}{2s}$$

$$\text{Area } ABV = GV \times GB = \frac{b^2}{4s}$$

$$OV = c + GV = c + \frac{b}{2s}$$

$$\text{Area } EODV = OV \times \frac{DL}{2} + OV \times \frac{ME}{2} = \left(c + \frac{b}{2s}\right)\frac{d_l + d_r}{2}$$

The triangle ABV is often called the "Grade Triangle"

$$A = EODV - ABV = \left(c + \frac{b}{2s}\right)\frac{d_l + d_r}{2} - \frac{b^2}{4s}$$

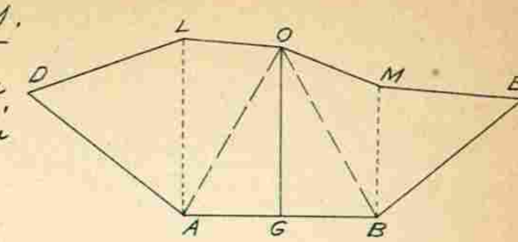
Let $D = d_l + d_r$.

$$A = \left(c + \frac{b}{2s}\right)\frac{D}{2} - \frac{b^2}{4s} \qquad (145.)$$

In using this formula for a *series* of cross sections of the same base and slope $\frac{b}{2s}$ and $\frac{b^2}{4s}$ are constants and the computation of A becomes simple and rapid.

(c) Five Level Sections.

Use notation the same as before; in addition let f_r = height MB, f_l = " LA



Then $A = AOB + DLOA + EMOB$

$$= \frac{cb}{2} + \frac{f_r d_r}{2} + \frac{f_l d_l}{2}$$

$$A = \frac{cb + f_r d_r + f_l d_l}{2} \qquad (146.)$$

(d.) Irregular Sections.

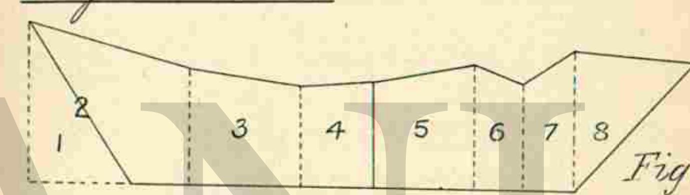


Figure 1.

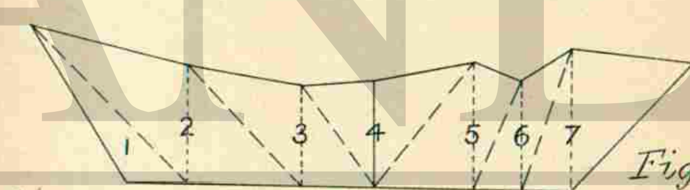


Figure 2.

The figure may be divided into trapezoids by vertical lines as in Fig. 1; or into triangles by vertical and diagonal lines as in Fig. 2.
The triangles in Fig. 2 can be figured in groups of two having a common base (vertical).
It will be seen that Fig. 1 requires 8 solutions and Fig. 2 only 7 solutions of trapezoids or triangles. The computations can be made with substantially equal simplicity in either case, and directly from the notes, with any necessity for a sketch.

Another method which has been used for calculating irregular cross sections is to plat them on cross section paper and get the area by planimeter. In very irregular cross sections this method would prove economical as compared with direct computation by ordinary methods, but by the use of suitable tables or diagrams (to be explained later) it is probable that in almost every case equal speed and equal precision could be obtained; for this reason the use of the planimeter is not recommended.

Having found the values of $\underline{A}$ for each cross section, S is found in each case by the formula above given

$$S = \frac{A_0 + A_1}{2} \cdot \frac{l}{27} \text{ (in cu. yds.)}$$

It is found that this formula is only approximately correct. Its simplicity and <u>substantial</u> accuracy in the majority of cases render it so valuable that it has become the formula in most common use. It gives results in general larger than the true solidity.

2. Prismoidal Formula.

"A prismoid is a solid having for its two ends, any dissimilar parallel plane figures of the same number of sides, and all the sides of the solid plane figures also"

Any prismoid may be resolved into prisms pyramids and wedges having as a common altitude the perpendicular distance between the two parallel end planes.

Let A_0 and A_1 = areas of end planes

M = area of middle section parallel to the end planes.

l = length of prismoid, or perpendicular distance between end planes

S = solidity of the prismoid.

Then it may be shown that

$$S = (A_0 + 4M + A_1)\frac{l}{6}$$

Let B = area of lower face or base of a prism, wedge or pyramid.

b = area of upper face.

m = middle area parallel to upper and lower faces.

a = altitude of prism, wedge or pyramid.

S = solidity " " " "

Then the area of the upper face b in terms of lower base B will be for

Prism $b = B$ Wedge $b = 0$ Pyramid $b = 0$

and the middle area m will be for

Prism $m = B$ Wedge $m = \frac{B}{2}$ Pyramid $m = \frac{B}{4}$

The Solidity S will be for

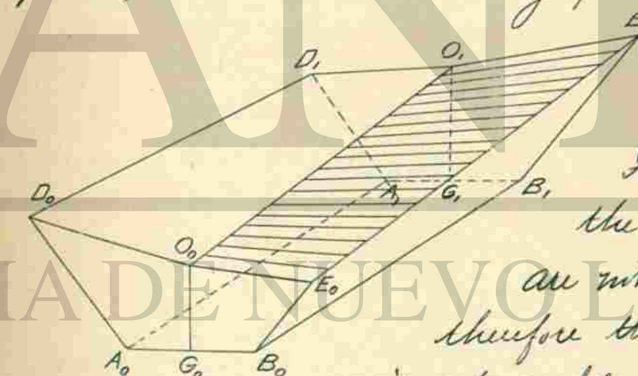
Prism $S = aB = \frac{a}{6} \cdot 6B = \frac{a}{6}(B + 4B + B) = \frac{a}{6}(B + 4m + b)$

Wedge $S = \frac{aB}{2} = \frac{a}{6} \cdot 3B = \frac{a}{6}(B + \frac{4B}{2} + 0) = \frac{a}{6}(B + 4m + b)$

Pyramid $S = \frac{aB}{3} = \frac{a}{6}\, 2B = \frac{a}{6}(B + \frac{4B}{4} + 0) = \frac{a}{6}(B + 4m + b)$

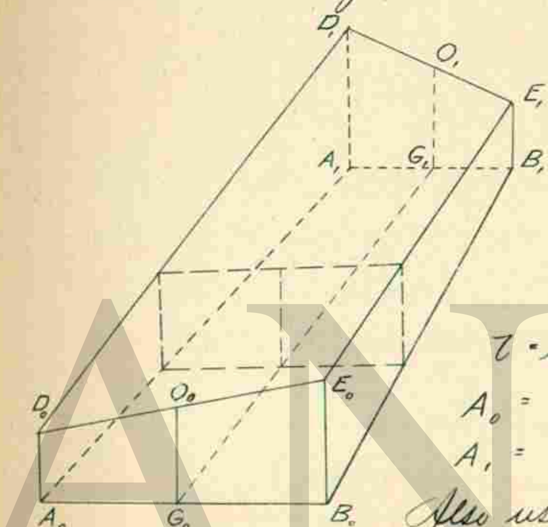
Since a prismoid is composed of prisms, wedges and pyramids, the same expression may apply to the prismoid, and this may be put in the general form $S = (A_0 + 4M + A_1)\frac{l}{6}$ (147.)
using the notation of the preceding page.

A regular section of Earthwork, having for its surface, a plane face, is a prismoid. Most sections of Earthwork have not their surface plane, and are not strictly prismoids.



In this figure the lines E_0O_0 and E_1O_1 are not parallel and therefore the surface $O_0O_1E_1E_0$ is not a plane. The most common assumption as to this surface, is that the lines O_0O_1 and E_0E_1 are right lines, and that the surface $O_0O_1E_1E_0$ is a warped surface, generated by a right line moving as a generatrix always parallel to the plane $O_0G_0B_0E_0$ and upon the lines O_0O_1 and E_0E_1 as directrices as indicated in the figure. The surface thus generated is a warped surface called a "hyperbolic paraboloid". It can be shown that the "prismoidal formula" applies also to this solid which is not however properly a prismoid.

In the following figure which has perpendicular sides $D_0A_0A_1D_1$ - $E_0B_0B_1E_1$ and the lines D_0E_0 and D_1E_1 right lines,



Let b_0 = base = A_0B_0

b_1 = " = A_1B_1

c_0 = center ht = O_0G_0 $= \dfrac{D_0A_0+E_0B_0}{2}$

c = center ht = O_1G_1 $= \dfrac{D_1A_1+E_1B_1}{2}$

l = length (altitude) of section = G_0G_1

A_0 = area of $D_0A_0B_0E_0$

A_1 = " " $D_1A_1B_1E_1$

Also use notation b_x - c_x - A_x for a section distant x from G_1

Then $A_0 = b_0c_0$ $\qquad A_1 = b_1c_1$

$$b_x = b_1 + (b_0-b_1)\frac{x}{l} \qquad c_x = c_1 - (c_1-c_0)\frac{x}{l}$$
$$= c + (c_0-c_1)\frac{x}{l}$$

$$A_x = b_xc_x = \left[b_1+(b_0-b_1)\frac{x}{l}\right]\left[c_1+(c_0-c_1)\frac{x}{l}\right]$$

Solidity $S = \int_0^l \left[b_1+(b_0-b_1)\frac{x}{l}\right]\left[c_1+(c_0-c_1)\frac{x}{l}\right]dx$

$$= b_1c_1l + \left[b_1(c_0-c_1)+c_1(b_0-b_1)\right]\frac{l^2}{2l} + \frac{(b_0-b_1)(c_0-c_1)l^3}{3l^2}$$

$$= \frac{l}{6}\left\{\begin{array}{l} 6b_1c_1 + 3b_1c_0 + 3b_0c_1 + 2b_0c_0 \\ - 3b_1c_1 - 2b_1c_0 - 2b_0c_1 \\ - 3b_1c_1 \\ + 2b_1c_1 \end{array}\right\}$$

$$= \frac{l}{6}(2b_1c_1 + 2b_0c_0 + b_1c_0 + b_0c_1 \qquad (148)$$

Apply the "Prismoidal Formula" to the same section. The base and center height of the middle section are:

$$b_m = \frac{b_0 + b_1}{2} \qquad c_m = \frac{c_0 + c_1}{2}$$

$$A_0 = b_0 c_0 \qquad A_1 = b_1 c_1$$

$$M = \frac{b_0 + b_1}{2} \cdot \frac{c_0 + c_1}{2} = \text{area of middle section}$$

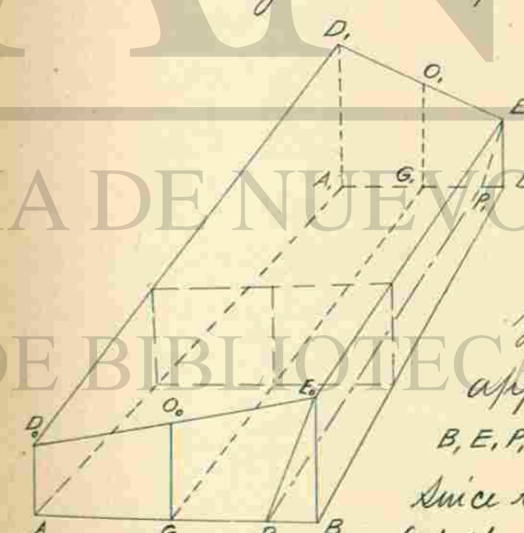
$$S' = (A_0 + 4M + A_1)\frac{l}{6}$$

$$= (b_0 c_0 + b_1 c_0 + b_0 c_1 + b_1 c_0 + b_1 c_1 + b_1 c_1)$$

$$= \frac{l}{6}(2 b_1 c_1 + 2 b_0 c_0 + b_1 c_0 + b_0 c_1) \qquad (149.)$$

This is the same as formula (148.) found above to be correct for the warped surface. Therefore the "Prismoidal Formula" applies to the section shown on page 146.

The sections of Earthwork commonly used in railroad work are bounded not by perpendicular sides, but by inclined planes.



In the figure, suppose a plane to be passed through the line $E_0 E_1$ cutting $A_0 B_0$ at P_0 and $A_1 B_1$ at P_1. The prismoidal formula applies to the solid $E_0 P_0 B_0 B_1 E_1 P_1$ cut out by this plane since the solid is a true prismoid. If the prismoidal formula applies to the entire solid and also to the part cut out, it must apply to the remaining solid $D_0 A_0 P_0 E_0 E_1 P_1 A_1 D_1$

and this represents in form one side of a regular three level section of Earthwork, in which $D_0 A_0$ is the center height, and $E_0 P_0$ the slope.
If the prismoidal formula applies to the section upon one side of the center, it applies also to the other side, and so to the entire section.
The "Prismoidal formula" is of wide application. Since it applies to prisms, wedges, pyramids, and to solids bounded by warped surfaces generated as described, it follows that it applies to any solid bounded by two parallel plane faces and defined by the surfaces generated by a right line moving upon the perimeters of these faces as directrices. It may also be stated here without demonstration, that it also applies to the frusta of all solids generated by the revolution of a conic section, as well as to the complete solids, for instance the sphere.
The prismoidal formula is generally accepted as correct for the computation of Earthwork and similar solids. The failure to use it results generally from the additional labor necessary for its use.
For three level sections of Earthwork, a result correct by the prismoidal formula may be secured, and the work simplified, by calculating the quantities first by the incorrect method of

"End areas", and then applying a correction which we may call "The Prismoidal Correction"

Let S_E = Solidity by "End Areas

S_p = " " Prismoidal formula

Then $C = S_E - S_p$ = Prismoidal Correction

In the figure p. 147

$$S_p = \text{by formula (148.)} = \frac{l}{6}(2b_1c_1 + 2b_0c_0 + b_1c_0 + b_0c_1)$$

$$S_E = \frac{l}{2}(b_1c_1 + b_0c_0) = \frac{l}{6}(3b_1c_1 + 3b_0c_0)$$

$$C = S_E - S_p = \frac{l}{6}(b_1c_1 + b_0c - b_1c_0 - b_0c_1)$$

$$= \frac{l}{6}(b_1 - b_0)(c_1 - c_0)$$

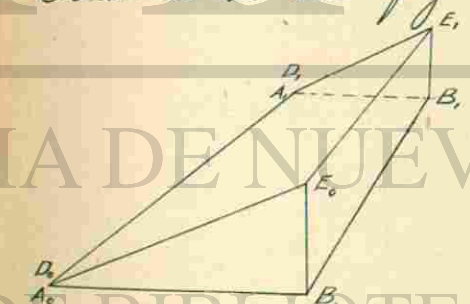
Let $D_0A_0 = h_0'$ $\qquad D_1A_1 = h_1'$

$E_0B_0 = h_0$ $\qquad E_1B = h_1$

Then $$C = \frac{l}{6}(b_1 - b_0)\left(\frac{h_1 + h_1'}{2} - \frac{h_0 + h_0'}{2}\right)$$

$$= \frac{l}{12}(b_1 - b_0)(h_1 + h_1' - h_0 - h_0')$$

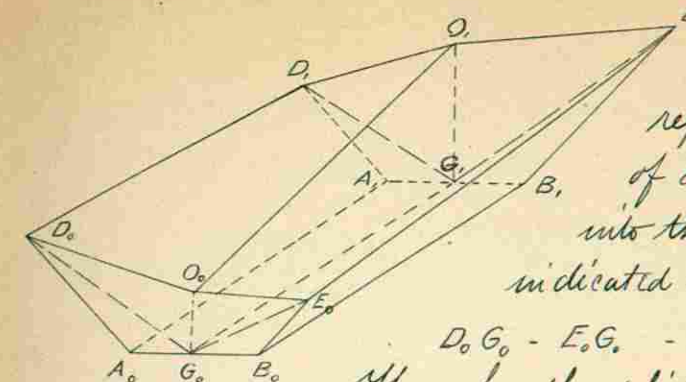
When the solid assumes a triangular cross-section as in the figure.



$h_0' = 0 \qquad h_1' = 0$

$$C = \frac{l}{12}(b_1 - b_0)(h_1 - h_0) \quad (150)$$

If any solid be divided into a number of solids each of triangular cross section, the above correction may be applied to each such triangular solid, and the sum of the corrections will be the correction for the entire solid.



Let this figure represent a section of Earthwork divided into three parts as indicated by the lines

$D_0G_0 - E_0G_0 - D_1G_1 - E_1G_1$

Then for the solid $O_0D_0G_0E_0E_1G_1D_1O_1$

$$C = \frac{l}{12}\left[(c_1 - c_0)(d_{l_1} - d_{l_0}) + (c_1 - c_0)(d_{r_1} - d_{r_0})\right]$$

$$= \frac{l}{12}(c_1 - c_0)(d_{l_1} + d_{r_1} - d_{l_0} - d_{r_0})$$

Let $D_1 = d_{l_1} + d_{r_1}$ and $D_0 = d_{l_0} + d_{r_0}$

$$C = \frac{l}{12}(c_1 - c_0)(D_1 - D_0)$$

For the solid $G_0B_0E_0E_1B_1G_1$

$$C = \frac{l}{12}(b_1 - b_0)(h_{r_1} - h_{r_0}) = \frac{l}{12}(0)(h_{r_1} - h_{r_0}) = 0$$

Similarly for the solid $A_0G_0D_0D_1G_1A_1$ $C = 0$

Hence for the entire solid $A_0B_0E_0O_0D_0D_1O_1E_1B_1A_1$

$$C = \frac{l}{12}(c_1 - c_0)(D_1 - D_0) \qquad (151.)$$

When $l = 100$

$$C = \frac{100}{12 \times 27}(c_1 - c_0)(D_1 - D_0)$$

$$= \frac{1}{3.24}(c_1 - c_0)(D_1 - D_0) \text{ in cu. yds} \qquad (152.)$$

Since $C = S_E - S_P$

$$S_P = S_E - C \qquad (153.)$$

When $(c_1 - c_0)(D_1 - D_0)$ is <u>positive</u>, the correction <u>C</u> is to be <u>subtracted</u> from S_E.

When $(c_1 - c_0)(D_1 - D_0)$ is <u>negative</u>, the arithmetical value of <u>C</u> is to be <u>added</u>.

The latter case occurs infrequently in practice

In general, for sections of Earthwork the prismoidal correction as given above, applies only when the width of base is the same at both ends of the section. There are certain special cases however which often occur, and which allow of the convenient use of this formula for prismoidal correction. Referring to the figure on page 136, and the corresponding notes on page 133, the correction can be correctly applied in the case of the Excavation from Sta 2+64 to 2+76 as follows:- Compute S_E and then apply C, using at Sta

2+64 $D_0 = 23.2$ and at Station

2+76 $D_1 = 11.9 = d_r$, or the distance out on one side only. This may readily be demonstrated to be true if the correction to the right of the center be taken using formula (151.) and the correction to the left using formula (150.) and the two corrections (right and left) be added.

Formula (150.) can also be used to find the correction for the triangular pyramids (for Excavation Sta 2+76 to 2+91 and Embankment 2+64 to 2+76) Each end of the pyramid being considered to have a triangular section. A much simpler way to find the correction for a pyramid is this

$C = S_E - S_P = \frac{1}{3} S_E$ as may readily be shown.

In the case of "Five Level Sections" as shown in the figure p. 142, the prismoidal correction may be computed for each of the triangular masses bounded by 1-AOB. 2.OBE. 3. OAD.

In the case of ACB the prismoidal correction will evidently be $= 0$ since $D_0 = 0 = D_1$ and therefore $D_0 - D_1 = 0$

The correction for the mass bounded on one end by OBE $= C = \frac{l}{12}(f_{r_0} - f_{r_1})(\bar{d}_{r_0} - \bar{d}_{r_1})$ and by ODA $= C = \frac{l}{12}(f_{l_0} - f_{l_1})(\bar{d}_{l_0} - \bar{d}_{l_1})$

OBE and ODA differ but little from regular sections of Earthwork in which $b = 0$.

In the case of "Irregular Sections", the prismoidal correction cannot with convenience be accurately employed. There are however several methods by which we may calculate a "prismoidal correction" which will be approximately correct.

For the purpose only of calculating the correction, either of the following methods may be employed.

1. Neglect all intermediate heights and figure correction from center and side heights.
2. Find level sections of equal area in each case and figure correction from the center heights and side distances of three level sections.

3. Having c and D of the irregular section, either
(a.) retain c and calculate D or
(b.) " D " " c for a
Regular Three Level Section of Equal area, and from these values calculate the correction.

4. Plat the cross-section on cross-section paper and equalize by a line or lines drawn in the most advantageous directions, and from data thus found, compute the correction.

Method No. 1 is most rapid and least accurate.
" " 2 is less accurate than 3 in most cases and probably no more rapid.
" " 3 is recommended as nearly equal to 4 in accuracy and far more rapid.
" " 4 would yield the most accurate results.

The value of these approximate methods cannot be properly appreciated until certain rapid methods of computation are explained, as will be seen later.

The results obtained by the methods shown above are approximate only, but in most cases the resulting error would be small, or a small fraction only of the entire correction which is itself generally small.

The method of calculating by averaging end areas and applying the prismoidal correction, will be found much more rapid than to calculate the middle area and apply the prismoidal formula directly. There is another advantage of importance in favor of the use of the prismoidal correction; in a majority of cases, the method of "end areas" is sufficiently accurate for all practical purposes, and from the use of the prismoidal correction the computer will soon learn to distinguish, by inspection merely, in what cases this correction need be applied.

3. Method of Middle Areas.

This consists in calculating the area of the middle section (not the mean of the end areas), and assuming the solidity to be that of a prism having a base equal to this middle area, and an altitude equal to the length of the section of Earthwork.

Let M = middle area

l = length of section.

Then $S = Ml$.

This method is not correct. It gives results generally less than the correct solidity. It is not sufficiently rapid to recommend it.

4. Method of Equivalent Level Sections.

This consists in finding level end sections of equal area with the actual end sections, from these calculating the level middle section, assuming the top surface connecting the level end sections to be a plane; and then calculating the solidity of this prismoid, by the prismoidal formula.

This method is not correct; it gives results less than the correct solidity. It is not sufficiently rapid to recommend it.

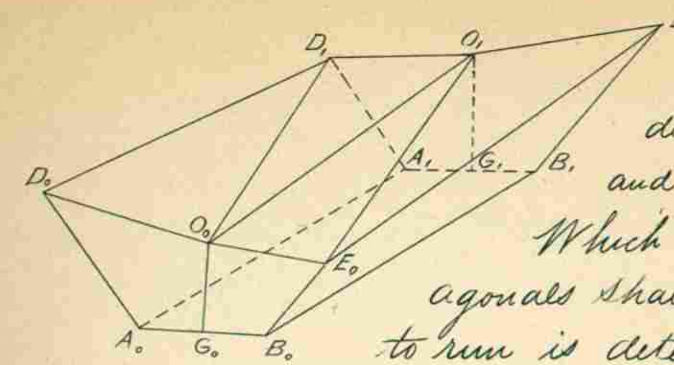
5. Method of Mean Proportionals.

This consists in assuming that the solid is the frustum of a pyramid, in which case all its sides would meet in one vertex.

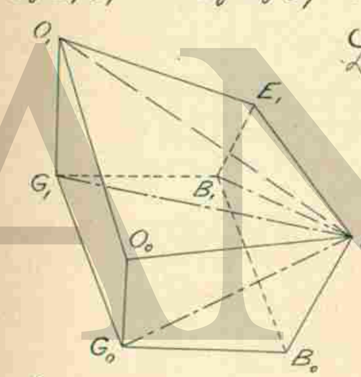
This method is not correct. It gives results always less than the correct solidity.

6. Henck's Method

In connection with the prismoidal formula, it was stated that the most common assumption was that the upper surface was a warped surface of a certain kind which was described. Henck's method assumes otherwise; that the upper surface is divided into plane surfaces by diagonals from the center height of one cross section to the side height of the next, as shown in the figure on the next page.



where the diagonals O_1E_0 and O_0D_1 are drawn. Which way the diagonals shall be assumed to run is determined on the ground by the shape of the surface in each case. The diagonals O_1E_0 and O_0D_1 divide the surface into four plane surfaces $D_0O_0D_1$, $O_0D_1O_1$ - $O_0E_0O_1$ - $E_0O_1E_1$.



Let this figure represent the right hand side of a section of Earthwork, with the diagonal assumed to run from E_0 to O_1.

Join E_0 with G_0 - B_1 and G_1

The entire solid may then be considered as composed of three pyramids having their vertices in a common point E_0

Using notation already familiar, the solidities of the three pyramids are as follows:-

S_1 = area $G_0B_0B_1G_1$ × Height at E_0 ÷ 3

$= \frac{1}{2} b l \times \frac{h_{r_0}}{3}$

S_2 = area $G_0O_0O_1G_1$ × distance out to E ÷ 3

$= \frac{c_0 + c_1}{2} l \times \frac{d_{r_0}}{3}$

S_3 = area $O_1G_1B_1E_1$ × length of section ÷ 3

$= \frac{1}{2}\left(\frac{b}{2} h_{r_1} + c_1 d_{r_1}\right) \times \frac{l}{3}$

Let S_r = solidity of this right half of section

$$S_r = S_1 + S_2 + S_3$$

$$= \frac{1}{6} b l h_{r_0} + \frac{1}{6} d_{r_0} l c_0 + \frac{1}{6} d_{r_0} l c_1 + \frac{1}{6} \frac{b h_{r_1} l}{2} + \frac{1}{6} c_1 d_{r_1} l$$

$$= \frac{l}{6}\left[b h_{r_0} + \frac{b h_{r_1}}{2} + d_{r_0} c_0 + c_1 d_{r_1} + c_1 d_{r_0}\right] \qquad (154.)$$

Let C = center height touched by diagonal.
H = side " " " .
D = distance out to side height touched by diag.

$$S_r = \frac{l}{6}\left[\frac{b}{2}(h_{r_0} + h_{r_1} + H_r) + c_0 d_{r_0} + c_1 d_{r_1} + C_r D_r\right]$$

Then $S_l = \frac{l}{6}\left[\frac{b}{2}(h_{l_0} + h_{l_1} + H_l) + c_0 d_{l_0} + c_1 d_{l_1} + C_l D_l\right]$

= solidity for left half of section.

$$S = S_r + S_l = \frac{l}{6}\left[\frac{b}{2}(h_{r_0} + h_{l_0} + h_{r_1} + h_{l_1} + H_r + H_l) + c_0(d_{r_0} + d_{l_0}) + c_1(d_{r_1} + d_{l_1}) + C_r D_r + C_l D_l\right] \qquad (155.)$$

An example will further show the application of this method.

Sta.	Surf. Elev.	Grade Elev.	Cross Sections		
1	123.0	121.00	$\frac{13.0}{+3.0}$	$+2.0$	$\frac{11.0}{+1.0}$
0	123.0	120.00	$\frac{15.0}{+5.0}$	$+3.0$	$\frac{11.0}{+1.0}$

The notes show the direction of the diagonal as taken on the ground.

In this case $b = 20$ $s = 1$ to 1.

Hence gives note book and calculations this form.

Sta	d_l	h_l	c	h_r	d_r	$d_l + d_r$	$(d_l+d_r)c$	D_r	D_l
1	13.0	3.0	2.0	1.0	11.0	24.0	48.0		
0	15.0	5.0	3.0	1.0	11.0	26.0	78.0	22.0	39.0

8.0 . . 8.0
4.0
$14.0 \times \frac{20}{2} =$ 140.0

48.0
78.0
22.0
39.0
140.0
6) 327.0
54.50. cu. ft.

The calculations could be conveniently made however from the notes as now generally taken, as is shown below:—

1 $\quad \frac{13.0}{+3.0} \quad +2.0 \quad \frac{11.0}{+1.0} \qquad 24.0 \times 2.0 = 48.0$

0 $\quad \frac{15.0}{+5.0} \quad +3.0 \quad \frac{11.0}{+1.0} \qquad 26.0 \times 3.0 = 78.0$

$\qquad\qquad\qquad\qquad\qquad\qquad 11.0 \times 2.0 = 22.0$

$\quad 8.0 \rightarrow \quad 2.0 \qquad 13.0 \times 3.0 = 39.0$

$\qquad\qquad 8.0$

$\qquad\qquad 4.0$

$\qquad\qquad 14.0 \times \frac{20}{2} \qquad = 140.0$

$\qquad\qquad\qquad\qquad 6 \,)\, 327.0$

$\qquad\qquad\qquad\qquad 5450.$ cu. ft.

The work of computation would not, in either of these cases, properly be done in the field note book, but rather in a calculation book, or other suitable place.

For a series of cross sections Henck systematizes the work, and reduces the labor noticeably from what is shown here.

Henck's method is strictly accurate, upon the assumption made as to the upper surface. In general railroad practice most engineers prefer to assume the upper surface a warped surface of the sort described.

Henck's Method is less rapid than that of averaging end areas and applying the prismoidal correction.

The method of averaging end areas and applying the prismoidal correction appears in point of accuracy and rapidity to meet the

Example.

Showing a comparison of various methods of calculating Earthwork.

Notes of Excavation Base 24 Slope $1\frac{1}{2}$ to 1.

Sta 1.	$\frac{13.5}{+1.0}$	$+1.0$	$\frac{16.5}{+3.0}$	$\frac{9\times30}{2} = 135$ grade triangle $= 96$ $A_1 = 39$
Sta 0.	$\frac{28.5}{+11.0}$	$+19.0$	$\frac{43.5}{+21.0}$	$\frac{27\times72}{2} = 972$ grade triangle $= 96$ $A_0 = 876$
The mid-section will be				
0+50	$\frac{21.0}{+6.0}$	$+10.0$	$\frac{30.0}{+12.0}$	$\frac{18\times51}{2} = 459$ 96 $A_{0+50} = 363$

Area of grade triangle $= \frac{24\times8}{2} = 96$

I. End Areas.

$S_E = 100 \times \frac{39+876}{2} = \frac{91500}{2} = 45750.$

Corr $= \frac{100}{12} \times 18 \times 42 = 6300.$

$S_P = 39450$

Error of $S_E = +6300 = 16$ per cent.

II. Prismoidal Formula.

$39 = A_1$

$363 \times 4 = 1452 = 4A_{0+50}$

$876 = A_0$

$6\,)\,236700$

$39450 = S_P$

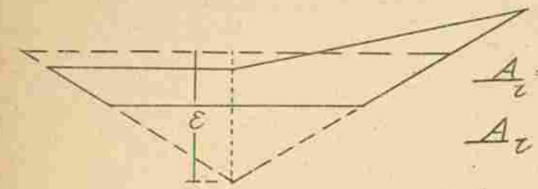
Middle Areas. III.

$S_m = 363 \times 100 = 36300$

S_m Error $= -3150 = 8$ per cent

S_E " $= +6300$

IV. Equivalent Level Sections.



A_l = Area of level Section

$A_l = 1\frac{1}{2} E^2$

$A_{0l} = 1\frac{1}{2} E_0^2 = 135$ (This includes grade triangle.)

$E^2 = 90$

$E = 9.5 \pm$

$A_{1l} = 1\frac{1}{2} E_1^2 = 972$

$E_1^2 = 648$

$E_1 = 25.5 \pm$

$E_m = \frac{25.5 + 9.5}{2} = 17.5 \pm$

$A_{ml} = 1\frac{1}{2} \times 17.5^2 = 457.8$

Grade Triangle $= 96.0$

361.8

$39. = A_1$

$4 \times 361.8 = 1447.2 = 4 A_m$

$876. = A_0$

$6 \overline{)2362.20}$

$39370 = S_l$ Error $= -80$

$= 0.2$ per cent.

V. Mean Proportionals.

$A_0 = 39$

$A_1 = 876$

$A_0 A_1 = 34164$

$\sqrt{A_0 A_1} = 184.8$

$39. = A_0$

$184.8 = \sqrt{A_0 A_1}$

$876. = A_1$

$3 \overline{)1099.80}$

$36660 = S_N = \frac{l}{3}(A_0 + \sqrt{A_0 A_1} + A_1)$

Error $-2790 = 7$ per cent.

VI. Henck's Method.

One system of diagonals as shown in notes

Sta.	d_l	h_l	c	h_r	d_r	$d_l + d_r$	$(d_l+d_r)c$	D'c	Dc
1	13.5	1.0	1.0	3.0	16.5	30.0	30.0		
0	28.5	11.0	19.0	21.0	43.5	72.0	1368.0	72.	
		12.0		24.0			72.0		
				12.0			816.0		
				11.0		6)	2286.		
				21.0			38100 = S_n		

$\frac{b}{2} \times$ 68.0
12.
816.

$S_p - S_n = 1350 = 3$ per cent

Opposite system of diagonals

Sta	d_l	h_l	c	h_r	d_r	$d_l + d_r$	$(d_l+d_r)c$	D'c	Dc
1	13.5	1.0	1.0	3.0	16.5	30.0	30.0		
0	28.5	11.0	19.0	21.0	43.5	72.0	1368.0	570.	
				24.0			570.0		
				12.0			480.0		
				1.0		6)	2448.0		
				3.0			40800 = S_n'		

$\frac{b}{2} \times$ 40.0
12
480.

38100 = S_n

$S_p - S_n' = -1350 = 3$ per ct. 2) 78900
39450
Mean value = S_p

requirements of modern railroad practice.

Some engineers whose opinions are entitled to careful consideration object to the use of the prismoidal formula or prismoidal correction in any form, some as an unnecessary refinement, and some on the ground that certain practical considerations render the results nearer the truth when the method of averaging end areas is used without applying the prismoidal correction. Probably the greater part of the best engineering practice favors the use of the prismoidal correction.

Correction for Curvature.

In the case of a curve, the ends of a section of Earthwork are not parallel, but are in each case normal to the curve. In calculating the solidity of a section of Earthwork we have heretofore assumed the ends parallel, and for curves this would be perpendicular to the chord of the curve between the two stations.

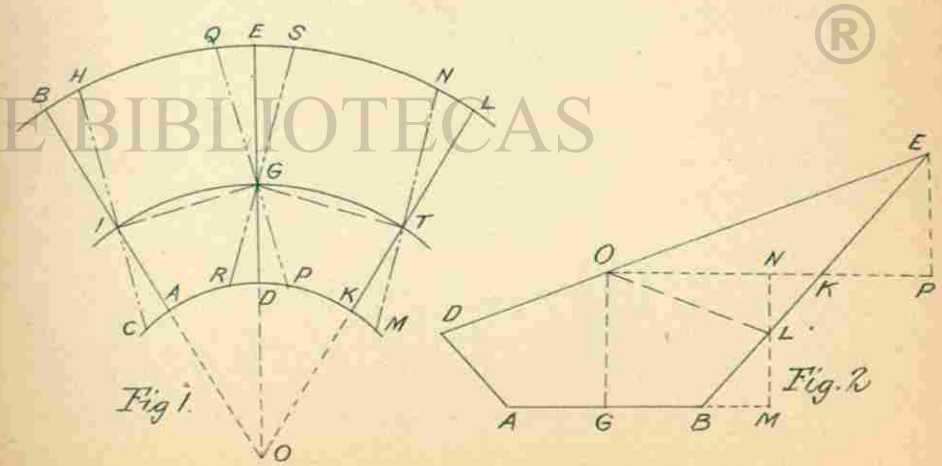


Fig 1. Fig. 2

Then as shown in Fig. 1 (where IG and GT are center line chords) the solidity (as above) of the sections IG and GT will be too great by the wedge shaped mass RGP, and too small by QGS. When the cross sections on each side of the center are equal, these masses balance each other. When the cross section on one side differs much in area from that on the other, the correction necessary may be considerable.

In Fig. 2. use $c\ h_l\ h_r\ d_l\ d_r\ b\ s$ as before
Let D = degree of curve.
Make BL = AD and join OL
Then ODAG balances OLBG and there remains an unbalanced area OLE
Draw OKP parallel to AB.

By the "Theorem of Pappus" (see Lanza Appl. Mech.)
"If a plane area lying wholly on the same side of a straight line in its own plane, revolves about that line, and thereby generates a solid of revolution, the volume of the solid thus generated is equal to the product of the revolving area, and of the path described by the center of gravity of the plane area during the revolution."

The correction for curvature, or the solidity developed by this triangle OLE (Fig. 2) revolving about OG as an axis, will be its area × this distance described by its center of gravity. The distance out (horizontal) to the center of gravity from the axis (center line) will be 2/3 of the mean of the distances out to E and to L or $= \frac{2}{3} \cdot \frac{d_l + d_r}{2}$ and the distance described will be $\frac{2}{3} \cdot \frac{d_l + d_r}{2} \cdot QGS$

The area $OLE = OK \times \frac{NL + PE}{2}$

$$= \left(\frac{b}{2} + sc\right) \frac{h_r - h_l}{2}$$

Therefore the correction for curvature

$$C = \left(\frac{b}{2} + sc\right) \cdot \frac{h_r - h_l}{2} \cdot \frac{d_r + d_l}{3} \cdot \text{angle } QGS$$

When 1G and GT are each an even station or 100 ft in length $QGS = D$ and

$$C = \left(\frac{b}{2} + sc\right) \cdot \frac{h_r - h_l}{2} \cdot \frac{d_r + d_l}{3} \cdot \text{angle } D$$

arc $1° = .01745$

$$C = \left(\frac{b}{2} + sc\right) \frac{h_r - h_l}{2} \cdot \frac{d_r + d_l}{3} \; .01745 D$$

$$= \left(\frac{b}{2} - sc\right)(h_r - h_l)(d_r + d_l) \times .00291 D \text{ (cu. ft) (156.)}$$

$$= \left(\frac{b}{2} - sc\right)(h_r - h_l)(d_r + d_l) \times .00011 D \text{ (cu. yds.) (157.)}$$

When 1G or GT or both are less than 100 ft.

Let $1G = t_0$

$GT = t_1$

Then $QGE = \frac{t_0}{100} \cdot \frac{D}{2}$ and $SGE = \frac{t_1}{100} \cdot \frac{D}{2}$

$$QGS = \frac{t_0 + t_1}{200} D$$

$$C = \left(\frac{b}{2} + sc\right)(h_r - h_l)(d_r + d_l) \frac{t_0 + t_1}{200} \times .00011 D \text{ (cu. yds) (158.)}$$

The correction C is to be added when the greater area is on the outside of the curve, and subtracted when the greater area is on the inside of the curve. When the center height is 0 as in Fig 3 we may consider this a regular section in which $c = 0$ $h_l = 0$ and $\bar{d}_l = \frac{b}{2}$; then

$$C = \left(\frac{b}{2} \cdot h_r \left(\bar{d}_r + \frac{b}{2}\right) \frac{l_0 + l_1}{200} \times .00011\, D \text{ (cu. yds)} \qquad (159.)$$

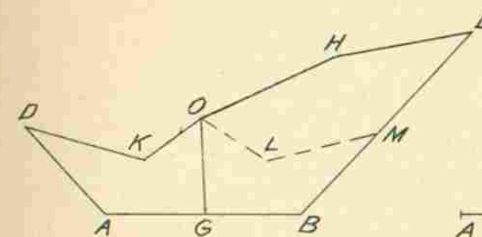


Fig. 4

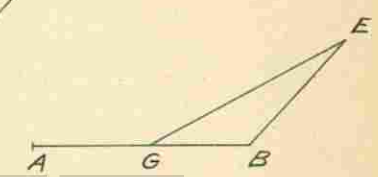
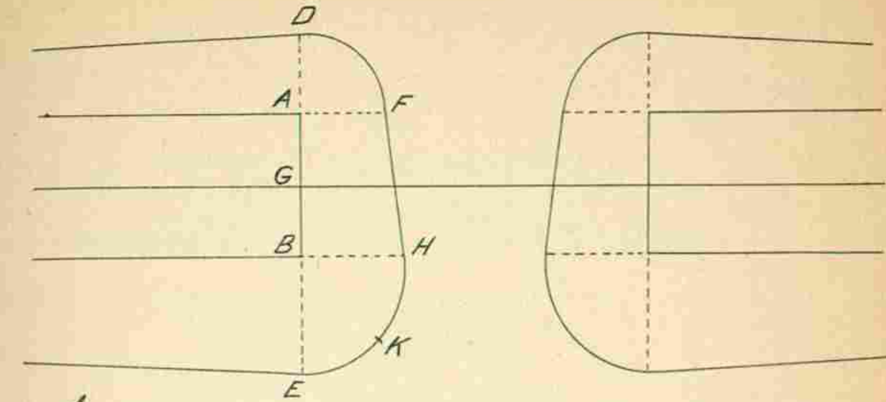


Fig. 3

In the case of irregular sections as shown in Fig. 4, the area and distance to center of gravity of OHEML may be found by any method available and the corrections figured accordingly. The correction for curvature is, in present railroad practice, more frequently neglected than used. Nevertheless, its amount is sufficient in many cases to fully warrant its use.

Opening in Embankment.



When an opening is left in an embankment there remains outside the regular sections, the mass DEKHF.

This must be calculated in 3 pieces,

ADF – BEKH – ABHF

Let b = base = AB

$d_l \; d_r$ = distances out right and left

p_r = BH } taken parallel to center line
p_l = AF

f_r = heights at B }
f_l = A

S_1 = solidity ADF; S_2 = BEKH; S_3 = ABHF

Then (approximately) following the "Theorem of Pappus"

S_1 = mean of triangles AD and AF × distance described by center of gravity.

$$\frac{\text{Area AD} + \text{area AF}}{2} = \frac{\frac{f_l}{2}\left(d_l - \frac{b}{2}\right) + \frac{f_l}{2}\, p_l}{2}$$

$$= \frac{f_l}{2} \cdot \frac{d_l + p_l - \frac{b}{2}}{2} = \text{mean area}$$

Distance described by center of gravity is found thus
length $AD = d_l - \frac{b}{2}$ and $AF = p_l$

$$\frac{\text{length } AD + \text{length } AF}{2} = \frac{d_l + p_l - \frac{b}{2}}{2} = \text{mean length.}$$

$$\text{distance out to center of gravity} = \frac{1}{3} \cdot \frac{d_l + p_l - \frac{b}{2}}{2}$$

$$\text{distance described by center of gravity} = \frac{d_l + p_l - \frac{b}{2}}{6} \cdot \frac{\pi}{2}$$

$$S_1 = \frac{f_l}{2} \cdot \frac{d_l + p_l - \frac{b}{2}}{2} \cdot \frac{d_l + p_l - \frac{b}{2}}{6} \cdot \frac{\pi}{2}$$

$$= f_l \left(d_l + p_l - \frac{b}{2}\right)^2 \frac{\pi}{48}$$

$$= \frac{f_l}{15}\left(d_l + p_l - \frac{b}{2}\right)^2 \text{ nearly} \qquad (160.)$$

$$S_2 = \frac{f_r}{15}\left(d_r + p_r - \frac{b}{2}\right)^2 \text{ nearly.} \qquad (161.)$$

The work of deriving these formulas is approximate throughout, but the total quantities involved are in general not large, and the error resulting would be unimportant.
There seems to be no method of accurately computing this solidity, which is adapted to general railroad practice.

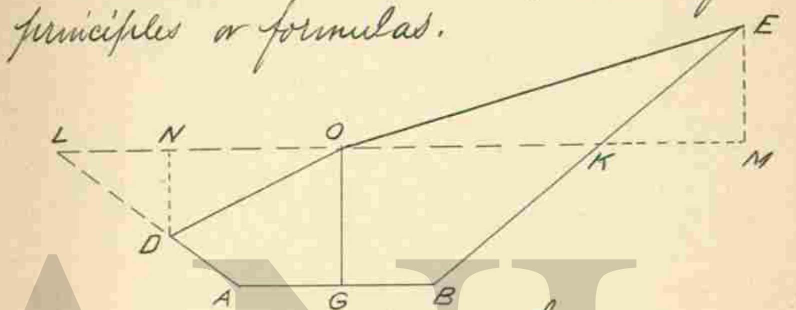
$$S_3 = \frac{\text{area } AF + \text{area } BH}{2} \times AB$$

$$= \frac{f_l p_l + f_r p_r}{4} \cdot b \qquad (162.)$$

Earthwork Tables.

The calculation of quantities can be much facilitated by the use of suitably arranged "Earthwork Tables".

For regular "Three Level Sections" very convenient tables can be calculated upon the following principles or formulas.



Use notation as before for

$c - h_l - h_r - d_l - d_r - s - l - A - S.$

Then $A = ABKL + OKE - ODL$

$$= c(b+sc) + \frac{OK \times EM}{2} - \frac{OL \times ND}{2}$$

$$= c(b+sc) + \frac{OK}{2}(EM - ND)$$

$$= c(b+sc) + \frac{\frac{b}{2}+sc}{2}(h_l - c - c + h_r)$$

$$A = c(b+sc) + \frac{\frac{b}{2}+sc}{2}(h_l + h_r - 2c) \quad (163.)$$

For a prism of base A and $l = 50$

$$S = 50\,A \text{ (cu. ft.)} = \frac{50}{27} A \text{ (cu. yds)}$$

$$S = \frac{50}{27} c(b+sc) + \frac{25}{27}\left(\frac{b}{2}+sc\right)(h_l + h_r - 2c) \quad (164.) \text{ cu. yds.}$$

For cross sections of a given base and slope, that is given b and s constant, we may calculate for successive values of c - and tabulate values of I and C as follows:—

	I	C
c	$\frac{50}{27} c(b+sc)$	$\frac{25}{27}(\frac{b}{2}+sc)$

I represents the solidity for the level Section. C is for use as a correction. The formula then adapts itself to this table for any desired values of $c - h_l - h_r$

$$S = I + C(h_l + h_r - 2c) \qquad (164.)$$

Having found for successive stations S_0 and S_1 (each for a prism $l = 50$), then for the full section by "End areas",

$S_{100} = S_0 + S_1$ for

$$S_{100} = \frac{A_0 + A_1}{2} \cdot \frac{100}{27} = \frac{50 A_0}{27} + \frac{50 A_1}{27}$$

$$S_{100} = \qquad S_0 + S_1 \qquad (165)$$

When l is less than 100

$$S_l = (S_0 + S_1)\frac{l}{100} \qquad (166.)$$

For level Sections $h_l = h_r = c$

$h_l + h_r - 2c = 0$ and the formula

$S = I + C(h_l + h_r - 2c)$ becomes

$S = I$ for level Sections $\qquad (167.)$

and the quantities for any given values of $\underline{c}$ can be directly taken from column I without any correction from column C.

In preliminary Estimates or wherever Center heights only are used, these tables are rapidly used.

Tables accompany these notes, calculated for
1 – $b = 20$ $s = 1\frac{1}{2}$ to 1
2 – $b = 14$ $s = 1\frac{1}{2}$ to 1
An example will illustrate their use.
$b = 14$ – $s = 1\frac{1}{2}$ to 1

Notes	Sta 1	$\frac{13.0}{-4.0}$	-3.7	$\frac{12.4}{3.6}$
	0	$\frac{10.6}{-2.4}$	-2.5	$\frac{10.3}{-2.2}$

Calculations

$$3.7 \quad I = 134.0 \qquad C = 11.6 \qquad h_l + h_r = 7.6$$
$$+\underline{\ \ 2.3} \qquad \underline{0.2} \qquad 2c = \underline{7.4}$$
$$S_1 = 136.3 \qquad 2.32 \qquad +\ 0.2$$

$$2.5 \quad I = 82.2 \qquad C = 10.0 \qquad h_l + h_r = 4.6$$
$$-\underline{\ \ 4.0} \qquad \underline{0.4} \qquad 2c = \underline{5.0}$$
$$S_0 = 78.2 \qquad 4.00 \qquad -\ 0.4$$

$$S_{100} = S_1 + S_0 = 214.5$$

Tables of the sort described have been published as follows:—
"The Civil Engineer's Excavation and Embankment Tables" by Clarence Rueller and Charles C. Chandler, published by the "J. M. W. Jones Stationery and Printing Co" Chicago.
Tables are calculated for $b = 12 - 14 - 16 - 18 - 20$
$s = \frac{1}{4} - \frac{1}{2} - 1 - 1\frac{1}{2}$

Accompanying these is also a "Table of Prismoidal Correction" calculated by the formula
$$C = \frac{1}{3.24}(c_0 - c_1)(D_0 - D_1)$$

In the Example on the preceding page

$C_0 - C_1 = 2.5 - 3.7 = -1.2$

$D_0 - D_1 = 20.9 - 25.4 = -4.5$

From Table find opp. 4.5 for 1 — 1.39 $\quad 10)\underline{2.78}$ $\;.28$

$$\begin{array}{rr} & 2 \\ \frac{2}{10} & \underline{0.28} \\ C = & 1.67 \end{array}$$

$$\begin{array}{rl} S_{100} = S_E = & 214.5 \\ C = & \underline{\quad 1.7} \\ S_p = & 212.8 \end{array}$$

When the section is less than 100 ft in length the $\left\{\begin{matrix}\text{prismoidal} \\ \underline{\text{correction}}\end{matrix}\right\}$ should be made before multiplying by $\frac{l}{100}$; that is

$$S_l = (S_0 + S_1 - C)\frac{l}{100} \qquad (168.)$$

For general calculation, adapted both to regular "Three Level Sections" and to "Irregular Sections", tables can be calculated upon the following principles and formulas.

These tables are in effect tables of "Triangular Prisms", in which, having given (in feet) the base $\underline{B}$ and altitude $\underline{\underline{a}}$ of any triangle, the tables give the solidity (in cu. yds.) for a prism of length $l = 50$; that is

$$S = \frac{aB}{2} \cdot \frac{50}{27} = \frac{50}{54}aB \qquad (169.)$$

Whenever the calculation can be brought into the form $S = \frac{50}{54}aB$, the result can be taken directly from the table.

Convenient Tables of this kind are "Tables for the Computation of Railway and other Earthwork" by C. L. Crandall C.E., and published by

In these tables, the formula $S = \frac{50}{54}\, aB$ takes form thus $S = \frac{50}{54} \times$ width $\times$ height and the tables are arranged as below.

	Heights
Widths	$\frac{50}{54}$ width × height.

The application to "Three Level Sections" is as follows:—

We have the formula p. 141.

$$A = \left(c + \frac{b}{2s}\right)\frac{D}{2} - \frac{b^2}{4s}$$

and for a prism 50 ft in length ($l = 50$)

$$S = A\,\frac{50}{27} = \frac{50}{54}\left(c + \frac{b}{2s}\right).D - \frac{50}{54}.\frac{b}{2s}.b$$

or S is the sum of two quantities, Each of which is in proper form for the use of the tables.

For cross-sections of a given base and slope (b and s constants) $\frac{b}{2s}$ is a constant, and also $\frac{50}{54}.\frac{b}{2s}.b$ is constant.

We may then calculate once for all, $\frac{b}{2s}$ and call this B (a constant); also $\frac{50}{54}.\frac{b}{2s}.b$ and call this a constant E.

Then $S' = \frac{50}{54}(c+B)D - E'$. (170.)

In using the tables $c+B$ = height
D = width

As in the previous tables having found S_0 and S_1,

$S_{100} = S_0 + S_1$ and

$S_x = (S_0 + S_1)\frac{x}{100}$

Example – using Crandall's Tables.

$b = 14$ $s = 1\frac{1}{2}$ to 1

Notes Sta 1 $\frac{13.0}{-4.0}$ -3.7 $\frac{12.4}{-3.6}$

0 $\frac{10.6}{-2.4}$ -2.5 $\frac{10.3}{-2.2}$

$\frac{b}{2s} = 4.7$
B

$\frac{50}{54} \times 4.7 \times 14.$
Tables - width 14 ht. 4 = 52.
correction ht. $\frac{7}{10}$ = 9.
E' = 61.

Sta. 1
$c = 3.7$
$B = 4.7$
ht. = 8.4
$D = 13.0 + 12.4 = 25.4$

Tables
Ht. 8 width 25 = 185.
correction ht. 4/10 = 9.
" width 4/10 = 3.
197.
E' = 61.
S_1' = 136.

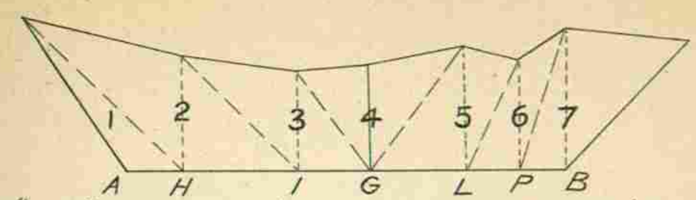
Sta. 0
$c = 2.5$
$B = 4.7$
ht = 7.2
$D = 10.6 + 10.3 = 20.9$

Tables
Ht. 7 width 20 = 130.
corr. ht. 2/10 = 4.
" width 9/10 = 6
140.
E' = 61.
S_1' = 79.

$S_{100} = S_0 + S_1 = 215.$

Crandall's Tables include tables of Prismoidal Correction

Irregular Sections.



An "Irregular Section" can be divided into triangular parts as in the figure. Taking generally two triangular parts together for purposes of calculation, we have

$$A_1 = \frac{h_2 \times (AG - \bar{d}_H)}{2} \qquad S_1 = \frac{50}{54} h_2 (AG - \bar{d}_H)$$

$$A_2 = \frac{h_H \times (\bar{d}_2 - \bar{d}_1)}{2} \qquad S_2 = \frac{50}{54} h_H (\bar{d}_2 - \bar{d}_1)$$

$$A_3 = \frac{h_1 \times (\bar{d}_H - 0)}{2} \qquad S_3 = \frac{50}{54} h_1 \bar{d}_H$$

$$A_4 = \frac{c \times (\bar{d}_1 + \bar{d}_L)}{2} \qquad S_4 = \frac{50}{54} c (\bar{d}_1 + \bar{d}_L)$$

$$A_5 = \frac{h_L \times (\bar{d}_P - 0)}{2} \qquad S_5 = \frac{50}{54} h_L \bar{d}_P$$

$$A_6 = \frac{h_P \times (\bar{d}_B - \bar{d}_L)}{2} \qquad S_6 = \frac{50}{54} h_P (\bar{d}_B - \bar{d}_L)$$

$$A_7 = \frac{h_B \times (\bar{d}_r - \bar{d}_P)}{2} \qquad S_7 = \frac{50}{54} h_B (\bar{d}_r - \bar{d}_P)$$

$$S' = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 \qquad (171.)$$

$$S_{100} = S_0 + S'_1$$

$$S_2 = (S_0 + S_1)\frac{2}{100}$$

The calculation of Irregular Sections in rough country becomes very laborious unless the best methods are used and this process should be thoroughly understood.

Earthwork Diagrams.

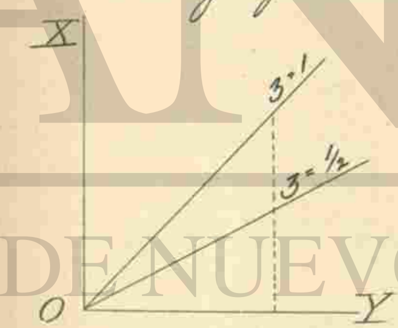
Computations of Earthwork may also be made by means of Diagrams from which results may be read by inspection merely.

The principle of their construction is explained as follows:

Given an Equation containing three variable quantities as $x = zy$

If we assume some value of z (making z a constant) the equation then becomes the equation of a right line.

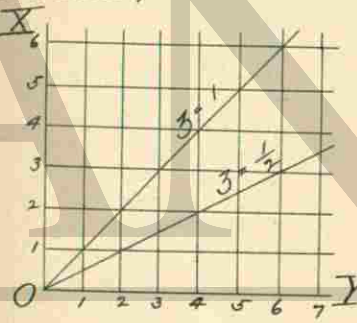
If this line be platted, using rectangular co-ordinates, (as the line $z = 1$ in the figure) then having given any value of y, the corresponding value of x may be taken off by scale. If a new value of z be assumed the equation is obtained of a new line which



may also be platted (as $z = \frac{1}{2}$ in the figure) and from which also, having given any value of y, the corresponding value of x may be determined by scale. Assuming a series of values of z and platting, we have a series of lines each representing a different value of z, and from any one of having given a value of y, we may by scale determine the value of x. Thus, having <u>given</u>, values of z and y; and <u>required</u>, x, we may find,

1. The line corresponding to the given value of z, and
2. Upon this line we may find the value of x corresponding to the given value of y.

Next, if instead of platting upon <u>lines</u> as co-ordinate axes, we plat upon cross-section paper, the cross-section lines form a scale, so that the values of x and y need not be <u>scaled</u>, but may be <u>read</u> by simple inspection as in the figure



If the equation be in the form $x = azy$ the same procedure is equally possible and the line representing any value of z will still be a right line.

If the equation be in the form $x = a(z+b)(y+c)+d$ in which a-b-c-d are constants, the same procedure is still possible, and the line representing a given value of z is a right line.

The use of Diagrams of this sort is therefore possible for the solution of Equations in the form of $x = a(z+b)(y+c)+d$ or in simpler modifications of this form.

Referring again to the figure on the preceding page, we may consider the horizontal lines to represent successive values of x and refer to them as the lines $x=0$; $x=1$; $x=2$ etc and similarly we may refer to vertical lines as the lines $y=0$; $y=1$; $y=2$ etc just as we refer to the inclined lines $z=\frac{1}{2}$; $z=1$ etc. Having given any two of the quantities x, y, z, the third may be found by inspection from the diagram by a process similar to that described.

Diagram for Prismoidal Correction.

Formula $C = \frac{1}{3.24}(c_0 - c_1)(D_0 - D_1)$

This has the form $x = a \times z \times y$

Construction of Diagram.

Assume (as we did for z) a series of values of $c_0 - c_1 = 0 - 1 - 2 - 3 - 4 - 5$ etc.

When $c_0 - c_1 = 0$ then $C = 0$

or, the line $c_0 - c_1$ coincides with the line $C = 0$.

When $c_0 - c_1 = 1$ the equation of the line $c_0 - c_1$ is $C = \frac{1}{3.24}(D_0 - D_1)$

To plat this right line, we must find two or more points on the line.

(Cross section paper is generally warped somewhat, and it is best to take a number of points not more than 3 or 4 inches apart, to get the lines sufficiently exact.)

For convenience, take values of $D_0 - D_1$ as follows:— (this is when $(c_0 - c_1) = 1$

$D_0 - D_1 = 0 - 3.24 - 6.48 - 9.72 - 12.96 - 16.20$ etc

and $C = 0 - 1 - 2 - 3 - 4 - 5$ etc

When $c_0 - c_1 = 2$

$D_0 - D_1 = 0 - 3.24 - 6.48 - 9.72 - 12.96 - 16.20$ etc

$C = 0 - 2 - 4 - 6 - 8 - 10$ etc.

In like manner a table may be constructed.

	0	3.24	6.48	9.72	12.96	16.20	19.44	22.68	26.92	$D_0 - D_1$
0	0	0	0	0	0	0	0	0	0	
1	0	1	2	3	4	5	6	7	8	
2	0	2	4	6	8	10	12	14	16	
3	0	3	6	9	12	15	18	21	24	
4	0	4	8	12	16	20	24	28	32	
5	0	5	10	15	20	25	30	35	40	
6	0	6	12	18	24	30	36	42	48	
7	0	7	14	21	28	35	42	49	56	
8	0	8	16	24	32	40	48	56	64	
9	0	9	18	27	36	45	54	63	72	
10	0	10	20	30	40	50	60	70	80	
$c_0 - c_1$										

It will be noticed that when $D_0 - D_1 = 0$, $C = 0$.

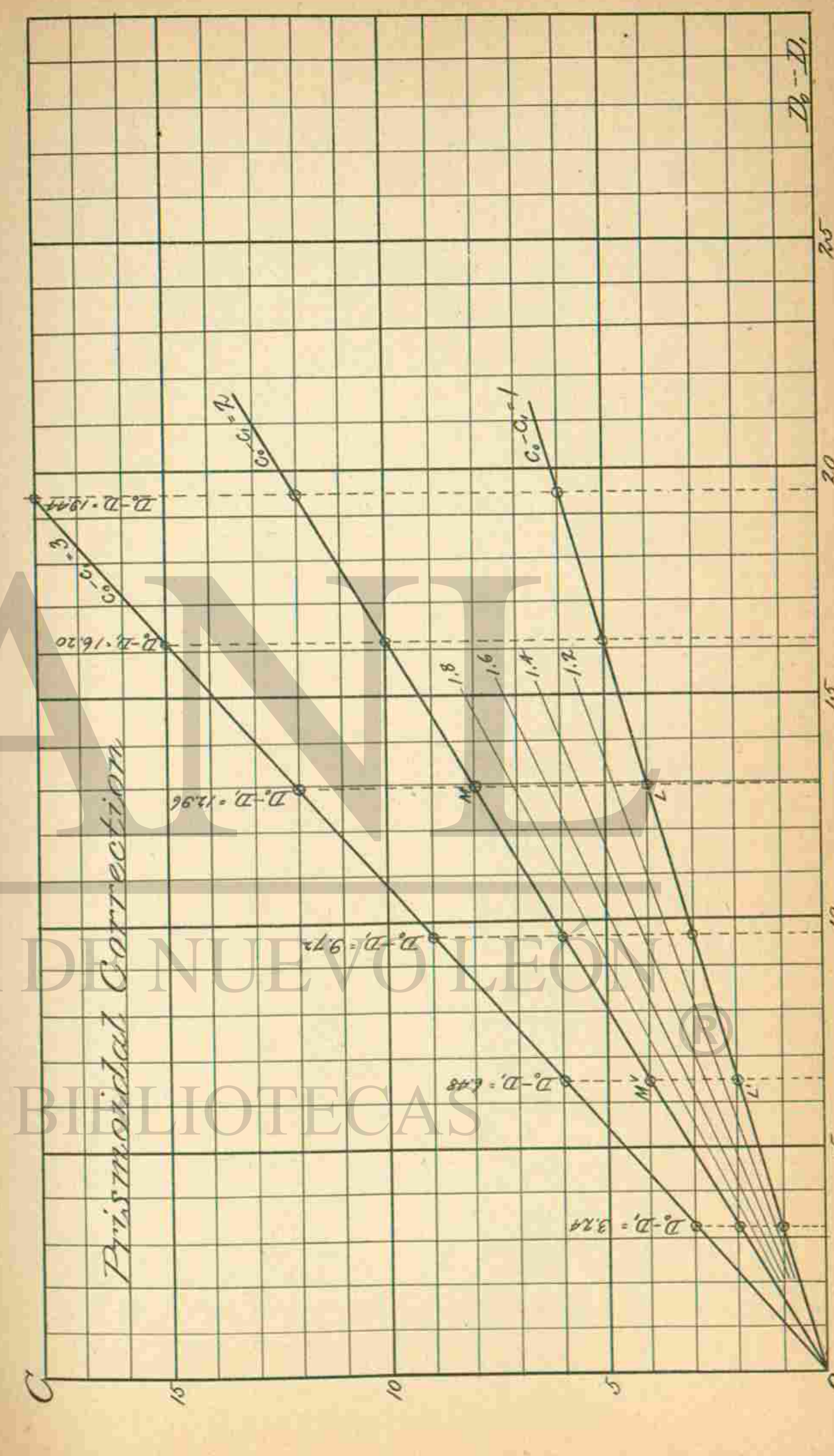
Therefore for all values of $c_0 - c_1$ the lines pass through the origin.

We may proceed to plat the lines $c_0 - c_1 = 1$; $c_0 - c_1 = 2$; $c_0 - c_1 = 3$ etc. from data shown in the above table, platting upon the lines $D_0 - D_1 = 3.24$ $D_0 - D_1 = 6.48$ etc. the points shown with circles around them in the cross section sheet p. 177.

Having the lines $c_0 - c_1 = 1$; $c_0 - c_1 = 2$ - 3 platted, intermediate lines can be interpolated mechanically, upon the principle that vertical lines would be proportionally divided (as ML is proportionally divided into 5 equal parts and points marked at a - b - c - d for the lines $c_0 - c_1 = 1.2 - 1.4 - 1.6 - 1.8$ -

The values of $c_0 - c_1$ are most conveniently for use, taken to every second tenth of a foot in interpolating, as is shown in the diagram p. 177 between 1 and 2. that is 1.2 - 1.4 - 1.6 - 1.8.

A complete diagram is shown at the back of the book.


Prismoidal Correction
C
$D_0 - D_1$
$D_0 - D_1 = 3.24$
$D_0 - D_1 = 6.48$
$D_0 - D_1 = 9.72$
$D_0 - D_1 = 12.96$
$D_0 - D_1 = 16.20$
$D_0 - D_1 = 19.44$
$C_0 - C_1 = 3$
$C_0 - C_1 = 2$
$C_0 - C_1 = 1$
1.8
1.6
1.4
1.2
M
L
M'
L'
0
5
10
15
20
25

For Use.

Find the diagonal line corresponding to the given value of $c_0 - c_1$; follow this up until the vertical line representing the given value of $D_0 - D_1$ is reached and this intersection thus found. Then read off the value of C corresponding to this intersection

Example.

$c_0 - c_1 = 1.2$
$D_0 - D_1 = 11.0$ $\qquad C = 4. +$

again

$c_0 - c_1 = 1.7$
$D_0 - D_1 = 7.0$ $\qquad C = 3.6 \pm$

Diagram for Triangular Prisms.

Formula $S = \frac{50}{54} c D.$

A table may be constructed.

	0	5.4	10.8	16.2	21.6	27.0	D
0	0	0	0	0	0	0	
1	0	5	10	15	20	25	
2	0	10	20	30	40	50	
3	0	15	30	45	60	75	
4	0	20	40	60	80	100	
5	0	25	50	75	100	125	
6	0	30	60	90	120	150	
7	0	35	70	105	140	175	
8	0	40	80	120	160	200	
9	0	45	90	135	180	225	
10	0	50	100	150	200	250	
c							

From this a diagram can be constructed similar in form to that for Prismoidal Correction.

The lines for all values of c pass through the origin.

In constructing this table, any values of D might have been taken instead of those used here. Those used were selected because they give results simple in value, easily obtained, and readily plotted.

Diagram for Three Level Sections.

Formula $S = \frac{50}{54}\left(c + \frac{b}{25}\right)D - \frac{50}{54} \cdot \frac{b}{25} \cdot b$

A separate diagram will be required for each value (or combination of values) of b and s. Since b and s thus become constants, the formula assumes the form of

$x = a(z + b)y + d$

and the diagram will consist of a series of right lines.

A table can be made up by taking successive values of c = 0 - 1 - 2 - 3 - 4 - etc. and finding for each of these the value of S corresponding to different values of D using the above formula.

To make separate and complete computations directly by formula, would be quite laborious; there is however a method of systematizing the construction of the table, which can be shown better by example than in any other way.

Example.

$b = 14$ $s = 1\frac{1}{2}$ to 1.

Formula $S' = \frac{50}{54}\left(c+\frac{b}{2s}\right)D - \frac{50}{54}\cdot\frac{b}{2s}\cdot b$

becomes $S' = \frac{50}{54}\left(c+\frac{14}{3}\right)D - \frac{50}{54}\cdot\frac{14}{3}\cdot 14$

$$S' = \frac{50}{54}\left(c+\frac{14}{3}\right)D - 60.49 \qquad (171.)$$

A table has been prepared for successive values of $c = 0 - 1 - 2 - 3 - 4 - 5$ etc. and for $D = 14. - 16.2 - 21.6 - 27.0$ etc. These values of D are selected for the following reasons; $D = 14$ is the least possible value; $D = 16.2 - 21.6$ are desirable because they are multiples of of 5.4 and the factors in the formula show that the computations will be simplified by selecting multiples of 5.4 for the successive values of D.

	14	16.2	21.6	27.0	32.4	37.8	43.2	D
	12.963	15.	20.	25.	30.	35.	40.	constant diff.
0	0	9.51	32.84	56.18	79.51	102.84	126.18	
1	12.963	24.51	52.84	81.18	109.51	137.84	166.18	
2	25.926	39.51	72.84	106.18	139.51	172.84	206.18	
3	38.889	54.51	92.84	131.18	169.51	207.84	246.18	
4	51.852	69.51	112.84	156.18	199.51	242.84	286.18	
5	64.815	84.51	132.84	181.18	229.51	277.84	326.18	
6	77.778	99.51	152.84	206.18	259.51	312.84	366.18	
7	90.741	114.51	172.84	231.18	289.51	347.84	406.18	
8	103.704	129.51	192.84	256.18	319.51	382.84	446.18	
9	116.667	144.51	212.84	281.18	349.51	417.84	486.18	
10	129.630	159.51	232.84	306.18	379.51	452.84	526.18	
c								

When $C = 0$ $\quad S = \frac{50}{54}\cdot\frac{14}{3}\cdot D - 60.49$

when $D = 14$ $\quad S = \frac{50}{54}\cdot\frac{14}{3}\cdot 14 - 60.49$

$$= 60.49 - 60.49 = 0$$

when $D = 16.2$ we may again calculate

$$S = \frac{50}{54}\cdot\frac{14}{3}\cdot 16.2 - 60.49$$

or we may find how much greater S will be for $D = 16.2$ than for $D = 14.0$

We have $S = \frac{50}{54}\cdot\frac{14}{3}\cdot D - 60.49$

for any new value D'

$$S' = \frac{50}{54}\cdot\frac{14}{3}\cdot D' - 60.49$$

$$S' - S = \frac{50}{54}\cdot\frac{14}{3}(D' - D)$$

$D' = 16.2 \quad D = 14.0 \quad D' - D' = 2.2$

$$S' - S = \frac{50}{54}\cdot\frac{14}{3}\times 2.2 = 9.51$$

$S = 0$

$S' = 9.51$ which is entered in table.

Similarly

$$S'' - S' = \frac{50}{54}\cdot\frac{14}{3}(D'' - D')$$

$D'' = 21.6 \quad D' = 16.2 \quad D'' - D' = 5.4$

$$S'' - S' = \frac{50}{54}\times\frac{14}{3}\times 5.4$$

$= 23.333$

$S' = 9.51$

$S'' = 32.843$

Similarly $S''' - S'' = 23.333$

$S''' = 56.176$

$S^{iv} - S''' = 23.333$

$S^{iv} = 79.509$

$S^{iv} = 79.509$

23.333

$S^{v} = 102.842$

23.333

$S^{vi} = 126.175$

Constant increment is 23.333 for $D' - D = 5.4$

Each result is entered in the table in its proper place.

The final result for $c = 0$ and $D = 43.2$ should be calculated independently as a check

When $c = 0$ $\quad S' = \frac{50}{54} \cdot \frac{14}{3} \cdot D - 60.49$

When $D = 43.2$ $\quad S' = \frac{50}{54} \cdot \frac{14}{3} \times 43.2 - 60.49$

$$= 50 \times \frac{14}{3} \times 0.8 - 60.49$$

$$= \frac{560}{3} - 60.49$$

$$= 186.67 - 60.49$$

$$S' = 126.18$$

This checks exactly and all intermediate values are also checked by this process which is also more rapid than an independent calculation for each value of D

We now have values of S' for the various values of $D = 14. - 16.2 - 21.6$ etc. when $c = 0$

How much will these be increased when $c = 1$?

Formula $S' = \frac{50}{54}\left(c + \frac{14}{3}\right)D - 60.49$

for any new value c'

$$S'' = \frac{50}{54}\left(c' + \frac{14}{3}\right)D - 60.49$$

$$S'' - S' = \frac{50}{54}(c' - c)D$$

when $c' = 1$ $\quad c = 0$ $\quad c' - c = 1$

$$S'' - S' = \frac{50}{54} D$$

Similarly $S'' - S' = \frac{50}{54}(c'' - c')D$
when $c'' = 2$ and $c' = 1$ $c'' - c' = 1$

$S''' - S' = \frac{50}{54} D$

That is for any increase of 1 ft. in the value of c $S' - S = \frac{50}{54} D$

When $D = 14.$

$S' - S = \frac{50}{54} \times 14 = 12.963$

This we enter as the constant difference for column $D = 14$

We have already found $S_0 = 0$

$$\begin{array}{r} 12.963 \\ \hline S_1 = 12.963 \\ 12.963 \\ \hline S_2 = 25.926 \\ 12.963 \\ \hline S_3 = 38.889 \text{ Etc.} \end{array}$$

This gives column 14.

When $D = 16.2$

$S' - S = \frac{50}{54} D = \frac{50}{54} \times 16.2 = 50 \times 0.3$
$= 15$

Enter 15 as the constant diff. in col. 16.2

We already have $S_0 = 9.51$

$$\begin{array}{r} 15. \\ \hline S_1 = 24.51 \\ 15. \\ \hline S_2 = 39.51 \\ 15. \\ \hline S_3 = 54.51 \text{ Etc} \end{array}$$

This allows us to complete column 16.2

Similarly for $D = 21.6$ $S' - S = 20.$
Enter 20 as constant diff. in column 21.6 and compute column as shown in table.

Similarly fill out all the columns shown in the table.
The final result for $C = 10$ $D = 43.2$ should be calculated independently and directly from the formula, as a check

$$S = \frac{50}{54}\left(C + \frac{14}{3}\right)D - 60.49$$

$$C = 10 \qquad D = 43.2$$

$$\begin{aligned} S &= \frac{50}{54} \times 14.667 \times 43.2 - 60.49 \\ &= 50 \times 14.667 \times 0.8 - 60.49 \\ &= 40 \times 14.667 - 60.49 \\ &= 586.68 - 60.49 \\ S &= 526.19 \end{aligned}$$

The table gives 526.18. This checks sufficiently close to indicate that no error has been made. ($C + \frac{14}{3} = 14.6667$ would yield an exact check).

Note that for $C=10$ $D=43.2$ value $= 526.18$
$C=10$ $D=37.8$ " 452.84
diff $= 73.34$

Between $C=10$ $D = 37.8$ and $C=10$ $D = 32.4$ } diff $= 73.33$

In line $C=10$ a constant difference is found between successive values of D differing by 5.4 (It may be demonstrated to be $= 73.33$)
All values in the table except col. 14 are satisfactorily checked by applying this diff of 73.33 in line 10 together with the independent check of $C = 10$ $D = 43.2$

The value of $C = 10$ $D = 14$ can also be checked and shown to be O.K.
Having the table p. 68 completed, the construction of the diagram is simple. The "Diagram for Three Level Sections, Base 14 Slope $1\frac{1}{2}$ to 1" was calculated and constructed according to this table. The Diagram given shows a convenient arrangement of lines and figures for use. For rapid and convenient use, the diagram should be constructed upon cross section paper Plate G, and in this case the diagram will be upon a scale twice that of the diagram accompanying these notes.

A "curve of level section" has been platted on this diagram in this manner. For level sections when

$C = 0$ $D = 14$ $C = 1$ $D = 17.0$
$C = 1.4$ $D = 18.2$ $C = 2$ $D = 20.0$
$C = 6$ $D = 32.0$ etc.

The line passing through these points gives the "curve of level section".
Aside from the direct use of this curve of level section (for preliminary estimates or otherwise), it is very useful in tending to prevent any gross errors in the use

of the table, since in general the points, (intersections) used in the diagram will lie not far from the curve of level section.

Use of Diagram.

Find the diagonal line corresponding to the given value of C; follow this up until the vertical line representing the given value of D is reached, and this intersection found. Then read off the value of S corresponding to this intersection.

Example.

Notes	Sta 1	$\frac{13.0}{-4.0}$	-3.7	$\frac{12.4}{-3.6}$	136.
	Sta 0	$\frac{10.6}{-2.4}$	-2.5	$\frac{10.3}{-2.2}$	78.
					214.

For Sta 1 $c = 3.7$ $D = 25.4$

$c = 3.7$ is the middle of the space between 3.6 and 3.8 — follow this up until the vertical line 25.4 is reached.

The intersection lies upon the line $S = 136$.

Enter this above opposite Sta 1.

For Sta 0 $c = 2.5$ $D = 20.9$

$C = 2.5$ is the middle of space between 2.4 and 2.6 — follow this up until the middle of space between 20.8 and 21.0 is reached.

The intersection lies just above the line $S_0 = 78$. Enter this opposite Sta 0

$S_{100} = S_0 + S_1$

$= 136 + 78 = 214$ cu. yds.

The prismoidal correction may be applied if desired.

It should be noticed that in each case the intersection was quite close to the "curve of level section"

Diagrams may be constructed in this way, that will give results to a greater degree of precision than is warranted by the precision reached in taking the measurements on the ground.

In point of rapidity diagrams are much more rapid than tables for the computation of "Three Level Sections"

For "Triangular Prisms" and for "Prismoidal" Correction", the diagrams are somewhat more rapid.

For "Level Sections" the Pullen and Chandler tables are at least equally rapid.

A book "Computation from Diagrams of Railway Earthwork" by Arthur M. Wellington, published by D. Appleton & Co. N.Y.

explains the application and construction of certain other tables in addition to those given here. "Wellington's Diagrams" as there published are upon a scale differing from that used here, and they do not allow of as great precision, but are on the contrary arranged to cover a large number of tables differing some as to base and slope.

The use of approximate methods for applying the prismoidal correction to irregular sections (pp. 152-153) will be rendered practicable by the use of these "Diagrams for Three Level Sections".

Method 1 - No use of diagrams is necessary.

Method 2 - Having found for any irregular section (by triangular prisms or any other method) the solidity for 50 ft in length = S, find upon the diagram the line corresponding to this value of S - follow this line to the curve of level section and read off the value of C (for a level section) which corresponds, and also the value of D for the same section.

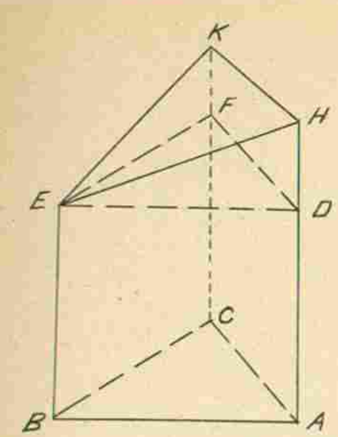
Method 3. Having found in any way the value of S; if c is given, find the value of D to correspond; if D is given find the value of c to correspond.

Method 4 – The use of diagrams is not needed.

Borrow Pits.

In addition to the ordinary work of Excavation and Embankment for railroads, Earth is often "borrowed" from outside the limits of the work proper, and in such Excavations, called "borrow-pits," it is common to prepare the work by dividing the surface into squares, rectangles, or triangles; taking levels at every corner upon the original surface; and again after the excavation of the borrow-pit is completed, the points are reproduced and levels again taken. The excavation is thus divided into a series of vertical prisms, having square, rectangular or triangular cross-sections. These prisms are are commonly truncated top and bottom, The lengths or altitudes of the vertical edges of these prisms are given by the difference in levels taken 1st on the original surface and 2nd after the excavation is completed. This method of measurements is very generally used, and for many purposes.

Truncated Triangular Prisms.



Let A = area of right section EFD of a truncated prism the base ABC being a right section.

h_1 = height AH

h_2 = " BE

h_3 = " CK

a = altitude of triangle EFD dropped from E to FD

Let S = Solidity of prism ABCKHE

S_t = " " " ABCFDE

S_u = " " pyramid FDEHK

Then $S_t = A \times AD = A \times \frac{3AD}{3} = A \times \frac{AD+BE+CF}{3}$

$$S_u = \text{area } DFKH \times \frac{a}{3}$$

$$= \frac{KF+HD}{2} \times FD \times \frac{a}{3}$$

$$= \frac{KF+HD}{3} \times FD \times \frac{a}{2}$$

$$= \frac{KF+HD}{3} \times A$$

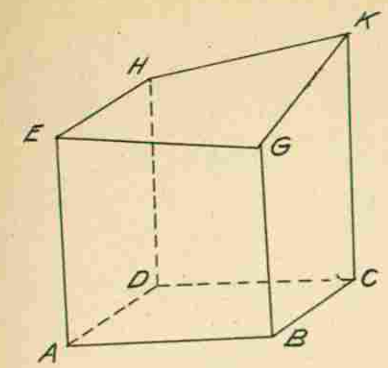
$$S = S_t + S_u = A\left(\frac{AD+BE+CF}{3} + \frac{KF+HD}{3}\right)$$

$$= A\,\frac{(AD+HD)+BE+(CE+KF)}{3}$$

$$= A\,\frac{h_1+h_2+h_3}{3} \qquad (172.)$$

If the prism be truncated top and bottom, the same reasoning holds and the same formula applies.

Truncated Rectangular Prism.



Let A = area of right section ABCD of a rectangular prism truncated on top (base is ABCD)

h_1 = height AE

h_2 = " BG

h_3 = " KC

h_4 = " HD

S = Solidity of prism

$b = AD = BC$

$a = AB = DC$

Then using method of End areas

$$S = \frac{AEHD + BGKC}{2} \times a$$

$$= \frac{b\,\frac{h_1+h_4}{2} + b\,\frac{h_2+h_3}{2}}{2} \times a$$

$$= ab\,\frac{h_1+h_2+h_3+h_4}{4}$$

$$S = A\,\frac{h_1+h_2+h_3+h_4}{4} \quad (\text{cu. ft.}) \qquad (173)$$

$$S = \frac{A}{27}\,\frac{h_1+h_2+h_3+h_4}{4} \quad (\text{cu. yds.}) \qquad (174)$$

We may find S correct by the prismoidal formula if we apply the prismoidal correction. The prismoidal correction $C = 0$ since $D_0 - D_1 = 0$ (or in this case $AD - BC = 0$) The formula therefore remains unchanged. It is evident from this then that the solution holds good and the formula is correct, not only when the surface EHKG is a plane, but also when it is a warped surface, generated by a right line moving always parallel to the plane ADHE and upon EG and HK as directrices.

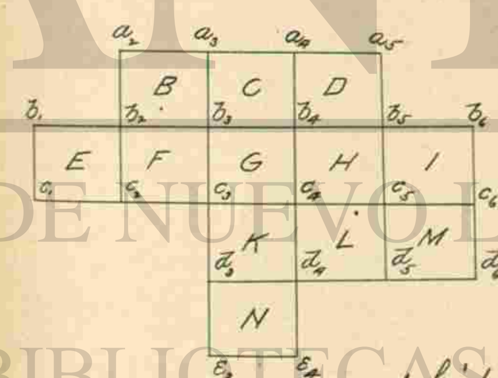
Some engineers prefer to cross section in rectangles of base 15'×18'. In this case

$$S = \frac{15' \times 18'}{27} \cdot \frac{h_1 + h_2 + h_3 + h_4}{4} \text{ (cu. yds)}$$

$$= 10 \; \frac{h_1 + h_2 + h_3 + h_4}{4} \text{ (cu. yds.)} \quad (175.)$$

Other convenient dimensions will suggest themselves, as 10'×13.5' or 20'×13.5 or 20'×27'

Assembled Prisms.



In the case of an assembly of prisms of equal base, it is not necessary to separately calculate each prism but the solidity of a number of prisms may be calculated in one operation.

In the prism B – $S_B = A \; \frac{a_2 + a_3 + b_3 + b_2}{4}$

$S_C = A \; \frac{a_3 + a_4 + b_4 + b_3}{4}$ etc

From inspection it will be seen, taking

A as the common area of base of a single prism, and taking the sum of the solidities, that the heights $a_2 - a_5$ enter into the calculation of one prism only; $a_3 - a_4$ into two prisms each; $b_1 - b_6$ one only; $b_2 - b_5$ into three prisms; $b_3 - b_4$ into four prisms; and similarly throughout.

Let t_1 = sum of hts. common to one prism.

t_2 = " " " " " two prisms.

t_3 = " " " " " three "

t_4 " " " " " four "

Then the total solidity

$$S_t = A\,\frac{t_1 + 2t_2 + 3t_3 + 4t_4}{4} \quad (\text{cu. ft.}) \quad (176.)$$

$$S_t = \frac{A}{27}\,\frac{t_1 + 2t_2 + 3t_3 + 4t_4}{4} \quad (\text{cu. yds.}) \quad (177.)$$

Haul.

When material from Excavation is hauled to be placed in Embankment, it is customary to pay to the contractor a certain sum for every cubic yard hauled. Oftentimes it is provided that no payment shall be made for material hauled less than a specified distance. In the East a common limit of "free haul" is 1000 feet. Often in the West 100 feet is the limit of "free haul".

A common custom is to make the unit for payment of haul, one yard hauled 100 feet; the price paid will often be from 1 to 2 cents per cubic yard hauled 100 feet.

The price paid for "haul" is small, and therefore the standard of precision in calculation need not be quite as fine as in the calculation of the quantities of Earthwork.

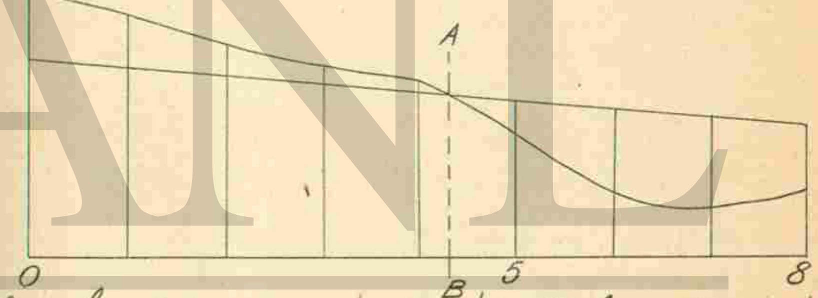
The total "haul" will be the product of (1.) the total amount of Excavation hauled, and (2.) the average length of haul.

The average length of haul is the distance between the center of gravity of the material as found in Excavation, and the center of gravity as deposited. It would in general not be simple to find the center of gravity of the entire mass of Excavation hauled, and the most convenient way is to take each Section of Earthwork by itself. The "haul" for Each Section is the product of the

(1.) number of cu. yds in that Section, and

(2.) distance between the center of gravity in Excavation, and the center of gravity as deposited.

When excavation is placed in Embankment, there may be some difficulty in determining just where any given Section of Excavation will be placed, and where its center of gravity will be in Embankment.



In hauling Excavation into Embankment, there is some plane, as indicated by AB, to which all excavation must be hauled on its way to be placed in Embankment, and (another way of putting it), from which all material placed in Embankment must be hauled on its way from excavation. We may figure the total haul as the sum of

(1.) total "haul" of Excavation to AB and

(2.) total "haul" of Embankment from AB.

The total "haul" of Excavation to AB and the total "haul" of Embankment from AB will most conveniently be calculated as the sum of the hauls of the several Sections of Earthwork.

For each section the haul is the product of (1.) the Solidity S of that Section, and (2.) distance from center of gravity of that section to the plane AB.

When the two end areas are equal, the center of gravity will be midway between the two end planes. When the two end areas are not equal in value, the center of gravity of the Section will be at a certain distance from the mid-section, as shown by the formula

$$x_g = \frac{l^2}{12} \cdot \frac{A_1 - A_0}{S}$$ in which

x_g = distance center of gravity from mid-section

Referring to the figure and demonstration for Solidity S on page 146, let us proceed to find the distance of the center of gravity from A, B, E, D, and let this $= x_c$

Then for any Elementary Section of thickness $\underline{dx}$ and distance x from A, B, E, D, its moment will be

$$\left[b_1 + (b_0 - b_1)\frac{x}{l}\right]\left[c_1 + (c_0 - c_1)\frac{x}{l}\right] x\,dx$$

$$S \cdot x_c = \int_0^l \left[b_1 + (b_0 - b_1)\frac{x}{l}\right]\left[c_1 + (c_0 - c_1)\frac{x}{l}\right] x\,dx$$

$$S \cdot x_c = \frac{b_1 c_1 l^2}{2} + \frac{b_1(c_0 - c_1) l^3}{3l} + \frac{c_1(b_0 - b_1) l^3}{3l} + \frac{(c_0 - c_1)(b_0 - b_1) l^4}{4 l^2}$$

$$= \frac{l^2}{12}\left[\begin{array}{l} 6 b_1 c_1 + 4 b_1 c_0 + 4 b_0 c_1 + 3 b_0 c_0 \\ - 4 b_1 c_1 - 3 b_1 c_0 - 3 b_0 c_1 \\ - 4 b_1 c_1 \\ + 3 b_1 c_1 \end{array}\right]$$

$$S \cdot x_c = \frac{l^2}{12}\left(b_1 c_1 + b_1 c_0 + b_0 c_1 + 3 b_0 c_0 \right)$$

$$x_c = \frac{l^2}{12} \cdot \frac{b_1 c_1 + b_1 c_0 + b_0 c_1 + 3 b_0 c_0}{S}$$

What is wanted is x_g rather than x_c

$$x_g = \frac{l}{2} - x_c$$

$$S x_g = S \frac{l}{2} - S x_c$$

$$S = \frac{l}{6}\left(2 b_1 c_1 + 2 b_0 c_0 + b_1 c_0 + b_0 c_1\right)$$

$$S \cdot \frac{l}{2} = \frac{l^2}{12}\left(2 b_1 c_1 + 2 b_0 c_0 + b_1 c_0 + b_0 c_1\right)$$

$$S \cdot x_c = \frac{l^2}{12}\left(b_1 c_1 + 3 b_0 c_0 + b_1 c_0 + b_0 c_1\right)$$

$$S \cdot x_g = \frac{l^2}{12}\left(b_1 c_1 - b_0 c_0\right)$$

$$= \frac{l^2}{12}\left(A_1 - A_0\right)$$

$$x_g = \frac{l^2}{12} \cdot \frac{A_1 - A_0}{S} \quad (S \text{ in cu. ft.}) \qquad (178.)$$

$$x_g = \frac{l^2}{12 \times 27} \cdot \frac{A_1 - A_0}{S} \quad (S \text{ in cu. yds}) \qquad (179.)$$

The formula above applies to the solid shown in the figure, which has trapezoidal ends, but it will apply also when D_0A_0 - D_1A_1 are each = 0, and therefore applies to such solids having triangular ends, and since any

Section of Earthwork with parallel ends may be divided into a number of such solids with triangular ends, it applies to all ordinary sections of railroad Earthwork; since it applies to the parts of which it is made up.

It may readily be shown that in fact this formula is correct for prisms, wedges and pyramids.

The formula $$x_g = \frac{l^2}{12\times 27}\cdot\frac{A_1 - A_0}{S}$$
is not in form convenient for use, because we have not found the values of A_1 and A_0, but instead have calculated directly from the tables or diagrams, the values of S_1 and S_0 for 50 ft. in length, where

$$S_1 = \frac{50}{27}A_1 \quad \text{or} \quad A_1 = \frac{27 S_1}{50} \quad \text{and} \quad A_0 = \frac{27}{50}S_0$$

Substituting $$x_{g100} = \frac{100\times 100}{12\times 27}\cdot\frac{S_1 - S_0}{S}\cdot\frac{27}{50}$$

$$x_{g100} = \frac{100}{6}\cdot\frac{S_1 - S_0}{S} \qquad (180.)$$

This formula is in shape convenient for use and results correct to the nearest foot can be calculated with rapidity.

For a section of length l less than 100 ft.

$$x_{gl} = \frac{l^2}{12\times 27}\cdot\frac{A_1 - A_0}{S_l} = \frac{l^2}{12\times 27}\cdot\frac{A_1 - A_0}{S_{100}\times\frac{l}{100}}$$

$$= \frac{100\,l}{12\times 27}\cdot\frac{A_1 - A_0}{S_{100}}$$

$$x_{g100} = \frac{100\times 100}{12\times 27}\cdot\frac{A_1 - A_0}{S_{100}}$$

$$x_{gl} = \frac{l}{100}\cdot x_{g100} \qquad (181.)$$

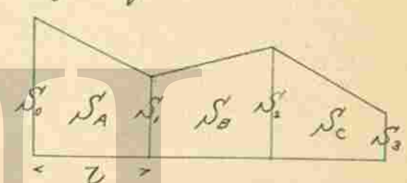
It is not however necessary to calculate the position of the center of gravity of each station or to calculate for each station the correction x_g. It is easy to calculate for a series of sections a correction to be applied to obtain the center of gravity of the entire mass.

To find the position of the center of gravity of the entire mass — let

$\underline{X}_c$ = cent. of grav. for entire mass (approximated) using for each section c.g. at $\frac{l}{2}$

$\underline{X}$ = true dist to c.g. of entire mass

$\underline{X}_g = \underline{X} - \underline{X}_c$



$S_0 - S_1 - S_2 - S_3 = \frac{50}{27}A_0 - \frac{50}{27}A_1$ etc as taken from tables or diagrams.

When all sections are of uniform length $= l$

$$\underline{X}_c S = \frac{l}{2}(S_A + 3S_B + 5S_C) \quad \text{in figure above} \tag{182.}$$

$$\underline{X}\, S = S_A\left(\frac{l}{2} + x_{gA}\right) + S_B\left(\frac{l}{2} + x_{gB}\right) + S_C\left(\frac{l}{2} + x_{gC}\right)$$

$$S(\underline{X} - \underline{X}_c) = S_A x_{gA} + S_B x_{gB} + S_C x_{gC}$$

from (180.)

$$= \frac{100}{6}\left[S_A \frac{S_0 - S_1}{S_A} + S_B \frac{S_1 - S_2}{S_B} + S_C \frac{S_2 - S_3}{S_C}\right)$$

$$S\underline{X}_g = \frac{100}{6}(S_0 - S_3) \quad \text{or in general}$$

$$\underline{X}_g = \frac{100}{6} \cdot \frac{S_0 - S_n}{S} \tag{183.}$$

Where S is the solidity of the entire mass.

Mass Diagram.

Many questions of "haul" may be very usefully treated by means of a graphical method known to some as "Mass Levelling", in which is used a diagram sometimes called a "Mass Profile", but which will be referred to here as the "Mass Diagram".

The construction of the "Mass Diagram" will be more clearly understood from an example than from a general description.

Let us consider the earthwork, shown by the profile on page 201, consisting of alternate "cut" and "fill". To show the work of constructing the "diagram" in full, we will calculate the quantities throughout, but for convenience and simplicity, will use "level sections and disregard prismoidal correction. In a case in actual practice, the solidities will have been calculated for the actual notes taken. In the table p. 202, the

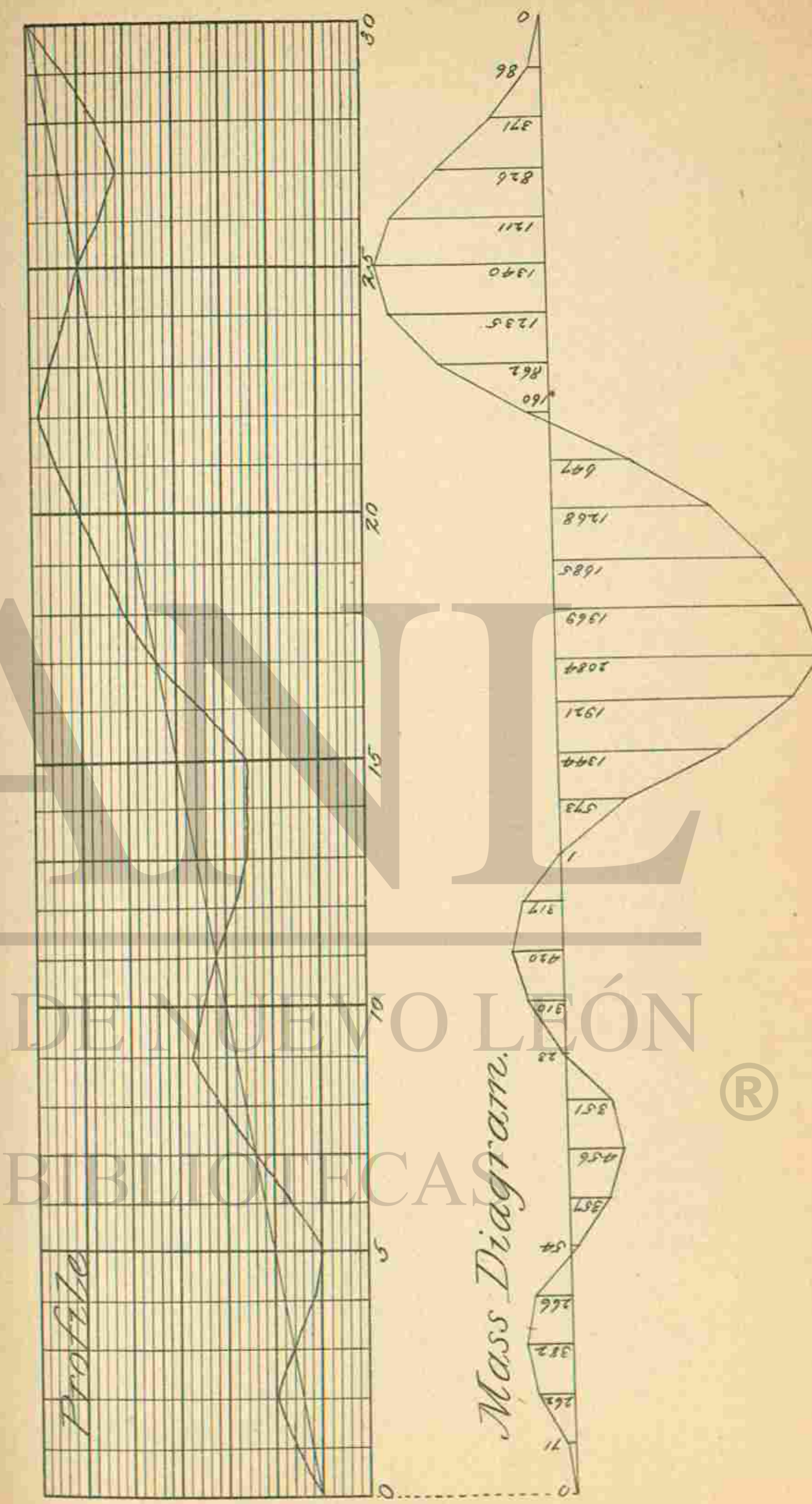
1st column gives the stations

2nd " " center heights

3rd " " values of S from tables

4th " " values of S for each section — and with sign + for cut or − for fill

5th column gives the total or the sum of


Profile
Mass Diagram.
0
5
10
15
20
25
30

Station	Center Heights	Solidity for 50' due to center ht. given (Taken from Tables)	Solidity for Full Section	Solidity Totals
0	0	0		0
1	+1.7	71.	+ 71.	+ 71.
2	+2.7	120.	+ 191	+ 262.
3	0	0.	+ 120.	+ 382.
4	−3.3	116.	− 116.	+ 266.
5	−5.1	204.	− 320.	− 54.
6	−2.9	99.	− 303.	− 357.
7	0	0	− 99.	− 456.
8	+2.4	105.	+ 105.	− 351.
9	+4.5	223.	+ 328.	− 23.
10	+2.5	110.	+ 333.	+ 310.
11	0	0	+ 110.	+ 420.
12	−3.0	103.	− 103.	+ 317.
13	−5.3	215.	− 318.	− 1.
14	−7.6	357.	− 572.	− 573.
15	−8.4	414.	− 771.	− 1344.
16	−4.3	163.	− 577.	− 1921.
17	0	0	− 163.	− 2084.
18	+2.6	115.	+ 115.	− 1969.
19	+3.6	169.	+ 284.	− 1685.
20	+4.9	248.	+ 417.	− 1268.
21	+6.7	373.	+ 621.	− 647.
22	+7.5	434.	+ 807.	+ 160.
23	+5.2	268.	+ 702.	+ 862.
24	+2.4	105.	+ 373.	+ 1235.
25	0	0	+ 105.	+ 1340.
26	−3.6	129.	− 129.	+ 1211.
27	−6.0	256.	− 385.	+ 826.
28	−5.0	199.	− 455.	+ 371.
29	−2.6	86.	− 285.	+ 86.
30	0	0	− 86.	0

solidities up to each station; and in getting this total, each + solidity is added and each − solidity is subtracted, as appears in the table from the results obtained.

Having completed the table, the next step is the construction of the "Mass Diagram" page 201. In the figure shown there, each station line is projected down, and the value from column 5, corresponding to any station, is platted to scale as an offset from the base line at that station, all + quantities above the line, and all − quantities below the line. The points thus found are joined, and the result is the "Mass Diagram".

It will follow from the methods of calculation and construction used, that the "Mass Diagram" will have the following properties, which can be understood by reference to the profile and diagram p. 201.

1. Grade points of the profile correspond to maximum and minimum points of the diagram.
2. In the diagram, ascending lines mark excavation, and descending lines embankment.
3. The difference in length between any two vertical ordinates of the diagram is the solidity between the points (stations) at which the ordinates are erected.

4. Between any two points where the diagram is intersected by any horizontal line, excavation equals embankment.
5. The area cut off by any horizontal line is the measure of the "haul" between the two points cut by that line.

It may be necessary to explain the latter point at somewhat greater length.

Any quantity (such as dimension weight or volume) is often represented graphically by a line; in a similar way, the product of two quantities (such as volume into distance or as foot pounds) may be represented or measured by an area. In the case of a figure other than a rectangle, the value or product measured by this area may be found by cutting up the area by lines, and these lines may be vertical lines representing volumes or horizontal lines representing distance. The result will be the same in either case. An Example will illustrate.

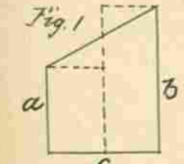
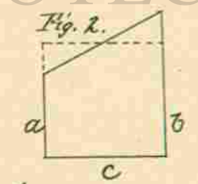
Fig. 1


Fig. 2


In the two figures let $\underline{a}$ and $\underline{b}$ represent lbs. $\underline{c}$ " feet. and the area of the trapezoid represent a certain number of foot pounds. The trapezoid may be resolved into rectangles by the use of a vertical line as shown in Fig 1, or by a horizontal line as in Fig. 2.

In Fig 1, the area is $a \times \frac{c}{2} + b \times \frac{c}{2}$

" Fig 2. " " " $\frac{a+b}{2} \times c$

the result being the same in both cases.

In an entirely similar way, the area ABC (p. 206) represents the "haul" of Earthwork (in cu. yds. moved 1 ft.) between A and C, and this area may be calculated by dividing it by a series of vertical lines representing solidities, as shown in the Fig. p. 201. That this area represents the haul between A and C may appear as follows. Take any elementary solidity dS at D (p. 206). Project this down upon the diagram at F and draw the horizontal lines FG. Between the points F and G (or between D and I) therefore, excavation equals embankment, and the mass dS must be hauled a distance FG, and the amount of "haul" on dS will be $dS \times FG$. Similarly with any other elementary dS. The total "haul" between A and C will be measured by the area ABC. This area is most conveniently measured by the trapezoids formed by the vertical lines representing solidities. The average length of haul will be


Profile
Mass Diagram
0
5
10
15
20
25
30
A
B
C
D
F
G
I

this area divided by the total solidity (represented in this case on p. 201 by the longest vertical line 2084).

The construction of the "Mass Diagram" as a series of trapezoids, involves the assumption that the center of gravity of a section of Earthwork lies at its mid-section, which is only approximately correct. If the line joining the ends of the vertical lines be made a curved line, the assumption becomes more closely accurate, and if the area be calculated by "Simpson's Rule" or by planimeter, results closely accurate will be reached.

It will be further noticed that in the "diagram", hill sections represent haul forward; and valley sections, haul backward (see p. 206). The mass diagram may therefore be used to indicate the methods by which the work shall be performed; whether excavation at any point shall be hauled forward or backward; and more particularly, to show the point where backward haul shall cease, and forward haul begin, as in the figure p. 206, which shows a very simple case, the cuts and fills being evenly balanced, and no haul over


Profile
0
5
10
15
20
25
30
Mass Diagram
O
C
S
E
V
A
B
T
K
N
X
W
M
G
H
U
Y
R
L
D
F
Z

over 900 feet. In the figure p. 208, the excavation from Sta. 0 to 14 is very much in excess of Embankment, and vice versa from Sta 14 to 30. The mass diagram indicates a haul of nearly 3000 ft. for a large mass of Earthwork as shown by the ordinate AB. It will not be Economical to haul the material 3000 ft; it is better to "waste" some of the material near Sta. 0, and to "borrow" some near Sta. 30 if this be possible as is commonly the case. If we draw the line CD, the cut and fill between C and D will still be equal, and the volume of cut measured by CE can be wasted, and the equal volume of fill measured by DF can be borrowed, to advantage. It can be seen that there is still a haul of nearly 2000 ft. (from A to D) on the large mass of Earthwork measured by GH. It is probable that it will not pay to haul the mass GH, or any part of it, as far as AD. We must find the limit beyond which it is unprofitable to haul material, rather than borrow and waste.

Let c = cost of 1 cu. yd. Excavation or Embankment.

h = cost of haul on 1 cu. yd. 100 ft.

n = length of haul in "Stations" of 100 ft. each.

Then when 1 cu. yd. of Excavation is wasted, and
1 cu. yd of Embankment is borrowed
the cost = $2c$

When 1 cu. yd. of Excavation is hauled into Embankment the cost = $c + nh$

The limit of profitable haul is reached when $2c = c + nh$ or
when $n = \frac{c}{h}$

Example. When Excavation or Embankment is 18 cents per cu. yd. and haul is $1\frac{1}{2}$ cts.
$n = \frac{18}{1.5} = 12$ stations.
When $c = 16$ and $h = 2$ $n = 8$ stations.

In the former case 1200 ft. haul, we should draw in mass diagram (p. 208), the line KGL. KG is less than 1200 ft. The line should not be lower than G, for in this case, the haul would be as great as KL or more than 1200 ft.

In the latter case (800 ft. haul) the line would be carried up to a point where NM = 800 ft. The masses between N and A also C–O can be better wasted than hauled, and the masses between M–G also L–Z can be better borrowed than hauled (always provided that there are suitable places at hand for borrowing and wasting). Next, produce NM to R. The number of yards borrowed will be the same R.Z or M-G + L-Z. That arrangement of work which gives the smallest "haul" (product of cu. yds x distance hauled), is the best arrangement. The "haul" in one case is measured by GLRYU and in the other by MGU + URY. If MGU is less than GLRU, then it is cheaper to borrow (a.) R_Z rather than (b.) M_G and L_Z. In a similar way, material N_T and S_O can be wasted more economically than N_A and O_O. The most economical position for the line MR is when MU = UR. For ST when SV = VT. Any change from these positions of MR and ST will show an increase of area representing "haul".

The case is often not as simple as that here given. Very often the material borrowed or wasted has to be hauled beyond the limit of "free haul". The limit beyond which it is unprofitable to haul will vary according to the length of haul on the borrowed or wasted material; the limit will in general be increased by the length of haul on the borrowed or wasted material. The haul on wasted or borrowed material as NT may be shown graphically by NTXW – where NW = TX shows the length of haul and NTXW the "haul" (mass x distance).

The mass diagram can be used also for finding the limit of "free haul" on the profile, and various applications will suggest themselves to those who become familiar with its use and the principles of its construction. Certainly one of its most important uses is in allowing "haul" and "borrow and waste" to be studied by a diagram giving a comprehensive view of the whole situation. There are few if any other available methods of accomplishing this result.

Table of Prismoidal Corrections.

$C_0 - C_1$ / $D_0 - D_1$	1	2	3	4	5	6	7	8	9	$C_0 - C_1$ / $D_0 - D_1$
0.1	.03	.06	.09	.12	.15	.19	.22	.25	.28	0.1
.2	.06	.12	.19	.25	.31	.37	.43	.49	.56	.2
.3	.09	.19	.28	.37	.46	.56	.65	.74	.83	.3
.4	.12	.25	.37	.49	.62	.74	.86	.99	1.11	.4
.5	.15	.31	.46	.62	.77	.93	1.08	1.23	1.39	.5
.6	.19	.37	.56	.74	.93	1.11	1.30	1.48	1.67	.6
.7	.22	.43	.65	.86	1.08	1.30	1.51	1.73	1.94	.7
.8	.25	.49	.74	.99	1.23	1.48	1.73	1.98	2.22	.8
.9	.28	.56	.83	1.11	1.39	1.67	1.94	2.22	2.50	.9
1.0	.31	.62	.93	1.23	1.54	1.85	2.16	2.47	2.78	1.0
.1	.34	.68	1.02	1.36	1.70	2.04	2.38	2.72	3.06	.1
.2	.37	.74	1.11	1.48	1.85	2.22	2.59	2.96	3.33	.2
.3	.40	.80	1.20	1.60	2.01	2.41	2.81	3.21	3.61	.3
.4	.43	.86	1.30	1.73	2.16	2.59	3.02	3.46	3.89	.4
.5	.46	.93	1.39	1.85	2.31	2.78	3.24	3.70	4.17	.5
.6	.49	.99	1.48	1.98	2.47	2.96	3.46	3.95	4.44	.6
.7	.52	1.05	1.57	2.10	2.62	3.15	3.67	4.20	4.72	.7
.8	.56	1.11	1.67	2.22	2.78	3.33	3.89	4.44	5.00	.8
.9	.59	1.17	1.76	2.35	2.93	3.52	4.10	4.69	5.28	.9
2.0	.62	1.23	1.85	2.47	3.09	3.70	4.32	4.94	5.56	2.0
.1	.65	1.30	1.94	2.59	3.24	3.89	4.54	5.19	5.83	.1
.2	.68	1.36	2.04	2.72	3.40	4.07	4.75	5.43	6.11	.2
.3	.71	1.42	2.13	2.84	3.55	4.26	4.97	5.68	6.39	.3
.4	.74	1.48	2.22	2.96	3.70	4.44	5.19	5.93	6.67	.4
.5	.77	1.54	2.31	3.09	3.86	4.63	5.40	6.17	6.94	.5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.01	4.81	5.62	6.42	7.22	.6
[illegible]	[illegible]	[illegible]	[illegible]	3.33	[illegible]	5.00	5.83	6.67	[illegible]	[illegible]
.8	.86	1.73	2.59	3.46	4.32	5.19	6.05	6.91	7.78	.8
.9	.90	1.79	2.69	3.58	4.48	5.37	6.27	7.16	8.06	.9
3.0	.93	1.85	2.78	3.70	4.63	5.56	6.48	7.41	8.33	3.0
.1	.96	1.91	2.87	3.83	4.78	5.74	6.70	7.65	8.61	.1
.2	.99	1.98	2.96	3.95	4.94	5.93	6.91	7.90	8.89	.2
.3	1.02	2.04	3.06	4.07	5.09	6.11	7.13	8.15	9.17	.3
.4	1.05	2.10	3.15	4.20	5.25	6.30	7.35	8.40	9.44	.4
.5	1.08	2.16	3.24	4.32	5.40	6.48	7.56	8.64	9.72	.5
.6	1.11	2.22	3.33	4.44	5.56	6.67	7.78	8.89	10.00	.6
.7	1.14	2.28	3.43	4.57	5.71	6.85	7.99	9.14	10.28	.7
.8	1.17	2.35	3.52	4.69	5.86	7.04	8.21	9.38	10.56	.8
.9	1.20	2.41	3.61	4.81	6.02	7.22	8.43	9.63	10.83	.9
4.0	1.23	2.47	3.70	4.94	6.17	7.41	8.64	9.88	11.11	4.0
.1	1.27	2.53	3.80	5.06	6.33	7.59	8.86	10.12	11.39	.1
.2	1.30	2.59	3.89	5.19	6.48	7.78	9.07	10.37	11.67	.2
.3	1.33	2.65	3.98	5.31	6.64	7.96	9.29	10.62	11.94	.3
.4	1.36	2.72	4.07	5.43	6.79	8.15	9.51	10.86	12.22	.4
.5	1.39	2.78	4.17	5.56	6.94	8.33	9.72	11.11	12.50	.5
.6	1.42	2.84	4.26	5.68	7.10	8.52	9.94	11.36	12.78	.6
.7	1.45	2.90	4.35	5.80	7.25	8.70	10.15	11.60	13.06	.7
.8	1.48	2.96	4.44	5.93	7.41	8.89	10.37	11.85	13.33	.8
.9	1.51	3.02	4.54	6.05	7.56	9.07	10.59	12.10	13.61	.9
5.0	1.54	3.09	4.63	6.17	7.72	9.26	10.80	12.35	13.89	5.0
$C_0 - C_1$	1	2	3	4	5	6	7	8	9	$C_0 - C_1$

Table of Prismoidal Corrections.

C_0-C_1	1	2	3	4	5	6	7	8	9	C_0-C_1
D_0-D_1										D_0-D_1
5.1	1.57	3.15	4.72	6.30	7.87	9.44	11.02	12.59	14.17	5.1
.2	1.60	3.21	4.81	6.42	8.02	9.63	11.23	12.84	14.44	.2
.3	1.64	3.27	4.91	6.54	8.18	9.81	11.45	13.09	14.72	.3
.4	1.67	3.33	5.00	6.67	8.33	10.00	11.67	13.33	15.00	.4
.5	1.70	3.40	5.09	6.79	8.49	10.19	11.88	13.58	15.28	.5
.6	1.73	3.46	5.19	6.91	8.64	10.37	12.10	13.83	15.56	.6
.7	1.76	3.52	5.28	7.04	8.80	10.56	12.31	14.07	15.83	.7
.8	1.79	3.58	5.37	7.16	8.95	10.74	12.53	14.32	16.11	.8
.9	1.82	3.64	5.46	7.28	9.10	10.93	12.75	14.57	16.39	.9
6.0	1.85	3.70	5.56	7.41	9.26	11.11	12.96	14.81	16.67	6.0
.1	1.88	3.77	5.65	7.53	9.41	11.30	13.18	15.06	16.94	.1
.2	1.91	3.83	5.74	7.65	9.57	11.48	13.40	15.31	17.22	.2
.3	1.94	3.89	5.83	7.78	9.72	11.67	13.61	15.56	17.50	.3
.4	1.98	3.95	5.93	7.90	9.88	11.85	13.83	15.80	17.78	.4
.5	2.01	4.01	6.02	8.02	10.03	12.04	14.04	16.05	18.00	.5
.6	2.04	4.07	6.11	8.15	10.19	12.22	14.26	16.30	18.33	.6
.7	2.07	4.14	6.20	8.27	10.34	12.41	14.48	16.54	18.61	.7
.8	2.10	4.20	6.30	8.40	10.49	12.59	14.69	16.79	18.89	.8
.9	2.13	4.26	6.39	8.52	10.65	12.78	14.91	17.04	19.17	.9
7.0	2.16	4.32	6.48	8.64	10.80	12.96	15.12	17.28	19.44	7.0
.1	2.19	4.38	6.57	8.77	10.96	13.15	15.34	17.53	19.72	.1
.2	2.22	4.44	6.67	8.89	11.11	13.33	15.56	17.78	20.00	.2
.3	2.25	4.51	6.76	9.01	11.27	13.52	15.77	18.02	20.28	.3
.4	2.28	4.57	6.85	9.14	11.42	13.70	15.99	18.27	20.56	.4
.5	2.31	4.63	6.94	9.26	11.57	13.89	16.20	18.52	20.83	.5
.6	2.35	4.69	7.04	9.38	11.73	14.07	16.42	18.77	21.11	.6
.7	2.38	4.75	7.13	9.51	11.88	14.26	16.64	19.01	21.39	.7
.8	2.41	4.81	7.22	9.63	12.04	14.44	16.85	19.26	21.67	.8
.9	2.44	4.88	7.31	9.75	12.19	14.63	17.07	19.51	21.94	.9
8.0	2.47	4.94	7.41	9.88	12.35	14.81	17.28	19.75	22.22	8.0
.1	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	.1
.2	2.53	5.06	7.59	10.12	12.65	15.19	17.72	20.25	22.78	.2
.3	2.56	5.12	7.69	10.25	12.81	15.37	17.93	20.49	23.06	.3
.4	2.59	5.19	7.78	10.37	12.96	15.56	18.15	20.74	23.33	.4
.5	2.62	5.25	7.87	10.49	13.12	15.74	18.36	20.99	23.61	.5
.6	2.65	5.31	7.96	10.62	13.27	15.93	18.58	21.23	23.89	.6
.7	2.69	5.37	8.06	10.74	13.43	16.11	18.80	21.48	24.17	.7
.8	2.72	5.43	8.15	10.86	13.58	16.30	19.01	21.73	24.44	.8
.9	2.75	5.49	8.24	10.99	13.73	16.48	19.23	21.97	24.72	.9
9.0	2.78	5.56	8.33	11.11	13.89	16.67	19.44	22.22	25.00	9.0
.1	2.81	5.62	8.43	11.23	14.04	16.85	19.66	22.47	25.28	.1
.2	2.84	5.68	8.52	11.36	14.20	17.04	19.88	22.72	25.56	.2
.3	2.87	5.74	8.61	11.48	14.35	17.22	20.09	22.96	25.83	.3
.4	2.90	5.80	8.70	11.60	14.51	17.41	20.31	23.21	26.11	.4
.5	2.93	5.86	8.80	11.73	14.66	17.59	20.52	23.46	26.39	.5
.6	2.96	5.93	8.89	11.85	14.81	17.78	20.74	23.70	26.67	.6
.7	2.99	5.99	8.98	11.98	14.97	17.96	20.96	23.95	26.94	.7
.8	3.02	6.05	9.07	12.10	15.12	18.15	21.17	24.20	27.22	.8
.9	3.06	6.11	9.17	12.22	15.28	18.33	21.39	24.44	27.50	.9
10.0	3.09	6.17	9.26	12.35	15.43	18.52	21.60	24.69	27.78	10.0
C_0-C_1 →	1	2	3	4	5	6	7	8	9	← C_0-C_1

Base 20' Slope $1\frac{1}{2}$ to 1. Table for Three Level Sections.

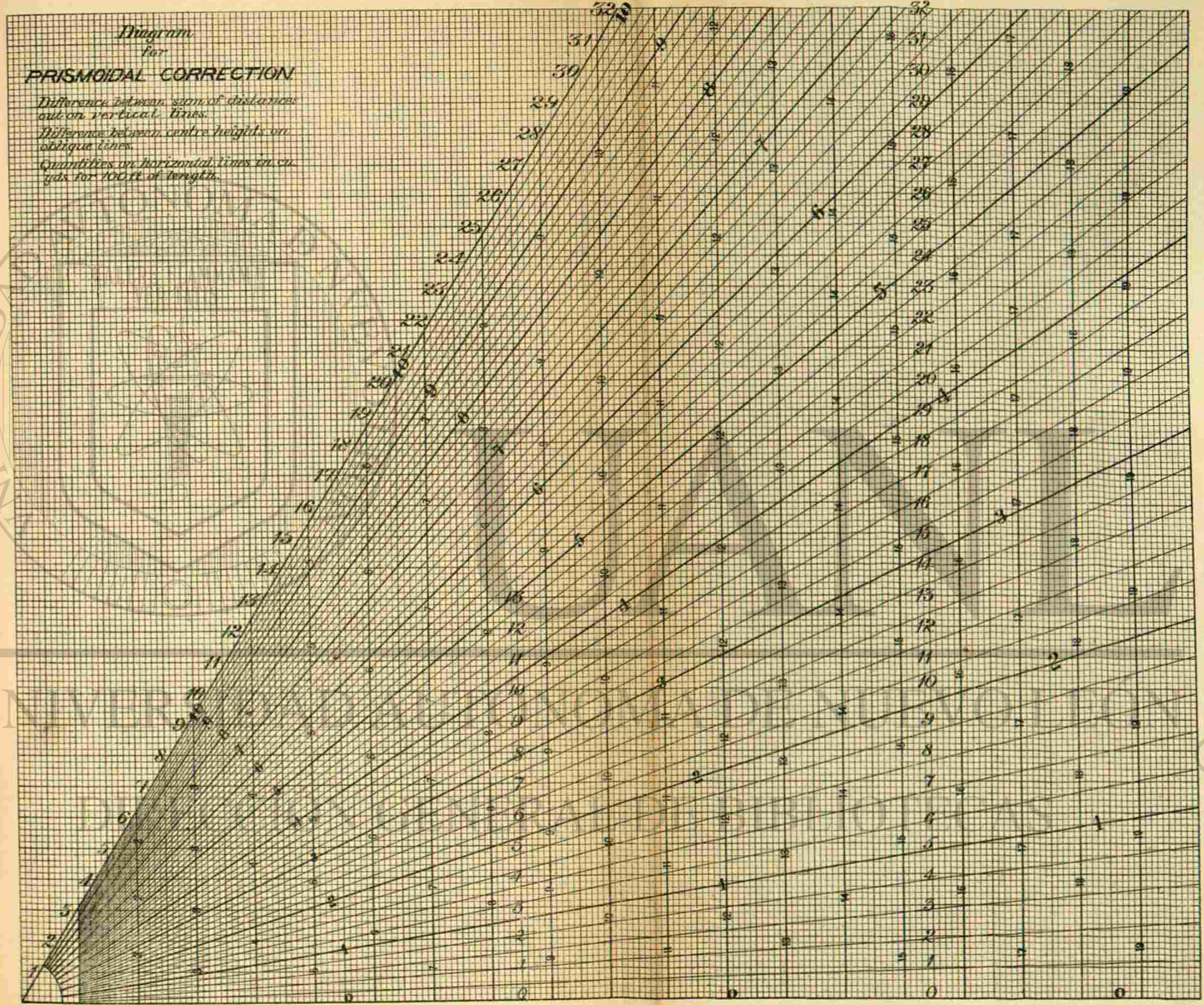
	0		.1		.2		.3		.4		
	L	C	L	C	L	C	L	C	L	C	
0	0	9.3	3.7	9.4	7.5	9.5	11.4	9.7	15.3	9.8	0
1	39.8	10.6	44.1	10.8	48.4	10.9	52.8	11.1	57.3	11.2	1
2	85.2	12.0	90.0	12.2	94.9	12.3	99.9	12.5	104.9	12.6	2
3	136.1	13.4	141.5	13.6	147.0	13.7	152.5	13.8	158.0	14.0	3
4	192.6	14.8	198.5	15.0	204.6	15.1	210.6	15.2	216.7	15.4	4
5	254.6	16.2	261.1	16.3	267.7	16.5	274.3	16.6	281.0	16.8	5
6	322.2	17.6	329.3	17.7	336.4	17.9	343.6	18.0	350.8	18.1	6
7	395.4	19.0	403.0	19.1	410.7	19.3	418.4	19.4	426.2	19.5	7
8	474.1	20.4	482.2	20.5	490.5	20.6	498.8	20.8	507.1	20.9	8
9	558.3	21.8	567.1	21.9	575.9	22.0	584.7	22.2	593.6	22.3	9
10	648.1	23.1	657.4	23.3	666.8	23.4	676.2	23.6	685.6	23.7	10
11	743.5	24.5	753.4	24.7	763.3	24.8	773.2	25.0	783.2	25.1	11
12	844.4	25.9	854.8	26.1	865.3	26.2	875.8	26.3	886.4	26.5	12
13	950.9	27.3	961.9	27.5	972.9	27.6	984.0	27.7	995.1	27.9	13
14	1063.0	28.7	1074.5	28.8	1086.0	29.0	1097.7	29.1	1109.3	29.3	14
15	1180.6	30.1	1192.6	30.2	1204.7	30.4	1216.9	30.5	1229.1	30.6	15
16	1303.7	31.5	1316.3	31.6	1329.0	31.8	1341.7	31.9	1354.5	32.0	16
17	1432.4	32.9	1445.6	33.0	1458.8	33.1	1472.1	33.3	1485.4	33.4	17
18	1566.7	34.3	1580.4	34.4	1594.2	34.5	1608.0	34.7	1621.9	34.8	18
19	1706.5	35.6	1720.8	35.8	1735.1	35.9	1749.5	36.1	1764.0	36.2	19
20	1851.9	37.0	1866.7	37.2	1881.6	37.3	1896.5	37.5	1911.6	37.6	20
21	2002.8	38.4	2018.2	38.6	2033.6	38.7	2049.1	38.8	2064.7	39.0	21
22	2159.3	39.8	2175.2	40.0	2191.2	40.1	2207.3	40.2	2223.4	40.4	22
23	2321.8	41.2	2337.8	41.3	2354.4	41.5	2371.0	41.6	2387.7	41.8	23
24	2488.9	42.6	2506.0	42.7	2523.1	42.9	2540.2	43.0	2557.5	43.1	24
25	2662.0	44.0	2679.7	44.1	2697.3	44.3	2715.1	44.4	2732.9	44.5	25
	0		.1		.2		.3		.4		

	.5		.6		.7		.8		.9		
	L	C	L	C	L	C	L	C	L	C	
0	19.2	10.0	23.2	10.1	27.3	10.2	31.4	10.4	35.6	10.5	0
1	61.8	11.3	66.4	11.5	71.0	11.6	75.7	11.8	80.4	11.9	1
2	110.0	12.7	115.1	12.9	120.2	13.0	125.5	13.1	130.8	13.3	2
3	163.7	14.1	169.3	14.3	175.1	14.4	180.9	14.5	186.7	14.7	3
4	222.9	15.5	229.1	15.6	235.4	15.8	241.8	15.9	248.2	16.1	4
5	287.7	16.9	294.5	17.0	301.4	17.2	308.3	17.3	315.2	17.5	5
6	358.1	18.3	365.4	18.4	372.8	18.6	380.3	18.7	387.8	18.8	6
7	434.0	19.7	441.9	19.8	449.9	20.0	457.9	20.1	466.0	20.2	7
8	515.5	21.1	524.0	21.2	532.5	21.3	541.0	21.5	549.7	21.6	8
9	602.5	22.5	611.6	22.6	620.6	22.7	629.7	22.9	638.9	23.0	9
10	695.1	23.8	704.7	24.0	714.3	24.1	724.0	24.3	733.7	24.4	10
11	793.3	25.2	803.4	25.4	813.6	25.5	823.8	25.6	834.1	25.8	11
12	897.0	26.6	907.7	26.8	918.4	26.9	929.2	27.0	940.0	27.2	12
13	1006.2	28.0	1017.5	28.1	1028.8	28.3	1040.1	28.4	1051.5	28.6	13
14	1121.1	29.4	1132.9	29.5	1144.7	29.7	1156.6	29.8	1168.5	30.0	14
15	1241.4	30.8	1253.8	30.9	1266.2	31.1	1278.6	31.2	1291.1	31.3	15
16	1367.4	32.2	1380.3	32.3	1393.2	32.5	1406.2	32.6	1419.3	32.7	16
17	1498.8	33.6	1512.3	33.7	1525.8	33.8	1539.4	34.0	1553.0	34.1	17
18	1635.9	35.0	1649.9	35.1	1664.0	35.2	1678.1	35.4	1692.2	35.5	18
19	1778.5	36.3	1793.0	36.5	1807.7	36.6	1822.3	36.8	1837.1	36.9	19
20	1926.6	37.7	1941.7	37.9	1956.9	38.0	1972.1	38.1	1987.4	38.3	20
21	2080.3	39.1	2096.0	39.3	2111.7	39.4	2127.5	39.5	2143.4	39.7	21
22	2239.6	40.5	2255.8	40.6	2272.1	40.8	2288.4	40.9	2304.8	41.1	22
23	2404.4	41.9	2421.2	42.0	2438.0	42.2	2454.9	42.3	2471.9	42.5	23
24	2574.8	43.3	2592.1	43.4	2609.5	43.6	2627.0	43.7	2644.5	43.8	24
25	2750.7	44.7	2768.6	44.8	2786.5	45.0	2804.6	45.1	2822.6	45.2	25
	.5		.6		.7		.8		.9		

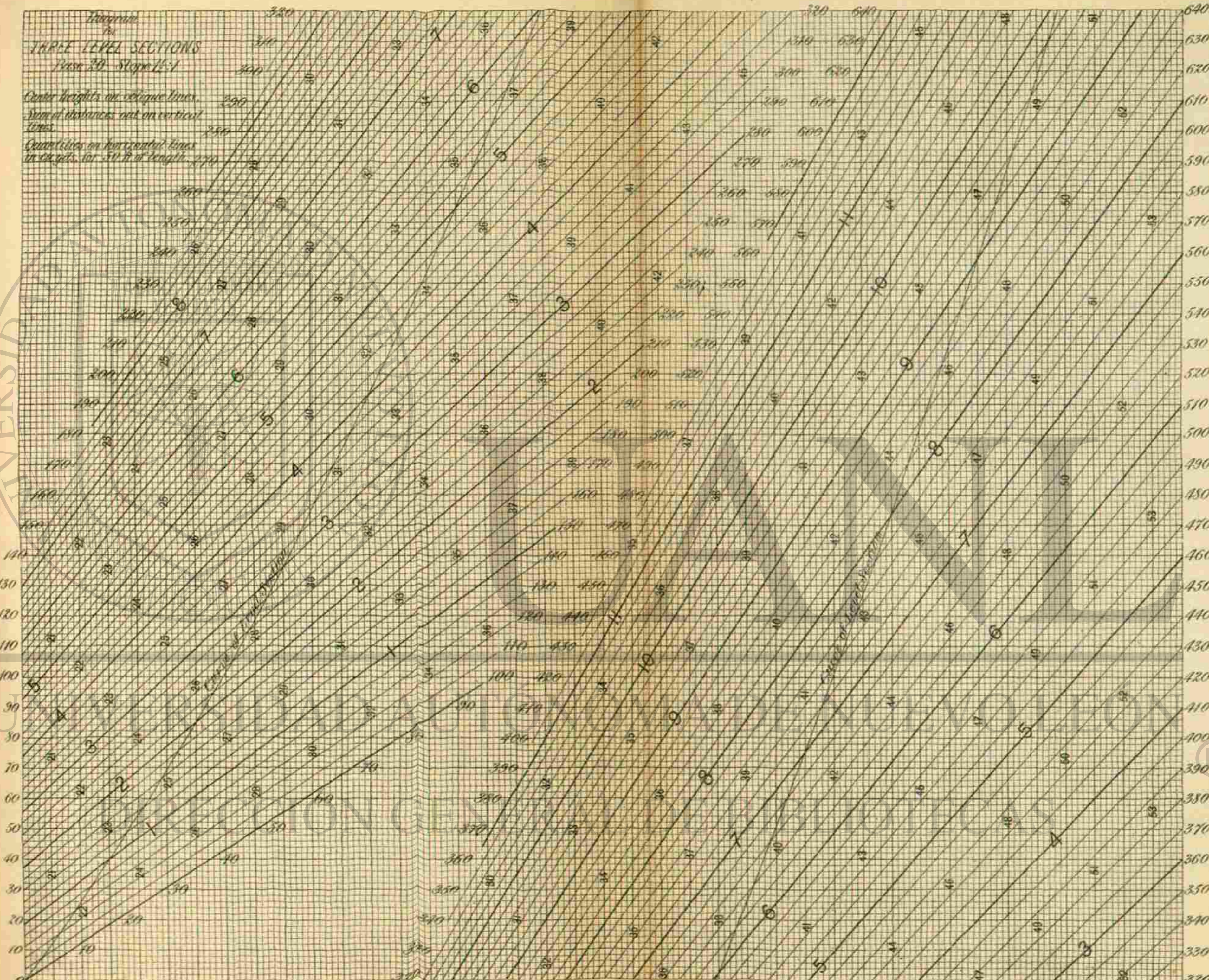
Base 14' Slope $1\frac{1}{2}$ to 1. Table for Three Level Sections.

	0		.1		.2		.3		.4		
	L	C	L	C	L	C	L	C	L	C	
0	0	6.5	2.6	6.6	5.3	6.8	8.0	6.9	10.8	7.0	0
1	28.7	7.9	31.9	8.0	35.1	8.1	38.4	8.3	41.7	8.4	1
2	63.0	9.3	66.7	9.4	70.5	9.5	74.3	9.7	78.2	9.8	2
3	102.8	10.6	107.1	10.8	111.4	10.9	115.8	11.1	120.3	11.2	3
4	148.1	12.0	153.0	12.2	157.9	12.3	162.8	12.5	167.9	12.6	4
5	199.1	13.4	204.5	13.6	209.9	13.7	215.4	13.8	221.0	14.0	5
6	255.6	14.8	261.5	15.0	267.5	15.1	273.6	15.2	279.7	15.4	6
7	317.6	16.2	324.1	16.3	330.7	16.5	337.3	16.6	344.0	16.8	7
8	385.2	17.6	392.2	17.7	399.4	17.9	406.5	18.0	413.8	18.1	8
9	458.3	19.0	466.0	19.1	473.6	19.3	481.4	19.4	489.1	19.5	9
10	537.0	20.4	545.2	20.5	553.4	20.6	561.7	20.8	570.1	20.9	10
11	621.3	21.8	630.0	21.9	638.8	22.0	647.7	22.2	656.6	22.3	11
12	711.1	23.1	720.4	23.3	729.7	23.4	739.1	23.6	748.6	23.7	12
13	806.5	24.5	816.3	24.7	826.2	24.8	836.2	25.0	846.2	25.1	13
14	907.4	25.9	917.8	26.1	928.3	26.2	938.8	26.3	949.3	26.5	14
15	1013.9	27.3	1024.8	27.5	1035.9	27.6	1046.9	27.7	1058.0	27.9	15
16	1125.9	28.7	1137.4	28.8	1149.0	29.0	1160.6	29.1	1172.3	29.3	16
17	1243.5	30.1	1255.6	30.2	1267.7	30.4	1279.9	30.5	1292.1	30.6	17
18	1366.7	31.5	1379.3	31.6	1392.0	31.8	1404.7	31.9	1417.5	32.0	18
19	1495.4	32.9	1508.5	33.0	1521.8	33.1	1535.1	33.3	1548.4	33.4	19
20	1629.6	34.3	1643.4	34.4	1657.1	34.5	1671.0	34.7	1684.9	34.8	20
21	1769.4	35.6	1783.7	35.8	1798.1	35.9	1812.5	36.1	1826.9	36.2	21
22	1914.8	37.0	1929.7	37.2	1944.6	37.3	1959.5	37.5	1974.5	37.6	22
23	2065.7	38.4	2081.1	38.6	2096.6	38.7	2112.1	38.8	2127.7	39.0	23
24	2222.2	39.8	2238.2	40.0	2254.2	40.1	2270.2	40.2	2286.4	40.4	24
25	2384.3	41.2	2400.8	41.3	2417.3	41.5	2434.0	41.6	2450.6	41.8	25
	0		.1		.2		.3		.4		

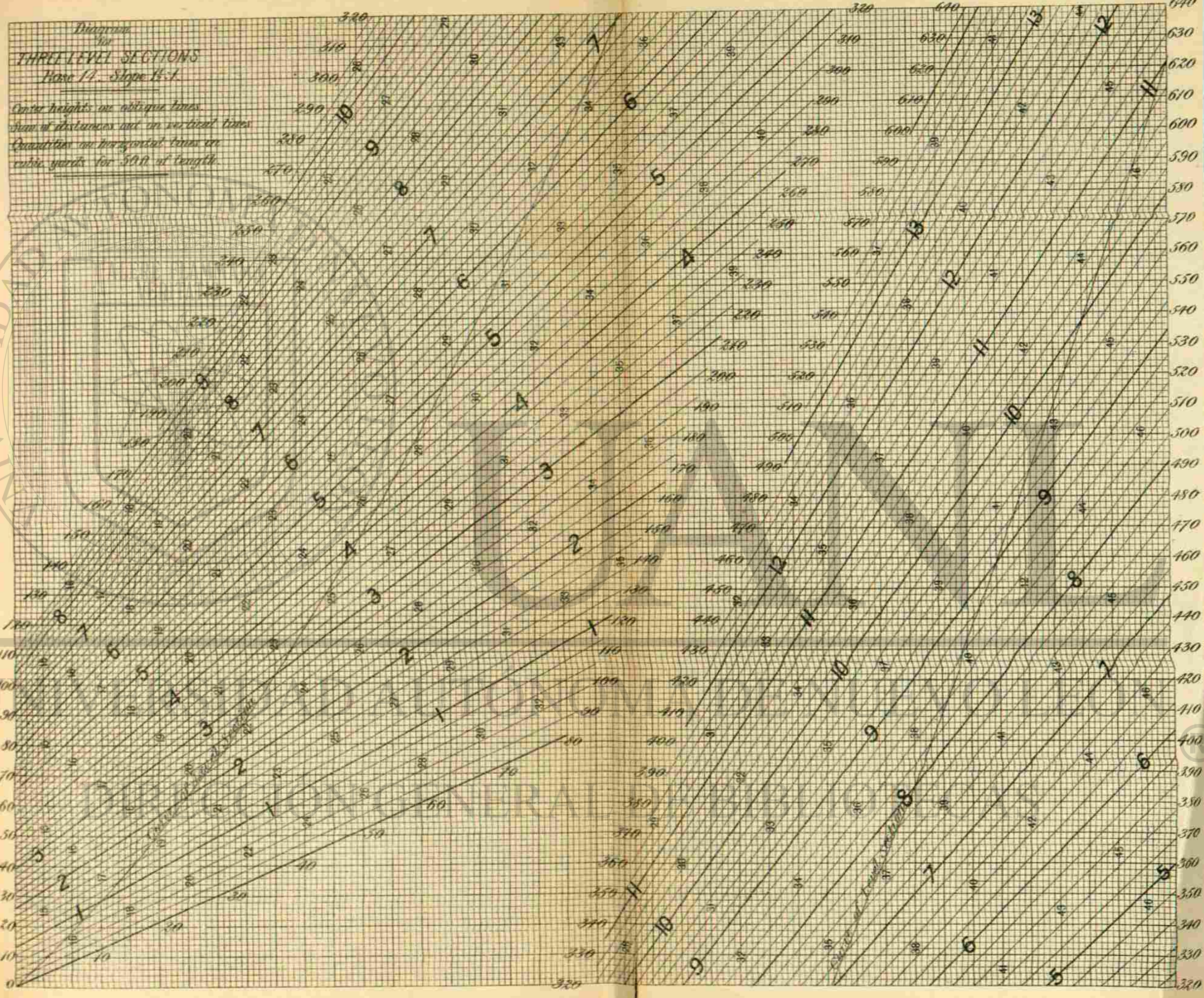
	.5		.6		.7		[illegible]		.9		
	L	C	L	C	L	C	L	C	L	C	
0	13.7	7.2	16.6	7.3	19.5	7.5	22.5	7.6	25.6	7.7	0
1	45.1	8.6	48.6	8.7	52.1	8.8	55.7	9.0	59.3	9.1	1
2	82.2	10.0	86.2	10.1	90.2	10.2	94.4	10.4	98.5	10.5	2
3	124.8	11.3	129.3	11.5	134.0	11.6	138.6	11.8	143.4	11.9	3
4	172.9	12.7	178.0	12.9	183.2	13.0	188.4	13.1	193.7	13.3	4
5	226.6	14.1	232.3	14.3	238.0	14.4	243.8	14.5	249.7	14.7	5
6	285.9	15.5	292.1	15.6	298.4	15.8	304.7	15.9	311.1	16.1	6
7	350.7	16.9	357.5	17.0	364.3	17.2	371.2	17.3	378.2	17.5	7
8	421.1	18.3	428.4	18.4	435.8	18.6	443.3	18.7	450.8	18.8	8
9	497.0	19.7	504.9	19.8	512.8	20.0	520.9	20.1	528.9	20.2	9
10	578.5	21.1	586.9	21.2	595.4	21.3	604.0	21.5	612.6	21.6	10
11	665.5	22.5	674.5	22.6	683.6	22.7	692.7	22.9	701.9	23.0	11
12	758.1	23.8	767.7	24.0	777.3	24.1	787.0	24.3	796.7	24.4	12
13	856.2	25.2	866.4	25.4	876.5	25.5	886.8	25.6	897.1	25.8	13
14	960.0	26.6	970.6	26.8	981.4	26.9	992.1	27.0	1003.0	27.2	14
15	1069.2	28.0	1080.4	28.1	1091.7	28.3	1103.1	28.4	1114.5	28.6	15
16	1184.0	29.4	1195.8	29.5	1207.7	29.7	1219.6	29.8	1231.5	30.0	16
17	1304.4	30.8	1316.7	30.9	1329.1	31.1	1341.6	31.2	1354.1	31.3	17
18	1430.3	32.2	1443.2	32.3	1456.2	32.5	1469.2	32.6	1482.2	32.7	18
19	1561.8	33.6	1575.3	33.7	1588.8	33.8	1602.3	34.0	1616.0	34.1	19
20	1693.8	35.0	1712.9	35.1	1726.9	35.2	1741.0	35.4	1755.2	35.5	20
21	1841.4	36.3	1856.0	36.5	1870.6	36.6	1885.3	36.8	1900.0	36.9	21
22	1989.6	37.7	2004.7	37.9	2019.9	38.0	2035.1	38.1	2050.4	38.3	22
23	2143.3	39.1	2159.0	39.3	2174.7	39.4	2190.5	39.5	2206.3	39.7	23
24	2302.5	40.5	2318.8	40.6	2335.1	40.8	2351.4	40.9	2367.8	41.1	24
25	2467.4	41.9	2484.1	42.0	2501.0	42.2	2517.9	42.3	2534.8	42.5	25
	.5		.6		.7		.8		.9		



Providence Lith. Co.



Providence Lith. Co.



Providence Lith. Co.