ELEMENTS

OF

WRITTEN ARITHMETIC.

BY CHARLES DAVIES, LL.D.,

AUTHOR OF A FULL COURSE OF MATHEMATICS

New York:

PUBLISHED BY BARNES & BURR,

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CHICAGO: GEORGE SHERWOOD.

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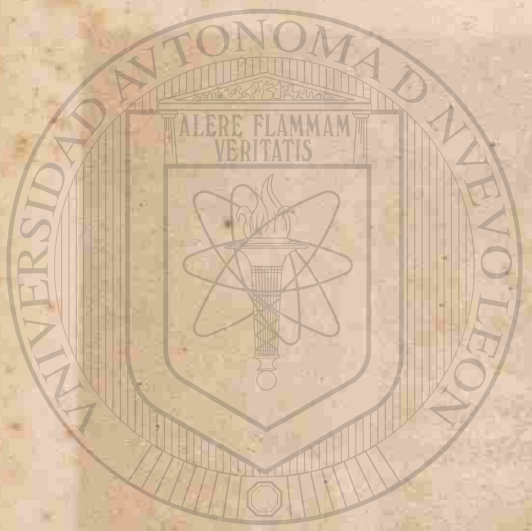

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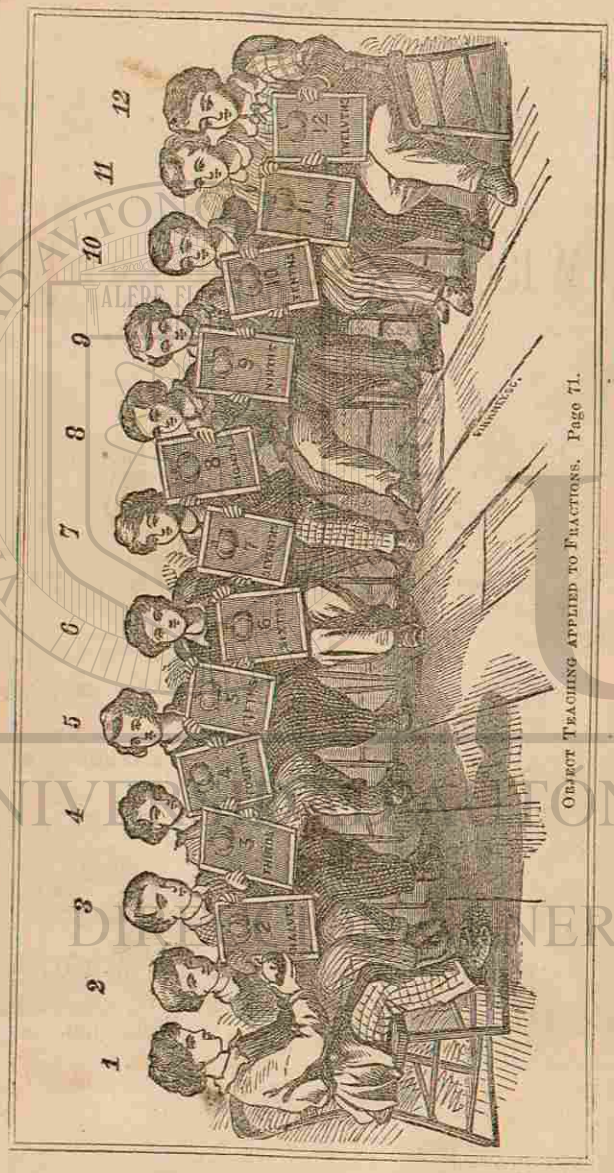

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PREFACE.

IT has become a settled principle in the science of teaching, that abstract principles and their elementary combinations must be first presented to the mind by the aid of sensible objects.

The eye is an active and an efficient agent in the acquisition of elementary knowledge. The elementary ideas of Number and Space, are acquired from things which are seen and handled, and the logical combinations of these elementary ideas make up the entire science of Mathematics.

It is the design of the present work, to present to the mind of the pupil the art, and to some extent, the science of Arithmetic, by a series of carefully constructed formulas of operation, with simple and concise rules. It is believed, that for beginners, the analysis, which explains the reasons of arithmetical operations, can, in most cases, be inferred from the operations themselves, and that elaborate explanations are hindrances, rather than aids.

The practical value of arithmetical instruction is dependent on the facility and accuracy of performing the operations. If, therefore, the operations are so arranged as to suggest the rules, the *practical* becomes the moving principle, and the rule, the consequence. This method of presenting the subject, suggests to the mind all the operations *through the eye*, and not *through the rule*. It is the method of *reading figures*, extended to the *reading of formulas*.

Although this book does not form a connecting link in the series, it should, nevertheless, be used after the Primary. It should, also, if convenient, be studied in connection with the Intellectual Arithmetic. Thus, the Formulas of Operation, the Rules, and the Analyses, will be presented separately, in their natural order, and in their proper connections.

FISHKILL LANDING,
July, 1863.

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ELEMENTS OF ARITHMETIC.

Definitions.

1. A Unit is a single thing, or one.

2. A Number is a unit, or a collection of units.

3. Arithmetic is the science of numbers and the art of computation.

4. An Operation is something done with numbers.

5. An Answer is the result of a correct operation.

6. A Rule is the direction for performing an operation.

Operations of Arithmetic.

7. There are five fundamental operations of Arithmetic: Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

NOTATION AND NUMERATION.

8. Notation is the method of expressing numbers, either by letters or figures.

Numeration is the art of reading, correctly, any number expressed by letters or figures.

There are two methods of Notation: one by letters, and one by figures. The method by letters is called, the *Roman Notation;* the method by figures is called, the *Arabic Notation.*

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Roman Notation.

9. The Roman notation employs seven capital letters. They express the following values:

I	V	X	L	C	D	M
One,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

All other numbers are expressed by combining these letters:

1. Every time a letter is repeated, the number which it denotes is repeated.

2. If a letter denoting a less number be written on the right of one denoting a greater, the number expressed will be denoted by the sum of the numbers.

3. If a letter denoting a less number be written on the left of one denoting a greater, the number expressed will be the difference of the numbers.

4. A dash (—), placed over a letter, increases the number for which it stands, a thousand times.

Roman Table.

I	. .	One.	LXXX	.	Eighty.
II	. .	Two.	XC	. .	Ninety.
III	. .	Three.	C	. .	One hundred.
IV	. .	Four.	CC	. .	Two hundred.
V	. .	Five.	CCC	. .	Three hundred.
VI	. .	Six.	CCCC	.	Four hundred.
VII	. .	Seven.	D	. .	Five hundred.
VIII	. .	Eight.	DC	. .	Six hundred.
IX	. .	Nine.	DCC	. .	Seven hundred.
X	. .	Ten.	DCCC	.	Eight hundred.
XX	. .	Twenty.	DCCCC	.	Nine hundred.
XXX	. .	Thirty.	M	. .	One thousand.
XL	. .	Forty.	MD	. .	Fifteen hundred.
L	. .	Fifty.	MM	. .	Two thousand.
LX	. .	Sixty.	$\overline{V}$	. .	Five thousand.
LXX	. .	Seventy.	$\overline{X}$	. .	Ten thousand.

Arabic Notation.

10. Arabic Notation is the method of expressing numbers by figures. Ten figures are used. They are,

0	1	2	3	4	5	6	7	8	9
Naught,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

All numbers are expressed by these figures, employed singly, or in combination. When in combination, the value of each is determined by the following principles:

1. That the same figure may express different values;

2. That it expresses its least value, when placed in the first place on the right;

3. That it expresses, when placed in the 2d place from the right, a value ten times as great as when in the 1st place; when put in the 3d place, a value 10 times as great as in the 2d place, and so on for every place to the left.

4. The names of the places are the following:

Billions.	Hundreds of Millions,	Tens of Millions,	Millions,	Hundreds of Thousands,	Tens of Thousands,	Thousands,	Hundreds,	Tens,	Units,
1	1	1	1	1	1	1	1	1	1

5. The value of 1 in each place is shown by the Table:

10 Units	. .	make	1 Ten.
10 Tens	. .	"	1 Hundred.
10 Hundreds	.	"	1 Thousand.
10 Thousands	.	"	1 Hundred thousand.
10 Hundred thousands		"	1 Million.
10 Millions	. .	"	1 Ten million.
&c.,			&c.

Express the following numbers by figures:

1. Forty-six. Also, ninety-nine. Also, five hundred.
2. Write four thousand. Also, six thousand and eleven.
3. Write nine thousand, eight hundred and seventy-six.

11. We have seen that, when a number is expressed by a single figure, as 8, it is read by its name, eight.

When a number is expressed by two figures, as 26, it is read from the right:

Tens.	Units.
2	6

that is, 6 units and 2 tens; and from the left, twenty-six.

When a number is expressed by three figures, as 375, it is read from the right:

Hundreds.	Tens.	Units.
3	7	5

that is, 5 units, 7 tens, and 3 hundreds; and from the left, three hundred and seventy-five.

When a number is expressed by four figures, as 6037, it is read:

Thousands.	Hundreds.	Tens.	Units.
6	0	3	7

that is, 7 units, 3 tens, no hundreds, and 6 thousands, or six thousand and thirty-seven.

1. Write all the numbers from three hundred and fifty-five, to three hundred and fifty-six, and read them.

2. Write all the numbers from six hundred and seventy, to six hundred and seventy-five, and read them.

3. Write all the numbers from one thousand and five, to one thousand and ten, and read them.

4. Write all the numbers from six thousand and eleven, to six thousand and twenty, and read them.

5. Write all the numbers from seven thousand and forty, to seven thousand and fifty, and read them.

Numeration Table.

6th Period. Quadrillions.			5th Period. Trillions.			4th Period. Billions.			3d Period. Millions.			2d Period. Thousands.			1st Period. Units.		
Hundreds of Quadrillions.			Hundreds of Trillions			Hundreds of Billions			Hundreds of Millions			Hundreds of Thousands			Hundreds .		
Tens of Quadrillions	.		Tens of Trillions	.		Tens of Billions	.		Tens of Millions	.		Tens of Thousands .			Tens .		
Quadrillions	.		Trillions	.		Billions	.		Millions	.		Thousands	.		Units		
																	5.
																4	9.
															3	0	7.
														6,	0	2	3.
													8	7,	4	2	1.
												4	2	4,	0	8	7.
											4,	0	0	1,	4	4	3.
										4	3,	2	1	0,	4	6	0.
									5	4	8,	0	0	0,	0	8	7.
								6,	2	4	5,	2	8	9,	4	2	1.
							7	2,	5	4	9,	1	3	6,	8	2	2.
						8	9	4,	6	0	2,	0	4	3,	2	8	8.
					7,	6	4	1,	0	0	0,	9	0	7,	4	5	6.
				8	4,	9	1	2,	8	7	6,	4	1	9,	2	8	5.
			9	1	2,	7	6	1,	2	5	7,	3	2	7,	8	2	6.
		6,	4	0	7,	2	1	2,	9	3	6,	8	7	6,	5	4	1.
	5	7,	2	8	9,	6	7	8,	5	4	1,	2	9	7,	3	1	3.
9	2	0,	3	2	3,	8	4	2,	7	6	8,	3	1	9,	6	7	5.

1. The first line, directly under units, is read, five; the second, forty-nine; the third, three hundred and seven; the fourth, six thousand and twenty-three, and so on.

2. Numbers expressed by more than three figures, are separated, by a comma, into periods of three figures each, beginning at the right.

3. Each period contains three figures, except the one at the left, which may contain one, two, or three figures.

4. The pupil should be required to commit, thoroughly, the names of the periods, and to read the figures fluently.

Exercises in Notation and Numeration.

1. Write 1 in each place of the first period, and read the number.

2. Write 3 in the 3d place of the first period, and 0's in the other places, and read the number.

3. Write 8 units, 5 tens, and 6 hundreds, and read the number.

4. Write 9 hundreds, 6 tens, and 7 units, and read.

5. Write 4 in each place of the first two periods, and read.

6. Write 3 in each place of the second period, and a 0 in each place of the first, and read.

7. Write 7 thousand, 6 hundred and fifty-five.

8. Write 5 in each place of the 3d period, 2 in each place of the 2d, and 1 in each place of the 1st, and read.

9. Write six hundred and four millions, ninety-five thousand, three hundred and forty-two.

10. Write 421 billions, a 0 in each place of the 3d period, and four thousand and sixteen, and read.

11. Write 0 in each place of the first three periods, and 5 in the first place of the 4th, and read the number.

Orders of Units.

1	is called . . .	a unit of the	1st order.
10, or 1 ten,	is called	" "	2d order.
100, or 1 hundred,	is	" "	3d order.
1000, or 1 thousand,	is	" "	4th order.

And so on for the higher numbers: hence,

Units of the first order are written in the first place, at the right;

Units of the second order, in the second place;

Units of the third order, in the third place;

Units of the fourth order, in the fourth place; and so on for places to the left.

Examples in Writing and Reading.

1. Write 6 units of the first order.

2. Write four units of the first order with five of the second.

3. Write nine units of the 3d order with 3 of the second and 1 of the first, and read.

4. Write 8 units of the 3d order with none of the second and six of the first.

5. Write 7 units of the 5th order with 2 of the second.

6. Write 5 units of the 6th order with 4 of the 4th, 3 of the 3d, and 1 of the 1st, and read.

7. Write 4 units of the 6th order with 5 units of the 1st order, and read.

8. Write 3 units of each order to the 6th, and read the number.

9. Write 9 units of each order to the 9th, and read the number.

Analysis of Numbers.

Let the pupil point off and read the following numbers; then write them in words.

1.	85	7.	50482	13.	275047078
2.	137	8.	602142	14.	4127043047
3.	6704	9.	8969797	15.	9730417071
4.	3678	10.	71462108	16.	10470621048
5.	30421	11.	104000009	17.	27049632101
6.	200410	12.	570478010	18.	31047021412

NOTE.—Let each of the above examples, after having been written on the blackboard, be analyzed as a class exercise; thus,

Ex. 1. How many tens in 85? How many units over?

2. In 137, how many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place? How many tens in the number? How many units?

3. In 6704, how many thousands in the thousands place? How many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place?

ADDITION.

12. Addition is the operation of finding the *sum* of two or more numbers.

The Sum contains as many units as there are in all the numbers added.

Addition Table.

2 and 0 are 2	3 and 0 are 3	4 and 0 are 4	5 and 0 are 5
2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6
2 and 2 are 4	3 and 2 are 5	4 and 2 are 6	5 and 2 are 7
2 and 3 are 5	3 and 3 are 6	4 and 3 are 7	5 and 3 are 8
2 and 4 are 6	3 and 4 are 7	4 and 4 are 8	5 and 4 are 9
2 and 5 are 7	3 and 5 are 8	4 and 5 are 9	5 and 5 are 10
2 and 6 are 8	3 and 6 are 9	4 and 6 are 10	5 and 6 are 11
2 and 7 are 9	3 and 7 are 10	4 and 7 are 11	5 and 7 are 12
2 and 8 are 10	3 and 8 are 11	4 and 8 are 12	5 and 8 are 13
2 and 9 are 11	3 and 9 are 12	4 and 9 are 13	5 and 9 are 14
2 and 10 are 12	3 and 10 are 13	4 and 10 are 14	5 and 10 are 15
6 and 0 are 6	7 and 0 are 7	8 and 0 are 8	9 and 0 are 9
6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
6 and 2 are 8	7 and 2 are 9	8 and 2 are 10	9 and 2 are 11
6 and 3 are 9	7 and 3 are 10	8 and 3 are 11	9 and 3 are 12
6 and 4 are 10	7 and 4 are 11	8 and 4 are 12	9 and 4 are 13
6 and 5 are 11	7 and 5 are 12	8 and 5 are 13	9 and 5 are 14
6 and 6 are 12	7 and 6 are 13	8 and 6 are 14	9 and 6 are 15
6 and 7 are 13	7 and 7 are 14	8 and 7 are 15	9 and 7 are 16
6 and 8 are 14	7 and 8 are 15	8 and 8 are 16	9 and 8 are 17
6 and 9 are 15	7 and 9 are 16	8 and 9 are 17	9 and 9 are 18
6 and 10 are 16	7 and 10 are 17	8 and 10 are 18	9 and 10 are 19

1 and 1 are how many?
2 added to 4, how many?
3 and 9 cents, how many?
9 and 10 are how many?
8 and 5 are how many?
7 and 4 are how many?
9 and 9 are how many?
8 and 5 are how many?
7 and 6 are how many?
8 and 7 are how many?
9 and 6 are how many?
7 and 10 are how many?
5 and 7 are how many?
6 and 8 are how many?

Of the Signs.

13. The sign, +, is called *plus*, which signifies, more. When placed between two numbers, it denotes that they are to be added.

The sign, =, is called the sign of equality. When placed between two numbers, it denotes that they are equal; that is, that they contain the same number of units. Thus, $3 + 2 = 5$, and is read, 3 plus 2 equals 5.

$3 + 7 =$ how many?
$1 + 2 + 3 =$ how many?
$3 + 4 + 5 + 1 =$ how many?
$1 + 0 + 2 + 3 + 3 =$ how many?

1. 1+3, are how many?
2. 1+5, are how many?
3. 6+0, are how many?
4. 7+9, are how many?
5. 8+7, are how many?

6. 1+2+3= how many?
7. 1+6+0= how many?
8. 9+5, are how many?
9. 10+5, are how many?
10. 1+9+10= how many?

21. 6+9= how many?
22. 7+5= how many?
23. 9+0+1= how many?
24. 0+3+12= how many?
25. 9+6, are how many?

26. 1+5+6, are how many?
27. 3+9= how many?
28. 7+5= how many?
29. 9+0= how many?
30. 6+5= how many?

31. How many fingers are 4 fingers and 2 fingers?

32. If an apple costs 3 cents, and an orange 5 cents, what is the cost of both?

33. What two numbers, added together, will make 8?

34. What two numbers, added together, will make 10?

35. A man earned 5 dollars on Monday, 6 dollars on Tuesday, and 7 dollars on Wednesday: how many dollars did he earn in the three days?

36. If a man spends 4 dollars for boots, 10 dollars for a coat, and 6 dollars for a hat, how much does he spend in all?

Exercises for the Slate or Blackboard.

14. Pupils in Arithmetic should be taught, from the very commencement, to *read* the figures. By reading, we mean the use of those words only which declare the final results.

For example, 2 and 2 are 4. The word *four* names the *result* arising from adding 2 and 2 together.

Having written the following, and similar examples, on the slate or blackboard, let the pupils, separately and in concert, pronounce the sum of each column.

1.	0	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1	1
2.	2	0	4	3	9	8	6	7	5	7
	3	4	5	9	0	4	3	8	9	0
3.	9	0	4	5	7	6	4	2	1	3
	7	4	6	9	8	5	3	0	2	1
4.	1	4	0	3	6	7	8	9	5	2
	2	3	4	5	6	4	3	8	0	9
5.	4	9	7	5	2	7	3	0	2	8
	3	0	6	0	1	9	7	6	0	1
6.	0	3	4	5	6	8	9	2	1	0
	3	9	0	3	8	7	4	6	0	5
	4	3	7	7	0	2	9	3	9	7
7.	9	8	6	5	0	4	2	3	1	9
	3	2	7	3	9	6	7	0	3	8
	4	8	0	6	7	8	4	9	6	7

1. What is the sum of 387 dollars, 579 dollars, and 793 dollars?

The numbers in this example are written down and added according to the following

OPERATION.

	3 8 7
	5 7 9
	7 9 3
	1 7 5 9

Rule.

I. *Write the numbers to be added, so that units of the same value shall fall in the same column; that is, units under units, tens under tens,* &c.

II. *Add the column of units; set down the units of the sum, and then add the tens, if any, to the next column.*

III. *Add each column in the same way, and set down the entire sum of the last column.*

Proof.

Begin at the top of the units column, and add all the columns downwards, carrying from one column to the other, as when the columns were added upwards. If the two results agree, the work is supposed to be right.

Examples.

(1.)	(2.)	(3.)	(4.)	(5.)
870	241	998	306	327
312	302	909	999	401
(6.)	(7.)	(8.)	(9.)	(10.)
604	407	974	49	409
709	369	302	906	599
999	421	981	429	999

(11.)	(12.)	(13.)	(14.)
3704	8476	27047	81434
2905	9703	29091	92069
6093	6949	67021	71470

(15.)	(16.)	(17.)	(18.)
67041	91224	87263	97046
3027	90712	9126	21049
975	3049	445	3627
69	274	371	294

(19.)	(20.)	(21.)	(22.)
814724	493279	940045	4096274
31270	210316	97214	670421
9089	97632	8526	29365
416	7421	914	8647
79	697	89	310

(23.)	(24.)	(25.)	(26.)
370414	38076	451321	5976428
279143	91742	910487	5849207
970492	6849	614823	6408302
380421	6927	712104	876041
91273	874	87415	292804
94870	674	9747	380496
69044	914	8741	390419
79050	815	9168	81468
60704	460	8704	81410
8912	860	304	71487
2704	29	963	87049
7047	27	603	6704
8433	41	974	327
7411	91	89	897
674	9	9	68

27. What is the sum of 297+496+3764+101+9056?

28. Find the sum of 56+479+2764, increased by the sum of 960+575+2300+100+205.

29. What is the sum of 54046 + 75 + 870423 + 999 + 87047 + 910468 + 874863 + 47049 + 372141?

30. What is the sum of 270999 + 310467 + 21 + 375 + 888880 + 9794967 + 73758941 + 47049 + 740416?

31. What is the sum of 67041 + 80046 + 97041 + 0 + 30967 + 814675 + 704069 + 70412704 + 90704 + 72304 + 99999?

32. Find the sum of four hundred and sixty-five; one thousand, three hundred and thirty-three; four hundred and twenty-nine thousand, eight hundred and eight; forty-four millions, nine hundred and ninety thousand and sixty; five hundred and sixteen; and seven hundred thousand, seven hundred and seventy-five.

33. Find the sum of twenty-seven thousand, nine hundred and sixteen; nine millions, nine thousand and nine; one hundred and fifty-six millions, eight hundred and twenty-six thousand, eight hundred and eighty-seven; twelve hundred and forty-nine; sixty-nine thousand and sixty-nine; thirty-four; and two hundred and sixty-two.

34. Find the sum of six hundred and seventy-three billions, three hundred and twenty millions, two thousand, six hundred and one; three billions, nine hundred and sixty-seven millions, eighty-nine thousand and six; eighty-seven thousand, nine hundred and twelve; and one hundred and eleven.

35. Find the sum of forty-nine quadrillions, two thousand billions, six hundred millions, four hundred and sixty-nine thousand and seventeen; ninety-five quadrillions, fifty-nine millions, four hundred and nine thousand, six hundred and fifty-nine.

Practical Questions.

1. If an apple costs 2 cents, an orange 6 cents, and a lemon 4 cents, what will the three cost?

2. What two numbers, added together, make 12?

3. If James pays 15 cents for a top, 75 cents for a knife, and 87 cents for a book, what does he pay in all?

4. John was born in the year 1840: in what year was he 21 years of age?

5. What is the cost of 3 city lots, the 1st costing 1457 dollars, the 2d 1259 dollars, and the 3d 965 dollars?

6. Add together five thousand nine hundred and sixty-five, 8759, and twenty thousand 846.

7. If two persons travel from the same point in opposite directions, the one 7 miles and the other 9 miles, how far apart will they then be?

8. James, after giving away 6 cents, spending 8 cents for a pie, and losing 9 cents, had 5 cents left: how much had he at first?

9. A boy who had bought a ball for 6 cents, wishes to sell it, so as to gain 4 cents: for what must he sell it?

10. A merchant bought a barrel of flour for 7 dollars, and a tub of butter for 9 dollars: for what must he sell the two, that he may gain 6 dollars on both?

11. A grocer bought some sugar for 8 dollars, and some tea for 7 dollars: what amount will he receive for the two, if he gains 2 dollars on the sugar and 3 dollars on the tea?

12. James and Joseph leave home in the morning, with an equal amount of money; during the day, James gains 10 dollars, and Joseph loses 7 dollars: at the close of the day, how much more has the one than the other?

13. James is 16 years old, and John is 24 years older: how old is John?

14. A merchant paid 450 dollars for sugar, 692 dollars for teas, 275 dollars for coffee, 3760 dollars for flour, and 105 dollars for soap: what did he pay for all?

15. Suppose a merchant has 3756 dollars in bank-bills, 4793 dollars in gold, 264 dollars in silver, and 5 dollars in cents: how much has he?

16. A farmer, wishing to build a barn, estimated its cost as follows: for the lumber, 490 dollars; carpenter's work, 360 dollars; hardware, 75 dollars, and painting, 124 dollars: what was the estimated cost of the barn?

17. If a person pays 750 dollars for a lot of ground, 3986 dollars for the house on it, 642 dollars for furniture, and 975 dollars for the library, what is the cost of the whole?

18. A man paid 275 dollars for a horse: for what must he sell it, that he may gain 45 dollars?

19. A person who had received a certain sum of money, paid out 675 dollars, and had 7835 dollars left: what amount did he receive?

20. What would be the wages of a year, if a person receives 75 dollars per month for 6 months, and 90 dollars per month for the remaining 6 months?

21. James was born in 1834, and Samuel was born 25 years after him: in what year was Samuel born?

22. Daniel, who was born in 1812, was 37 years old when he died; and Reuben died 10 years after Daniel: in what year did Reuben die?

23. A drover bought some cattle for 4395 dollars, and, after having kept them for 3 weeks at an expense of 175 dollars, sold them at a profit of 396 dollars: for what did he sell them?

24. A merchant bought 25 barrels of flour for 150 dollars; 72 barrels, for 376 dollars; 317 barrels, for 1698 dollars; 764 barrels, for 4379 dollars. How many barrels did he buy, and what did they cost?

25. A gentleman bought a horse, a carriage, and harness: for the harness he paid 75 dollars; for the horse, 65 dol-

lars more than for the harness; and for the carriage, 172 dollars more than for the horse: what was the cost of the three?

26. A flour merchant bought a quantity of flour in St. Louis, for 5600 dollars, and sent it to New York: the freight amounted to 275 dollars, cartage to 196 dollars, storage to 50 dollars, and insurance to 25 dollars: for what must the flour be sold, to gain 800 dollars?

27. A merchant has in store, merchandise worth 25642 dollars; he has debts due him, to the amount of 5719 dollars; he has in bank 7695 dollars; he owns two houses, each worth 4965 dollars, a ship worth 35450 dollars, a farm worth 11290 dollars, and a factory worth 26475 dollars: what is his fortune?

28. A father bequeathed his fortune in the following manner: to his wife, 10600 dollars; to each of three sons, 6750 dollars; to each of 2 daughters, 4975 dollars; 2763 dollars to pay all his debts; 565 dollars to the Bible Society, and to the Education Society 725 dollars: what was the fortune?

29. A person having neglected to make a record, wished to know how many bushels of potatoes he had bought at a certain time. He had sold of them 496 bushels, had thrown away 15 bushels, given away 36 bushels, and had 247 bushels left: how many bushels had he bought?

30. The distance from Jersey City to Port Jervis is 88 miles; from Port Jervis to Deposit, 88 miles; from Deposit to Great Bend, 24 miles; from Great Bend to Binghampton, 14 miles; from Binghampton to Elmira, 59 miles; from Elmira to Corning, 18 miles; from Corning to Hornellsville, 40 miles; from Hornellsville to Olean, 64 miles; and from Olean to Dunkirk, 64 miles: what is the distance from Jersey City to Dunkirk?

SUBTRACTION.

15. Subtraction is the operation of finding the difference between two numbers.

The Difference is such a number as, added to the less, will give the greater.

Table.

1 from 1 leaves 0	2 from 2 leaves 0	3 from 3 leaves 0
1 from 2 leaves 1	2 from 3 leaves 1	3 from 4 leaves 1
1 from 3 leaves 2	2 from 4 leaves 2	3 from 5 leaves 2
1 from 4 leaves 3	2 from 5 leaves 3	3 from 6 leaves 3
1 from 5 leaves 4	2 from 6 leaves 4	3 from 7 leaves 4
1 from 6 leaves 5	2 from 7 leaves 5	3 from 8 leaves 5
1 from 7 leaves 6	2 from 8 leaves 6	3 from 9 leaves 6
1 from 8 leaves 7	2 from 9 leaves 7	3 from 10 leaves 7
1 from 9 leaves 8	2 from 10 leaves 8	3 from 11 leaves 8
1 from 10 leaves 9	2 from 11 leaves 9	3 from 12 leaves 9
1 from 11 leaves 10	2 from 12 leaves 10	3 from 13 leaves 10
4 from 4 leaves 0	5 from 5 leaves 0	6 from 6 leaves 0
4 from 5 leaves 1	5 from 6 leaves 1	6 from 7 leaves 1
4 from 6 leaves 2	5 from 7 leaves 2	6 from 8 leaves 2
4 from 7 leaves 3	5 from 8 leaves 3	6 from 9 leaves 3
4 from 8 leaves 4	5 from 9 leaves 4	6 from 10 leaves 4
4 from 9 leaves 5	5 from 10 leaves 5	6 from 11 leaves 5
4 from 10 leaves 6	5 from 11 leaves 6	6 from 12 leaves 6
4 from 11 leaves 7	5 from 12 leaves 7	6 from 13 leaves 7
4 from 12 leaves 8	5 from 13 leaves 8	6 from 14 leaves 8
4 from 13 leaves 9	5 from 14 leaves 9	6 from 15 leaves 9
4 from 14 leaves 10	5 from 15 leaves 10	6 from 16 leaves 10
7 from 7 leaves 0	8 from 8 leaves 0	9 from 9 leaves 0
7 from 8 leaves 1	8 from 9 leaves 1	9 from 10 leaves 1
7 from 9 leaves 2	8 from 10 leaves 2	9 from 11 leaves 2
7 from 10 leaves 3	8 from 11 leaves 3	9 from 12 leaves 3
7 from 11 leaves 4	8 from 12 leaves 4	9 from 13 leaves 4
7 from 12 leaves 5	8 from 13 leaves 5	9 from 14 leaves 5
7 from 13 leaves 6	8 from 14 leaves 6	9 from 15 leaves 6
7 from 14 leaves 7	8 from 15 leaves 7	9 from 16 leaves 7
7 from 15 leaves 8	8 from 16 leaves 8	9 from 17 leaves 8
7 from 16 leaves 9	8 from 17 leaves 9	9 from 18 leaves 9
7 from 17 leaves 10	8 from 18 leaves 10	9 from 19 leaves 10

Examples.

1 from 2 how many?	0 from 4 how many?
3 from 4 how many?	2 from 6 how many?
2 from 9 how many?	3 from 7 how many?
1 from 8 how many?	3 from 8 how many?
2 from 4 how many?	4 from 7 how many?
3 from 5 how many?	5 from 6 how many?
1 from 7 how many?	7 from 8 how many?
2 from 6 how many?	3 from 9 how many?
4 from 9 how many?	5 from 7 how many?
0 from 7 how many?	3 from 6 how many?

4 from 5 how many?	5 from 10 how many?
1 from 6 how many?	8 from 16 how many?
7 from 8 how many?	7 from 14 how many?
8 from 9 how many?	4 from 14 how many?
4 from 8 how many?	6 from 11 how many?
9 from 9 how many?	7 from 12 how many?
7 from 9 how many?	6 from 16 how many?
0 from 6 how many?	7 from 12 how many?
2 from 8 how many?	3 from 13 how many?
5 from 9 how many?	8 from 18 how many?

1. John had 7 apples, and gave away 3 of them: how many had he left?

2. John has 9 cents, and Mary has 5: how many more has John than Mary?

3. A boy wishes to buy a ball worth 18 cents, but he has only 12 cents: how many cents more does he need?

4. If one man walks 16 miles in a day, and another 7 miles less, how many miles did the latter walk?

5. If a person earns 15 dollars in a week, and his expenses are 9 dollars, how much does he save?

6. A mechanic who earns 12 dollars per week, wishes to save 5 dollars: how much may he spend?

7. A merchant bought some cloth for 13 dollars, and sold it for 8 dollars: how much did he lose?

8. Two men are to pay, together, 16 dollars; one is to pay 9 dollars: what must the other pay?

9. A person bought sugar for 8 dollars, and sold it for 14 dollars: what did he gain?

10. If the number 14 be separated into two parts, one of which shall be 6, what will the other be?

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16. The MINUEND is the number subtracted from.

17. The SUBTRAHEND is the number subtracted.

18. The REMAINDER is what is left.

19. The sign, —, is called *minus*, and denotes subtraction: Thus,

$$5 - 3 = 2,$$

denotes that 5 is the minuend, 3 the subtrahend, and 2 the remainder; and is read, 5 minus 3 equals 2.

6 — 4 = how many?	1 — 1 = how many?
7 — 2 = how many?	8 — 4 = how many?
9 — 3 = how many?	9 — 9 = how many?
7 — 5 = how many?	8 — 6 = how many?
8 — 3 = how many?	4 — 1 = how many?
9 — 1 = how many?	9 — 1 = how many?

Let the pupil *read* the difference between each number and the one over it:

1.
	1	4	3	6	5	9	8	7	2
	1	1	1	1	1	1	1	1	1

2.
	4	5	7	3	2	6	9	8	7
	2	2	2	2	2	2	2	2	2

3.	5	4	9	8	7	6	3	5	9
	3	3	3	3	3	3	3	3	3
4.	6	7	9	8	4	5	6	8	7
	4	4	4	4	4	4	4	4	4
5.	7	6	5	8	9	10	11	12	13
	5	5	5	5	5	5	5	5	5
6.	8	6	10	12	13	9	7	16	17
	6	6	6	6	6	6	6	6	6
7.	8	7	9	10	14	15	12	10	17
	7	7	7	7	7	7	7	7	7
8.	9	8	10	12	14	16	18	11	17
	8	8	8	8	8	8	8	8	8
9.	10	12	9	14	15	17	18	19	20
	9	9	9	9	9	9	9	9	9

20. Write the subtrahend under the minuend: thus,

	(1.)	(2.)	(3.)	(4.)
Minuend,	9	12	13	15
Subtrahend,	5	4	8	6
Remainder,	4			

	(5.)	(6.)	(7.)	(8.)	(9.)
From	27	56	35	67	87
Take	13	21	25	26	63

	(10.)	(11.)	(12.)	(13.)	(14.)
From	436	375	679	974	899
Take	125	341	576	973	791

	(15.)	(16.)	(17.)	(18.)
From	570	290	695	809
Take	210	170	364	705

	(19.)	(20.)	(21.)	(22.)
From	8749	9999	8847	9097
Take	2647	1789	8746	2096

21. From 545 dollars take 194 dollars.

Write the less number under the greater, and perform the subtraction according to the following Rule.

Let the pupil *read* the result in each subtraction.

OPERATION.

5 4 5	Minuend.
1 9 4	Subtrahend.
3 5 1	Remainder.

Rule.

I. *Write the less number under the greater, so that units of the same value shall fall in the same column.*

II. *Begin at the right hand, and subtract each figure of the subtrahend from the one directly over it, when the upper figure is the greater.*

III. *When the upper figure is the less, add* 10 *to it, before subtracting, and then add* 1 *to the next figure of the subtrahend.*

Proof.

Add the remainder to the subtrahend. If the work is right, the sum will be equal to the minuend.

Examples.

	(1.)	(2.)	(3.)	(4.)
From	7041	10904	40709	59756
Take	2104	8759	27047	30093

	(5.)	(6.)	(7.)	(8.)
From	67045	27041	10000	10000
Take	10916	3709	3001	1

	(9. dollars.	(10. days.	(11. feet.	(12. yards.
From	47055	32704	60413	29041
Take	34093	10419	209	3098

	(13. inches.	(14. men.	(15. sheep.	(16. cows.
From	2741	97041	370456	919904
Take	1909	2074	909	99999

	(17. weeks.	(18. hours.	(19. pounds.	(20. rods.
From	4097	2749	8946	9049
Take	3209	1940	2078	2104

21. From 674187 take 2704. *Ans.* 671483.
22. From 2947049 subtract 21470. *Ans.* 2925579.
23. Subtract 97048 from 9704909. *Ans.* 9607861.
24. How many are 496087 — 22041? *Ans.* 474046.
25. 479630 — 29472 = how many? *Ans.* 450158.
26. 1100910 — 974609 = how many? *Ans.* 126301.
27. 100000 — 10999 = how many? *Ans.* 89001.
28. 6900760 — 294099 = how many? *Ans.* 6606661.
29. Subtract 910969 from 1000000. *Ans.* 89031.

Practical Questions.

1. Take twenty-five from twenty-five hundred.
2. 100000 — 444 = how many?
3. 1000000 — 404404 = how many?
4. From ten thousand take one.
5. 2360064 — 194506 = how many?
6. From a log 45 feet long, 37 feet were cut off: how many feet were left?
7. If one lot of ground costs 350 dollars, and another 315 dollars, how much more did one cost than the other?
8. A person bought a quantity of goods for 1860 dollars and sold them for 2512 dollars: how much was gained?
9. A merchant sold, for 2710 dollars, goods which had cost 1964 dollars: what was the gain?
10. A man paid 3645 dollars for a house and lot, and sold them for 2987 dollars: what was the loss?
11. A vessel that cost 7682 dollars, was sold for 6995 dollars: what was the loss?
12. A gentleman received in 1860 a salary of 3000 dollars, but now receives 495 dollars less: what is his salary?
13. A person sold his horse for 3750 dollars, and by so doing gained 968 dollars: how much had he paid for it?
14. A person in 1861 was seventy-five years old: in what year was he born?
15. A person was born in 1765 and died in 1810: how old was he when he died?
16. How many years elapsed between the landing of Columbus in 1492 and the era of the Revolution in 1775?
17. The difference of the ages of two persons was 49 years: the younger person was born in 1850: in what year was the older born?
18. The Revolutionary war began in 1775 and the Great Rebellion in 1861: how many years elapsed between these two events?

Examples combining Addition and Subtraction.

1. There were 27 pear-trees in one row, and 26 in another, and 15 were blown down: how many were left standing?

2. Laura has 75 cents in one hand, and 36 in the other; she buys an Arithmetic for 69 cents: how many cents has she left?

3. There are 106 scholars in the Primary department of a school, and 359 in the higher department; of the whole, 279 are boys: how many girls are there?

4. James has 87 cents; he pays 25 cents for a whistle, and 40 cents for a knife: how much has he left?

5. A merchant bought 1250 yards of cloth of one person, 3270 yards of another, and then sold 1459 yards: how many yards had he left?

6. A farmer has 425 sheep, 30 cows, 16 horses, 20 calves, and 6 colts; if he sells the sheep, how many animals will he have left?

7. By the census of 1850, the entire population of the United States was 23191876; the slave population 3204313; free colored 434495: what was the white population?

8. A man's income is 1849 dollars a year; he spends for food, 450 dollars; for clothing, 129 dollars; and for other things, 627 dollars: how much does he save?

9. A grocer bought a lot of flour for 216 dollars; some rye for 127 dollars; and some corn for 420 dollars; he sold the whole for 999 dollars: what did he make?

10. Mr. Jones owes his butcher, grocer, and baker 365 dollars; he owes his grocer 219 dollars: how much does he owe the other two?

11. James and John start from the same point and travel in opposite directions; James goes 20 miles, and John 17; how far are they then apart?

12. If two men start from the same place and travel in the same direction: how far will they be apart after one has travelled 55 miles, and the other 37 miles?

13. A father is 26 years older than his eldest son, and 52 years older than his youngest: what is the difference of the sons' ages?

14. A farmer has 50 sheep in one pasture, 38 in a second, and 25 in a third; if 9 sheep escape from the first, 3 from the second, and 1 from the third: how many sheep will be left?

15. From four thousand three hundred and twenty-seven, plus two hundred and thirty-one, subtract 287.

16. From three millions six hundred and five, plus 217, subtract one thousand and 9.

17. From one million, subtract one thousand plus 6.

18. From six millions, subtract 200+5.

19. A man gains 512 dollars, then loses 401 dollars; a second time he gains 512 dollars, and loses 104 dollars: how many dollars has he left?

20. A merchant bought 120 hogsheads of sugar for 6000 dollars, and paid 325 dollars freight; he then sold the whole for 7529 dollars: how much did he gain?

21. I agree to pay Mr. Squires 36 dollars for ploughing a piece of land; 167 dollars for fencing it, and 139 dollars for cultivating it: how much shall I owe him after paying him 287 dollars?

22. Mr. Jones has a yearly income of 6750 dollars; he pays 475 dollars for rent, 1325 dollars for family expenses, and 950 dollars for his horses and carriage: how much has he left?

23. Mr. James has a fortune of 37689 dollars, which he divides among his four sons; he gives John 10421 dollars, William 9875, and Charles 8751 dollars: how many dollars has Reuben?

MULTIPLICATION.

22. Multiplication is the operation of taking one number as many times as there are units in another.

23. The Multiplicand is the number to be taken.

24. The Multiplier is the number denoting how many times the multiplicand is to be taken.

25. The Product is the result of the operation.

26. The Multiplicand and Multiplier are called Factors.

27. The sign $\times$, is called the sign of multiplication. When placed between two numbers it denotes that they are to be multiplied together; thus:

$7 \times 5 = 35$; and is read, 5 times 7 are 35.

Multiplication Table.

Once	1	is	1	2	times	1	are	2	3	times	1	are	3
Once	2	is	2	2	times	2	are	4	3	times	2	are	6
Once	3	is	3	2	times	3	are	6	3	times	3	are	9
Once	4	is	4	2	times	4	are	8	3	times	4	are	12
Once	5	is	5	2	times	5	are	10	3	times	5	are	15
Once	6	is	6	2	times	6	are	12	3	times	6	are	18
Once	7	is	7	2	times	7	are	14	3	times	7	are	21
Once	8	is	8	2	times	8	are	16	3	times	8	are	24
Once	9	is	9	2	times	9	are	18	3	times	9	are	27
Once	10	is	10	2	times	10	are	20	3	times	10	are	30
Once	11	is	11	2	times	11	are	22	3	times	11	are	33
Once	12	is	12	2	times	12	are	24	3	times	12	are	36
4	times	1	are	4	5	times	1	are	5	6	times	1	are	6
4	times	2	are	8	5	times	2	are	10	6	times	2	are	12
4	times	3	are	12	5	times	3	are	15	6	times	3	are	18
4	times	4	are	16	5	times	4	are	20	6	times	4	are	24
4	times	5	are	20	5	times	5	are	25	6	times	5	are	30
4	times	6	are	24	5	times	6	are	30	6	times	6	are	36
4	times	7	are	28	5	times	7	are	35	6	times	7	are	42
4	times	8	are	32	5	times	8	are	40	6	times	8	are	48
4	times	9	are	36	5	times	9	are	45	6	times	9	are	54
4	times	10	are	40	5	times	10	are	50	6	times	10	are	60
4	times	11	are	44	5	times	11	are	55	6	times	11	are	66
4	times	12	are	48	5	times	12	are	60	6	times	12	are	72

7	times	1	are	7	8	times	1	are	8	9	times	1	are	9
7	times	2	are	14	8	times	2	are	16	9	times	2	are	18
7	times	3	are	21	8	times	3	are	24	9	times	3	are	27
7	times	4	are	28	8	times	4	are	32	9	times	4	are	36
7	times	5	are	35	8	times	5	are	40	9	times	5	are	45
7	times	6	are	42	8	times	6	are	48	9	times	6	are	54
7	times	7	are	49	8	times	7	are	56	9	times	7	are	63
7	times	8	are	56	8	times	8	are	64	9	times	8	are	72
7	times	9	are	63	8	times	9	are	72	9	times	9	are	81
7	times	10	are	70	8	times	10	are	80	9	times	10	are	90
7	times	11	are	77	8	times	11	are	88	9	times	11	are	99
7	times	12	are	84	8	times	12	are	96	9	times	12	are	108
10	times	1	are	10	11	times	1	are	11	12	times	1	are	12
10	times	2	are	20	11	times	2	are	22	12	times	2	are	24
10	times	3	are	30	11	times	3	are	33	12	times	3	are	36
10	times	4	are	40	11	times	4	are	44	12	times	4	are	48
10	times	5	are	50	11	times	5	are	55	12	times	5	are	60
10	times	6	are	60	11	times	6	are	66	12	times	6	are	72
10	times	7	are	70	11	times	7	are	77	12	times	7	are	84
10	times	8	are	80	11	times	8	are	88	12	times	8	are	96
10	times	9	are	90	11	times	9	are	99	12	times	9	are	108
10	times	10	are	100	11	times	10	are	110	12	times	10	are	120
10	times	11	are	110	11	times	11	are	121	12	times	11	are	132
10	times	12	are	120	11	times	12	are	132	12	times	12	are	144

3	times	5	are how many?	0	times	4	are how many?
6	times	7	are how many?	5	times	0	are how many?
4	times	5	are how many?	6	times	9	are how many?
4	times	9	are how many?	7	times	12	are how many?
2	times	10	are how many?	11	times	12	are how many?
6	times	5	are how many?	9	times	8	are how many?
5	times	9	are how many?	10	times	12	are how many?
9	times	6	are how many?	12	times	12	are how many?
10	times	11	are how many?	5	times	9	are how many?
12	times	12	are how many?	6	times	8	are how many?
12	times	7	are how many?	9	times	10	are how many?
11	times	5	are how many?	12	times	7	are how many?
12	times	8	are how many?	9	times	9	are how many?
6	times	5	are how many?	8	times	7	are how many?
9	times	9	are how many?	12	times	5	are how many?

Practical Questions.

1. If an orange is worth 6 cents, how many cents are 3 oranges worth?

2. What is the cost of 8 yards of cloth at 5 dollars a yard?

3. How much will a man earn in 6 days, if he earns 7 dollars per day?

4. If a man's expenses are 4 dollars per day, what are his expenses for 6 days?

5. If a horse travels 8 miles per hour, how far will he travel in 9 hours?

6. A bushel contains 4 pecks: how many pecks are there in 5 bushels?

7. If a pail holds 3 gallons of water, how many gallons are required to fill 8 pails?

8. If a father gives to each of 7 children 9 dollars, how many dollars will he give to all?

9. If 8 yards of cloth are required for a suit of clothes, how many yards will be required for 7 suits?

10. If a mechanic earns 15 dollars per week and spends 9 dollars, how much will he save in 6 weeks?

11. If a man earns 4 dollars per day, how much will he earn in 7 days?

12. If a sheet of paper can be folded in 8 leaves, how many leaves will 9 sheets make?

13. If a carpenter, in measuring a piece of timber, lays his measuring rod, which is 6 feet long, 10 times along the piece, how long is it?

14. In how many days can 1 man do as much work as 3 men can do in four days?

15. If 6 men can do a piece of work in 7 days, in how many days can 1 man do the same work?

16. If 5 men can build a wall in 9 days, how many men can build it in 1 day?

CASE I.

28. When the multiplier does not exceed 9.

1. What is the product of 325 multiplied by 5?

ANALYSIS.—Since 325 is to be taken 5 times, each order of its units must be taken 5 times: hence, the product must contain 25 units, 10 tens, and 15 hundreds:

OPERATION.

	3 2 5	
	5	
	2 5	units.
1 0		tens.
1 5		hundreds.
1 6 2 5		

Therefore, the product is . . 1625

In practice, the operation is performed thus:

Say, 5 times 5 are 25; set down the 5, and then say, 5 times 2 are 10 and 2 to carry are 12; set down the 2, and then say, 5 times 3 are 15 and 1 to carry are 16; set down the 16, and the product, as before, is

OPERATION.

3 2 5	Multiplicand.
5	Multiplier.
1 6 2 5	Product.

Rule.

Multiply each figure of the multiplicand by the multiplier, carrying and setting down as in Addition.

NOTE.—In performing the multiplication, the pupil should be taught to pronounce only the final result of each operation. Thus, in the example above, he should say, 25; then 12 (instead of 5 times 2 are 10 and 2 to carry are 12); then, 16.

Examples.

	(1.)	(2.)	(3.)	(4.)
Multiplicand,	2704	3049	1648	6146
Multiplier,	2	3	4	5
Product,	5408	9147	6592	30730

(5.)	(6.)	(7.)	(8.)
15049	30413	24604	41970
6	7	8	9

(9.)	(10.)	(11.)	(12.)
104700	970421	874909	919905
7	8	9	9

(13.)	(14.)	(15.)	(16.)
190804	987672	695143	794169
5	8	7	9

17. Multiply 30746 by 8.
18. Multiply 99099 by 9.
19. Multiply 670497 by 8.
20. Multiply 270496 by 7.
21. Multiply 490767 by 5.
22. Multiply 540992 by 7.
23. Multiply 888888 by 8.
24. Multiply 999999 by 9.

25. $3567064 \times 9 =$ what product?

26. $51606 \times 8 =$ what product?

27. Multiply sixty-five thousand six hundred and forty-eight, by eight.

28. Multiply seven hundred and eighty-six thousand nine hundred and ninety-five, by five.

29. Multiply the sum of 9756 and 2864, by 9.

30. Multiply the difference of 7050 and 4986, by 6.

31. The multiplicand is 25689 and the multiplier is 5: what is the product?

32. If one horse costs 175 dollars, what will be the cost of 6 horses at the same rate?

33. In one cord of wood there are 128 solid feet: how many solid feet are there in 8 cords?

34. What would be the cost of 7 houses, each costing 3759 dollars?

35. What is the product of 19700685 multiplied by 8?

CASE II.

29. When the multiplier contains two or more figures.

1. Multiply 3046 by 504.

ANALYSIS.—The multiplicand is to be taken 504 times: taking it 4 times, we obtain 12184. When we come to take it 5 hundred times, the *lowest order of units* in the product will be hundreds: hence, 0, the first figure of the product, must be written in the third place.

OPERATION.

3046	Multiplicand.
504	Multiplier.
12184	
15230	
1535184	Product.

NOTE.—The product obtained by multiplying by a single figure of the multiplier, is called *a partial product*. The sum of the partial products, is the required product.

Rule.

I. *Write the multiplier under the multiplicand, placing units of the same order in the same column.*

II. *Beginning with the units' figure, multiply the multiplicand by each significant figure of the multiplier, and write the first figure of each partial product directly under its multiplier.*

III. *Then add the partial products, and their sum will be the required product.*

Proof.

Write the multiplicand in the place of the multiplier, and find the product, as before. If the two products are the same, the work is supposed to be right.

Examples.

(1.)	(2.)	(3.)
7406	9046	9046
306	204	805
44436	36184	45230
22218	18092	72368
2266236	1845384	7282030

(4.)	(5.)	(6.)
564	8946	80269
23	18	16

(7.)	(8.)	(9.)
7692	62567	47619
19	25	32

(10.)	(11.)	(12.)
500675	951046	802706
65	79	68

(13.)	(14.)	(15.)
547654	940573	64609
85	125	246

16. Multiply the number 50687 by 264.
17. Multiply the number 67094 by 568.
18. Multiply the number 260407 by 2426.
19. Multiply the number 175905 by 15435.
20. Multiply the number 875606 by 404.
21. Multiply the number 470065 by 20605.
22. Multiply the number 471002 by 340106.

23. $450604 \times 6094 =$ how many?
24. $569074 \times 21702 =$ how many?
25. The multiplicand is 47568 and the multiplier is 5964: what is the product?
26. The multiplicand is 495 and the multiplier is 17964: what is the product?
27. What is the product of 16084 taken 1207 times?
28. Multiply one million eight hundred and sixty thousand five hundred and six, by one thousand and sixty-two.
29. Multiply two hundred and seven millions four hundred and sixteen thousand seven hundred and sixteen, by two thousand six hundred and forty-seven.
30. What is the product of 2845 and 796?
31. What is the product of 165, 962, and 175?
32. What is the product of 2, 45, 166, and 205?
33. Multiply 1009 by one thousand and nine.
34. Multiply five hundred and six by itself.
35. Multiply the number 3000 by 46985.
36. Multiply the number 8704057 by 6939484.

CONTRACTIONS.

30. A Composite Number is one which may be produced by multiplying together two or more numbers.

31. A Factor is any one of the numbers which, multiplied together, produce a composite number.

Thus, $2 \times 3 = 6$, 2 and 3 are the factors of the composite number 6.

Also, 12 is a composite number, $= 6 \times 2 = 3 \times 2 \times 2$, in which the factors are 6 and 2, or 3, 2, and 2.

What are the factors of 9? of 14? of 16? of 20?
What are the factors of 4? of 15? of 18? of 24?

CASE I.

32. When the multiplier is any composite number.

Rule.

I. *Separate the composite number into its factors:*

II. *Multiply the multiplicand by one factor, and the product by a second factor; and so on, till all the factors have been used; the last product will be the product required.*

Examples.

1. Multiply 321 by $16 = 4 \times 4 = 8 \times 2$.

321		321		321
4		8		16
1284	or	2568		1926
4		2		321
5136		5136		5136

2. Multiply 375 by 24.
3. Multiply 6095 by 49.
4. Multiply 7206 by 30.
5. Multiply 6810 by 63.
6. Multiply 19760 by 56.
7. Multiply 37568 by 40.
8. Multiply 20270 by 35.
9. Multiply 75670 by 36.

CASE II.

33. When the multiplier is 1, with any number of ciphers annexed; as, 10, 100, 1000, &c.

1. Multiply 376 by 10.

Analysis.—The operation is performed by simply annexing the 0's of the multiplier to the multiplicand.

OPERATION.

$376 \times 10 = 3760.$

Rule.

Annex to the multiplicand as many ciphers as there are in the multiplier, and the number so formed will be the required product.

Examples.

1. Multiply 2756 by 10; by 100.
2. Multiply 5680 by 1000.
3. Multiply 7690430 by 10000.
4. Multiply 4063 by 10; by 100.
5. Prove the last example by multiplying by the factors of 10, and of 100.
6. Multiply 37006 by 100; by 1000.
7. Multiply 570468 by 10, by 100, by 1000, and by 10000.

CASE III.

34. When there are ciphers on the right of one or both of the factors.

1. Multiply 520 by 360.

Multiply 52 by 36 and add two 0's to the product.

$520 \times 360 =$

520
360
312
156
187200

Rule.

I. *Omit the ciphers and multiply the significant figures:*

II. *Then place as many ciphers at the right hand of the product as there are in both factors.*

Examples.

1. Multiply 6750 by 20.
2. Multiply 80400 by 60.
3. Multiply 974000 by 700.
4. Multiply 230 by 9000.
5. Multiply 16930 by 2500.
6. Multiply 3750 by 52000.
7. Multiply 3000 by 3000.
8. Multiply 2500 by 2500.
9. Multiply 3007 by 7000.
10. Multiply 190 by 109900.

Practical Questions.

1. If one pound of butter costs 23 cents, what will be the cost of 9 pounds at the same rate?

2. If one horse costs 125 dollars, what will be the cost of 24 horses at the same rate?

3. How many miles will a ship sail in 16 days, if it sails 245 miles a day?

4. If a gentleman's annual expenses amount to 2765 dollars, to what will they amount in 7 years?

5. In a barrel of flour there are 196 pounds: how many pounds are there in 160 barrels?

6. What is the product, when the multiplier is 3500 and the multiplicand 296400?

7. How much money is required to buy 164 barrels of flour at 7 dollars a barrel?

8. What amount of money will enable me to give 35 dollars to each of 245 laborers?

9. If a horse eats 12 bushels of oats in a month, how many bushels would 6 horses eat in 4 months?

10. How much hay would be required to feed 8 horses for 9 weeks, if one horse ate 175 pounds in one week?

11. In how many days could one man dig a trench, if 25 men can dig it in 16 days?

12. If 40 men could build a wall in 30 days, how many men would be required to do it in one day?

13. How many days are there in 1000 common years of 365 days each?

14. Suppose a book to have 360 pages, each page to contain 35 lines, and each line to contain 11 words: how many words are there in the book?

15. If a chest of tea contains 75 pounds, and is worth 62 cents per pound, what would be the value of 45 chests of the same size and at the same price?

Examples, combining the previous Rules.

1. The cost of a horse is 145 dollars; of a carriage 275 dollars; and of a set of harness 75 dollars: what is the entire cost?

2. The income of a person is 4725 dollars per year, and his expenses are 2460 dollars for the same time: how much could he, at this rate, save in 26 years?

3. A person bought 275 horses at 100 dollars per head, and sold them at 135 dollars per head: what did he gain?

4. A farmer has 3 flocks of sheep, numbering respectively 50, 96, and 140 head. If, at shearing, each yields 4 pounds of wool, what will be its value at 24 cents per pound?

5. If a person, out of his monthly salary, spends 30 dollars for board, 8 dollars for washing, and saves 5 dollars, what is his monthly salary?

6. In a factory, 45 hands receive each 30 dollars per month; 24 receive each 25 dollars, and 15 receive each 20 dollars: what is the amount of the monthly payment?

7. A merchant sold 156 yards of cloth, which cost 4 dollars a yard, at 6 dollars per yard; and 256 barrels of flour, which cost 5 dollars, at 7 dollars per barrel: how much did he gain?

8. Suppose an orchard to contain 16 rows of apple-trees, and each row to have 27 trees in it: how many bushels will the orchard produce, if 30 bushels are gathered from each tree?

9. A farmer has a farm of 175 acres, the whole of which was sown with wheat: what will be the yield, if each acre produces 35 bushels?

10. If a lot of ground, that cost 375 dollars, were sold for 250 dollars, what would be the loss in selling 294 lots at the same rate?

11. Two men are 950 miles apart. If they travel towards

each other, one at the rate of 30 miles, and the other of 42 miles per day, how many miles will they be apart at the end of 8 days?

12. If two men, at the same time, leave the same place, and travel in opposite directions, one at the rate of 26 miles, and the other of 19 miles per day, how far apart will they be at the end of 19 days?

13. A drover bought 180 head of cattle, in Illinois, at 25 dollars a head, and sent them to New York where he sold them at a profit of 7 dollars a head: what did he receive for the drove?

14. A lady purchased, at a dry goods store, 9 yards of cloth at 3 dollars per yard; 15 yards of satin at 2 dollars; 7 yards of merino at 1 dollar per yard; she paid for the above articles 3 twenty-dollar bills and 1 ten-dollar bill: what balance was due her?

15. A person contracted to do a piece of work for 5000 dollars. He hired 5 cartmen for 60 days at 5 dollars each, per day; 25 laborers for 75 days at 1 dollar each, per day, and two overseers for 80 days at 3 dollars each, per day: what amount did the contractor save for himself?

16. In one year, a farmer sold the produce of his farm, as follows: 300 bushels of wheat at 60 cents a bushel; 500 bushels of corn at 35 cents a bushel; 150 bushels of oats at 45 cents a bushel: what amount did he receive?

17. A fortune of 25000 dollars is so divided that each of 4 sons is to receive 3725 dollars, and each of two daughters 2575 dollars, and the widow the remainder: what is the widow's share?

DIVISION.

1. When a number is divided into 2 equal parts, each part is called, *one-half of the number.*

What is one-half of 4 apples? What is one-half of 4? How many times is 2 contained in 4?

2. When a number is divided into 3 equal parts, each part is called, *one-third of the number.*

What is one-third of 9 apples? What is one-third of 9? How many times is 3 contained in 9?

3. When a number is divided into 4 equal parts, each part is called, *one-fourth of the number.*

What is one-fourth of 12 pears? What is one-fourth of 12?

4. When a number is divided into 5 equal parts, each part is called, *one-fifth of the number.*

What is one-fifth of 10 marbles? What is one-fifth of 10?

5. When a number is divided into 6 equal parts, each part is called, *one-sixth of the number.*

35. Division is the operation of dividing a number into equal parts; or, of finding how many times one number is contained in another.

36. The Dividend is the number to be divided.

37. The Divisor is the number by which we divide.

38. The Quotient is the result of the division.

39. The Remainder is what is left after the operation.

40. There are three signs used to denote Division:

$18 \div 4$, expresses that 18 is to be divided by 4.

$\frac{18}{4}$, expresses that 18 is to be divided by 4.

4) 18, expresses that 18 is to be divided by 4.

Division Table.

1 in 1 1 time	2 in 2 1 time	3 in 3 1 time
1 in 2 2 times	2 in 4 2 times	3 in 6 2 times
1 in 3 3 times	2 in 6 3 times	3 in 9 3 times
1 in 4 4 times	2 in 8 4 times	3 in 12 4 times
1 in 5 5 times	2 in 10 5 times	3 in 15 5 times
1 in 6 6 times	2 in 12 6 times	3 in 18 6 times
1 in 7 7 times	2 in 14 7 times	3 in 21 7 times
1 in 8 8 times	2 in 16 8 times	3 in 24 8 times
1 in 9 9 times	2 in 18 9 times	3 in 27 9 times
4 in 4 1 time	5 in 5 1 time	6 in 6 1 time
4 in 8 2 times	5 in 10 2 times	6 in 12 2 times
4 in 12 3 times	5 in 15 3 times	6 in 18 3 times
4 in 16 4 times	5 in 20 4 times	6 in 24 4 times
4 in 20 5 times	5 in 25 5 times	6 in 30 5 times
4 in 24 6 times	5 in 30 6 times	6 in 36 6 times
4 in 28 7 times	5 in 35 7 times	6 in 42 7 times
4 in 32 8 times	5 in 40 8 times	6 in 48 8 times
4 in 36 9 times	5 in 45 9 times	6 in 54 9 times
7 in 7 1 time	8 in 8 1 time	9 in 9 1 time
7 in 14 2 times	8 in 16 2 times	9 in 18 2 times
7 in 21 3 times	8 in 24 3 times	9 in 27 3 times
7 in 28 4 times	8 in 32 4 times	9 in 36 4 times
7 in 35 5 times	8 in 40 5 times	9 in 45 5 times
7 in 42 6 times	8 in 48 6 times	9 in 54 6 times
7 in 49 7 times	8 in 56 7 times	9 in 63 7 times
7 in 56 8 times	8 in 64 8 times	9 in 72 8 times
7 in 63 9 times	8 in 72 9 times	9 in 81 9 times
10 in 10 1 time	11 in 11 1 time	12 in 12 1 time
10 in 20 2 times	11 in 22 2 times	12 in 24 2 times
10 in 30 3 times	11 in 33 3 times	12 in 36 3 times
10 in 40 4 times	11 in 44 4 times	12 in 48 4 times
10 in 50 5 times	11 in 55 5 times	12 in 60 5 times
10 in 60 6 times	11 in 66 6 times	12 in 72 6 times
10 in 70 7 times	11 in 77 7 times	12 in 84 7 times
10 in 80 8 times	11 in 88 8 times	12 in 96 8 times
10 in 90 9 times	11 in 99 9 times	12 in 108 9 times

5 in 10 how many times?	6 in 48 how many times?
8 in 16 how many times?	8 in 96 how many times?
9 in 27 how many times?	9 in 36 how many times?
6 in 54 how many times?	8 in 48 how many times?

Practical Questions.

1. In how many days, at 2 dollars a day, will a man earn 16 dollars?

2. How many hats, at 3 dollars each, may be bought for 27 dollars?

3. If it takes 6 yards of cloth for a suit, how many suits may be made from 42 yards?

4. How many boxes, each holding 3 pounds, would be filled by 24 pounds of sugar?

5. A fathom is 6 feet: how many fathoms of depth are there in a river that is 24 feet deep?

6. In how many days could a man walk 63 miles, if he walked 9 miles per day?

7. In what time could a mechanic earn 42 dollars, if he earned 7 dollars per week?

8. In how many days will Daniel and Samuel together earn 72 cents, if Daniel earns 5 cents, and Samuel 3 cents per day?

9. If two men, who are 63 miles apart, should walk towards each other, the one at the rate of 3 miles, and the other of 4 miles per hour, in how many hours would they meet?

10. How many yards of cloth, at 4 dollars a yard, will pay for 6 barrels of flour at 6 dollars a barrel?

11. How many weeks' labor, at 9 dollars a week, will pay for 6 barrels of flour at 6 dollars a barrel?

12. Paid 24 dollars for 4 barrels of flour: what was the cost of one barrel?

13. If 8 men gained 40 dollars, how much did each gain?

14. If 54 dollars will buy 9 barrels of flour, how much will buy one barrel?

15. A person received 30 dollars for 6 days' labor: at what rate per day was he paid?

SHORT DIVISION.

41. When the divisor does not exceed 12.

Let the pupil divide, in each of the following examples, and *read* the result in each case.

1)	1	4	3	2	5	8	7	6	9	12	11	10
2)	2	8	6	4	10	16	14	12	18	24	22	20
3)	3	12	9	6	15	24	21	18	27	36	33	30
4)	4	16	12	8	20	32	28	24	36	48	44	40
5)	5	20	15	10	25	40	35	30	45	60	55	50
6)	6	24	18	12	30	48	42	36	54	72	66	60
7)	7	28	21	14	35	56	49	42	63	84	77	70
8)	8	32	24	16	40	64	56	48	72	96	88	80
9)	9	36	27	18	45	72	63	54	81	108	99	90
10)	10	40	30	20	50	80	70	60	90	120	110	100
11)	11	44	33	22	55	88	77	66	99	132	121	110
12)	12	48	36	24	60	96	84	72	108	144	132	120

1. Divide 48 into 2 equal parts.

Divide each figure separately, by the divisor.

OPERATION.

Divisor. 2) 4 8 Dividend.

2 4 quotient.

2. Divide 31254 by 6.

We first say, 6 in 3 we can't; then 6 in 31, 5 times and 1 over; then 6 in 12, twice; then 6 in 5, 0 times; then 6 in 54, 9 times.

OPERATION.

6) 3 1 2 5 4

5 2 0 9

3. Divide 327 by 8.

We divide as before, and find a remainder of 7. We write 8 under 7, and the quotient is 40 and 7 divided by 8.

OPERATION.

8) 3 2 7

4 0—7 rem.

4 0 $\frac{7}{8}$ quotient.

Rule.

I. *Write the divisor on the left of the dividend. Begin at the left hand and divide each figure of the dividend by the divisor, and set each quotient figure under its dividend.*

II. *If there is a remainder after any division, annex to it the next figure of the dividend and divide as before.*

III. *If any dividend is less than the divisor, write* 0 *for the quotient figure, and annex the next figure of the dividend for a new dividend.*

IV. *If there is a remainder, after dividing the last figure, set the divisor under it, and annex the result to the quotient.*

Proof.

Multiply the quotient by the divisor, and to the product add the remainder, if any: if the work is right, the result will be equal to the dividend.

Examples.

(1.)	(2.)	(3.)
4) 3276	6) 4167	8) 27458
Answer, 819	$694\frac{3}{6}$	3432—2 rem.
4	6	8
Proof, 3276	4167	27458

(4.)	(5.)	(6.)
6) 28743	7) 97476	9) 840460

7. Divide	129 by 2.	20. Divide	19507034 by 7.
8. Divide	9856 by 4.	21. Divide	16039004 by 5.
9. Divide	79604 by 8.	22. Divide	91001100 by 9.
10. Divide	647617 by 6.	23. Divide	42006004 by 9.
11. Divide	819647 by 5.	24. Divide	22002200 by 8.
12. Divide	391764 by 3.	25. Divide	36709967 by 6.
13. Divide	1064720 by 7.	26. Divide	47000047 by 3.
14. Divide	10066100 by 6.	27. Divide	56700958 by 8.
15. Divide	16435420 by 2.	28. Divide	68704960 by 7.
16. Divide	90010555 by 2.	29. Divide	78490473 by 5.
17. Divide	14769647 by 9.	30. Divide	45704905 by 6.
18. Divide	39500047 by 4.	31. Divide	99904708 by 8.
19. Divide	25046700 by 6.	32. Divide	91979998 by 9.

Practical Questions.

1. If 12 apples be equally divided among 4 boys, how many will each have?

2. If 24 peaches be equally divided among 6 boys, how many will each have?

3. A man has 32 miles to walk, and can travel 4 miles an hour: how many hours will it take him?

4. A farmer receives 245 dollars for 7 cows: how much is that apiece?

5. How many lead pencils could you buy for 396 cents, if they cost 6 cents apiece?

6. How many oranges could you buy for 8496 cents, if they cost 6 cents apiece?

7. A trader wishes to pack 768 hats in boxes, and can put but 8 hats in a box: how many boxes does he need?

8. If a man could build 7 rods of fence in a day, how long will it take him to build 847 rods?

9. If a man pays 56 dollars for seven yards of cloth, how much is that a yard?

10. Nine men receive 1296 dollars for doing a piece of of work: how much does each one receive?

11. A merchant has 1344 dollars with which he is going to buy cloth at 8 dollars a yard: how many yards can he purchase?

12. James is to learn forty-two verses of Scripture in a week: how much must he learn each day?

13. A man has 994 pounds of butter, and wishes to put 7 pounds in a box: how many boxes does he need?

14. James goes to school for 6 weeks, and receives 264 credit marks: how many does he get each week?

15. An estate, worth 7212 dollars, is to be equally divided among 4 sons and 2 daughters: what will be the part of each?

16. If 5 bushels of wheat make 1 barrel of flour, how many barrels will 13080 bushels make?

17. If 8568 marbles are divided equally among 9 boys, how many marbles will each boy have?

18. How many barrels of flour, at 8 dollars a barrel, can be bought for 3496 dollars?

19. If an estate, worth 69741 dollars, is to be equally divided among 7 sons and 2 daughters, what is the portion of each?

LONG DIVISION.

42. When the divisor exceeds 9.

1. Divide the number 4564 into 14 equal parts.

OPERATION.

	Thous.	Hunds.	Tens.	Units.	Hunds.	Tens.	Units.
		4	5	6	4		
14		4	2				
			3	6			
			2	8			
					8	4	
					8	4	

The operation in this, and in all similar examples, is performed according to the following Rule.

NOTE.—The numbers 45, 36, and 84 are called *partial dividends*.

Rule.

I. *Write the divisor on the left of the dividend.*

II. *Note the fewest figures of the dividend, at the left, that will contain the divisor, and set the quotient figure at the right of the dividend.*

III. *Multiply the divisor by the quotient figure, subtract the product from the first partial dividend, and to the remainder annex the next figure of the dividend, forming a second partial dividend.*

IV. *Find, in the same manner, the second and succeeding figures of the quotient, till all the figures of the dividend are brought down.*

Proof.

Multiply the divisor by the quotient, and to the product add the remainder. If the work is right, the sum will be the same as the dividend.

NOTE 1.—There are five operations in Division: 1st, To write down the numbers; 2d, Divide, or find how many times; 3d, Multiply; 4th, Subtract; 5th, Bring down, to form the partial dividend.

2. The product of a quotient figure by the divisor must never be larger than the corresponding partial dividend; if it is, the quotient figure is too large, and must be diminished.

3. When any one of the remainders is greater than the divisor, the quotient figure is too small, and must be increased.

4. The unit of any quotient figure is the same as that of the partial dividend from which it is obtained. The pupil should always name the unit of every quotient figure.

5. The unit of a remainder is the same as that of the dividend.

Examples.

1. Divide 32641 dollars into 24 equal parts.

OPERATION.

Divisor.	Dividend.	Quotient.
24	32641	1360 $\frac{1}{24}$
	24	
	86	
	72	
	144	
	144	
	01 Rem.	

PROOF.

1360
24
5440
2720
32640
1 Rem.
32641

2. Divide 874614 by 75.

OPERATION.

75) 874614 (11661 $\frac{39}{75}$
75
124
75
496
450
461
450
114
75
39 Rem.

3. Divide 409059 by 96.

OPERATION.

96) 409059 (4261 $\frac{3}{96}$
384
250
192
585
576
99
96
3 Rem.

4. Divide 875 by 15.
5. Divide 9506 by 16.
6. Divide 804967 by 18.
7. Divide 954678 by 15.
8. Divide 479604 by 19.
9. Divide 327084 by 21.
10. Divide 697047 by 14.
11. Divide 591678 by 13.
12. Divide 927041 by 11.
13. Divide 900672 by 18.
14. Divide 9657723 by 627.
15. Divide 1320796 by 201.
16. Divide 2147735 by 156.
17. Divide 3372630 by 544.
18. Divide 89582619 by 119.
19. Divide 90604 by 18.
20. Divide 691645 by 25.
21. Divide 1406275 by 31.
22. Divide 25672916 by 35.
23. Divide 19647062 by 41.
24. 9647569 ÷ 245 =
25. 37964756 ÷ 296 =
26. 467980126 ÷ 384 =
27. 547690476 ÷ 1352 =
28. 691464280 ÷ 3045 =
29. 87467047 ÷ 240 =
30. 45848043 ÷ 328 =
31. 69274216 ÷ 416 =
32. 71364891 ÷ 5943 =
33. 81486974 ÷ 3742 =

CONTRACTIONS.

43. Contractions, in Division, are short methods of finding the quotient when the divisor is a composite number.

CASE I.

44. When the divisor is any composite number.

1. Divide 18576 dollars equally among 24 men.

$24 = 2 \times 3 \times 4$.

OPERATION.

2) 1 8 5 7 6
3) 9 2 8 8
4) 3 0 9 6
7 7 4

Rule.

Divide by the factors of the divisor, in succession, and the last quotient will be the quotient required.

Examples.

1. Divide the number 19152 by $72 = 8 \times 9$.
2. Divide the number 3780 by $12 = 3 \times 4$.
3. Divide the number 19296 by $48 = 3 \times 4 \times 4$.
4. Divide the number 92880 by $48 = 2 \times 2 \times 2 \times 2 \times 3$.
5. Divide the number 111456 by $144 = 4 \times 4 \times 9$.
6. Divide the number 308736 by $24 = 3 \times 2 \times 2 \times 2$.

To find the true remainder, if any.

7. Divide 3274 by the number $24 = 2 \times 3 \times 4$.

OPERATION.

2) 3 2 7 5
3) 1 6 3 7 .. 1
4) 5 4 5 .. 2
1 3 6 .. 1

1
$2 \times 2 = 4$
$1 \times 3 \times 2 = 6$

True remainder, 11

Rule.

Multiply each remainder by all the preceding divisors except its own, and add the products to the first remainder.

8. Divide the number 5088 by $16 = 2 \times 2 \times 2 \times 2$.
9. Divide the number 8974 by $15 =$
10. Divide the number 6446 by $18 = 2 \times 3 \times 3$.
11. Divide the number 312176 by $24 =$
12. Divide the number 902725 by $21 = 3 \times 7$.
13. Divide the number 307697 by $35 =$
14. Divide the number 7406342 by $81 = 9 \times 9$.
15. Divide the number 9468933 by $45 =$
16. Divide the number 2346912 by $64 = 2 \times 4 \times 8$.
17. Divide the number 1000000 by $72 =$
18. Divide the number 9479564 by $36 = 6 \times 6$.
19. Divide the number 4709063 by $42 = 2 \times 3 \times 7$.

CASE II.

45. When the divisor is 10, 100, 1000, &c.

1. From 6272 marbles, how many piles can be formed, of 100 each?

Rule.

I. *From the right hand, cut off by a line, as many figures as there are ciphers in the divisor.*

II. *The figures at the left will be the quotient, and those at the right, the remainder.*

OPERATION.

$1|00\ )\ 62|72$

62—72 rem.

$62\frac{72}{100}$, quotient.

Examples.

1. Divide 6747 by 10.
2. Divide 270460 by 100.
3. Divide 927000 by 1000.
4. Divide 704963 by 1000.
5. Divide 2394704 by 10000.
6. Divide 97469 by 1000.
7. Divide 967000 by 10000.
8. Divide 490400 by 1000.
9. Divide 6972004 by 1000.
10. Divide 4970906 by 1000.

CASE III.

46. When the divisor contains significant figures, with ciphers on the right of them.

1. Divide 77256 by 3700.

Rule.

I. *Cut off the ciphers by a line, and cut off the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor; and if there is no remainder, the figures cut off will be the true remainder. But if there is a remainder, annex to it the figures cut off, and the result will be the true remainder.*

OPERATION.

$37|00\ )\ 772|56\ (20\frac{3256}{3700}$

74

3256 rem.

$20\frac{3256}{3700}$, quotient.

Examples.

1. Divide the number 561754 by $20 = 2 \times 10$.
2. Divide the number 6934756356 by $200 =$
3. Divide the number 5810636 by $5000 = 5 \times 1000$.
4. Divide the number 349275 by $47000 =$
5. Divide the number 71692 by $6400 = 64 \times 100$.
6. Divide 1000664300 by $125000 = 125 \times 1000$.

Practical Questions.

1. If an individual earns 3 dollars per day, in what time will he earn 450 dollars?
2. If a person pays, per week, to his laborers, 4563 dollars, at the rate of 9 dollars each, how many does he employ?
3. How many cattle, at 35 dollars per head, can be bought for 1015 dollars?
4. If 45 men earn in one year 27675 dollars, how much does each earn?
5. What is the price of one boat, if 25 boats cost 900 dollars?
6. A farmer has 6944 pounds of butter, which he wishes to put into firkins, each to contain 56 pounds: how many firkins does he require?
7. A barrel of flour contains 196 pounds: how many barrels are required to hold 37436 pounds?
8. If a vessel sails 5678 miles in 34 days, how many miles does it sail per day?
9. In how many days will a vessel reach her destined port, which is 2960 miles distant from the place of sailing, if she sails 216 miles per day?
10. If one man can earn a certain sum of money in 28 days, how long would it take 7 men to earn the same amount of money?

11. If one man can dig a trench in 279 days, in how many days could 9 men have dug the same?

12. A man has sufficient feed to keep one horse 224 days: how many days would the feed last 7 horses?

13. A contractor engages to do a piece of work in 60 days. Supposing that one man can do it in 1320 days, how many men must be employed to finish the work in time?

14. Suppose that, in a month, a mechanic labors 26 days, and receives 3750 cents: how much does he receive a day?

15. How many times would a measure, 18 feet long, be laid down in measuring a distance of 882 feet.

16. How many times would a surveyor lay down a chain, 66 feet long, in measuring a distance of 47520 feet?

17. How many dozen eggs are there in a barrel, containing 2760 eggs; and how many baskets would contain them, if each basket held 30?

18. A farmer wishes to fence, with rails, his fields to the length of 5280 feet. Suppose a panel to be 7 feet long, how many panels will there be?

19. Massachusetts, by the census of 1850, had a population of about 990600 on an area of 7800 square miles: what was the population to the square mile?

20. By the census of 1850, Maine had a population of about 570000, and an area of 30000 square miles: what was the population to the square mile?

21. New York had a population of about 3087695 upon an area of 46085 square miles: what was the population to the square mile.

22. If the divisor be 3675 and the dividend 926100, what is the quotient?

23. What is the divisor when the dividend is 1732800, and the quotient 380?

24. A grocer has 1380 pounds of buckwheat, which he wishes to put into bags, each containing 25 pounds: how many bags would be required?

Examples in the preceding Rules.

1. If one person receives 36 dollars per month, and another 45 dollars per month, in what time will they together earn 1944 dollars?

2. A gentleman died possessed of a fortune of 50000 dollars; he directed that after paying his debts, amounting to 6000 dollars. the remainder should be equally divided among his four children: what was the portion of each?

3. How many pounds of coffee, worth 18 cents a pound, must be given for 368 pounds of sugar, worth 9 cents a pound?

4. James has 14 calves, worth 4 dollars each; 40 sheep, worth 3 dollars each; he gives them all for a horse worth 150 dollars: does he make or lose, and how much?

5. How many barrels of flour, at 5 dollars per barrel, will pay for 20 barrels of potatoes, at 3 dollars per barrel?

6. If a suit of clothes requires 9 yards of cloth, how many suits may be made from 12 pieces of cloth, each containing 45 yards?

7. A young man having engaged his services at the rate of 756 dollars a year, left his employer at the end of 8 months: what should he receive, at the rate agreed on?

8. A person buys a house and lot for 3500 dollars, of which sum he pays 1850 dollars cash, and agrees to pay the remainder in annual instalments of 150 dollars each: in how many years will the debt be paid?

9. A person bought 65 cattle for 1950 dollars, and sold them for 2795 dollars, what was the gain on each?

10. I bought 600 barrels of flour for 3500 dollars, and wish to gain 700 dollars, after paying 100 dollars for charges: at what price per barrel must they be sold?

11. If a drover asks 1125 dollars for a flock of 375 sheep, how many head would 159 dollars buy at that rate?

12. A person bought a farm of 192 acres for 11520 dollars, and sold 60 acres for 6000 dollars: what did he gain on each acre sold?

13. The product of two numbers is 6843020, and one of them is 15695: what is the other?

14. If the dividend is 2884476, the quotient 471, and the remainder 72, what is the divisor?

15. A merchant sold 45 barrels of flour at 6 dollars per barrel, and 25 barrels of potatoes at 2 dollars per barrel; and took in payment, butter at 8 dollars per firkin: how many firkins did he receive?

16. A person paid, at a hotel, 12 dollars for the first week's board, 15 dollars for the second, 18 dollars for the third, and 23 dollars for the fourth: what was the equal or average price per week?

17. A grocer has five boxes of sugar, containing respectively 175 pounds, 250 pounds, 523 pounds, 365 pounds, and 872 pounds: had each box contained an equal quantity of the sugar, what would have been the weight in each?

18. A shopkeeper sold on Monday to the amount of 37 dollars; on Tuesday, 49 dollars; Wednesday, 105 dollars; Thursday, 66 dollars; Friday, 56 dollars; and on Saturday, 125 dollars: what was the average sale of each day?

19. In a certain school, the attendance was, on Monday, 154; on Tuesday, 170; on Wednesday, 190; on Thursday, 184; on Friday, 172: what was the average attendance?

20. If a man's salary is 1600 dollars a year, and his expenses 850 dollars, in what time will he be worth 20000 dollars, if he is worth 5000 when his salary begins?

21. Two persons counting their money, found that together they had 684 dollars; but one had 28 dollars more than one-half of it: how much had the other?

22. What is the cost of 32568 oranges, at the rate of 25 cents per dozen? and how many pineapples, at the rate of 3 for 15 cents, will pay for the oranges?

CANCELLATION.

47. CANCELLATION is a process of shortening Arithmetical operations in Division, by omitting, or *cancelling*, factors common to the dividend and divisor.

It depends on the principle that,

If the dividend and divisor be both divided by the same number, the quotient will not be changed.

1. Divide 42 by 14.

ANALYSIS.—Resolve the dividend and divisor into factors, then cancel those which are common, and mark the cancelled figures.

OPERATION.

$$\frac{42}{14} = \frac{6 \times \cancel{7}}{2 \times \cancel{7}} = \frac{6}{2} = 3.$$

2. In 5 times 35 how many times 7?

ANALYSIS.—Resolve 35 into the two factors 7 and 5, then cancel the 7.

OPERATION.

$$\frac{35 \times 5}{7} = \frac{\cancel{7} \times 5 \times 5}{\cancel{7}} = 25.$$

3. In 18 times 15 how many times 45?

ANALYSIS.—We see that 15 is a factor of 45. Divide by this factor, write the quotient 3 under 45, and cancel 15 and 45. Again, 3 is a factor of 18: divide by it, and write the quotient 6 above 18, and cancel 18 and 3: hence, the quotient is equal to 6.

OPERATION.

$$\frac{\overset{6}{\cancel{15} \times \cancel{18}}}{\underset{3}{\cancel{45}}} = \frac{6}{1} = 6.$$

Rule.

I. *Write the dividend above a horizontal line, and the divisor below it.*

II. *Cancel all the factors that are common to the dividend and divisor, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTES.—1. If one of the numbers contains a factor equal to the product of two or more factors of the other, they may all be cancelled.

2. If the product of two or more factors of the dividend is equal to the product of two or more factors of the divisor, they may all be cancelled.

3. If all the factors of the dividend are cancelled, the quotient 1 must be put for the factor last cancelled.

Examples.

1. Divide the product of $6\times8\times12\times15$ by $3\times4\times9$.

We cancel the factor 3, in 3 and 6; then 4, in 4 and 8; then 3, in 9 and 12; then 3, in 3 and 15.

OPERATION.

$$\frac{\overset{2}{\cancel{6}}\times\overset{2}{\cancel{8}}\times\overset{4}{\cancel{12}}\times\overset{5}{\cancel{15}}}{\underset{3}{\cancel{3}}\times4\times\underset{3}{\cancel{9}}}=80.$$

2. Divide the product of $8\times9\times5\times6$ by $3\times5\times2$.

3. Divide the product of $9\times8\times7\times6$ by $2\times3\times4\times5$.

4. Divide the product of $6\times5\times3\times7\times8$ by $4\times3\times7\times9$.

5. Divide the product of $4\times5\times6\times8\times9$ by $7\times5\times4\times3$.

6. Divide the product of $10\times5\times4\times3$ by $4\times9\times12$.

7. Multiply $8\times7\times5$ by $3\times6\times5$, and divide the product by $3\times4\times6$.

8. Divide the product of $15\times20\times24$ by $8\times5\times4$.

9. If the product of 20 by 12, be divided by the product of 1, 2, 3 and 4, what will be the quotient?

10. How many loads of wood, at 4 dollars a load, must be given for 14 yards of broadcloth at 6 dollars a yard?

11. How many bushels of buckwheat, at 45 cents a bushel, must be given for 120 bushels of corn at 60 cents a bushel?

12. How many bushels of oats, at 48 cents a bushel, must be given for 6 boxes of raisins, each containing 52 pounds, at 12 cents a pound?

13. How many pounds of butter, at 25 cents a pound, will pay for 60 pounds of tea at 75 cents a pound?

14. How many sheep, at 6 dollars a piece, must be given for a drove of 36 cows at 49 dollars a piece?

15. How many lambs, at 3 dollars each, must be given for 20 tons of hay at 18 dollars a ton?

16. How much butter, at 26 cents a pound, will pay for 8 boxes of sugar, each weighing 216 pounds, if the sugar is valued at 13 cents per pound.

17. A man bought 136 yards of cloth, at 4 dollars a yard, and paid for it in hay, at 16 dollars a ton: how many tons did he give?

18. A worked for B, 36 days at 6 shillings a day, and took his pay in wheat at 9 shillings a bushel: how many bushels did he receive?

19. How many boxes of tea, each containing 48 pounds, worth 75 cents a pound, must be given for 6 barrels of sugar, each containing 200 pounds, at 9 cents a pound?

20. If sugar can be bought for 11 cents a pound, how many bushels of oats, at 66 cents a bushel, must be given for 396 pounds?

21. A person worked 125 days for 6 shillings a day, and received in payment corn at 5 shillings a bushel: how much corn did he receive?

22. How many chickens, at 3 shillings a piece, must be given for 185 turkeys, worth 9 shillings a piece?

COMMON DIVISOR.

48. An Exact Divisor of a number, is a divisor which will divide it without a remainder.

49. A Common Divisor of two or more numbers, is a divisor which will divide each, separately, without a remainder.

50. The Greatest Common Divisor of two or more numbers, is the greatest number that will divide each, separately, without a remainder.

To find the greatest common Divisor.

1. What is the greatest common divisor of 112 and 144?

Rule.

Divide the greater number by the less; and then divide the divisor by the remainder; and continue the operation till nothing remains. The last divisor will be the greatest common divisor of the two numbers.

OPERATION.

112) 144 (1
112
32) 112 (3
96
16) 32 (2
32

2. What is the greatest common divisor of 75 and 275?
3. What is the greatest common divisor of 420 and 510?
4. Find the greatest common divisor of 216 and 316.
5. Find the greatest common divisor of 24 and 1956.
6. Find the greatest common divisor of 39 and 192.
7. What is the greatest common divisor of 1728 and 5000?
8. What is the greatest common divisor of 3750 and 5495?
9. What is the greatest common divisor of 960 and 1920?
10. What is the greatest common divisor of 376 and 495?
11. What is the greatest common divisor of 96 and 360?
12. What is the greatest common divisor of 113 and 7650?

COMMON FRACTIONS.

51. A Unit is a single thing; as, 1 apple, 1 chair, 1 pound of tea; and is denoted by 1.

If a unit be divided into two equal parts, each part is called, *one-half.*

If a unit be divided into three equal parts, each part is called, *one-third.*

If a unit be divided into four equal parts, each part is called, *one-fourth.*

If a unit be divided into twelve equal parts, each part is called, *one-twelfth;* and if it be divided into *any number* of equal parts, we have a like expression for each part.

The parts are thus written:

$\frac{1}{2}$	is read,	one-half.	$\frac{1}{7}$	is read,	one-seventh.
$\frac{1}{3}$	. .	one-third.	$\frac{1}{8}$	. .	one-eighth.
$\frac{1}{4}$	. .	one-fourth.	$\frac{1}{10}$	. .	one-tenth.
$\frac{1}{5}$	. .	one-fifth.	$\frac{1}{15}$	. .	one-fifteenth.
$\frac{1}{6}$	. .	one-sixth.	$\frac{1}{50}$	. .	one-fiftieth.

52. The Unit of a Fraction is the single thing that is divided into equal parts.

53. A Fractional Unit is one of the equal parts of the unit that is divided.

54. A Fraction is a fractional unit, or a collection of fractional units.

1. If an apple be divided into 30 equal parts, write the fractional unit.

2. If a pear be divided into 29 equal parts, write the fractional unit.

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1. If an apple be divided into 30 equal parts, write the fractional unit.

2. If a pear be divided into 29 equal parts, write the fractional unit.

3. If a barrel of flour be divided into 19 equal parts, write the fractional unit.

4. If a yard of cloth be divided into 37 equal parts, write the fractional unit.

Writing Fractions.

$\frac{2}{3}$	is read,	2 thirds.	$\frac{9}{10}$	is read,	9 tenths.
$\frac{3}{4}$	. .	3 fourths.	$\frac{11}{13}$	. .	11 thirteenths.
$\frac{4}{5}$	. .	4 fifths.	$\frac{4}{17}$	. .	4 seventeenths.
$\frac{5}{6}$	. .	5 sixths.	$\frac{8}{9}$	. .	8 ninths.
$\frac{7}{8}$	. .	7 eighths.	$\frac{7}{16}$	. .	7 sixteenths.

55. The DENOMINATOR is the number written below the line, and shows into how many equal parts the unit of the fraction is divided.

56. The NUMERATOR is the number written above the line, and shows how many fractional units are taken.

57. The TERMS of a fraction are the numerator and denominator, taken together; hence, every fraction has two terms.

58. A whole number may be expressed fractionally, by writing 1 under it for a denominator. Thus,

3	may be written	$\frac{3}{1}$	and is read, 3 ones.
5	. . . . .	$\frac{5}{1}$	. . . 5 ones.
6	. . . . .	$\frac{6}{1}$	. . . 6 ones.
8	. . . . .	$\frac{8}{1}$	. . . 8 ones.

59. Properties of Fractions.

1. All the parts of 1, however it may be divided, make up the unit itself; hence, any fractional unit, multiplied by the number of parts, is equal to 1.

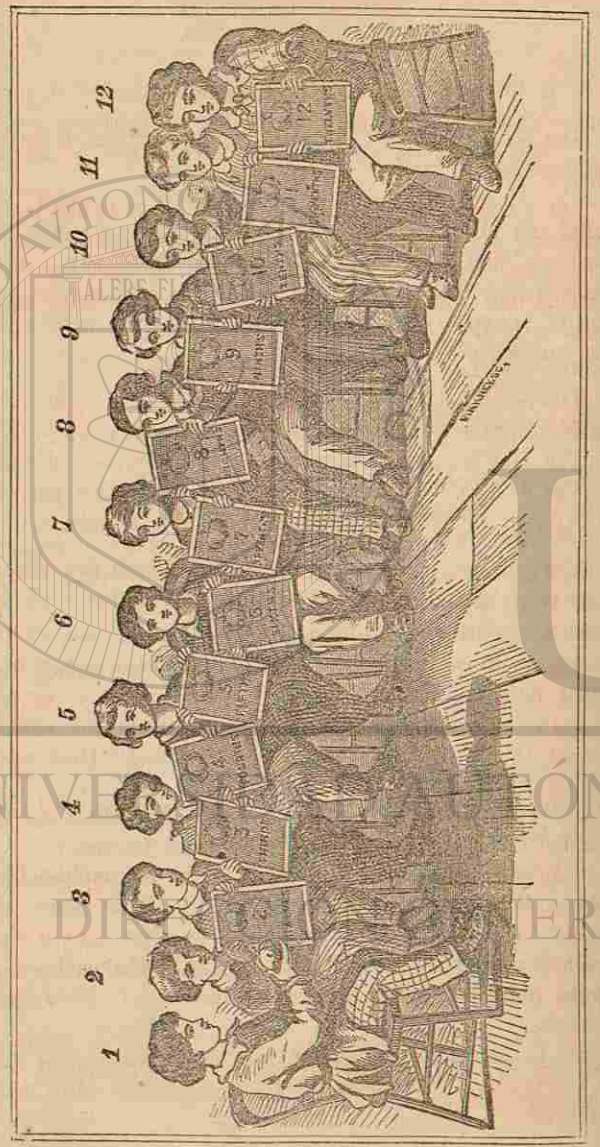
2. If the numerator is less than the number of parts, the value of the fraction is less than 1.

3. If the numerator is greater than the number of parts, some of the fractional units must have come from a second unit; and hence, the value of the fraction will be greater than 1.

60. To ANALYZE a fraction, is to name its unit, its fractional unit, and the number of fractional units taken: Thus, in the fraction $\frac{3}{4}$ of an apple, the unit of the fraction is 1 apple; the fractional unit, $\frac{1}{4}$ of an apple; and the number of fractional units taken, is 3.

Writing and Reading.

1. Write seven-eighths. Write three-fourths.
2. Write six-ninths. Write seven-fifteenths.
3. Write four-twelfths. Write nine-fifteenths.
4. Write seven-fourteenths. Write five-fortieths.
5. Write six-elevenths. Write nine-twelfths.
6. In six-fifths of an orange, what is the unit of the fraction? What is the fractional unit? How many fractional units are taken?
7. In twelve-fifteenths of a dollar, what is the unit of the fraction? What is the fractional unit? How many are taken?
8. Write eleven-thirteenths of a pound. What is the fractional unit? What is the unit of the fraction?
9. In nine-tenths of a yard, what is the unit of the fraction? What is the fractional unit? How many are taken?
10. Write fifteen-twentieths of a pear. What is the unit of the fraction? What is the fractional unit? How many are taken?
11. In nineteen-twentieths of an hour, what is the unit of the fraction? What is the fractional unit? How many are taken?



61. Object Teaching.

The class of boys, represented in the Diagram, is used to teach fractions in the following manner:

Let a class be numbered from the first boy to the highest number. Suppose each boy to have an apple of exactly the same size; and suppose the apple of each boy to be divided into a number of equal parts, corresponding to his number in the class: then,

The first boy will have the entire apple;
The second boy will have the apple in two equal parts;
The third boy will have the apple in three equal parts;
The fourth boy will have the apple in four equal parts;
The fifth boy will have the apple in five equal parts;
And so on, to the highest number of the class.

The parts of the apple held by the fourth boy may be derived from those of the second, by dividing each half into 2 equal parts, giving 4 fourths.

The parts held by the sixth boy may be derived from those of the second, by dividing each part by 3; or from those of the third, by dividing each part by 2.

The parts of the apple held by the eighth boy may be derived from those of the second, by dividing by 4; and from those of the fourth, by dividing by 2.

Q. From what boys may the parts of the apple held by the ninth boy be derived?

A. From the first, by dividing the apple into 9 equal parts; and from the third boy, by dividing each of his equal parts into 3 equal parts.

Q. From what numbers may the parts of the tenth boy be derived? Of the twelfth boy? Of the fourteenth? Of the sixteenth, &c.

Questions.

1. Which boy has the unit of the fraction?
2. What is the fractional unit corresponding to the second boy? How many has he?
3. What is the fractional unit corresponding to the third boy? How many has he?
4. What is the fractional unit corresponding to the fourth boy? How many has he?
5. What is the fractional unit corresponding to the tenth boy? How many has he?
6. What is the fractional unit corresponding to the twelfth boy? How many has he?

Writing the Fractions.

1. Write one of the equal parts of the boy number two.
2. Write two of the equal parts of number three.
3. Write five of the equal parts of number six.
4. Write nine of the equal parts of number ten.
5. Write twelve of the equal parts of number fourteen.
6. Write fifteen of the equal parts of number twenty.
7. Write thirty-nine of the equal parts of number fifty.
8. Write thirty-six of the equal parts of number 37.
9. Write sixty of the equal parts of number seventy-five.
10. Write forty-nine of the equal parts of number fifty.
11. Write sixty-nine of the equal parts of number 70.
12. Write thirty-eight of the equal parts of number 90.
13. Write 100 of the equal parts of number 100.
14. Write sixty-nine of the equal parts of number 75.
15. Write seventy-seven of the equal parts of number 80.
16. Write fifty-nine of the equal parts of number 60.
17. Write ninety-nine of the equal parts of number 101.
18. Write forty-nine of the equal parts of number 70.

62. Six Kinds of Fractions.

1. A Proper Fraction is one whose numerator is less than the denominator.

The following are proper fractions:

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{4}{7}$, $\frac{5}{8}$.

2. An Improper Fraction is one whose numerator is equal to, or exceeds the denominator.

The following are improper fractions:

$\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{4}$, $\frac{9}{8}$.

3. A Simple Fraction is one whose numerator and denominator are both whole numbers.

The following are simple fractions:

$\frac{1}{4}$, $\frac{3}{2}$, $\frac{5}{6}$, $\frac{8}{7}$, $\frac{9}{2}$, $\frac{8}{3}$.

Note.—A simple fraction may be either proper or improper.

4. A Compound Fraction is a fraction of a fraction, or several fractions connected by the word *of*, or $\times$.

The following are compound fractions:

$\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{6} \times 3$, $\frac{1}{7} \times \frac{1}{8} \times 4$.

5. A Mixed Number is the sum of a whole number and a fraction.

The following are mixed numbers:

$3\frac{1}{2}$, $4\frac{1}{3}$, $6\frac{2}{8}$, $5\frac{3}{5}$, $6\frac{5}{8}$, $3\frac{1}{7}$.

6. A Complex Fraction is one whose numerator or denominator is fractional; or, in which both are fractional.

63. **Properties of Fractions, deduced from the Diagram.**

Let us now see what use may be made of the diagram in illustrating the principles of Fractions: Remember,

1. That each boy of the class has the entire apple, divided into as many equal parts as are marked by his number;

2. That the apple is the unit of the fraction;

3. That each equal part of any apple, is a fractional unit;

4. That the denominator of *any fraction* will denote the number of some boy in a class;

5. That the numerator of such fraction will show how many fractional units are taken;

6. That we may pass from a larger to a smaller fractional unit, by multiplying the denominator, and from a smaller to a larger, by dividing the denominator.

64. By the aid of the above principles, the Diagram, and a sensible object to denote the unit of the fraction, the teacher can readily show to the class,

1. *That multiplying the numerator, multiplies the value of the fraction as many times as there are units in the multiplier:*

2. *That multiplying the denominator, diminishes the value of the fraction as many times as there are units in the multiplier:*

3. *That dividing the numerator, diminishes the value of the fraction as many times as there are units in the divisor:*

4. *That dividing the denominator, multiplies the value of the fraction as many times as there are units in the divisor:*

5. *That multiplying the numerator and denominator by the same number, does not alter the value of the fraction:* and

6. *That dividing the numerator and denominator by the same number, does not alter the value of the fraction.*

REDUCTION.

67. Reduction of Fractions is the operation of changing the fractional unit, without altering the value of the fraction.

1. How many halves are there in 2 units? Write them.
2. How many halves are there in 5 units? Write them.
3. How many thirds are there in 7 units? Write them.
4. How many sixths are there in 3 units? Write them.
5. How many eighths are there in 6 units? Write them.
6. How many twentieths are there in 2 units? Write them.
7. How many thirds are there in $2\frac{1}{3}$? Write them.
8. How many fourths are there in 3 and $\frac{3}{4}$? Write them.
9. How many sixths are there in 2 and $\frac{5}{6}$? Write them.
10. How many ninths are there in 3 and $\frac{4}{9}$? Write them.

CASE I.

68. **To reduce a whole number to a fraction having a given denominator.**

1. Reduce 7 to a fraction whose denominator shall be 5.

Rule.—*Multiply the whole number by the required denominator, and write the product over the denominator.*

OPERATION.

$5 \times 7 = 35.$

Ans. $\frac{35}{5}$.

Examples.

1. How many twentieths are there in 15?
2. Reduce 25 to sixteenths.
3. Reduce 47 to thirtieths.
4. How many fortieths are there in 75?
5. Reduce 29 to a fraction whose denominator shall be 18.
6. Reduce 112 to a fraction whose denominator shall be 63.

CASE II.

69. To reduce a mixed number to an improper fraction.

1. Reduce $4\frac{5}{6}$ to an improper fraction.

Rule.—*Multiply the whole number by the denominator of the fraction, to the product add the numerator, and place the sum over the denominator.*

OPERATION.

$$4\frac{5}{6}$$
$$6$$
$$24$$
$$5$$
$$\frac{29}{6} =$$

Examples.

1. Reduce $5\frac{1}{7}$ to an improper fraction.
2. Reduce $6\frac{2}{3}$ to an improper fraction.
3. Reduce $10\frac{4}{9}$ to an improper fraction.
4. Reduce $16\frac{7}{9}$ to an improper fraction.
5. What fraction is equal to $18\frac{7}{8}$?
6. What fraction is equal to $25\frac{5}{6}$?
7. Reduce $37\frac{5}{8}$ yards to eighths of a yard.
8. Reduce $63\frac{7}{10}$ dollars to tenths of a dollar.
9. Reduce $45\frac{4}{11}$ and $28\frac{9}{10}$ to improper fractions.
10. Reduce $300\frac{4}{7}$ and $400\frac{7}{20}$ to improper fractions.
11. Reduce $25\frac{3}{4}$ and $16\frac{3}{5}$ to improper fractions.
12. Reduce $60\frac{7}{9}$ and $59\frac{11}{7}$ to improper fractions.

CASE III.

70. To reduce an improper fraction to a mixed number.

1. Reduce $\frac{14}{5}$ to a mixed number.

Rule.—*Divide the numerator by the denominator, and the quotient will be the whole or mixed number.*

OPERATION.

$$14 \div 5 = 2\frac{4}{5}.$$

Examples.

1. How many units are there in $\frac{8}{4}$? In $\frac{12}{6}$?
2. How many units are there in $\frac{16}{8}$? In $\frac{64}{32}$?
3. How many units are there in $\frac{32}{4}$? In $\frac{30}{15}$?
4. Reduce $\frac{25}{6}$ to a mixed number.
5. Reduce $\frac{38}{7}$ to a mixed number.
6. Reduce $\frac{54}{8}$ to a mixed number.
7. Reduce $\frac{73}{16}$ to a mixed number.
8. Reduce $\frac{112}{18}$ to a mixed number.
9. Reduce $\frac{750}{26}$ to a mixed number.
10. Find the value of $\frac{990}{160}$.
11. Find the value of $\frac{196}{24}$.
12. How many miles are equal to $\frac{725}{18}$ miles?
13. How many barrels are equal to $\frac{2586}{7}$ barrels?
14. What number is equal to $\frac{12600}{430}$?
15. What number is equal to $\frac{8160}{84}$?
16. Reduce $\frac{38960}{2620}$ to a mixed number.
17. Find the value of $\frac{894750}{800}$ dollars.
18. Reduce $\frac{20020}{91}$ to a mixed number.

CASE IV.

71. To reduce a fraction to its lowest terms.

1. Reduce $\frac{16}{48}$ to its lowest terms.

Rule.—*Divide the numerator and denominator by any number that will exactly divide them; divide the quotients in the same manner, until no number greater than 1 will exactly divide them:*

Or, *Divide both terms of the fraction by their greatest common divisor.*

1ST OPERATION.

$$2\,)\,16 \quad 2\,)\,48 = \frac{8}{24}.$$
$$8\,)\,8 \quad 8\,)\,24 = \frac{1}{3}.$$

2D OPERATION.

$$16\,)\,16 \quad 16\,)\,48 = \frac{1}{3}.$$

Examples.

1. Reduce $\frac{18}{24}$.
2. Reduce $\frac{24}{30}$.
3. Reduce $\frac{56}{64}$.
4. Reduce $\frac{28}{84}$.
5. Reduce $\frac{56}{98}$.
6. Change $\frac{112}{144}$.
7. Reduce $\frac{160}{950}$.
8. Reduce $\frac{144}{1728}$.
9. Change $\frac{192}{486}$.
10. Reduce $\frac{510}{5610}$.
11. Reduce $\frac{75}{1000}$.
12. Reduce $\frac{160}{9000}$.
13. Reduce $\frac{175}{812}$.
14. Reduce $\frac{240}{875}$.
15. Reduce $\frac{419}{637}$.
16. Reduce $\frac{897}{1495}$.

CASE V.

72. To reduce a compound fraction to a simple one.

1. What is the value of $\frac{5}{6}$ of $\frac{3}{7}$?

Rule.—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

OPERATION.

$$\frac{5\times3}{6\times7}=\frac{15}{42}.$$

NOTE.—If there are mixed numbers, reduce them to their equivalent improper fractions.

By Cancellation.

2. Reduce $\frac{3}{7}$ of $\frac{7}{8}$ of $\frac{5}{3}$ to its lowest terms.

Rule.—*Cancel like factors in the numerator and denominator.*

OPERATION.

$$\frac{\cancel{3}}{\cancel{7}}\text{ of }\frac{\cancel{7}}{8}\text{ of }\frac{5}{\cancel{3}}=\frac{5}{8}.$$

Examples.

1. Reduce $\frac{3}{4}$ of $\frac{2}{6}$.
2. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{3}{7}$.
3. Reduce $\frac{2}{3}$ of $\frac{3}{9}$ of $\frac{1}{8}$.
4. Reduce $2\frac{1}{2}$ of $\frac{4}{8}$.
5. Reduce $4\frac{1}{3}$ of $\frac{5}{7}$.
6. Reduce $5\frac{8}{6}$ of $\frac{2}{9}$.
7. Reduce $\frac{4}{9}$ of $\frac{5}{6}$.
8. Reduce $\frac{5}{8}$ of $\frac{9}{10}$.

9. A boy having $\frac{5}{8}$ of a dollar, gave away $\frac{2}{3}$ of his money: how much did he give away?
10. A cask holding $\frac{8}{9}$ of a hogshead, lost by leakage $\frac{2}{5}$ of its contents: how much was lost?
11. A man having $3\frac{3}{4}$ dollars, gave $\frac{2}{9}$ of his money for a dinner: what was the cost of his dinner?

CASE VI.

73. To reduce fractions having different denominators, to fractions having a common denominator.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$ to a common denominator.

Rule.—*Multiply the numerator of each fraction by all the denominators except its own, for the new numerators, and all the denominators together for a common denominator.*

OPERATION.

$1\times4\times6=24$ 1st num.
$3\times2\times6=36$ 2d num.
$5\times4\times2=40$ 3d num.
$2\times4\times6=48$ com. den.

Ans. $\frac{24}{48}$, $\frac{36}{48}$, and $\frac{40}{48}$.

NOTES.—1. Before multiplying, reduce all fractions to simple fractions.

2. When the numbers are small, the work may be performed mentally: Thus:

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{5}$, $=\frac{20}{40}$, $\frac{10}{40}$, $\frac{16}{40}$.

Examples.

1. Reduce $\frac{4}{7}$, $\frac{5}{6}$, and $\frac{1}{3}$.
2. Reduce $\frac{2}{5}$, $\frac{3}{2}$, and $\frac{2}{3}$.
3. Reduce $\frac{1}{7}$, $\frac{3}{8}$, and $\frac{1}{2}$.
4. Reduce $2\frac{1}{2}$ and $4\frac{1}{4}$.
5. Reduce $5\frac{1}{3}$ and $2\frac{3}{4}$.
6. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ and $\frac{4}{5}$ of 6.
7. Reduce $\frac{3}{2}$ of 4 and $\frac{1}{3}$ of $5\frac{1}{2}$.
8. Reduce $\frac{6}{7}$ of 2 and 5.
9. Reduce $7\frac{1}{3}$ of 2 and 6.
10. Reduce $\frac{4}{2}$, $\frac{3}{9}$, and $\frac{4}{7}$.
11. Reduce $8\frac{1}{2}$ of 3 and $5\frac{1}{3}$.
12. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{4}{5}$.
13. Reduce $\frac{4}{7}$, $\frac{6}{9}$, and $\frac{1}{5}$.
14. Reduce $\frac{4}{8}$, $\frac{9}{12}$, and $\frac{3}{4}$.

ADDITION OF FRACTIONS.

74. ADDITION OF FRACTIONS is the operation of finding the sum of two or more fractions.

CASE I.

75. When the fractions have the same denominator.

1. What is the sum of $\frac{1}{3}$, $\frac{4}{3}$, and $\frac{7}{3}$?

Rule.—*Add the numerators, and place their sum over the common denominator.*

OPERATION.

$1 + 4 + 7 = 12.$

Ans. $\frac{12}{3} = 4.$

Examples.

1. Add $\frac{3}{4}$, $\frac{5}{4}$, and $\frac{7}{4}$.
2. Add $\frac{1}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$.
3. Add $\frac{3}{5}$, $\frac{5}{5}$, $\frac{6}{5}$, and $\frac{9}{5}$.
4. Add $\frac{13}{9}$, $\frac{14}{9}$, and $\frac{17}{9}$.
5. Add $\frac{6}{15}$, $\frac{9}{15}$, $\frac{12}{15}$, and $\frac{17}{15}$.
6. Add $\frac{10}{13}$, $\frac{7}{13}$, $\frac{9}{13}$, and $\frac{15}{13}$.
7. Add $\frac{5}{8}$, $\frac{15}{8}$, $\frac{19}{8}$, and $\frac{17}{8}$.
8. Add $\frac{15}{11}$, $\frac{11}{11}$, and $\frac{12}{11}$.
9. Add $\frac{17}{16}$, $\frac{19}{16}$, $\frac{20}{16}$, and $\frac{11}{16}$.
10. Add $\frac{14}{20}$, $\frac{15}{20}$, and $\frac{30}{20}$.

CASE II.

76. When the denominators are unlike.

1. Add $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{3}{4}$ together.

Rule.—I. *Reduce the fractions to a common denominator:*

II. *Add the numerators, and place their sum over the common denominator.*

OPERATION.

$3 \times 2 \times 4 = 24$ 1st num.
$2 \times 5 \times 4 = 40$ 2d num.
$3 \times 2 \times 5 = 30$ 3d num.
$5 \times 2 \times 4 = 40$ com. den.

$24 + 40 + 30 = 94$: hence, sum $= \frac{94}{40} = 2\frac{7}{20}$.

Examples.

1. Add $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{7}{8}$.
2. Add $\frac{1}{9}$, $\frac{4}{7}$, and $\frac{1}{2}$.
3. $\frac{5}{6} + \frac{7}{9} =$ what?
4. $\frac{1}{10} + \frac{3}{4} + \frac{11}{12} =$ what?
5. Add $\frac{2}{7}$ of $\frac{3}{5}$ and $\frac{7}{2}$ of $\frac{4}{5}$.
6. Add $\frac{5}{9}$ of $\frac{3}{4}$ and $\frac{2}{3}$ of $\frac{3}{6}$.
7. Add $\frac{3}{5}$, $\frac{6}{6}$, and $\frac{4}{10}$.
8. Add $\frac{5}{2}$ and $\frac{3}{4}$.
9. Add $\frac{2}{3}$, $\frac{3}{7}$, and $\frac{5}{8}$.
10. Add $\frac{5}{9}$, $\frac{6}{7}$, and $\frac{3}{2}$.
11. Add $\frac{4}{7}$, $\frac{3}{6}$, and 2.
12. Add $\frac{5}{9}$, $\frac{6}{2}$, and $\frac{1}{3}$.
13. Add $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{3}{7}$.
14. Add $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{1}{7}$.

CASE III.

77. When mixed numbers are to be added.

1. Add $4\frac{1}{2}$, $5\frac{1}{3}$, and $6\frac{1}{4}$ together.

Rule.—*Add the whole numbers and fractions separately, and then unite their sums.*

OPERATION.

$4 + 5 + 6 = 15.$
$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}.$
Ans. $15\frac{13}{12} = 16\frac{1}{12}.$

Examples.

1. Add $4\frac{1}{3}$, $5\frac{1}{2}$, and $\frac{1}{4}$.
2. Add $3\frac{1}{5}$, $6\frac{1}{7}$, and $3\frac{2}{5}$.
3. Add $\frac{3}{8}$, $5\frac{4}{9}$, and $6\frac{7}{8}$.
4. Add $4\frac{3}{8}$, $6\frac{2}{9}$, and $\frac{1}{7}$.
5. Add $\frac{4}{5}$, $6\frac{7}{8}$, and $5\frac{6}{7}$.
6. Add $5\frac{3}{7}$, $6\frac{1}{3}$, and $\frac{6}{7}$.
7. Add $3\frac{1}{7}$, $5\frac{1}{8}$, and $\frac{1}{4}$.
8. Add $2\frac{1}{2}$, $4\frac{1}{7}$, and 4.
9. Add $7\frac{7}{8}$, $\frac{1}{2}$, and $6\frac{4}{5}$.
10. Add $9\frac{4}{7}$, $3\frac{1}{6}$, and $\frac{4}{9}$.

Practical Questions.

1. James pays $\frac{3}{7}$ of a dollar for a pair of gloves, and $\frac{3}{8}$ of a dollar for a handkerchief: how much do they cost him?
2. Nancy buys a work-box for $\frac{7}{8}$ of a dollar, a pair of gloves for $\frac{3}{8}$ of a dollar, and a comb for $\frac{2}{16}$ of a dollar: how much do they all cost?
3. Jane buys a yard of ribbon for $\frac{2}{7}$ of a dollar, a gold

pin for $\frac{5}{2}$ of a dollar, and an inkstand for $\frac{3}{14}$ of a dollar: how much did she pay in all?

4. William buys a kite for $\frac{2}{3}$ of a dollar, and a string for $\frac{4}{15}$ of a dollar: how much did he pay?

5. Three ducks cost $\frac{4}{5}$ of a dollar, two fowls $\frac{2}{3}$ of a dollar, and two geese $\frac{9}{6}$ of a dollar: what is the entire cost?

6. Two sheep cost $\frac{17}{2}$ of a dollar, a calf $\frac{9}{4}$ of a dollar, and a lamb $\frac{5}{4}$ of a dollar: what is the entire cost?

7. Three yards of shirting cost $\frac{4}{9}$ of a dollar, a handkerchief $\frac{7}{9}$ of a dollar, and a pair of gloves $\frac{1}{3}$ of a dollar: what is the entire cost?

8. A person paid $\frac{4}{5}$ of a dollar for butter, $\frac{3}{8}$ of a dollar for tea, and $\frac{9}{16}$ of a dollar for coffee: what was paid for the three articles?

9. A person received $5\frac{4}{5}$ dollars on Monday, $8\frac{3}{8}$ dollars on Tuesday, and $7\frac{2}{3}$ dollars on Wednesday: how much did he receive during the three days?

10. A man who had spent $9\frac{7}{8}$ dollars for a coat, and $2\frac{3}{4}$ dollars for a vest, had $6\frac{2}{7}$ dollars remaining: how much had he at first?

11. In doing a piece of work, a laborer was employed $9\frac{1}{5}$ days, a second was employed $8\frac{2}{3}$ days, and a third was engaged $5\frac{5}{6}$ days: how many days were the three laborers employed?

12. A person who had $4\frac{2}{5}$ dollars, earned $6\frac{4}{7}$ dollars: if, in addition, he had received a present of $5\frac{1}{4}$ dollars, how much would he then have had?

13. A person paid $17\frac{4}{5}$ dollars for groceries; he lost $10\frac{5}{12}$ dollars in going home, and had $16\frac{7}{8}$ dollars remaining: how much had he at first?

14. A scholar, in adding several fractions, obtained $2\frac{5}{6}$ for the sum, but he omitted to add $1\frac{9}{10}$: what was the correct sum?

15. A person paid his butcher $9\frac{1}{5}$ dollars, which was $2\frac{5}{8}$ dollars too little: what was the amount due?

SUBTRACTION.

78. Subtraction is the operation of finding the difference between two fractions.

CASE I.

79. When the denominators are the same.

1. What is the difference between $\frac{5}{4}$ and $\frac{3}{4}$?

Rule.—*Subtract the less numerator from the greater, and place the difference over the common denominator.*

OPERATION.

$5 - 3 = 2$.

Ans. $\frac{2}{4}$.

Examples.

1. From $\frac{15}{8}$ take $\frac{7}{8}$.
2. From $\frac{6}{9}$ take $\frac{2}{9}$.
3. From $\frac{16}{30}$ take $\frac{12}{30}$.
4. From $\frac{27}{50}$ take $\frac{9}{50}$.
5. From $\frac{19}{54}$ take $\frac{17}{54}$.
6. From $\frac{17}{45}$ take $\frac{15}{45}$.
7. From $\frac{19}{27}$ take $\frac{6}{27}$.
8. From $\frac{120}{150}$ take $\frac{115}{150}$.

CASE II.

80. When the denominators are different.

1. What is the difference between $\frac{5}{8}$ and $\frac{3}{7}$?

Rule.

I. *Reduce the fractions to a common denominator.*

II. *Subtract the less numerator from the greater, and place their difference over the common denominator.*

OPERATION.

$5 \times 7 = 35$ 1st num.

$3 \times 8 = 24$ 2d num.

$7 \times 8 = 56$ com. den.

$\frac{5}{8} - \frac{3}{7} = \frac{35}{56} - \frac{24}{56} =$

$= \frac{11}{56}$, *Ans.*

Note.—If there are mixed numbers, reduce them to improper fractions, and reduce all compound fractions to simple ones.

Examples.

1. From $\frac{5}{6}$ take $\frac{3}{4}$.
2. From $\frac{7}{9}$ take $\frac{3}{5}$.
3. From $\frac{8}{11}$ take $\frac{4}{9}$.
4. From $6\frac{1}{2}$ take $5\frac{2}{3}$.
5. From $1\frac{1}{2}$ subtract $\frac{9}{16}$.
6. From $\frac{16}{19}$ take $\frac{12}{15}$.
7. From $\frac{18}{21}$ take $\frac{15}{20}$.
8. From $1\frac{1}{5}$ take $\frac{2}{15}$.
9. From $\frac{1}{2}$ of $\frac{5}{6}$ take $\frac{9}{25}$.
10. Take $\frac{3}{4}$ of $\frac{5}{11}$ from $4\frac{4}{27}$.
11. Take $6\frac{3}{5}$ from 8.
12. Take $7\frac{2}{3}$ from $9\frac{3}{7}$.
13. From $\frac{8}{13}$ take $\frac{3}{4}$ of $\frac{4}{27}$.
14. Subtract $\frac{3}{16}$ from $\frac{7}{15}$.
15. Subtract $\frac{7}{20}$ from $\frac{7}{10}$.
16. From $3\frac{7}{30}$ take $\frac{9}{100}$.

Practical Questions.

1. James gave $\frac{7}{8}$ of a shilling for a top, and $\frac{1}{3}$ of a shilling for an orange: how much more did he give for the top than for the orange?

2. John gave $\frac{7}{8}$ of a dollar for a pair of gloves, and $\frac{3}{5}$ of a dollar for a pocket handkerchief: how much more did his gloves cost him than his handkerchief?

3. From the sum of $\frac{5}{6}$ and $2\frac{3}{4}$ take $\frac{4}{5}$ of $\frac{4}{5}$.

4. What is the difference of $2\frac{7}{8}$ dollars and $\frac{2}{5}$ of $1\frac{1}{2}$ dollar?

5. What is the difference of $\frac{11}{20}$ of a pound and $\frac{5}{21}$ of a pound?

6. A person, who had $5\frac{3}{5}$ dollars, spent $4\frac{5}{6}$ dollars for groceries: what had he left?

7. A person paid $4\frac{2}{3}$ dollars for a barrel of flour, and sold it for $6\frac{1}{8}$ dollars: how much was gained?

8. If from a barrel, that contains $31\frac{1}{2}$ gallons of wine, $15\frac{5}{8}$ gallons are drawn: how much remains in it?

9. How much is $6\frac{3}{7}$ greater than $\frac{3}{4}$ of $4\frac{2}{5}$?

10. A cap is valued at $1\frac{5}{8}$ dollars, but the buyer has only $1\frac{1}{5}$ dollars: how much does he require, to pay for it?

MULTIPLICATION.

81. Multiplication of Fractions is the operation of taking one number as many times as there are units in another, when one or both are fractional.

CASE I.

82. To multiply a fraction by a whole number.

1. If one yard of cloth costs $\frac{3}{10}$ of a dollar, what will 5 yards cost?

Rule.

Multiply the numerator, or divide the denominator.

1st operation.

$$\frac{3}{10} \times 5 = \frac{3 \times 5}{10} = \frac{15}{10} = 1\frac{1}{2}.$$

2d operation.

$$\frac{3}{10} \times 5 = \frac{3}{10 \div 5} = \frac{3}{2} = 1\frac{1}{2}.$$

Examples.

1. Multiply $\frac{7}{8}$ by 4.
2. Multiply $\frac{6}{9}$ by 8.
3. Multiply $\frac{6}{32}$ by 16.
4. Multiply $\frac{25}{24}$ by 13.
5. Multiply $\frac{14}{16}$ by 16.
6. Multiply $\frac{2}{5}$ of $\frac{6}{7}$ by 9.
7. Multiply $\frac{6}{5}$ of $\frac{3}{11}$ by 12.
8. Multiply $\frac{5}{8}$ of $\frac{2}{7}$ by 18.
9. Multiply $\frac{3}{7}$ of $\frac{2}{9}$ by 17.
10. Multiply $\frac{6}{15}$ of 2 by 15.

CASE II.

83. To multiply a whole number by a fraction.

1. If 1 yard of cloth costs 6 dollars, what will $\frac{7}{8}$ of a yard cost?

Rule.—*Multiply the whole number by the numerator, and divide the product by the denominator.*

operation.

$$6 \times \frac{7}{8} = \frac{6 \times 7}{8} = \frac{42}{8} = 5\frac{1}{4}.$$

Examples.

1. Multiply 6 by $\frac{3}{6}$.
2. Multiply 10 by $\frac{7}{9}$.
3. Multiply 16 by $\frac{4}{5}$.
4. Multiply 20 by $\frac{11}{15}$.
5. Multiply 35 by $\frac{15}{7}$.
6. Multiply 32 by $\frac{16}{17}$.
7. Multiply 100 by $\frac{9}{12}$.
8. Multiply 117 by $\frac{18}{19}$.

9. If 1 month's wages amount to 45 dollars, what are the wages for $\frac{9}{15}$ of a month?

10. What will $\frac{7}{8}$ of a yard of cloth cost, if one yard costs 125 cents?

11. What will $\frac{9}{16}$ of a ton of iron be worth, if one ton is valued at 46 dollars?

CASE III.

84. To multiply a whole number by a mixed number.

1. Multiply 16 by $5\frac{2}{3}$.

Rule.—*First multiply by the fractional part, and then by the whole number, and add the products.*

OPERATION.

$16 \times \frac{2}{3} = \frac{32}{3} = 10\frac{2}{3}$

$16 \times 5 = 80$

Ans. $90\frac{2}{3}$.

Examples.

1. Multiply 15 by $5\frac{3}{4}$.
2. Multiply 18 by $5\frac{4}{9}$.
3. Multiply 60 by $6\frac{3}{5}$.
4. Multiply 32 by $9\frac{7}{8}$.
5. Multiply 45 by $6\frac{1}{9}$.
6. Multiply 84 by $7\frac{7}{12}$.
7. Multiply 64 by $6\frac{3}{8}$.
8. Multiply 96 by $4\frac{3}{12}$.

9. If one barrel of flour costs 7 dollars, what will $6\frac{2}{5}$ barrels cost?

10. What must be paid for $9\frac{3}{4}$ yards of cloth, at 6 dollars a yard?

11. If the wages for a month are 36 dollars, what are the wages for $9\frac{5}{6}$ months?

CASE IV.

85. To multiply one fraction by another.

1. If a bushel of corn costs $\frac{7}{8}$ of a dollar, what will $\frac{3}{7}$ of a bushel cost?

Rule.—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

OPERATION.

$\frac{7}{8} \times \frac{3}{7} = \frac{7 \times 3}{8 \times 7} = \frac{21}{56} = \frac{3}{8}$.

Examples.

1. Multiply $\frac{7}{9}$ by $\frac{2}{3}$.
2. Multiply $\frac{5}{6}$ by $\frac{4}{7}$.
3. Multiply $\frac{7}{8}$ by $\frac{3}{5}$.
4. Multiply $\frac{5}{12}$ by $\frac{13}{4}$.
5. Multiply $4\frac{1}{2}$ by $2\frac{1}{4}$.
6. Multiply $\frac{2}{5}$ of $\frac{6}{7}$ by $\frac{1}{8}$.
7. Multiply $\frac{6}{7}$ of $\frac{4}{5}$ by $\frac{3}{9}$.
8. Multiply $\frac{1}{2}$ of $\frac{9}{16}$ by $\frac{4}{9}$.

Practical Questions.

1. If 1 yard of cloth costs $1\frac{3}{4}$ dollars, what will $\frac{7}{8}$ of a yard cost?

2. If 1 pound of tea costs $\frac{8}{9}$ of a dollar, what will $2\frac{1}{2}$ pounds cost?

3. At $8\frac{1}{2}$ cents a dozen, what will be the cost of $3\frac{1}{3}$ dozen of apples?

4. James paid $9\frac{3}{4}$ cents for a top: how much must he pay for 9 tops?

5. At $2\frac{5}{9}$ dollars a head, what would be the cost of 15 sheep?

6. If 1 pair of gloves costs $\frac{7}{8}$ of a dollar, what will 8 pair cost?

7. If 1 bushel of barley costs $\frac{7}{8}$ of a dollar, what will $6\frac{3}{4}$ bushels cost?

8. If one goose costs $\frac{3}{4}$ of a dollar, what will 12 geese cost?

9. What will $3\frac{1}{2}$ pounds of butter cost, at $\frac{1}{5}$ of a dollar a pound?

10. Multiply $\frac{1}{2}$ of $\frac{3}{7}$ of 6 by $\frac{5}{7}$ of $9\frac{1}{2}$.

11. What is the product of the three fractions, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{9}{10}$?

12. What is the product of the fractions, $\frac{1}{2}$, $\frac{2}{13}$, $\frac{4}{5}$, and $\frac{9}{11}$?

13. If a bushel of apples costs $\frac{3}{4}$ of a dollar, what will $\frac{2}{3}$ of a bushel cost?

14. If a yard of muslin is worth $\frac{3}{10}$ of a dollar, what is the worth of $\frac{7}{8}$ of a yard?

15. What will be the cost of $7\frac{2}{5}$ pounds of butter, at $\frac{2}{5}$ of a dollar per pound?

16. If a person travels $3\frac{7}{8}$ miles per hour, how far will he travel in $4\frac{5}{6}$ hours?

17. If John earns $1\frac{5}{8}$ dollars per day, how much will he earn in $\frac{7}{12}$ of a day?

18. A father has five children, and gives to each $\frac{3}{7}$ of a dollar: how much does he give them?

19. If Lucy can hem $1\frac{3}{4}$ yards in 1 hour, how much can she hem in $2\frac{1}{2}$ hours?

20. If 1 pound of tea costs $\frac{5}{7}$ of a dollar, what will be the cost of $2\frac{1}{4}$ pounds?

21. If it requires $3\frac{1}{4}$ yards of cloth for a suit of clothes, how much will be necessary to make 6 suits?

22. If James can earn $4\frac{1}{5}$ cents in one hour, how much can he earn in $3\frac{1}{4}$ hours?

23. If muslin is $13\frac{4}{5}$ cents a yard, what will be the cost of $5\frac{3}{4}$ yards?

24. If apples are $2\frac{7}{8}$ dollars a barrel, what will be the cost of $5\frac{1}{2}$ barrels?

DIVISION.

86. Division of Fractions is the operation of finding how many times one number is contained in another, when one or both, are fractional.

CASE I.

87. To divide a fraction by a whole number.

1. If 5 yards of muslin cost $\frac{10}{11}$ of a dollar, what will 1 yard cost?

Analysis.—1 yard will cost one-fifth as much as 5 yards.

Operation.

$$\frac{10}{11} \div 5 = \frac{10 \div 5}{11} = \frac{2}{11}.$$

Multiplying the denominator by 5 will produce the same result.

$$\frac{10}{11} \div 5 = \frac{10}{11 \times 5} = \frac{10}{55} = \frac{2}{11}.$$

Note.—Divide the numerator, when it is exactly divisible by the divisor: when it is not, multiply the denominator.

Rule.

Divide the numerator, or multiply the denominator, by the divisor.

Examples.

1. Divide $\frac{8}{17}$ by 2.
2. Divide $\frac{6}{11}$ by 3.
3. Divide $\frac{9}{10}$ by 7.
4. Divide $\frac{12}{15}$ by 6.
5. Divide $\frac{7}{8}$ by 10.
6. Divide $\frac{16}{19}$ by 8.
7. Divide $\frac{3}{4}$ by 4.
8. Divide $\frac{2}{25}$ by 8.
9. Divide $\frac{3}{14}$ by 10.
10. Divide $\frac{15}{16}$ by 9.
11. Divide $\frac{12}{7}$ by 7.
12. Divide $\frac{1}{2}$ of $\frac{7}{8}$ by 6.
13. Divide $\frac{1}{3}$ of 4 by 9.
14. Divide $\frac{3}{4}$ of $\frac{3}{7}$ by 8.
15. Divide $\frac{6}{9}$ of $\frac{3}{4}$ by 10.
16. Divide $\frac{5}{6}$ of $\frac{3}{7}$ by 8.
17. Divide $\frac{3}{6}$ of $\frac{2}{5}$ by 5.
18. Divide $\frac{4}{7}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of 2.
19. Divide $\frac{3}{7}$ of $\frac{2}{5}$ by $\frac{4}{9}$ of 9.
20. Divide $\frac{6}{5}$ of $\frac{2}{9}$ by $\frac{2}{3}$ of 3.

When the dividend is a mixed number.

21. Divide $4\frac{1}{3}$ by 6.

Rule.

Reduce the mixed number to an improper fraction, and then divide as before.

OPERATION.

$4\frac{1}{3} = \frac{13}{3}$.

$\frac{13}{3} \div 6 = \frac{13}{18}$.

22. Divide $2\frac{3}{7}$ by 3.	28. Divide $18\frac{2}{5}$ by 7.
23. Divide $6\frac{4}{5}$ by 5.	29. Divide $3\frac{1}{10}$ by 16.
24. Divide $7\frac{2}{3}$ by 6.	30. Divide $5\frac{6}{11}$ by 12.
25. Divide $3\frac{1}{7}$ by 5.	31. Divide $19\frac{1}{5}$ by 8.
26. Divide $7\frac{2}{2}$ by 8.	32. Divide $6\frac{2}{5}$ by 9.
27. Divide $3\frac{4}{7}$ by 9.	33. Divide $8\frac{1}{4}$ by 15.

34. How many barrels of flour can be bought for $60\frac{3}{4}$ dollars, at 6 dollars a barrel?

35. If a suit of clothes requires 5 yards of cloth, how many suits may be cut from a piece of $35\frac{1}{4}$ yards?

36. If $25\frac{1}{3}$ dollars be equally divided among 4 persons, how much will each receive?

37. If James earns $37\frac{1}{2}$ cents in 9 hours, how many cents does he earn in 1 hour?

CASE II.

88. **To divide a whole number by a fraction.**

1. Divide 6 by $\frac{3}{4}$.

Rule.—*Invert the terms of the divisor, and multiply the whole number by the new fraction.*

OPERATION.

$6 \div \frac{3}{4} = \frac{6 \times 4}{3} = \frac{24}{3} = 8$.

Examples.

1. Divide 5 by $\frac{3}{4}$.	9. Divide 15 by $\frac{4}{7}$.
2. Divide 9 by $\frac{2}{3}$.	10. Divide 20 by $\frac{5}{9}$.
3. Divide 12 by $\frac{1}{6}$.	11. Divide 18 by $\frac{2}{9}$.
4. Divide 8 by $\frac{3}{7}$.	12. Divide 28 by $\frac{6}{5}$.
5. Divide 7 by $\frac{4}{9}$.	13. Divide 30 by $\frac{2}{3}$ of $\frac{7}{8}$.
6. Divide 14 by $\frac{9}{10}$.	14. Divide 16 by $\frac{3}{5}$ of $\frac{1}{6}$.
7. Divide 18 by $\frac{16}{11}$.	15. Divide 18 by $\frac{3}{5}$ of $\frac{2}{3}$.
8. Divide 16 by $\frac{4}{6}$.	16. Divide 17 by $\frac{3}{8}$ of $\frac{8}{5}$.

17. How many gallons of molasses, at $\frac{3}{8}$ of a dollar per gallon, can be bought for 7 dollars?

18. Suppose a boy earns $\frac{9}{10}$ of a dollar per day: in how many days will he earn 12 dollars?

19. If an arithmetic costs $\frac{7}{20}$ of a dollar, how many can be bought for 21 dollars?

If the divisor is a mixed number, reduce it to an improper fraction, and divide as before.

20. Divide 5 by $7\frac{1}{4}$.	25. Divide 11 by $6\frac{2}{7}$.
21. Divide 7 by $9\frac{1}{7}$.	26. Divide 7 by $5\frac{3}{4}$.
22. Divide 6 by $8\frac{4}{5}$.	27. Divide 21 by $6\frac{1}{3}$.
23. Divide 10 by $7\frac{5}{9}$.	28. Divide 16 by $2\frac{1}{2}$.
24. Divide 18 by $4\frac{2}{9}$.	29. Divide 24 by $2\frac{1}{6}$.

CASE III.

89. **To divide one fraction by another.**

1. Divide $\frac{3}{4}$ by $\frac{6}{7}$.

Rule.—*Invert the terms of the divisor, and multiply the dividend by the new fraction.*

OPERATION.

$\frac{3}{4} \div \frac{6}{7} = \frac{3}{4} \times \frac{7}{6} = \frac{21}{24} = \frac{7}{8}$.

NOTE.—Cancel all common factors in every operation of fractions.

Examples.

1. Divide $\frac{5}{9}$ by $\frac{3}{5}$.
2. Divide $\frac{6}{11}$ by $\frac{7}{9}$.
3. Divide $\frac{4}{9}$ by $1\frac{1}{12}$.
4. Divide $\frac{8}{9}$ by $\frac{8}{9}$.
5. Divide $\frac{2}{3}$ by $\frac{7}{8}$.
6. Divide $\frac{5}{7}$ by $\frac{9}{10}$.
7. Divide $\frac{2}{5}$ by $\frac{7}{18}$.
8. Divide $\frac{4}{13}$ by $\frac{3}{7}$.
9. Divide $\frac{14}{15}$ by $2\frac{1}{2}$.
10. Divide $\frac{2}{7}$ of $\frac{3}{4}$ by $4\frac{1}{5}$.
11. Divide $8\frac{1}{3}$ by $\frac{3}{4}$.
12. Divide $6\frac{1}{3}$ by $\frac{7}{8}$.
13. Divide $\frac{3}{4}$ of $\frac{7}{9}$ by $2\frac{1}{2}$.
14. Divide $\frac{3}{7}$ of $\frac{1}{3}$ by $3\frac{1}{2}$.
15. Divide $\frac{1}{3}$ of $\frac{3}{4}$ by $\frac{1}{6}$.
16. Divide $\frac{8}{9}$ of 3 by $4\frac{1}{6}$.
17. Divide $3\frac{1}{3}$ by $5\frac{1}{2}$.
18. Divide $2\frac{1}{5}$ by $3\frac{3}{7}$.
19. Divide $9\frac{1}{3}$ by $3\frac{1}{2}$.
20. Divide $6\frac{2}{3}$ by $5\frac{1}{5}$.

21. At $\frac{3}{4}$ of a dollar per gallon, how many gallons of molasses may be bought for $5\frac{3}{5}$ dollars?

22. In how many months can Samuel save $10\frac{1}{2}$ dollars, if he saves $\frac{5}{8}$ of a dollar per month?

23. How much flour can be bought for $\frac{9}{10}$ of a dollar, if one barrel costs $6\frac{1}{4}$ dollars?

24. A laborer owes $6\frac{2}{5}$ dollars: how many days, at $1\frac{1}{8}$ dollars a day, must he labor to pay that debt?

25. There are $5\frac{1}{2}$ yards in a rod: how many rods are there in $28\frac{1}{6}$ yards?

26. By what number must $\frac{7}{9}$ be multiplied, that the product may be $1\frac{14}{15}$?

27. How many times $\frac{6}{11}$ are equal to $\frac{5}{12}$?

28. If a ton of coal costs $6\frac{6}{25}$ dollars, how much can be bought for $\frac{15}{16}$ of a dollar?

29. How much silk can be bought for $\frac{1}{2}$ of a dollar, if one yard costs $\frac{13}{16}$ of a dollar?

Examples in the preceding Rules.

1. What is the sum of $4\frac{1}{3}$, $6\frac{7}{8}$, and $5\frac{3}{7}$?

2. What is the sum of $9\frac{1}{2}$, $4\frac{6}{11}$, and $\frac{1}{12}$?

3. If James pays $\frac{3}{4}$ of a dollar for a penknife, and $\frac{3}{8}$ of a dollar for an algebra, what does he pay for both?

4. A merchant cut off, for a customer, $3\frac{4}{7}$ yards of cloth from one piece, $6\frac{1}{7}$ yards from another, and $5\frac{3}{4}$ yards from a third piece: how much did he cut off in all?

5. Mr. Jones gave John $4\frac{7}{8}$ dollars, and to Charles half that sum: how much more had John than Charles?

6. What is the difference between $5\frac{4}{9}$ and $3\frac{5}{11}$?

7. If I pay $250\frac{7}{11}$ dollars for a horse, and $175\frac{4}{15}$ dollars for a wagon, how much more do I pay for the horse than for the wagon?

8. If 1 dollar will buy $\frac{3}{7}$ of a cord of wood, how much will 12 dollars buy?

9. What will $2\frac{1}{2}$ pounds of tea cost, if 1 pound costs $\frac{9}{10}$ of a dollar?

10. If a family consume $8\frac{4}{7}$ barrels of flour in 1 year, how much will they consume in $2\frac{1}{2}$ years?

11. If I own $\frac{3}{7}$ of a farm, and sell $\frac{5}{6}$ of it, what part of it have I left?

12. At $\frac{5}{9}$ of a dollar a pound, what will be the cost of $2\frac{1}{5}$ pounds of tea?

13. If a knife costs $\frac{3}{4}$ of a dollar, and a slate $\frac{3}{5}$ as much, what is the difference of their cost?

14. If $\frac{3}{5}$ of $\frac{2}{3}$ of a dollar will pay for 1 pound of tea, what will be the cost of $5\frac{7}{9}$ pounds?

15. What will $12\frac{5}{9}$ cords of wood cost, at $3\frac{2}{5}$ dollars a cord?

16. If 8 yards of ribbon cost $\frac{6}{7}$ of a dollar, what will 1 yard cost?

17. If 10 men consume $15\frac{3}{7}$ pounds of meat in 1 day, how much will 1 man consume?

18. If Charles walks 15 miles in $\frac{3}{5}$ of a day, how far can he walk in 1 day?

19. If the dividend is $2\frac{5}{16}$ and the divisor $6\frac{1}{3}$, what is the quotient?

20. If 29 bushels of wheat cost $29\frac{3}{7}$ dollars, what will be the cost of 1 bushel?

21. If $\frac{7}{9}$ of a ton of hay is worth $15\frac{1}{3}$ dollars, what is the value of 1 ton?

22. If a bushel of apples costs $\frac{4}{5}$ of a dollar, and was sold for $\frac{7}{8}$ of a dollar, what would be the gain on 6 bushels?

23. If a mechanic received $12\frac{3}{5}$ dollars per week, and paid $3\frac{1}{2}$ dollars for board, how much will he save in 5 weeks?

24. What is the difference of $3\frac{1}{2}$ times $\frac{5}{6}$, and $\frac{2}{3}$ times $4\frac{1}{4}$?

25. One man earns $1\frac{3}{8}$ dollars in a day, and another earns $1\frac{3}{4}$ dollars: how much do both earn in 3 days?

26. If a person pays $\frac{3}{8}$ of a dollar per yard for linen, and sells it for $\frac{4}{7}$ of a dollar per yard, how much would he gain on 3 yards?

27. From the sum of $\frac{6}{7}$ and $\frac{9}{10}$ take $\frac{5}{6}$, and multiply the remainder by $\frac{2}{9}$: what will be the result?

28. If sugar costs $\frac{1}{10}$ of a dollar a pound, coffee $\frac{2}{5}$ of a dollar, and tea $\frac{7}{8}$ of a dollar, what will be the total cost of 7 pounds of each?

29. If a pair of pantaloons requires $2\frac{1}{2}$ yards of cloth, and a vest $\frac{7}{8}$ of a yard, how much will be left from a piece of 35 yards, after cutting off 3 suits?

30. How much of 50 dollars was left, after paying John for 16 days' work, at $1\frac{1}{2}$ dollar per day, and William for 15 days' work, at $\frac{7}{8}$ of a dollar per day?

DECIMAL FRACTIONS.

90. A Decimal Fraction is one in which the unit is divided into *tenths, hundredths, thousandths,* &c.

When the unit is divided into 10 equal parts, there are 10 such parts of the unit, and each part is called, *one-tenth.*

If each tenth be divided into 10 equal parts, there will be 100 equal parts of the unit, and each part will be $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$.

If each hundredth be divided into 10 equal parts, there will be 1000 equal parts of the unit, and each part will be $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$; and smaller parts may be obtained, by dividing continually by 10.

Notation and Numeration.

91. A period (.), called the *decimal point*, written before a figure, denotes the decimal division of the unit:

Thus,	.1	is read,	1 tenth	$= \frac{1}{10}$.
	.4	"	4 tenths	$= \frac{4}{10}$.
	.7	"	7 tenths	$= \frac{7}{10}$.
	&c.,		&c.	

The second place from the decimal point, is the place of *hundredths:*

Thus,	.01	is read,	1 hundredth	$= \frac{1}{100}$.
	.04	"	4 hundredths	$= \frac{4}{100}$.
	.07	"	7 hundredths	$= \frac{7}{100}$.
	&c.,		&c.	

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20. If 29 bushels of wheat cost $29\frac{3}{7}$ dollars, what will be the cost of 1 bushel?

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26. If a person pays $\frac{3}{8}$ of a dollar per yard for linen, and sells it for $\frac{4}{7}$ of a dollar per yard, how much would he gain on 3 yards?

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	.04	"	4 hundredths	$= \frac{4}{100}$.
	.07	"	7 hundredths	$= \frac{7}{100}$.
	&c.,		&c.	

The third place is the place of *thousandths:*

Thus, .001 is read, 1 thousandth = $\frac{1}{1000}$.
.004 " 4 thousandths = $\frac{4}{1000}$.
.007 " 7 thousandths = $\frac{7}{1000}$.
.008 " 8 thousandths = $\frac{8}{1000}$.
.009 " 9 thousandths = $\frac{9}{1000}$.
&c., &c.

The fourth place is the place of *ten-thousandths;* the fifth, of *hundred-thousandths;* the sixth, of *millionths,* &c.

92. We numerate from the decimal point to the right, and name the *lowest fractional unit of the decimal.*

Thus, 3 tenths 0 hundredths 4 thousandths, are, *three hundred and four thousandths.*

Tenths.	Hundredths.	Thousandths.
. 3	0	4

93. A Mixed Number is composed of a whole number and a decimal: Thus, 27.047 is a mixed number, and is read, twenty-seven, and forty-seven thousandths.

Numeration Table.

Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands	Thousands.	Hundreds.	Tens.	Units.	*Decimal point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.
3	0	4	2	1	0	6	9	.	4	0	3	6	7	0	4

Examples.

1. Express six-tenths in figures.
2. Write, in figures, forty-one hundredths.
3. Write, in figures, fifty-nine thousandths.
4. Write, in figures, forty-seven ten-thousandths.
5. Write, in figures, ninety-five thousandths.
6. Write, in figures, eighty ten-thousandths.
7. Write three hundred and twenty-seven thousandths.
8. Write, in figures, forty-nine millionths.
9. Write, in figures, nineteen ten-thousandths.
10. Write, in figures, sixty ten-thousandths.
11. Write, in figures, forty-one millionths.

Numerate and express in words the following decimals:

(12.)	(13.)	(14.)	(15.)	(16.)
.045	.6704	.0049	.21046	.1049
(17.)	(18.)	(19.)	(20.)	(21.)
.4704	.2147	.0412	.00497	.40264

22. Express, in figures, four, and twenty-five hundredths.
23. Write twenty-one, and forty-seven hundredths.
24. Write sixty, and one thousandth.
25. Write three hundred, and forty-nine thousandths.
26. Write six hundred, and six hundredths.
27. Write twenty-nine, and forty-one thousandths.

Numerate the following mixed numbers:

(28.)	(29.)	(30.)	(31.)
87.0471	904.27040	3601.0004	72045.20413
(32.)	(33.)	(34.)	(35.)
6274.0470	274.0416	167.0416	1874.04132

Principles of Decimal Notation.

1. *That annexing ciphers to a decimal does not change its value.*

Thus, $.4 = .40 = .400 = .4000$, &c.

2. *That prefixing a cipher to a decimal diminishes its value ten times.*

Thus, if we prefix a cipher to .4, it becomes .04, which is one-tenth of 4 tenths.

3. *That the unit of any place, is one-tenth of the unit of the place next to the left—the same as in whole numbers.*

4. *That the denominator of a decimal fraction, though not written, is* 1 *with as many ciphers annexed as there are figures in the decimal.*

Examples.

Write the following numbers decimally:

(1.)	(2.)	(3.)	(4.)
$\frac{3}{10}$	$\frac{5}{100}$	$\frac{6}{1000}$	$\frac{14}{100}$
(5.)	(6.)	(7.)	(8.)
$\frac{15}{1000}$	$14\frac{6}{100}$	$12\frac{5}{10}$	$15\frac{7}{100}$
(9.)	(10.)	(11.)	(12.)
$20\frac{15}{1000}$	$19\frac{56}{100}$	$9\frac{105}{1000}$	$10\frac{16}{1000}$
(13.)	(14.)	(15.)	(16.)
$9\frac{19}{1000}$	$\frac{150}{10000}$	$\frac{450}{100}$	$\frac{6540}{1000}$

Numerate the following decimals:

(17.)	(18.)	(19.)	(20.)
.27	.041	.0291	.1672

(21.)	(22.)	(23.)	(24.)
.04049	.04190	.2704	.67029
(25.)	(26.)	(27.)	(28.)
.00046	.04121	.0496	.270496

Numerate the following:

(29.)	(30.)	(31.)	(32.)
159.04704	169.5704	1327.0493	12704.41214
(33.)	(34.)	(35.)	(36.)
214.67049	214.0493	14.04704	16.416704

Write the following numbers in figures, and then numerate them: write, also, and name the denominator of each decimal.

37. Fifty-nine, and three-tenths.
38. Forty-five, and sixteen hundredths.
39. Sixty-four, and four thousandths.
40. Sixty-nine ten-thousandths.
41. Fifty-four one hundred thousandths.
42. Four hundred, and twenty-nine thousandths.
43. Five, and seven millionths.
44. Four thousand and six, and forty-nine millionths.
45. Fifty-six, and six ten-thousandths.
46. Fifteen hundred, and fifteen ten-millionths.
47. Thirty-nine, and six hundred and forty thousandths.
48. Five thousand, and five thousandths.
49. Thirty-six millions, and thirty-six millionths.
50. Thirty-one thousand, and forty-nine millionths.
51. Seventy-five hundred-thousandths.
52. Fifty-one, and fifty-one millionths.
53. Sixty thousand, and sixty-thousandths.

ADDITION.

94. Addition of Decimals is the operation of finding the sum of two or more decimal numbers.

1. What is the sum of 3.04 2.81, and 86.36?

Analysis.—Place the decimal points in the same column: this brings units of the same value in the same column: then add as in whole numbers.

OPERATION.

	3.0 4
	2.8 1
8	6.3 6
9	2.2 1

Rule.

I. *Write the numbers to be added, so that units of the same value shall fall in the same column:*

II. *Add as in whole numbers, and place the decimal point in the sum directly under the points in the numbers added.*

Proof.—The same as in simple numbers.

Examples.

(1.)	(2.)	(3.)
3.0493	27.7249	50.07049
7.02	8.049	9.97
3.2704	9.60	7.6

4. Add 25.625; 37.125; 187.1875; 96.1372; 1.625.

5. Add 6.6; 17.17; 29.05; 275.875; 181.62; .2647.

6. Add .5725; .6875; .125; 5.27625; 19.687; 27.4726.

7. Add .05; .275; .17; .8; 2.8375; .1875; .00125; .5.

8. What is the sum of 4.2+16.02+27.002+99.99+8.8?

9. Add 1.75; 179.875; 64.32; 28.9375; 28.28.

10. Add 100.95; 111.919; 229.619; 77.75625; .29; .167.

11. Add the following decimals: Twenty-seven hundredths; two, and fifty-seven hundredths; four hundred and twelve, and one hundred and twenty-two thousandths.

12. Add thirteen, and five-tenths; one, and ninety-six hundredths; sixty-six, and five thousandths; eighty, and one hundred and thirty-nine thousandths; five hundred and sixty-four, and twenty-four millionths.

13. What is the sum of 1.4+4.1+.04+19.006+217.5?

14. A person bought a horse for 175.375 dollars, a carriage for 296.875 dollars, a set of harness for 116.1875 dollars, and a quantity of feed for 38.3125 dollars: what was the cost of the whole?

15. The expenses of a person, per week, are 5.25 dollars for board, 1.75 dollars for washing, .625 of a dollar for fuel and light, .60 of a dollar for travelling, .24 of a dollar for newspapers, and 2.25 dollars for incidental charges: what are his weekly expenses?

16. Four persons, together, purchase a ship: the first pays 2165.50 dollars; the second pays 1563.75 dollars more than the first; the third pays 5625.1875 dollars, and the fourth pays 765.6875 dollars more than the third: what was paid for the ship?

17. A gentleman bought a house for 3762.75 dollars; he paid 167.25 dollars for repairs, 112.625 dollars for painting, and 119.315 dollars for gas-fixtures; after which he sold the house, and gained 565 dollars: what did he receive for it?

18. A drover had 6 horses, for which he asked the following prices: 1st, 275.50 dollars; 2d, 196.875 dollars; 3d, 216.25 dollars; 4th, 317.315 dollars; 5th, 375 dollars; and 6th, 225.75 dollars: what was the total value of the horses?

SUBTRACTION.

95. Subtraction of Decimals is the operation of finding the difference between two decimal numbers.

1. What is the difference between 37.049 and 12.8704?

Rule.—I. *Write the subtrahend so that its decimal point shall fall under that of the minuend:*

II. *If the decimal places in the numbers are not equal, make them so by annexing ciphers:*

III. *Then, subtract as in whole numbers, and place the decimal point, in the remainder, under that of the subtrahend.*

Proof.—Same as in whole numbers.

OPERATION.

	37.0490
	12.8704
	24.1786

Examples.

	(1.)	(2.)	(3.)
From	27.049	61.047	169.47041
Take	3.149	21.9927	21.072
Rem.			

4. What is the difference of 87.306 and 49.978?

5. From 3765.0075 take 896.87.

6. From 1245.1875 take 750.375.

7. How much does 67.875 exceed 49.9375?

8. How much is 305.15 greater than 87.875?

9. How much must be added to 15.25 to make it 31.315?

10. A person who had 159.37 dollars, lost 85.79 dollars: how much had he left?

11. A traveller had 97.5 miles to go: after having travelled 69.875 miles, what distance yet remained?

12. A person owes a debt of 246.125 dollars: should he pay 198.1875 dollars, how much would remain unpaid?

13. How much does 25.0625 dollars exceed 19.1875 dollars?

14. A farmer owes a merchant 47.5625 dollars; he pays 29.625 dollars in flour, and the rest in cash: how much cash does he pay?

15. It is proposed to raise 180.75 dollars by subscription; four persons subscribe 149.125 dollars, on condition that Mr. Jones will subscribe the remainder: how much must Mr. Jones subscribe?

16. Two boxes of sugar together weigh 39.475 hundredweight; one weighs 23.9875 hundredweight: what does the other weigh?

17. If two lots of ground contain 6745.25 square feet and one contains 3796.78965 square feet, how much does the other contain?

18. If I sell a house, which cost me 4716.6875 dollars, for 5910.16 dollars, what shall I gain?

19. The difference of two numbers is 27.965, and the larger one is 31.4761: what is the smaller one?

20. Thomas gained 57.625 dollars more than James: how much did James gain, if Thomas gained 82.175 dollars?

21. If two fields contain 641.847 acres, and the larger one contains 375.04 acres, how many acres will there be in the smaller field?

22. Mr. James bought 37.047 pounds of tea, and gave away 12.10904 pounds to a sick family: how much had he left?

Examples in Addition and Subtraction.

1. From the sum of one tenth and one hundredth, take the sum of one hundredth and one thousandth.

2. From the sum of five, and sixty-nine thousandths, take nine ten-thousandths.

3. If from two bags of salt, each containing 375.041 pounds, you take one bag containing 275.4708 pounds, how much will be left?

4. If from three bags of coffee, each containing 97.946 pounds, one bag be filled containing 98.075 pounds, how many pounds will be left?

5. A gentleman received from one person 67.75 dollars, and from another 89.25 dollars; he then paid 113.18 dollars: what had he remaining?

6. A piece of cloth measured 35.375 yards; from it the tailor cut three suits, requiring respectively 6.5 yards, 7.3125 yards, and 7.875 yards: how much of the piece remained?

7. A gentleman owned a lot of land, containing 1675 acres; he sold out of this lot, at various times, 275.75 acres, 164.375 acres, 396.875 acres, and 186.25 acres: how many acres remained?

8. A farmer gathered from one field 762.5 bushels of wheat, and from a second field 234.75 bushels: he sold at various times, 45.6 bushels, 176.15 bushels, and 260.875 bushels: how much then remained?

9. How much must we add to the sum of 475.75 and 296.875, to make 1062.9125?

10. A person owed 2563.625 dollars; he paid at different times, 156.75 dollars, 579.63 dollars, 492.16 dollars, and 297.74 dollars: how much remained unpaid?

11. If from the sum of 475.65+192.6325+99.1645, you take the sum of 16.9725+43.1645+186.375, what will be the remainder?

MULTIPLICATION.

96. Multiplication of Decimals is the operation of taking one of two decimal numbers as many times as there are units in another.

1. Multiply 20.048 by 3.21.

The multiplier is placed under the multiplicand, and the multiplication is performed as in simple numbers. The decimal point is so placed in the product, that there shall be as many decimal places as there are in both factors.

OPERATION.

```
  2 0.0 4 8
      3.2 1
  ---------
      2 0 0 4 8
    4 0 0 9 6
  6 0 1 4 4
  ---------
  6 4.3 5 4 0 8
```

Rule.

I. *Write the multiplier under the multiplicand, and multiply as in whole numbers:*

II. *Point off in the product, from the right hand, as many places for decimals as there are decimal places in both factors; if there be not so many in the product, supply the deficiency by prefixing ciphers.*

Note.—To multiply a decimal number by 10, 100, 1000, &c., remove the decimal point as many places to the right, as there are 0's in the multiplier.

Examples.

(1.)	(2.)	(3.)	(4.)
67.043	21.0497	69.1041	8.7509
.04	.047	.446	.0041

(5.)	(6.)	(7.)	(8.)
10.078	549.063	.04704	6.9743
10	100	1000	1000

9. Multiply 25.04 by .04.
10. Multiply .3704 by .005.
11. Multiply 97.079 by 3.049.
12. Multiply .6703 by .0496.
13. Multiply .75 by .005.
14. Multiply 87.04 by .0005.
15. Multiply 65.01 by .0001.
16. Multiply 45.049 by 10.
17. Multiply .045 by 100.
18. Multiply 2.4903 by 1000.

19. Multiply the mixed number 1976.4625 by 2.7.
20. Multiply the mixed number 2364.9775 by 1.62.
21. Multiply the decimal .75646 by .6.
22. Multiply the mixed number 47.69636 by .87.
23. Multiply the mixed number 269.456 by .065.
24. Multiply the mixed number 1847.6235 by 2.007.
25. Multiply the decimal .00675 by 4.625.
26. What is the product of .1725 and .0625?
27. Multiply .5 by .5; also, .07 by .07.
28. Multiply the mixed number 117.675 by .06125.
29. Multiply the mixed number 694.68325 by 1000.
30. Multiply the mixed number 1564.375 by 600.
31. The multiplicand is 675.8725, and the multiplier is .875: what is the product?
32. If in a month a person earns 267.625 dollars, what will he earn in 7.9 months?
33. If a vessel sails 215.65 miles per day, how far will it sail in 24.75 days?
34. If, in selling a barrel of flour, a merchant gains .625 dollars, how much would he gain in selling 2000 barrels, at the same rate?
35. If a barrel of apples weighs 116.25 pounds, how much would 26.75 barrels weigh, at the same rate?
36. In a franc there are 18.75 cents: how many cents are there in 250 francs?
37. If 3.75 dollars will pay for a cord of pine wood, how much will pay for .875 of a cord?

38. How many are 675.625 times 1.87635?
39. What will be the cost, at .1875 of a dollar per yard, of 15 pieces of calico, each measuring 37.5 yards?
40. If each box contains 1897.75 pounds of sugar, how many pounds are contained in 29 boxes?
41. If 7.875 yards of cloth are required for one suit of clothes, how many yards are required to furnish 3 regiments, each comprising 1200 men?

DIVISION.

97. Division of Decimals is the operation of finding how many times one decimal number is contained in another.

1. Divide 28.9170 by 1.05.

Rule.—*Divide as in whole numbers, and from the right of the quotient, point off as many places for decimals as the decimal places in the dividend exceed those in the divisor; if there be not so many in the quotient, supply the deficiency by prefixing ciphers.*

OPERATION.

```
1.05 ) 28.9170 ( 27.54
       210
       ---
        791
        735
        ---
         567
         525
         ---
          420
          420
          ---
```

Notes.—1. If the divisor has *more* decimal places than the dividend, make the number equal by annexing ciphers to the dividend; *all the figures of the quotient will then be whole numbers.*

2. To divide by 10, 100, 1000, &c., remove the decimal point as many places to the left as there are ciphers in the divisor.

3. If the division does not *terminate*, write + after the quotient, which shows that it may be continued.

Examples.

(1.) $.8) .664$ — .83

(2.) $.8) 4.896$ — 6.12

(3.) $.06) .384$ — 6.4

(4.) $.004) 49.052$ — 12263

(5.) $10) 27.046$ — 2.7046

(6.) $100) 61.046$ — .61046

(7.) $1000) 470.43$ — .47043

8. Divide 78.964 by 4.5.
9. Divide 10.643 by 2.9.
10. Divide 47.1065 by .75.
11. Divide 874.625 by .08.
12. 375.643278 ÷.006=.
13. 48.9167562 ÷.012=.
14. Divide .96147 by 10.
15. Divide 5000.5 by .5.
16. Divide 12456 by .625.
17. Divide .875 by 875.
18. Divide 2 by .16.
19. Divide 14.75 by 9.5.
20. .36872567 ÷.0025=.
21. Divide 1764 by .1764.
22. Divide 2.567 by 100.
23. Divide .5 by .005.

24. The dividend is 45.675, and the divisor is 3.95: what is the quotient?

25. If 51.26 be divided by 1.68, what will be the quotient?

26. If 45 barrels of flour cost 327.1875 dollars, what will be the cost of one barrel?

27. If one box holds 63.75 pounds of tea, how many boxes will be required to hold 956.25 pounds?

28. If 6.5 bushels of oats are required to feed one horse for one month, how many horses would 318.16 bushels feed?

29. If a journey of 617.5 miles is performed in 16.25 hours, what was the rate per hour?

Miscellaneous Examples in the preceding Rules.

1. What is the sum of one-tenth and one-hundredth?
2. What is the difference between five-tenths and five-hundredths?
3. From six thousand take six-thousandths.
4. Multiply five-tenths by five-thousandths.
5. Divide one by one-tenth.
6. Divide 10 by one-hundredth.
7. From one-tenth take one-millionth.
8. Two persons are 37.6325 miles apart, and travelling towards each other; one at the rate of 3.25 miles an hour, and the other of 4.125 miles: how far will they be apart, after travelling 4 hours?
9. A person has a journey to perform of 456.75 miles. After travelling 15.375 hours, at the rate of 24.6 miles per hour, how far will he yet have to travel?
10. If 6 pounds of sugar cost .84 of a dollar, what will be the cost of one pound?
11. If 9 barrels of flour cost 57.33 dollars, what will 8 barrels cost?
12. At 12.5 dollars a ton, how much hay can be bought for 203.75 dollars?
13. A steam-ship makes the same distance every day, and in 12.3125 days goes 172.375 miles: what is her daily rate?
14. The divisor is 96.4, the quotient 162.82, and the remainder .419: what is the dividend?
15. If 1 man can build a wall 9.045 rods long in 4 days, how much wall can 5 men build in 1 day?
16. What will 37.47 yards of cloth cost, at 4.04 dollars a yard?
17. Multiply 30.0046 by 100.
18. Divide 1 by one-millionth.
19. If 1 yard of cloth costs 1.25 dollars, what will be the cost of 75 yards?

98. **To change a common to a decimal fraction.**

1. Reduce $\frac{3}{8}$ to a decimal.

Rule.—*Annex decimal ciphers to the numerator, and then divide by the denominator; pointing off as in division of decimals.*

OPERATION.

8) 3 . 0 0 0
.3 7 5

Examples.

1. Reduce $\frac{1}{2}$ to a decimal fraction.
2. Reduce $\frac{5}{8}$ to a decimal fraction.
3. Reduce $\frac{2}{5}$, $\frac{5}{16}$, and $\frac{9}{15}$ to decimal fractions.
4. What decimal fraction is equal to $\frac{16}{25}$?
5. What decimal fraction equals $\frac{21}{24}$?
6. Reduce $\frac{11}{32}$ to a decimal fraction.
7. Reduce $\frac{7}{40}$ and $\frac{2}{3}$ to decimal fractions.
8. Change $\frac{17}{50}$ and $\frac{16}{80}$ to decimal fractions.
9. Express $\frac{2}{5}$, $\frac{6}{16}$, and $\frac{9}{80}$ in decimal fractions.
10. Reduce $\frac{240}{1250}$ and $3\frac{7}{8}$ to decimals.
11. Reduce $\frac{16}{975}$ and $\frac{36}{450}$ to decimals.
12. Reduce $\frac{5}{160}$, $\frac{16}{90}$, and $\frac{17}{200}$ to decimals.

99. **To change a decimal to the form of a common fraction.**

1. Change .88 to the form of a common fraction.

Rule.—*Write the denominator of the decimal, and reduce the fraction to its lowest terms.*

OPERATION.

$.88 = \frac{88}{100} = \frac{44}{50} = \frac{22}{25}$.

Examples.

1. Change .47 to the form of a common fraction.
2. Change 4.69 to the form of a common fraction.
3. Change 3.004 to the form of a common fraction.
4. Change 64.0049 to the form of a common fraction.
5. Change 87.490 to the form of a common fraction

UNITED STATES CURRENCY.

100. Currency is the money of a country, established by law. It is composed of paper money and coins.

Coins.

101. Coins are pieces of metal, whose values are fixed by law.

The coins of the United States are the following:

1. Gold: Eagle, double-eagle, half-eagle, three-dollars, quarter-eagle, dollar.
2. Silver: Dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece.
3. Copper: Cent, half-cent.
4. Nickel: Cent.

102. The Dollar is the unit of United States Currency. It is divided decimally, for the denominations which are less than a dollar, and multiplied by 10 for those which are greater, according to the following

Table.

10 Mills (*m.*)	.	make 1 Cent,	.	marked *ct.*
10 Cents	.	" 1 Dime,	.	" *d.*
10 Dimes	.	" 1 Dollar,	.	" $.
10 Dollars	.	" 1 Eagle,	.	" *E.*

Hence, a dime is *one-tenth* of a dollar; a cent, *one-tenth* of a dime; and a mill, *one-tenth* of a cent. Therefore, in writing,

The dollars fall at the left of the decimal point, the

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Hence, a dime is *one-tenth* of a dollar; a cent, *one-tenth* of a dime; and a mill, *one-tenth* of a cent. Therefore, in writing,

The dollars fall at the left of the decimal point, the

dimes in the first place at the right of it, the cents in the second place, and the mills in the third place. Thus,

$4.875

expresses, 4 dollars, 8 dimes, 7 cents, and 5 mills. But the dimes are generally read with the cents; thus we say, four dollars 87 cents and 5 mills. If there are no dimes, the 0 occupies the dimes' place.

Express the following sums of money decimally:

1. Five dollars 4 dimes and 3 cents.
2. Twenty-seven dollars 6 dimes and 7 cents.
3. Forty dollars 8 dimes 2 cents and 9 mills.
4. Thirty dollars forty-three cents, and 2 mills.
5. One hundred and five dollars 6 dimes 4 cents and 4 mills.
6. Three dimes 7 cents and 8 mills.
7. Sixty-five cents and 7 mills.
8. One dollar one cent and one mill.
9. Twenty-five cents.
10. Three dollars and seventy-five cents.
11. Eight cents and eight mills.
12. Twelve dollars twelve cents and nine mills.
13. Nine mills.
14. Two cents and two mills.
15. Sixty dollars and five cents.
16. Forty-nine dollars four dimes and six mills.
17. Two hundred dollars eight dimes and three cents.

Read the following numbers:

$5.625;	$16.147;	$23.492;	$72.169;	$1.196.
$1.064;	$.75;	$.045;	$.006;	$.107.
$67.041;	$30.470;	$.047;	$87.401;	$61.414.

REDUCTION.

103. Reduction is the operation of changing the unit of a number, without altering the value of a number.

104. **To reduce from a greater unit to a less.**

1. In 5 dollars, how many dimes, how many cents, and how many mills?

Rule.—I. *To change from any denomination to the next less, multiply by* 10.

II. *To change from any denomination to the second less, multiply by* 100.

III. *To change from any denomination to the third less, multiply by* 1000.

OPERATION.

$5 = 5 × 10 = 50 dimes.

$5 = 50 × 10 = 500 cents.

$5 = 500 × 10 = 5000 mills.

Note.—If there be no decimal point in the number, perform the operation by annexing ciphers. If there is a decimal point, observe the rule for multiplying a decimal by 10, 100, or 1000.

Examples.

1. Reduce $15.25 to cents. *Ans.* 1525.
2. Reduce $47.375 to mills. *Ans.* 47375.
3. How many dimes are there in $96?
4. How many cents are there in $87.50?
5. In 7 dimes, how many mills are there?
6. Reduce $160 to dimes; to cents; and to mills.
7. Reduce $3.03 to dimes; to cents; and to mills.
8. Reduce $16 to cents; and to mills.
9. Change $10.109 to dimes; to cents; and to mills.
10. Change $400.754 to dimes; to cents; and to mills.
11. Change $7.046 to dimes; to cents; and to mills.
12. Change $12.419 to dimes; to cents; and to mills.

105. **To change from a less unit to a greater.**

1. Reduce 4672 mills to dollars, cents, and mills.

Rule.—I. *To change from any denomination to the next higher, divide by* 10:

II. *To change from any denomination to the second higher, divide by* 100:

III. *To change from any denomination to the third higher, divide by* 1000.

OPERATION.

10) 4 6 7 2
10) 4 6 7.2 dimes.
10) 4 6.7 2 cents.
$4.6 7 2

Examples.

1. Reduce 5672 cents to dollars.
2. Change 72356 mills to dollars.
3. Reduce 3674 mills to dollars.
4. Reduce 12745 cents to dollars.
5. Change 26945 cents to dollars.
6. In 649 dimes, how many dollars?
7. How many dimes are there in 2469 mills?
8. How many dollars are there in 476 dimes?
9. Reduce 57 cents to dollars.
10. How many dollars and cents are there in 157 cents? How many in 75 cents? In 127 cents?
11. How many dollars, cents, and mills are there in 6749 mills? How many in 37049 mills?
12. In 40409 mills, how many dollars, cents, and mills? How many in 904607 mills?
13. How many dollars, cents, and mills in 10460 mills? In 270460 mills, how many?
14. How many dollars, cents, and mills are there in 874904 mills? In 47049 mills, how many?

ADDITION.

106. Addition of United States Money is performed in the same manner as addition of decimals.

1. What is the sum of $37.027, $12.49, and $15.379?

Rule.—*Write the numbers so that units of the same value shall fall in the same column, and then add as in decimal fractions.*

Proof.—Same as in decimals.

OPERATION.

3 7.0 2 7
1 2.4 9
1 5.3 7 9
$6 4.8 9 6

Examples.

Add the following:

1. 16 dollars 15 cents 7 mills; $16.25, and $19.004.
2. 17 dollars 4 dimes 6 cents; $25 19 cents 6 mills; and $75 and 25 cents.
3. $16.125 + $296.875 + 75 cents + 10 dollars 16 cents 3 mills.
4. $96.476 + $179.06 + 37½ cents + 18¾ cents + $1.956.
5. $2716.149 + 17 cents 8 mills + $1.129 + $62.62½ + 5 cents.
6. $9 + 4 dollars 50 cents + $16.16 + $8.08 + 25 dollars + 25 cents.
7. 37½ cents + 12½ cents + 7 cents 8 mills + 2 dollars + 2 cents.
8. A man went to a grocery and purchased a barrel of flour for 6 dollars 25 cents, a barrel of potatoes for 3 dol-

lars 50 cents, a box of raisins for 2 dollars 12½ cents, and a box of candles for 5 dollars 75 cents: what was the cost of all the articles?

9. A boy was sent to the grocery to purchase various articles. He purchased ½ pound of tea for 37 cents, 1 pound of coffee for 35 cents, 7 pounds of sugar for 87½ cents, 1 ham for 1 dollar 62½ cents, and 1 pound of butter for 27 cents: what was the entire cost?

10. A person bought a hat for 3 dollars 75 cents, a coat for 9 dollars 87½ cents, a pair of pantaloons for 4 dollars 37½ cents, and a pair of boots for 4 dollars 18 cents: what did he pay for all?

11. A lady purchased, for a dress, 20 yards of silk for 18 dollars 18 cents, trimmings for 6 dollars 37 cents 5 mills, lining for 96 cents 6 mills, and paid 3 dollars 15 cents for making it: what did the dress cost?

12. A gentleman bought some cloth for a coat, for which he paid 16 dollars 25 cents, some lining for 3 dollars 12½ cents, buttons for 62½ cents, 1 dollar 12½ cents for cutting, and 2 dollars 62½ cents for making it: what did the coat cost?

13. A wood-dealer paid 362 dollars 88 cents for wood still lying in the forest; he paid 49 dollars 27 cents for freight, and 27 dollars for cartage: for what must he sell the wood, that he may gain 62 dollars 62½ cents?

14. James, during the day, earned 87 cents, and found 4 dollars 25 cents: how much would he then have, supposing that he had in the morning 10 dollars 10 cents?

15. A boy wished to buy a set of school-books, which cost as follows: Arithmetic, 50 cents; Reader, 62½ cents; Geography, 87½ cents; Speller, 12 cents; Grammar, 65 cents; History, 49 cents, and Etymology, 37 cents: what must he pay for the set?

SUBTRACTION.

107. SUBTRACTION OF UNITED STATES CURRENCY is performed in the same manner as subtraction of Decimal Fractions.

1. From 169 dollars 27 cents and 6 mills, take 97 dollars 89 cents and 9 mills.

Rule.	OPERATION.
Write the numbers and make the subtraction as in Decimal Fractions.	$1 6 9.2 7 6 9 7.8 9 9 $7 1.3 7 7

Proof.—Same as in Decimals.

Examples.

1. From 212 dollars 16 cents 4 mills, take 97 dollars 29 cents 8 mills.

2. From 47 dollars 2 cents 3 mills, take 9 dollars 19 cents 6 mills.

3. Subtract 118 dollars 25 cents, from 250 dollars 12½ cents.

4. Subtract 1 dollar 87 cents 5 mills, from 7 dollars 10 cents.

5. How much less is 5 dollars 75 cents, than 6 dollars 18 cents?

6. Take 6 dollars 37 cents 5 mills, from 10 dollars.

7. If a person having 12 dollars 62 cents, spends 7 dollars 81 cents, what will he have left?

8. If a gentleman buys a barrel of flour for 6 dollars 38 cents, and hands the seller a ten-dollar bill, how much should be returned to him?

9. If I buy cloth for 37 dollars 35 cents, and sell it for 51 dollars 5 mills, what do I gain?

10. A lad wishes to buy a sled, the price of which is 1 dollar 62 cents; he has only 95 cents: how much more does he need?

11. A barrel of flour costs 5 dollars 75 cents, and a barrel of potatoes 2 dollars 88 cents: what is the difference in the price of the two?

12. A person owing 15 dollars 27 cents, pays 9 dollars 75 cents: what does he still owe?

13. A mechanic, out of 25 dollars 25 cents due him, receives 18 dollars 48 cents: what is still due him?

14. A grocer bought coal to the amount of 28 dollars 50 cents, and paid for it with groceries to the amount of 19 dollars 38 cents, and the rest in cash: how much cash did he pay?

Examples in Addition and Subtraction.

1. A farmer bought sugar for $\$2.62\frac{1}{2}$, tea for 1 dollar 50 cents, coffee for 75 cents, cheese for $\$0.87\frac{1}{2}$, and molasses for 96 cents; he gave in payment a quantity of potatoes that were worth $4.20, and paid the rest in cash: what amount of cash did he pay?

2. A country merchant starts for New York city with $1000 to buy goods. When in the city, he bought dry goods to the amount of $379.16, groceries to the amount of $262.71, boots and shoes for $160, and drugs for $72.15; his expenses for board and travelling were $26.75: what had he left on his arrival at home?

3. A merchant, during a certain day, received the following sums: $47.61, $115.23, $416.37, $12.72, and $0.71: he paid out on the same day $96.17, $49.12, $139.97, $5.17, and $95.95: how much did the receipts exceed the payments?

4. A pedler, at the beginning of the week, had goods to the value of $25.50; he bought, during the week, goods to the value of $21.16, and sold to the value of $31.27; at the end of the week, his goods were worth $30.60: what had he gained during the week?

5. A drover bought a horse for $160: after keeping it for 6 months at an expense of $70, he sold it for $212.50: did he gain or lose, and how much?

6. A farmer sold a horse for 95 dollars 25 cents, and a cow for 47 dollars 36 cents, and received in payment a wagon worth $165.75; the value of the wagon exceeded that of the horse and cow, and he gave another cow, which just paid the balance: what was the value of the second cow?

MULTIPLICATION.

108. Multiplication of United States Currency is performed in the same manner as Multiplication of Decimal Fractions.

1. Multiply 125 dollars 7 dimes 6 cents and 5 mills, by 8.

Rule.—*Express the sum of money in decimals of a dollar, and multiply as in multiplication of decimals.*

Proof.—Same as in decimals.

OPERATION.

$\$1\ 2\ 5.7\ 6\ 5$
$\times\ 8$
$\$1\ 0\ 0\ 6.1\ 2\ 0$

Examples.

1. Multiply 275 dollars 18 cents 3 mills, by 25.

2. Multiply $116, 7 dimes 8 cents 9 mills, by 46.

3. Multiply 1693 dollars 3 cents and 7 mills, by 83.

4. Multiply 37 cents 8 mills, by $28\frac{1}{2}$.

5. Multiply 98 dollars and 9 mills, by $36\frac{1}{2}$.

6. If one barrel of flour costs 6 dollars 25 cents, what will be the cost of 12 barrels?

7. What must be paid for 23 days' work, at $\$1.62\frac{1}{2}$ per day?

8. What would be the cost of 18 yards of cloth, at \$2.75 per yard?

9. At $18\frac{3}{4}$ cents per yard, what would be the cost of $37\frac{1}{2}$ yards of French calico?

10. If board costs \$4.625 per week, how much must be paid for 13 weeks?

11. If the wages of one man be $\$12.12\frac{1}{2}$ for one week, what will be the wages of 26 men for 4 weeks?

12. If the transportation, by railroad, of one ton of merchandise costs \$16, what would be the cost of 21.75 tons?

13. The cost of constructing a railroad is \$27695 60 per mile: what is the cost of construction for $65\frac{3}{8}$ miles?

14. What is the product of \$27.65, by $37\frac{1}{2}$?

15. What will 126.5 yards of muslin cost, at the rate of $27\frac{1}{2}$ cents per yard?

16. How much money must a person have, to give \$3.47 to each of 27 poor families?

17. How much will a person save in 16 weeks, if he saves $\$2.37\frac{1}{2}$ per week?

18. A drover bought a drove of cattle, comprising 35 head, at \$32.14 per head: what did the drove cost?

19. A merchant sold 67 barrels of flour, at a price by which he lost $\$1.87\frac{1}{2}$ on each barrel: what was his entire loss?

Examples in the preceding Rules.

1. A merchant bought 96 barrels of flour at \$5.25 per barrel, and sold them all for \$600: what did he gain?

2. A lady bought $12\frac{1}{2}$ yards of muslin at 18 cents per yard, and 14 yards of calico at $12\frac{1}{2}$ cents per yard; she handed in payment a five-dollar bill: what change should be returned?

3. A gentleman agreed to buy 8 lots of ground at \$674.375 per lot, and to pay \$3715.875 in cash, and to give a bond and mortgage for the remainder: what was the amount of the bond?

4. A person owed \$2716 on his house; at one time he paid \$475.62; at another time, \$675.625; at another time, \$276.375; and subsequently he made 3 equal payments of \$276.75 each: how much is still due?

5. A merchant bought a ship for \$37160.25, and a steamer for \$107645.50; he gave, in payment, a block of 12 houses, each valued at \$4750; 37 lots of ground, valued at \$425.25 per lot, and the remainder in cash: how much cash was paid?

6. A merchant bought 346 yards of calico at 17 cents per yard, $85\frac{1}{2}$ yards of cloth at \$1.16 per yard, and 63 yards of silk at 73 cents per yard; for the goods he paid \$125 cash, and 25 yards of cloth worth \$65.45: what was still due?

7. A laborer worked on a farm 9 months, at \$11.75 per month; in payment he had received 6 bushels of potatoes at $37\frac{1}{2}$ cents per bushel, 2 barrels of flour at $\$6.12\frac{1}{2}$ per barrel, 125 pounds of Indian meal at $1\frac{1}{2}$ cents per pound, one hog, weighing 144 pounds, at $4\frac{5}{8}$ cents per pound, and the remainder in cash: what cash did he receive?

8. A farmer sold, for cash, 25 bushels of potatoes at $62\frac{1}{2}$ cents per bushel, 15 bushels of turnips at 18 cents per

bushel, and 50 heads of cabbage at 3 cents each: out of the proceeds, he bought 20 yards of muslin at 14 cents per yard, 36 yards of calico at 24 cents per yard, and 28 pounds of sugar at 8½ cents per pound: what cash had he remaining?

Bills.

BROOKLYN, Feb. 12th, 1863.

9. *Mr. James Smith*

Bought of Samuel Wells:

25 pounds of sugar, at 9 cents per pound . . $
3 pounds of tea, at 75 cents per pound . . .
5 pounds of Java coffee, at 34 cents per pound .
3 gallons of molasses, at 63 cents per gallon . .
7 pounds of cheese, at 12 cents per pound . . .

$

Received Payment for Samuel Wells.
Charles Ferguson.

What was the amount of the above bill?

NEW YORK, Feb. 13th, 1863.

10. *Mr. Andrew Biere*

Bought of Thomas Shears:

15 yards of muslin, at 16 cents per yard . . $
12 yards of calico, at 18 cents per yard. . . .
16 yards of silk, at 87½ cents per yard . . .
6 yards of white flannel, at 50 cents per yard .
18 yards of Canton flannel, at 27 cents per yard .
4 spools of cotton thread, at 6 cents each. . .
2 papers of pins, at 10 cents each . . . . .

$

Received Payment.

CHICAGO, Feb. 19th, 1863.

11. *Mr. Seth Williams*

Bought of John Spencer:

5 boys' caps, at $1.87½ each . . . . . $
6 boys' suits, at $8.75 each . . . . . .
9 coats, at $3.65 each . . . . . .
10 pair of pantaloons, at $1.69 a pair . . . .
12 dozen of shirts, at $9.50 per dozen . . . .

$

Received Payment.

ALBANY, Feb. 17th, 1863.

12. *Mr. Sylvester Thomas*

Bought of James Spinner:

6 Practical Arithmetics, at 50 cents each . . $
12 National Readers No. 4, at 56 cents each . .
9 Bullion's Grammars, at 48 cents each . . .
18 Intellectual Arithmetics, at 25 cents each . .
3 dozen Beer's Copy-books, at $1.08 per dozen .
1 Geometry, at $1.25 . . . . . .

$

Received Payment.

BROOKLYN, Feb. 24th, 1863.

13. *Mr. John Thompson*

To Philip Shelf, Dr.

For 2 pair of shoes, at $1.65 per pair . . . $
" 3 pair of boots, at $3.62½ per pair . . . .
" 3 pair of slippers, at $.95 per pair . . . .
" 6 yards of superfine cloth, at $2.12½ per yard
" 7 pair of India-rubber shoes, at $.96 per pair

$

March 2d. *Credit by Cash* . . . . . $ 7.75

What is the balance due?

DIVISION.

109. Division of United States Currency is performed in the same manner as division of decimals.

1. Divide 296 dollars 27 cents by 26.

Rule.

Write the numbers and divide as in division of decimals.

Proof.

Same as in decimals.

OPERATION.

```
26 ) 296.270 ( 11.395
     26
     ---
      36
      26
      --
      102
       78
       --
       247
       234
       ---
        130
        130
        ---
```

Examples.

1. Divide 472 dollars 16 cents 5 mills by 28.
2. Divide 1173 dollars 87 cents 7 mills by 37.
3. Divide 567 dollars 29 cents by 45.
4. Divide 2761 dollars 17 cents by 5 dollars 16 cents.
5. Divide 616 dollars 8 cents 5 mills by 4 dollars 17 cents 5 mills.
6. Divide 2030 dollars 6 mills by 156.
7. How many yards of cloth at $1.75 per yard, can be bought for $39.20?
8. If 8 yards of muslin cost $2.16, what will 1 yard cost?
9. If a man for 15 days' work receives $28.75, what was his daily wages?

10. How many barrels of flour, at 4 dollars 37½ cents per barrel, can be bought for 567 dollars 25 cents?
11. What would be the cost of one arithmetic, if $162.96 were paid for 400 copies?
12. How many horses at $82.50 each, can be bought for $6187.50?
13. It is desired to raise by subscription, for a benevolent object, $846: if each subscriber pays $2.25, how many subscribers will be necessary to raise the amount?
14. A merchant finds that by selling calico at $.18 per yard, he has received in one day cash to the amount of $450.75: how many yards did he sell?
15. A father, at his death, left a fortune of $25650 to be divided equally among his 5 children, after deducting one-third of it for his widow: what was the share of the widow, and what of each child?

Promiscuous Examples.

1. A farmer sold 16 bushels of potatoes at 62½ cents per bushel, and took his pay in sugar at 9½ cents per pound: how much sugar did he get?
2. A person bought 162 cords of oak wood, at $3.25 per cord; he paid $250 in cash, and the remainder in coal at $4.60 per ton: how many tons of coal were given?
3. Two farmers agreed to exchange their farms: one farm comprised 175 acres, and was valued at $85 per acre; and the other comprised 218 acres, valued at [illegible] per acre: the difference of value was paid in cash: how much cash was paid?
4. A was in debt to B to the amount of $916.75; in payment he gave one lot of ground, valued at $345.60; cash, $216.90; 15 boxes of oranges, at $2.75 each; and 40 boxes of lemons, at $2.16 each: what was still due?

5. A laborer was employed for 5 months, at $27.50 per month; he received, each month, $12.25 in cash and $9.75 in groceries: at the end of the time, what had he saved?

6. A poor man bought a barrel of flour for $6.50; 7 pounds of sugar, at 9 cents per pound; 28 pounds of Indian meal, at 3 cents per pound; 4 pounds of butter, at 23½ cents per pound; and 15 pounds of ham, at 9 cents per pound; he paid $5 in cash, and agreed to pay for the remainder in labor at $1.25 per day: how many days must he labor?

7. If I pay $96 for 25 hats, how much must I pay for 63 hats at the same rate?

8. If 36 men can be hired for $50.40 for one day, how many men could be hired for the same time for $133.00?

9. Find $\frac{7}{8}$ of 679 dollars 19 cents 6 mills.

10. A war-vessel captured a prize, which was afterwards sold for $37650; $\frac{4}{5}$ of this sum was to be equally divided among 250 men: what was the share of each man?

11. How much is $\frac{11}{12}$ of $56412.60?

12. In how many weeks could a father and son together earn $65.75, if the father earns $10.60 and the son $3.75, per week?

13. A family, consisting of father, mother, and 4 children, desires to board in the country during the summer, and can afford to pay $162: how many weeks can they remain, if the board of each parent is $4.50, and of each child $2.25?

14. A gentleman bought a farm of 160 acres, at $75 per acre, and sold it for $19000: what was the entire gain, and how much per acre?

15. If a merchant buys coal at the rate of $3.75, and sells it at $5 per ton, how many tons must he sell in order to gain $1500?

DENOMINATE NUMBERS.

110. An Abstract Number is one whose unit is not named.

111. A Denominate Number is one whose unit is named, as 3 pounds, 4 horses, &c.

112. A Simple Number is a collection of units of the same kind, whether abstract or denominate.

113. A Compound Denominate Number is one expressed by two or more denominations.

114. A Scale is a series of numbers expressing the law of relation between the different units of any number.

Kinds of Units.

There are eight different Units of Arithmetic:

I. Units of Abstract Number;
II. Units of Currency;
III. Units of Length;
IV. Units of Surface;
V. Units of Volume, or Capacity;
VI. Units of Weight;
VII. Units of Time;
VIII. Units of Circular Measure.

I. ABSTRACT NUMBERS.

Table.

10 Units	make	1 Ten.
10 Tens		1 Hundred.
10 Hundred		1 Thousand.
10 Thousand		1 Ten-thousand.
&c.		&c.

5. A laborer was employed for 5 months, at $27.50 per month; he received, each month, $12.25 in cash and $9.75 in groceries: at the end of the time, what had he saved?

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14. A gentleman bought a farm of 160 acres, at $75 per acre, and sold it for $19000: what was the entire gain, and how much per acre?

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10 Tens		1 Hundred.
10 Hundred		1 Thousand.
10 Thousand		1 Ten-thousand.
&c.		&c.

Table Reversed.

				Hund.		Ten.		Units.
				1	=	1	=	10.
			Thous.	1	=	10	=	100.
		Ten-thous.	1	=	10	=	100	= 1000.
1	=	10	=	100	=	1000	=	10000.

SCALE.—The *steps*, or units of the scale, are all equal, and each 10: hence, the scale is uniform.

II. CURRENCY.

I. UNITED STATES CURRENCY.

115. The United States Currency is the Decimal Currency established by a law of Congress.

Table.

10 Mills	(*m.*)	. .	make	1 Cent,	.	marked *ct.*
10 Cents	.	. .		1 Dime,	.	. *d.*
10 Dimes	.	. .		1 Dollar,	.	. $.
10 Dollars	.	. .		1 Eagle,	.	. *E.*

Table Reversed.

				d.		ct.		m.
				1	=	1	=	10.
			$.	1	=	10	=	100.
		E.	1	=	10	=	100	= 1000.
1	=	10	=	100	=	1000	=	10000.

SCALE.—The steps of the scale are each 10: hence, the scale is uniform.

II. ENGLISH CURRENCY.

116. English Currency is the Currency of Great Britain.

Table.

4 Farthings	(*far.*)	make	1 Penny,	.	marked *d.*	
12 Pence	. . .		1 Shilling	.	. . *s.*	
20 Shillings	. .		1 Pound, or sovereign,		£	
21 Shillings	. . .		1 Guinea.			

Table Reversed.

				s.		d.		far.
				1	=	1	=	4.
			£	1	=	12	=	48.
1	=	20	=	240	=	960.		

NOTES.—1. The steps, or units of the scale, beginning at the lowest, are 4, 12, and 20. If we begin at the highest unit, the order is reversed, and the units are 20, 12, and 4. The step or connecting link between any two denominations, is, however, the same in both cases.

2. The steps of the scale are equal only in abstract and decimal numbers: hence, these numbers alone have uniform scales.

3. Farthings are generally expressed in fractions of a penny: Thus, 1far. = ¼d.; 2far. = ½d.; 3far. = ¾d.

3. By reading the second table from left to right, we can see the value of any unit expressed in each of the lower denominations. Thus, 1d. = 4far.; 1s. = 12d. = 48far.; £1 = 20s. = 240d. = 960far.

III. UNITS OF LENGTH.

I. LONG MEASURE.

117. This Measure is used to measure distances, lengths, breadths, heights, depths, &c.

Table.

12 Inches	(*in.*) . .	make	1 Foot,	. .	marked *ft.*
3 Feet	. . .		1 Yard,	. . .	*yd.*
5½ Yards, or 16½ Feet	.		1 Rod,	. . .	*rd.*
40 Rods	. . . .		1 Furlong,	. . .	*fur.*
8 Furlongs, or 320 Rods	.		1 Mile,	. . .	*mi.*
3 Miles	. . . .		1 League,	. . .	*L.*
69½ Statute Miles (nearly), or 60 Geographical Miles	.		1 Degree of the Equator,	.	*deg.* or °
360 Degrees	. . .		a Circumference of the Earth.		

Table Reversed.

						yd.		ft.		in.
						1	=	1	=	12.
				rd.		1	=	3	=	36.
		fur.		1	=	$5\frac{1}{2}$	=	$16\frac{1}{2}$	=	198.
mi.		1	=	40	=	220	=	660	=	7920.
1	=	8	=	320	=	1760	=	5280	=	63360.

NOTES.—1. A fathom is a length of six feet, and is generally used to measure the depth of water. A pace is three feet.

2. A hand is 4 inches, used to measure the height of horses.

3. SCALE.—The steps, or units of the scale, beginning at the lowest, are 12, 3, $5\frac{1}{2}$, 40, and 8.

4. The geographical mile is equal to a minute of one of the great circles of the earth.

II. SURVEYORS' MEASURE.

118. The Surveyors' or Gunter's Chain is generally used in surveying land. It is 4 rods, or 66 feet in length, and is divided into 100 links.

Table.

7.92 Inches	. . make	1 Link, . .	marked *l.*
100 Links, or 66 feet,		1 Chain, ...	*c.*
80 Chains . . .		1 Mile, ...	*mi.*

Table Reversed.

								l.		in.
				c.		ft.		1	=	7.92.
m.				1	=	66	=	100	=	792.
1	=			80	=	5280	=	8000	=	63360.

SCALE.—The steps, or units of the scale, beginning at the lowest, are 7.92, 100, and 80.

III. CLOTH MEASURE.

119. CLOTH MEASURE is used for measuring all kinds of cloth, ribbons, and other things sold by the yard.

Table.

$2\frac{1}{4}$ Inches	(*in.*) . make	1 Nail, . .	marked *na.*
4 Nails	. . .	1 Quarter of a yard	. *qr.*
3 Quarters	. . .	1 Ell Flemish, . .	*E. Fl.*
4 Quarters	. . .	1 Yard, ...	*yd.*
5 Quarters	. . .	1 Ell English . .	*E. E.*
6 Quarters	. . .	1 Ell French . . .	*E. F.*

Table Reversed.

										na.		in.
								qr.		1	=	$2\frac{1}{4}$.
						yd.		1	=	4	=	9.
				E. Fl.		1	=	4	=	16	=	36.
		E. E.		1	=	$\frac{3}{4}$	=	3	=	12	=	27.
	E. F.	1	=	$1\frac{2}{3}$	=	$1\frac{1}{4}$	=	5	=	20	=	45.
1	=	$1\frac{1}{5}$	=	2	=	$1\frac{1}{2}$	=	6	=	24	=	54.

SCALE.—1. The steps, or units of the scale, beginning at the lowest, and then reckoning from the quarter-yard, are $2\frac{1}{4}$, 4, 4, 3, 5, and 6.

2. The yard of Cloth Measure, is the yard of Long Measure, and is equal to 36 inches.

IV. UNITS OF SURFACE.

I. SQUARE MEASURE.

120. SQUARE MEASURE is used in measuring surfaces, which combine length and breadth.

The unit of this measure, is a square, constructed on the unit of length.

A square, is a figure bounded by four equal sides, at right angles to each other. If each side be one foot, the figure is called, a *square foot*.

Table.

144	Square Inches (*sq. in.*) make	1	Square Foot, marked *sq. ft.*
9	Square Feet . . .	1	Square Yard, . *sq. yd.*
30¼	Square Yards . . .	1	Square Rod, or Perch, . *P.*
40	Square Rods or Perches .	1	Rood, ... *R.*
4	Roods	1	Acre, ... *A.*
640	Acres	1	Square Mile . . . *M.*

Table Reversed.

			sq. yd.	sq. ft.	sq. in.
		P.	1	= 9	= 1296.
	R.	1	= 30¼	= 272¼	= 39204.
A.	1	40	= 1210	= 10890	= 1568160.
1	= 4	= 160	= 4840	= 43560	= 6272640.

SCALE.—The steps, or units of the scale, beginning at the lowest, are 144, 9, 30¼, 40, and 4.

II. SURVEYORS' MEASURE.

121. Surveyors estimate the area of land in Square Miles, Acres, Roods, and Perches.

Table.

16	Perches (*P.*) . . .	make 1	Square Chain, . *sq. ch.*
40	Perches, or 2½ Square Chains .	1	Rood, . . *R.*
4	Roods	1	Acre, . . *A.*
640	Acres	1	Square Mile, . *sq. mi.*

Table Reversed.

			sq. ch.	P.
		R.	1	= 16.
	A.	1	= 2½	= 40.
sq. mi.	1	= 4	= 10	= 160.
1	= 640	= 2560	= 6400	= 10240.

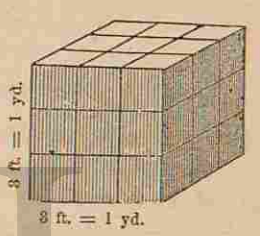
SCALE.—The steps, or units of the scale, beginning at the lowest, are 16, 2½, 4, and 640.

V. UNITS OF VOLUME OR CAPACITY.

I. CUBIC MEASURE.

122. CUBIC MEASURE is used for measuring solids; as stone, timber, earth, and other things, in which the three dimensions of length, breadth, and thickness, are considered.

The unit of this measure is a cube whose edge is the unit of length.



A cube is a figure bounded by six equal squares, called *faces;* the sides of the square are called *edges.*

A cubic foot is a cube, each of whose faces is a square foot; its edges are each 1 foot.

A cubic yard is a cube, each of whose edges is 1 yard.

Table.

1728	Cubic Inches (*cu. in.*) make	1	Cubic Foot, . marked *cu. ft.*
27	Cubic Feet	1	Cubic Yard, . . *cu. yd.*
40 Feet of round, or 50 Feet of hewn Timber,		1	Ton, ... *T.*
42	Cubic Feet	1	Ton of Shipping, . *T.*
16	Cubic Feet	1	Cord Foot, . . . *c. ft.*
8 Cord Feet, or 128 Cubic Feet,		1	Cord, ... *C.*

NOTES.—1. A cord of wood is a pile 4 feet wide, 4 feet high, and 8 feet long.

2. A cord foot is 1 foot in length of the pile which makes a cord.

3. A ton of *round timber*, when square, is supposed to produce 40 cubic feet; hence, *one-fifth is lost by squaring.*

Table Reversed.

				Cu. ft.		cu. in.
			C. ft.	1	=	1728.
		Cu. yd.	1	= 16	=	27648.
	T. rd. T.	1	=	27	=	46656.
T. hewn T.	1	=	$2\frac{1}{2}$	= 40	=	69120.
T. ship.	1	=	$3\frac{1}{5}$	= 50	=	86400.
Cord.	1	=	$2\frac{5}{8}$	= 42	=	72576.
1	=		3	= 128	=	221184.

II. LIQUID MEASURE.

123. LIQUID MEASURE is used for measuring all liquids. Formerly some of them were measured by Beer Measure; but that measure is now not much used.

Table.

4 Gills (*gi.*)	make	1 Pint,	marked *pt.*
2 Pints		1 Quart,	*qt.*
4 Quarts		1 Gallon,	*gal.*
31½ Gallons		1 Barrel,	*bar.* or *bbl.*
2 Barrels, or 63 Gallons		1 Hogshead,	*hhd.*
2 Hogsheads		1 Pipe,	*pi.*
2 Pipes		1 Tun,	*tun.*

Table Reversed.

					pt.		gi.
				qt.	1	=	4.
			gal.	1	= 2	=	8.
		bar.	1	= 4	= 8	=	32.
	hhd.	1	= $31\frac{1}{2}$	= 126	= 252	=	1008.
pt.	1	= 2	= 63	= 252	= 504	=	2016.
tun. 1	= 2	= 4	= 126	= 504	= 1008	=	4032.
1 = 2	= 4	= 8	= 252	= 1008	= 2016	=	8064.

NOTE.—The *standard unit*, or gallon of Liquid Measure, in the United States, contains 231 cubic inches.

III. DRY MEASURE.

124. DRY MEASURE is used in measuring all dry articles, such as grain, fruit, salt, coal, &c.

Table.

2 Pints (*pt.*)	make	1 Quart,	marked *qt.*
8 Quarts		1 Peck,	*pk.*
4 Pecks		1 Bushel,	*bu.*
36 Bushels		1 Chaldron,	*ch.*

Table Reversed.

					qt.		pt.
				pk.	1	=	2.
			bu.	1	= 8	=	16.
		ch.	1	= 4	= 32	=	64.
1	= 36	=	144	=	1152	=	2304.

SCALE.—The steps, or units of the scale, beginning with the lowest, are 2, 8, 4, and 36.

NOTES.—1. The standard bushel of the United States is the Winchester bushel of England. It is a circular measure, $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains $2150\frac{3}{5}$ cubic inches, nearly.

2. A gallon, Dry Measure, contains $268\frac{4}{5}$ cubic inches.

VI. UNITS OF WEIGHT.

I. AVOIRDUPOIS WEIGHT.

125. By this weight all articles are weighed, except gold, silver, jewels, and liquors.

Table.

16 Drams (*dr.*)	make	1 Ounce,	marked *oz.*
16 Ounces		1 Pound,	*lb.*
25 Pounds		1 Quarter,	*qr.*
4 Quarters		1 Hundredweight,	*cwt.*
20 Hundredweight		1 Ton,	*T.*

Table Reversed.

				oz.		dr.
			lb.	1	=	16.
		qr.	1	= 16	=	256.
	cwt.	1	= 25	= 400	=	6400.
T	1	= 4	= 100	= 1600	=	25600.
1	= 20	= 80	= 2000	= 32000	=	512000.

SCALE.—The steps, or units of the scale, beginning with the lowest, are 16, 16, 25, 4, and 20.

NOTES.—1. The standard Avoirdupois pound is the weight of 27.7015 cubic inches of distilled water.

2. By the old method of weighing, adopted from the English system, 112 pounds were reckoned for a hundredweight; but now, the laws of most of the States, as well as general usage, fix the hundredweight at 100 pounds.

3. A ton of coal at the mines, is reckoned at 2240 lbs., but at the yards, at 2000 lbs.

II. TROY WEIGHT.

126. Gold, silver, jewels, and liquors, are weighed by Troy weight.

Table.

24 Grains (*gr.*)	. make	1 Pennyweight,	marked *pwt.*
20 Pennyweights	. .	1 Ounce, . . .	*oz.*
12 Ounces	. . .	1 Pound, . . .	*lb.*

Table Reversed.

			pwt.		gr.
		oz.	1	=	24.
	lb.	1	= 20	=	480.
1	= 12	=	240	=	5760.

SCALE.—The steps, or units of the scale, beginning with the lowest, are 24, 20, and 12.

NOTE.—The standard Troy pound is the weight of 22.794377 cubic inches of distilled water. It is less than the pound Avoirdupois.

III. APOTHECARIES' WEIGHT.

127. This weight is used by apothecaries and physicians in mixing their medicines. But medicines are generally sold, in the quantity, by avoirdupois weight.

Table.

20 Grains (*gr.*)	. make	1 Scruple, .	marked ℈.
3 Scruples . .	.	1 Dram, . .	ʒ.
8 Drams . .	.	1 Ounce, . .	℥.
12 Ounces . .	.	1 Pound, . .	℔.

Table Reversed.

				℈		gr.
			ʒ	1	=	20.
		℥	1	= 3	=	60.
	℔	1	= 8	= 24	=	480.
1	= 12	=	96	= 288	=	5760.

SCALE.—The steps, or units of the scale, beginning with the lowest, are 20, 3, 8, and 12.

NOTE.—The pound and ounce are the same as the pound and ounce in Troy weight.

VII. UNITS OF TIME.

128. TIME is a part of duration. The time in which the earth revolves on its axis is called a *day*. The time in which it goes round the sun is 365 days and 6 hours, nearly, and is called a *solar year*.

Time is divided into parts according to the following

Table.

60 Seconds	(*sec.*)	make	1 Minute,	marked *m.*
60 Minutes			1 Hour,	*hr.*
24 Hours			1 Day,	*da.*
7 Days			1 Week,	*wk.*
52 Weeks (nearly)			1 Year,	*yr.*
365 Days			1 Common Year,	*yr.*
366 Days			1 Leap Year,	*yr.*
12 Calendar Months			1 Year,	*yr.*
100 Years			1 Century	*C.*

Table Reversed.

						m.		sec.
					hr.	1	=	60.
			da.	1	=	60	=	360.
		wk.	1	=	24	=	1440	= 86400.
		1	=	7	=	168	= 10080	= 604800.
yr.	m.		365	=	8760	=	525600	= 31536000.
1	= 12		366	=	8784	=	527040	= 31622400.

SCALE.—The steps, or units of the scale, beginning with the lowest, are 60, 60, 24, 7, 52, and 12.

Calendar Year.

WINTER,	1st	Month,	January,	has 31 days.
	2d	"	February,	" 28 or 29 days.
SPRING,	3d	"	March,	" 31 days.
	4th	"	April,	" 30 days.
	5th	"	May,	" 31 days.
SUMMER,	6th	"	June,	" 30 days.
	7th	"	July,	" 31 days.
	8th	"	August,	" 31 days.
AUTUMN,	9th	"	September,	" 30 days.
	10th	"	October,	" 31 days.
	11th	"	November,	" 30 days.
WINTER,	12th	"	December,	" 31 days.

365 days *in* a year.

NOTES.—1. The years are numbered from the beginning of the Christian Era. The year is divided into 12 calendar months, numbered from January; the days are numbered from the beginning of the month; hours from 12 at night and 12 at noon.

2. The length of the solar year is 365 da. 5 hr. 48 m. 48 sec., nearly; but it is reckoned at 365 days 6 hours.

3. Since the length of the year is computed at 365 days and 6 hours, the odd 6 hours, by accumulating for 4 years, make 1 day, so that every fourth year contains 366 days. This is called Bissextile or Leap Year. The leap years are exactly divisible by 4: 1864, 1868, 1872, 1876 will be leap years.

4. The additional day, when it occurs, is added to the month of February, so that this month has 29 days in the leap year.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

VIII. CIRCULAR MEASURE.

129. CIRCULAR MEASURE is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies.

The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*. Each degree is divided into 60 minutes, and each minute into 60 seconds.

Table.

60 Seconds (″)	make	1 Minute,	marked ′.
60 Minutes		1 Degree,	°
15 Degrees		1 Hour Angle,	*hr. an.*
30 Degrees		1 Sign,	*s.*
12 Signs, or 360 Degrees		1 Circle,	*c.*

Table Reversed.

										′		″
									°	1	=	60.
							hr. an.	1	=	60	=	3600.
						s.	1	=	15	=	900	= 54000.
					c.	1	=	2	=	30	=	1800 = 108000.
1	=	12	=	24	=	360	=	21600	=	1296000.		

SCALE.—The steps, or units of the scale, beginning at the lowest, are 60, 60, 15, 30, and 12.

Miscellaneous Tables.

COUNTING.

12 Units, or things,	make	1 Dozen.
12 Dozen		1 Gross.
12 Gross		1 Great Gross.
20 Things		1 Score.

LENGTH.

18 Inches	1 Cubit.
22 Inches, nearly,	1 Sacred Cubit.

WEIGHT.

100 Pounds	1 Quintal of fish.
196 Pounds	1 Barrel of flour.
200 Pounds	1 Barrel of pork.
14 Pounds of iron or lead	1 Stone.
21½ Stones	1 Pig.
8 Pigs	1 Fother.

PAPER.

24 Sheets	1 Quire.
20 Quires	1 Ream.
2 Reams	1 Bundle.
2 or more Bundles	1 Bale.

BOOKS.

The terms *folio, quarto, octavo, duodecimo,* &c., indicate the number of leaves into which a sheet of paper is folded.

A sheet folded in	2 leaves,	is called a folio.
A sheet folded in	4 leaves,	a quarto, or 4to.
A sheet folded in	8 leaves,	an octavo, or 8vo.
A sheet folded in	12 leaves,	a 12mo.
A sheet folded in	16 leaves,	a 16mo.
A sheet folded in	18 leaves,	an 18mo.
A sheet folded in	24 leaves,	a 24mo.
A sheet folded in	32 leaves,	a 32mo.

Aliquot Parts.

130. AN ALIQUOT PART of a number, is any part that will divide the number without a remainder.

Aliquot Parts of One Dollar.

50 cents = ½ of 1 dollar.	12½ cents = ⅛ of 1 dollar.
33⅓ cents = ⅓ of 1 dollar.	10 cents = 1/10 of 1 dollar.
25 cents = ¼ of 1 dollar.	8⅓ cents = 1/12 of 1 dollar.
20 cents = ⅕ of 1 dollar.	6¼ cents = 1/16 of 1 dollar.
16¾ cents = ⅙ of 1 dollar.	5 cents = 1/20 of 1 dollar.

Aliquot Parts of Time.

6 months = ½ of 1 year.	15 days = ½ of 1 month.
4 months = ⅓ of 1 year.	10 days = ⅓ of 1 month.
3 months = ¼ of 1 year.	6 days = ⅕ of 1 month.
2 months = ⅙ of 1 year.	5 days = ⅙ of 1 month.
1 month = 1/12 of 1 year.	1 day = 1/30 of 1 month.

REDUCTION.

131. REDUCTION is the operation of changing the unit of a number, without altering the value of the number.

132. REDUCTION DESCENDING is the operation of changing the unit from one of a greater to one of a less value.

133. REDUCTION ASCENDING is the operation of changing the unit from one of a less to one of a greater value.

Reduction Descending.

1. Reduce £25 16s. and 6d. to pence.

Rule.

I. *Multiply the number in the highest denomination by the units of the scale which connect it with the next lower, and add to the product the units of that denomination, if any.*

II. *Proceed with this result, in the same manner, through all the denominations, until the required denomination is reached.*

OPERATION.

£25 16s. 6d.
20
516s.
12
Ans. 6198d.

Examples.

1. Reduce 3 bu. 3 pk. 2 qt. to pints.

OPERATION.

3 bu. 3 pk. 2 qt.
4
15 pk.
8
122 qt.
2
244 pints, *Answer.*

2. In 5 da. 6 hr. 30 min., how many seconds?

OPERATION.

5 da. 6 hr. 30 min.
24
126
60
7590 minutes, *Answer.*

3. How many ounces are there in 6 lb. 5 oz. Troy?
4. In 45 rods 5 yards, how many feet?
5. In £12 8s. 9½d., how many farthings?
6. How many inches are there in 40 rods 2 feet?
7. How many yards are there in 5 miles?

8. How many feet are there in 29 furlongs?
9. In 6 mi. 6 fur. 36 rods, how many rods?
10. In 6 yards 6 feet 6 inches, how many inches?
11. In £4 8s. 9½d., how many farthings?
12. In 6 gallons 5 quarts 1 pint, how many pints?
13. Reduce 10 bushels 1 pk. 6 quarts to pints.
14. Reduce 7 C. ft. 14 cu. ft. to cubic inches.
15. How many pounds in 3 T. 5 cwt. and 1 qr.?
16. Change 6 ℔ 3 ℥ 5 ʒ 2 ℈ to grains.
17. How many feet are there in 5 mi. 7 fur. 3 rd.?
18. How many feet are there in 69 chains?
19. How many minutes are there in 6 s. 17° 27′?
20. In 67 cords of wood, how many cubic feet?
21. In 57 reams of paper, how many sheets?
22. How many single things are there in 55 score?
23. How many single things are there in 44 great gross?
24. How many sheets of paper in 12 reams 6 quires?
25. How many seconds are there in 27° 30′?
26. Change 40 sq. rd. 15 sq. yd. 8 sq. ft. to square inches.
27. How many hours are there in the Winter months?
28. How many minutes in the Summer months?
29. How many pints are there in 7 bbl. 14 gal. 3 qt.?
30. Change 6 T. 5 cwt. 3 qr. 4 lb. to pounds.
31. Change 8 ℔ 9 ℥ 5 ʒ 3 ℈ 5 gr. to grains.
32. How many seconds are there in 45° 39′ 27″?
33. In 1 common year, 320 da. 6 hr. 5 min., how many minutes?
34. In 5 leap years, 27 da. 6 hr. 5 min., how many minutes?
35. In 5 A. 3 R. 5 P., how many square yards?
36. In 4 leap years and one common year, how many minutes?

Reduction Ascending.

1. Reduce 1392 inches to rods.

Rule.

I. *Divide the given number by the units of the scale which connect it with the denomination next higher, and set aside the remainder, if any:*

II. *Divide the quotient by the units of the scale which connect it with the next higher denomination, and so on till the required denomination is reached. The last quotient, with the several remainders annexed, will be the answer.*

OPERATION.

12)1392
3)116 ft.
5½)38 .. 2 ft.
11)76
6 .. 5 yd.

Ans. 6 rd. 5 yd. 2 ft.

Examples.

1. In 15204 seconds, how many degrees?

OPERATION.

60)15204
60)253 .. 24″
4 .. 13′

Ans. 4° 13′ 24″.

2. In 37469 pints, how many bushels?

OPERATION.

2)37469
8)18734 .. 1 pt.
4)2341 .. 6 qt.
585 .. 1 pk.

Ans. 585 bu. 1 pk. 6 qt. 1 pt.

Proof.

134. The proof of Reduction, either Ascending or Descending, is made by reversing the operation.

1. In 12 cu. yd. 15 cu. ft. 12 cu. in., how many cubic inches?

OPERATION.

cu. yd.	cu. ft.	cu. in.
12	15	12
27		
99		
24		
339 cu. ft.		

cu. in.
339 × 1728 + 12 = 585804

2. In 585804 cu. in., how many cubic yards, &c.?

OPERATION.

1728)585804 cu. in.
27)339 .. 12 cu. in.
12 .. 15 cu. ft.

	cu. yd.	cu. ft.	cu. in.
Ans.	12	15	12

Examples.

1. How many pounds are there in 8445 pence?
2. How many shillings are there in 49742 farthings?
3. In 87049 inches, how many rods?
4. In 4704609 feet, how many miles?
5. How many yards are there in 87408 inches?
6. Reduce 690492 square inches to square rods.
7. Reduce 870496 square feet to acres.
8. Reduce 588967 perches to square miles.
9. Reduce 57409 square feet to square chains.
10. In 678569 cubic inches, how many cubic yards?
11. In 87496 cord feet, how many cords?
12. In 4521624 cubic inches, how many tons of hewn timber?
13. In 78757 pints, how many barrels?
14. In 874904 quarts, how many pipes?
15. Reduce 6874049 quarts to tuns.
16. In 387049 pints, how many bushels?
17. In 886604 quarts, how many bushels?
18. In 72411 bushels, how many chaldrons?
19. In 27416 drams, how many quarters?

20. In 47409 ounces, how many hundredweight?
21. Reduce 875604 ounces to tons.
22. Change 4704967109 pounds to tons.
23. In 41049 grains, Troy, how many ounces?
24. Reduce 94099 pennyweights to pounds.
25. Reduce 610476 grains, Apothecaries' weight, to lbs
26. Reduce 45046 scruples, Apothecaries', to pounds.
27. Change 84049 drams, Apothecaries', to pounds.
28. Change 589405 grains, Apothecaries', to pounds.
29. How many hours are there in 654604 seconds?
30. How many days are there in 869504 minutes?
31. Change 3870469 seconds to months of 30 days.
32. Reduce 6549047 minutes to common years.
33. How many leap years in 8704926 hours?
34. How many common years in 974605 minutes?
35. How many months, of 31 days each, in 87049 minutes?
36. How many degrees in 870493 seconds?
37. How many hour angles in 764904 minutes?
38. How many signs in 270493 minutes?
39. How many circumferences in 429072 minutes?
40. How many degrees are there in 99804 seconds?
41. How many circumferences in 80493063 seconds?

Miscellaneous Examples.

1. Reduce 7 fur. 39 rd. 4 yd. 1 ft. 11 in. to inches.
2. In 74 wk. 6 da. 15 hr., how many seconds are there?
3. In 11 s. 14° 49′, how many seconds are there?
4. In a box of sugar, weighing 15 cwt. 1 qr. 16 lb., how many ounces are there?
5. If a person has walked 24 miles, how many inches has he walked?

6. What will be the cost of 37 cwt. 2 qr. 20 lb. of sugar, at $7\frac{1}{2}$ cts. per lb.?
7. A grocer has 16 cwt. 1 qr. 13 lb. of sugar, and wishes to put it into bags, each containing 7 lb.: how many bags will he require?
8. In 3 yr. 161 da. 13 hr., how many minutes are there?
9. In 19 wk. 4 da., how many half-hours are there?
10. How many weeks has a man labored, who has not worked on Sundays, and been employed 4964 hours?
11. How many minutes has a man lived, whose age is 47 years, supposing 11 of them to have been leap years?
12. In 476949 perches, how many acres?
13. How many cords of wood are there in a pile containing 674969 cubic inches?
14. Reduce 47 cu. yd. 15 cu. ft. 162 cu in. to cubic inches.
15. Reduce 45 ch. 18 bu. to pecks.
16. How many more feet are there in 16 miles than in 14 mi. 7 fur. 29 rd.?
17. How many pints are there in a cask of wine, containing 76 gallons?
18. How many gills are there in 17 hhd. 49 gal. 1 qt.?
19. How many bottles, containing 6 gills each, can be filled from a cask of wine, containing 48 gal.?
20. How much must be paid for 3 pi. 1 hhd. 47 gal. 3 qt. 1 pt. of wine at 34 cents per pint?
21. In 15 tun 1 pi. 1 hhd. 61 gal., how many gills are there?
22. How many grains in a bar of gold, whose weight is 2 lb. 7 oz. 14 pwt. 13 gr.?
23. What must be given for 145 ch. 26 bu. of coal at 25 cents per bushel?
24. How much will be paid for the labor of 42 weeks,

allowing 6 days to the week, and 10 hours to the day, at the rate of $0.25 per hour?

25. Reduce 17 tons to quarter pounds.

26. Reduce 9 weeks to minutes and to sixths of minutes.

27. How many ounces are there in 2 hogsheads of tobacco, each weighing 14 cwt. 3 qr. 15 lb.?

28. How many pints are there in 6 casks of wine, each containing 1 hhd. 39 gal. 3 qt. 1 pt.?

29. Reduce 56746896 drams, Avoirdupois, to tons.

30. What will be the cost of 37608 eggs at 9 cents per dozen?

31. If a ship has on board 1525 bales of cotton, each weighing 675 pounds, how many tons has it?

32. If one book requires 350 sheets of paper, how many reams would be required to print 850 copies?

33. How many yards of cloth are there in 6752 nails? and what is its value at $2.25 per yard?

34. Reduce 265 yd. 3 qr. 1 na. 1 in. to inches.

35. Reduce 569646 inches to yards.

36. Change 476 ells French to inches.

37. How many feet are there in a telegraphic wire that reaches a distance of 256 mi. 5 fur. 17 rd.?

38. How many inches from each other are two cities that are 63 miles apart?

39. How many yards of cloth at $1.75 per yard, can be bought for $47.50?

40. How many pair of pantaloons, each requiring $2\frac{1}{2}$ yd., can be made from $37\frac{1}{2}$ yd. of cloth?

41. If a suit of clothes requires 4 yd. 3 qr. of cloth, how many suits can be made from 76 yd.?

42. If the circumference of a wheel is 16 ft. 3 in., how many times will it turn in going a distance of 36 miles?

ADDITION.

135. Addition of Denominate Numbers is the operation of finding a the sum of two or more denominate numbers.

1. What is the sum of £6 8s. 9d., £7 5s. 7d., and £3 15s. 10d.?

Rule.—I. *Write the numbers to be added, so that units of the same name shall stand in the same column:*

II. *Begin with the lowest denomination, and add as in simple numbers; divide the sum of each column by the units of the scale, and add the quotient to the next column.*

OPERATION.

£.	s.	d.
6	8	9
7	5	7
3	15	10
£17	10	2

Proof.—The same as in simple numbers.

Examples.

(1.)

£.	s.	d.
4	15	8
3	4	7
2	1	1
10	1	4

(2.)

yd.	ft.	in.
14	2	10
7	2	11
3	2	9
26	2	6

(3.)

£.	s.	d.	far.
8	5	7	3
5	3	4	2
4	12	9	2
18	1	9	3

(4.)

bu.	pk.	qt.
13	3	6
14	2	7
13	3	5

(5.)

lb.	oz.	dr.
15	12	15
13	15	6
14	9	3

(6.)

°	′	″
27	36	37
5	15	20
6	45	50

(7.)

£.	s.	d.
5	17	9
6	13	4
2	11	6

(8.)

£.	s.	d.
49	19	$10\frac{1}{4}$
63	10	$11\frac{1}{2}$
71	8	$8\frac{3}{4}$

(9.)

mi.	fur.	rd.
69	7	39
91	6	10
47	1	18

(10.)

T.	cwt.	qr.	lb.
96	15	3	16
74	12	1	10
68	17	2	12

(11.)

cwt.	qr.	lb.	oz.
75	1	17	9
64	3	14	12
30	1	15	15

(12.)

lb.	oz.	pwt.	gr.
36	9	16	23
97	11	20	19
60	10	19	7

(13.)

mi.	fur.	rd.	yd.
19	6	15	4
74	7	34	4
56	1	25	4
17	5	16	3

(14.)

R.	P.	sq. yd.	sq. ft.	sq. in.
3	26	16	8	120
1	39	12	7	136
3	21	9	1	112
1	32	7	5	132

15. A mechanic worked on Monday, 14 hr. 15 min.; on Tuesday, 12 hr. 3 min.; on Wednesday, 12 hr. 45 min.; on Thursday, 17 hr. 16 min.; on Friday, 16 hr. 25 min.; on Saturday, 14 hr. 35 min.: how much time did he work during the six days?

16. What amount of sugar in 5 boxes, weighing as follows: 1st, 17 cwt. 3 qr. 18 lb.; 2d, 12 cwt. 1 qr. 17 lb.; 3d, 15 cwt. 2 qr. 4 lb.; 4th, 9 cwt. 19 lb.; and 5th, 13 cwt. 3 qr. 13 lb.?

17. What quantity of oats in a bin into which has been put 6 bu. 3 pk. 7 qt.; 9 bu. 2 pk. 6 qt.; 14 bu. 3 pk.; 25 bu. 1 pk. 3 qt.; 17 bu.; 35 bu. 1 pk. 6 qt.; and 27 bu. 1 pk. 5 qt.?

18. What quantity of cloth is in the following pieces: 1st, 31 yd. 3 qr. 3 na.; 2d, 37 yd. 1 qr.; 3d, 42 yd. 3 qr. 3 na.; 4th, 32 yd. 2 qr. 2 na.; and 5th, 35 yd. 1 qr. 3 na.?

19. What is the sum of 275 da. 11 hr. 50 min. 30 sec.; 106 da. 13 hr. 40 min. 40 sec.; 300 da. 18 hr. 18 min. 25 sec.; 212 da. 6 hr.; 65 da. 30 min. 30 sec.; and 1 da. 1 hr. 1 min.?

20. A gentleman bought 4 pieces of land: the first contained 85 A. 3 R. 14 P.; the second, 62 A. 1 R. 15 P.; the third, 14 A. 3 R. 13 P.; and the fourth, 25 A. 35 P.: how much land did he purchase?

21. A silver spoon weighs 13 pwt. 16 gr.; a knife, 18 pwt. 12 gr.; a cup, 7 oz. 5 pwt. 10 gr.; a napkin ring, 1 oz. 10 pwt.; a candlestick, 10 oz. 15 pwt. 20 gr.: what is the entire weight of these 5 articles?

22. A ship sails, on the 1st day, 219 mi. 6 fur. 32 rd.; the 2d, 230 mi. 3 fur. 30 rd.; on the 3d, 196 mi. 5 fur. 20 rd.; on the fourth, 212 mi.; on the 5th, 216 mi. 7 fur. 27 rd.; and on the 6th day, 225 mi. 5 fur. 29 rd.: how far did it sail in the 6 days?

23. A merchant received 6 casks of molasses, marked as follows: 1st, 1 hhd. 23 gal. 3 qt.; 2d, 2 hhd. 43 gal.; 3d, 49 gal. 3 qt.; 4th, 2 hhd. 35 gal. 1 qt.; 5th, 1 hhd. 51 gal. 2 qt. 1 pt.; and 6th, 2 hhd. 42 gal.: what quantity was received?

24. What is the value, at 6 cents per pound, of the following lots of sugar: 1st, 3 cwt. 3 qr. 3 lb.; 2d, 10 cwt. 1 qr. 10 lb.; 3d, 14 cwt. 2 qr. 13 lb.; 4th, 16 cwt. 2 qr. 20 lb.?

25. A farmer raised from one field, 39 bu. 3 pk. 6 qt. of wheat; from 2d, 45 bu. 2 pk. 3 qt.; from 3d, 26 bu. 1 pk. 5 qt.; from 4th, 35 bu. 3 pk. 5 qt.; and from 5th, 46 bu. 3 pk. 2 qt.: what was raised from all the fields?

26. A person offers £495 11s. 6d. for a house; the owner offers to sell it at an advance of £26 9s. 9d. on the amount offered: what price was demanded for the house?

SUBTRACTION.

136. Subtraction of Denominate Numbers is the operation of finding the difference between two denominate numbers.

1. What is the difference between 8 bu. 2 pk. 6 qt., and 5 bu. 3 pk. 7 qt.?

Rule.—I. *Set down the less number under the greater, placing units of the same value in the same column.*

II. *Begin with the lowest denomination, and subtract as in simple numbers, borrowing and carrying when necessary, according to the scale.*

Proof.—The same as in simple numbers.

OPERATION.

bu.	pk.	qt.
8	2	6
5	3	7
2	2	7

Examples.

(1.)

	£.	s.	d.
From	127	14	5
Take	89	17	9
	37	16	8

(2.)

	£.	s.	d.	far.
	292	17	3	1
	169	18	10	3
	122	18	4	2

(3.)

	£.	s.	d.
	1	9	$6\frac{1}{2}$
	0	19	$10\frac{3}{4}$
		9	$7\frac{3}{4}$

(4.)

	Tons.	cwt.	qr.	lb.
From	262	15	1	16
Take	197	17	2	24

(5.)

	cwt.	qr.	lb.	oz.
	7	0	5	11
	5	3	17	15

(6.)

	yd.	qr.	na.
	461	2	1
	279	3	3

(7.)

	mi.	fur.	rd.	yd.	ft.
From	161	1	19	2	1
Take	79	6	27	4	2

(8.)

Tuns.	pt.	hhd.	gal.	qt.	pt.	gills.
226	1	1	21	3	0	1
179	1	1	39	3	1	3

9. What is the difference of £21 14s. 6d. and £19 19s. 11d.?

10. How much is 161 lb. 3 oz. 11 pwt. 16 gr. greater than 98 lb. 7 oz. 15 pwt. 21 gr.?

11. Find the difference of 120 A. 1 R. 29 P. 16 sq. yd. 6 sq. ft., and 65 A. 3 R. 39 P. 20 sq. yd. 8 sq. ft.?

12. If, from a piece of cloth, containing 32 yd. 3 qr., a tailor cuts 14 yd. 3 qr. 2 na., how much will be left?

13. From a cask of wine containing 1 hhd. 15 gal., there were drawn 39 gal. 3 qt. 1 pt.: what quantity was left?

14. A mason was engaged to put up a wall of 37 yd. 2 ft. 6 in. in length: after building 19 yd. 2 ft. 9 in., how much remained to be built?

15. A grocer took 9 cwt. 1 qr. 15 lb. from a box of sugar which contained 16 cwt. 10 lb.: how much was left?

16. How much does 116 ch. 16 bu. 2 pk. exceed 89 ch. 29 bu. 3 pk.?

17. If 17 ℔ 11 ℥ 7 ʒ 1 ℈ 16 gr. be taken from 21 ℔ 6 ℥ 3 ʒ 1 ℈ 5 gr., what will be left?

18. Subtract 29 lb. 10 oz. 14 pwt. from 51 lb. 1 oz. 10 pwt. 6 gr.

19. Subtract 19 tons 17 cwt. 1 qr. 16 lb. from 21 tons 15 cwt.

20. If a quantity of flour, which cost £123 16s. 7d., be sold for £131 6s. 6d., what will be the gain?

21. A cask can hold 2 hhd. 15 gal.: 59 gal. 1 qt. 1 pt. have been put in it: how much more will it hold?

22. One piece of calico contains 36 yd. 1 qr.; another piece 32 yd. 3 qr. 2 na.: how much more in one piece than in the other?

MULTIPLICATION.

137. MULTIPLICATION OF DENOMINATE NUMBERS is the operation of taking a denominate number as many times as there are units in the multiplier.

1. Multiply 6 T. 14 cwt. 2 qr. 15 lb. by 3.

Rule.—I. *Write down the denominate number, and set the multiplier under the lowest denomination:*

II.—*Multiply as in simple numbers, and in passing from one denomination to another, divide by the units of the scale, set down the remainder, and carry the quotient to the next product.*

OPERATION.

T.	cwt.	qr.	lb.
6	14	2	15
			3
20	3	3	20

Examples.

(1.)

£.	s.	d.
3	4	9
		4
12	19	0

(2.)

bu.	pk.	qt.
6	3	6
		5
34	2	6

(3.)

cwt.	qr.	lb.
2	3	20
		7
20	2	15

(4.)

yd.	ft.	in.
9	2	8
		8

(5.)

mi.	fur.	rd.
17	6	30
		9

(6.)

wk.	da.	hr.
15	6	19
		6

7. Multiply 3 A. 5 R. 18 P. by 6.

8. How many gallons in 3 casks, each containing 30 gal. 3 qt. 1 pt.?

9. Multiply 27 bu. 3 pk. 4 qt. 1 pt. by 7.

10. What is the product of 5 ℔ 8 ℥ 4 ʒ 6 ℈ 12 gr. multiplied by 9?

11. Multiply 7 wk. 6 da. 9 hr. 10 min. by 5.

12. If 1 piece of calico costs £1 8s. 6½d., what will 8 pieces cost?

13. If a pipe discharges 169 gal. 3 qt. 1 pt. of water in 1 hour, how much will it discharge in 5 hours?

14. If each of 9 plots of ground contains 3 A. 1 R. 25 P., how much in all the plots?

15. Multiply 6 bu. 3 pk. 2 qt. 1 pt. by $24 = 4 \times 6$.

OPERATION.

bu.	pk.	qt.	pt.
6	3	2	1
			4
27	1	2	0

bu.	pk.	qt.	pt.
27	1	2	0
			6
163	3	4	0

NOTE.—When the multiplier is a composite number, multiply by the factors separately.

16. How much calico in 12 pieces, each of which contains 377 yd. 3 qr.?

17. Ten men have been employed in a factory 14 da. 15 hr. 30 min.: how long must one man be employed to do an equal amount of work?

18. What is the product of £3 8s. 9d. 3far. by 14.

19. Multiply 16 da. 8 hr. 15 min. by 28.

20. How much water will 14 casks contain if one cask holds 31 gal. 2 qt. 1 pt.?

21. If, to make one book, 1 ream 6 quires and 14 sheets of paper are required, how much will be necessary for 100 books?

22. If a vessel can carry 175 tons 1 cwt. 3 qr. 18 lb. of railroad iron, how much can 7 such vessels carry?

23. A man divided his farm among 5 sons, giving to each 121 A. 1 R. 35 P.: how large was the farm?

24. If a man works at his trade 9 hr. 45 sec. per day, how much does he work in 2 weeks 4 days?

25. A horse car makes 6 trips per day over a road 3 mi. 1 fur. 29 rd. in length: how far does it run?

26. How many bushels in 24 barrels of potatoes, if each barrel contains 1 bu. 2 pk. 4 qt.?

27. If a bottle of cider contains 1 pt. 3 gi., how much will 4 dozen bottles contain?

28. If a man can mow 1 A. 2 R. 39 P. of grass in 1 day, how much can he mow in 11 days?

29. If a hogshead of molasses contains 61 gal. 2 qt. 1 pt., how much will 14 hogsheads contain?

30. If a vessel sails 14 L. 1 mi. 6 fur. 17 rd. in 1 day, how far will it sail in the month of January?

31. If a person sleeps 7 hr. 15 min. 15 sec. daily, how much will he sleep in 3 weeks?

32. How many yards of cloth in 36 pieces, if each piece contains 27 yd. 3 qr.?

33. If 1 silver spoon weighs 1 oz. 11 pwt. 12 gr., what will be the weight of 1 dozen of the same kind?

34. The earth revolves 0° 15′ of space in 1 minute of time: how far does it revolve in 1 hour?

35. If 1 silver cup weighs 9 oz. 10 pwt. 16 gr., how much will 15 such cups weigh?

36. The multiplier is 18, and the multiplicand 7 bu. 2 pk. 5 qt., what is the product?

37. What is the weight of 9 boxes of sugar, if each weighs 17 cwt. 1 qr. 16 lb.?

38. What is the product of 56 A. 3 R. 21 P. by 6?

39. If 4 tons. 15 cwt. 1 qr. 10 lb. of hay will last a horse for one year, how much would 15 horses require?

DIVISION.

138. Division of Denominate Numbers is the operation of dividing a denominate number into equal parts; or, of finding how many times one denominate number is contained in another.

1. Divide 16 cwt. 3 qr. 21 lb. by 3.

Rule.

I. *Begin with the highest denomination, and divide as in simple numbers:*

II. *Reduce the remainder, if any, to the next lower denomination, and add in the units of that denomination, for a new dividend:*

III. *Proceed in the same manner, through all the denominations.*

OPERATION.

	cwt.	qr.	lb.
3)	16	3	20
	5	2	15

Proof.—Multiply the quotient by the divisor.

Notes.—1. If the divisor is a composite number, we may divide by the factors in succession, as in simple numbers.

2. Each quotient figure has the same unit as the dividend from which it was derived.

Examples.

(1.)

	£.	s.	d.
4)	12	19	0
	3	4	9

(2.)

	bu.	pk.	qt.
5)	34	2	6
	6	3	6

(3.)

	cwt.	qr.	lb.
7)	20	2	15
	2	3	20

(4.)

	mi.	fur.	rd.
6)	56	5	4

	s.
3)	27

(5.)

	°	′	″
	28	22	45

(6.)

	yr.	mo.	da.	hr.
5)	32	3	18	18

(7.)

	T.	cwt.	qr.	lb.	oz.	dr.
8)	53	3	2	16	4	8

8. If 7 loads of wood contain 8 C. 6 C. ft., what will load contain?

9. What will be the weight of 1 tierce of rice, if 7 tierces weigh 1 ton 19 cwt. 2 qr. 12 lb.?

10. If 4 equal packages of medicines weigh 13 lb 7 ʒ 2 ʒ 1 ℈ 4 gr., what will be the weight of each.

11. How far will a man travel in 1 day, if in 5 days he travels 122 mi. 4 fur. 23¾ rd.?

12. If 9 equal fields contain 111 A. 2 R. 25 P., how much is there in each field?

13. If 9 equal pieces of calico contain 267 yd. 0 qr. 3 na., how much is there in each piece?

14. If a vessel running at an equal rate, sails 47 L. 1 mi. 7 fur. 8 rd., in 8 days, how far does she sail in 1 day?

15. If a steamer moves at the rate of 15 mi. 2 fur. 40 rd. per hour, what is the rate per minute?

16. A cartman carried 117 cords 110 feet of wood in 100 loads: how much did he carry at each load?

17. If a quantity of provisions will last one man for 2 weeks 6 days, how long will it last 50 men?

18. A person wishes to perform a journey of 165 mi. in 16 hours: at what rate must he travel?

19. How many suits of clothes, each requiring 7 yd. 2 qr., can be cut from a piece of cloth containing 67 yd. 2 qr.?

20. A ship has 468 T. 2 cwt. 2 qr. of railroad iron, and a wagon can carry 2 tons 3 cwt. 3 qr.: how many wagon-loads in the entire cargo?

ANALYSIS.

139. An ANALYSIS is an examination of the separate parts of a question, and of their connection with each other.

In analyzing, we reason from a *given number* to its *unit*, and then from this unit to the *required* number, or answer.

The processes are indicated by the relations which exist between the given and required numbers, and are pursued, step by step, independently of set rules.

CASE I.

140. To find the cost of several things, when the price of a single thing is an aliquot part of 1 dollar.

1. What is the cost of 75 yards of cotton cambric, at 33⅓ cents per yard?

ANALYSIS.—33⅓ cents = ⅓ of a dollar: 75 yards, at $1 a yard, would cost $75; at ⅓ of a dollar a yard, it would cost ⅓ of $75, which is $25: Hence,

OPERATION.

3) 7 5

$ 2 5

Rule.

Take such a part of the number of things, as the price of a single thing is of $1.

Examples.

1. What is the cost of 200 yards of cambric, at 25 cents a yard?

2. What will be the cost of 300 pencils, at 12½ cents each?

3. What will be the cost of 150 tops, at 6¼ cents apiece?

4. What will 500 melons cost, at 5 cents apiece?

5. What will be the cost of 150 gallons of molasses, at 33⅓ cents per gallon?

(6.)

	yr.	mo.	da.	hr.
5)	32	3	18	18

(7.)

	T.	cwt.	qr.	lb.	oz.	dr.
8)	53	3	2	16	4	8

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OPERATION.

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3. What will be the cost of 150 tops, at 6¼ cents apiece?

4. What will 500 melons cost, at 5 cents apiece?

5. What will be the cost of 150 gallons of molasses, at 33⅓ cents per gallon?

CASE II.

141. To find the cost, when the price of 1, and the number of things are given.

1. What is the cost of 48 lemons, at 3 cents apiece?

ANALYSIS.—Since 1 lemon costs 3 cents, 48 lemons will cost 48 times 3 cents, or 3 times 48 cents, which is 144 cents: Hence,

OPERATION.

4 8
3
$1.4 4

Rule.

Multiply the price of 1 by the number of things, or the number of things by the price of 1, and the product will be the cost.

Examples.

1. What will 75 hats cost, at $4.25 each?
2. If wheat is $1.60 a bushel, what will be the cost of 13.5 bushels?
3. What is the cost of 4204 yards of cloth, at $3.37½ a yard?
4. What will 3704 pair of shoes cost, at $2.75 a pair?
5. If 1 cheese costs $3.75, what will be the cost of 324?

CASE III.

142. To find the cost of things sold by the 100 or 1000.

1. What is the cost of 8781 feet of lumber, at $4 per hundred feet?

Rule.—*Multiply the number of things and price together, and point off in the product, two places of decimals more than there are in both factors, when sold by the hundred, and three places more, when sold by the thousand.*

OPERATION.

8 7 8 1
4
$3 5 1.2 4

Examples.

1. What will be the cost of 54704 bricks, at 50 cents per hundred?
2. What will 1347 feet of lumber cost, at $2.25 per C?
3. What will be the cost of 15758 feet of boards, at $10.62 per M?
4. What is the value of 57046 feet of lathing, at $7 per M?
5. What will be the value of 560 chickens, at $33 per C?
6. What is the value of 4704 pounds of butter, at $23 per hundred?

CASE IV.

143. To find the cost of articles sold by the ton of 2000 pounds, when the price of a ton is known.

1. What is the cost of 6528 pounds of hay, at $18.50 per ton?

Rule.—*Divide the price by 2, and then find the cost of the quantity by the last Case.*

OPERATION.

2)1 8.5 0
$9.2 5

Examples.

1. What will 57045 pounds of plaster cost, at $4.25 per ton?
2. What is the cost of transportation of 87415 pounds of iron from Buffalo to New York, at $7 per ton?
3. What is the cost of 75049 pounds of coal, at $7.75 per ton?
4. What is the cost of transporting 785674 pounds of coal from Albany to Boston, at $2.70 per ton?

CASE V.

144. When the number of things is known, and their cost, to find the price of 1 thing.

1. If 36 pounds of tea cost $52.20, what is the price per pound?

ANALYSIS.—1 pound will cost one thirty-sixth as much as 36 pounds; one thirty-sixth of $52.20 is $1.45: therefore, 1 pound will cost $1.45.

OPERATION.

$$
\begin{array}{r|l}
36 & 52.20 \ (1.45 \\
 & 36 \\
\hline
 & 162 \\
 & 144 \\
\hline
 & 180 \\
 & 180 \\
\hline
\end{array}
$$

Rule.—*Divide the entire cost by the number of things.*

Examples.

1. Divide 1884.625 into 25 equal parts.
2. A farmer purchased 758 sheep for $3750: how much did he pay per head?
3. A merchant bought 30 bales of goods, for which he paid $2000: what did they cost him per bale?
4. A drover paid $2500 for 400 sheep: what must he sell them for apiece, that he may neither make nor lose?

CASE VI.

145. When the cost of a number of things is given, and the price of 1, to find the number.

1. If I pay $6.50 for a ton of coal, how much can I buy for $97.50?

ANALYSIS.—As many tons as $6.50 is contained times in $97.50, which is 15.

OPERATION.

$$
\begin{array}{r|l}
650 & 9750 \ (15 \text{ tons.} \\
 & 650 \\
\hline
 & 3250 \\
 & 3250 \\
\hline
\end{array}
$$

Rule.—*Divide the entire cost by the cost of 1 thing.*

Examples.

1. If 1 acre of land costs $77.50, how much can be bought for $27125?
2. How many sheep will $396 buy, at $4.12\frac{1}{2}$ each?
3. At $4.25 a yard, how much cloth can be bought for $136?

RATIO AND PROPORTION.

146. A RATIO is the quotient obtained by dividing one number by another.

147. The terms of a ratio are the divisor and dividend: hence, every ratio has two terms.

148. The divisor is called the ANTECEDENT.

149. The dividend is called the CONSEQUENT.

150. The ratio of one number to another is generally expressed by a colon; thus, 3 : 12; and is read, 3 is to 12, or 12 divided by 3.

151. The terms of a ratio, taken together, are called a COUPLET.

Examples.

1. What is the ratio of 2 feet to 8 feet?
2. What is the ratio of 4 yards to 12 yards?
3. What is the ratio of 6 to 18?
4. What is the ratio of 9 to 27?
5. What is the ratio of 12 to 48?
6. What is the ratio of 1 to 15?
7. What is the ratio of 10 to 100?
8. If the antecedent is 6 and the consequent 12, what is the ratio?
9. If the antecedent is 9, and the consequent 18, what is the ratio?
10. If the consequent is 16, and the antecedent 2, what is the ratio?
11. If the consequent is 24, and the antecedent 12, what is the ratio?

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 & 650 \\
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\hline
\end{array}
$$

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1. What is the ratio of 2 feet to 8 feet?
2. What is the ratio of 4 yards to 12 yards?
3. What is the ratio of 6 to 18?
4. What is the ratio of 9 to 27?
5. What is the ratio of 12 to 48?
6. What is the ratio of 1 to 15?
7. What is the ratio of 10 to 100?
8. If the antecedent is 6 and the consequent 12, what is the ratio?
9. If the antecedent is 9, and the consequent 18, what is the ratio?
10. If the consequent is 16, and the antecedent 2, what is the ratio?
11. If the consequent is 24, and the antecedent 12, what is the ratio?

PROPORTION.

152. A PROPORTION is the comparison of the terms of two equal ratios.

Thus, the ratio of 4 : 8, is 2; and the ratio of 6 : 12, is 2; and we compare the terms by writing a double colon between the couplets; thus,

4 : 8 :: 6 : 12;

which is read, 4 is to 8, as 6 to 12. Hence, *every proportion has two couplets and four terms.*

153. The first and fourth terms of a proportion are called the *extremes:* the second and third terms, the *means.* Thus, in the proportion,

4 : 8 :: 6 : 12,

4 and 12 are the *extremes,* and 8 and 6 the *means.*

154. In any proportion, the product of the means is equal to the product of the extremes: Hence,

1st. *Either extreme is equal to the product of the means divided by the other extreme;* and

2d. *Either mean is equal to the product of the extremes divided by the other mean.*

Examples.

1. The terms of the first couplet are 6 and 8, and the antecedent of the second couplet, 4: what is the consequent?

2. The 1st, 2d, and 3d terms of a proportion, are 4, 6, and 12: what is the fourth term?

3. The 1st, 3d, and 4th terms of a proportion, are 6, 18, and 24: what is the second term?

4. The 2d, 3d, and 4th terms of a proportion, are 12, 24, and 72: what is the first term?

5. The 1st, 2d, and 3d terms of a proportion, are 6, 18, and 24: what is the 4th term?

6. The 1st, 3d, and 4th terms of a proportion, are 5, 15, and 60: what is the second term?

RULE OF THREE.

155. THE RULE OF THREE shows us how to find, from three given numbers, a fourth, to which one of the given numbers shall have the same ratio as exists between the other two.

1. If 1 barrel of flour costs $7, what will 8 barrels cost?

ANALYSIS.—It is plain that 8 barrels will cost 8 times as much as 1 barrel. Hence,

OPERATION.

bar. bar. $ $
1 : 8 :: 7 : 56

2. If 5 yards of cloth cost $20, what will 15 yards cost, at the same rate?

ANALYSIS.—The quantity, 5 yards, is to the quantity, 15 yards, as $20, the cost of 5 yards, is to $60, the cost of 15 yards; and, generally,

OPERATION.

5 : 15 :: 20 : 60

15
5)300(60 *Ans.*

Quantity : *quantity* :: *cost* : *cost.*

NOTES.—1. The unit is always the same in both terms of the first couplet: in the first example, it is 1 barrel; in the second, it is 1 yard.

2. The units of both terms of the second couplet are also always alike: in the first example, each unit is 1 dollar, and in the second example it is also 1 dollar.

3. Only one class of cases is considered

Rule.

I. *Write that term which is of the same unit value with the answer sought, in the third place; the term mentioned in connection with it, in the first place; and the remaining term, in the third place:*

II. *Then multiply the second and third terms together, and divide their product by the first term: the quotient will be the answer.*

Examples.

1. If 5 pounds of raisins cost 80 cents, how many cents will 20 pounds cost?

ANALYSIS.—Since the answer is to be cents, 80 cents are written in the 3d place: then 5 (mentioned in connection with 80) is written in the first place, and 20, the remaining number, in the second place.

OPERATION.

lb.	lb.	cts.
5 :	20 ::	80
		20
	5)	1600
		320 cts. = $3.20.

Writing the numbers thus, is called the *statement*.

2. If James can walk 12 miles in 4 hours, how far can he walk in 20 hours?

3. If 9 hats cost $36, how much will 40 hats cost?

4. If a family consumes 100 pounds of meat in 20 days, how much would they consume in 3 months, of 30 days each?

5. If 30 yards of cloth cost $150, what will be the cost of 96 yards?

6. If a flock of 40 sheep yield 240 pounds of wool, how much would be produced by a flock of 160?

7. If 20 gallons of molasses cost $8, what will be the cost of 135 gallons?

8. If a man travels at the rate of 32 miles in 4 hours, how far will he travel in 16 hours?

9. If 20 yards of tape cost 75 cents, what will be the cost of 160 yards?

10. What is the cost of 8 bushels of coal, if 9 bushels cost $2.25?

11. If 2 pipes will fill a cistern of 126 hogsheads in 5 hours, how many hogsheads would 9 pipes fill in the same time?

12. If 12 men consume 24 barrels of flour in 1 year, how much will 60 men consume?

13. If 27 pounds of butter will buy 18 pounds of sugar, how much butter will 36 pounds of sugar buy?

14. If 4 calves are worth 37 dollars, what will be the cost of 44 calves?

15. What will be the cost of 47 yards of cloth, if one-quarter of a yard costs $2.40.

16. If 24 yards of cloth cost $67.25, what will be the cost of 72 yards?

17. If 2000 feet of boards cost $65, what will be the cost of 5000 feet?

18. To what number has 6 the same ratio, as 5 has to 20?

19. To what number of yards of cloth has 6 yards the same ratio, as exists between the numbers 5 and 30?

20. What will 150 bushels of wheat cost, if 4 bushels and 3 pecks cost $14.

21. If a man travels 25 miles a day, and rests every Sunday, how far will he travel in the month of July, when the month begins on Sunday?

22. If 24 men can be boarded 1 week for $79.16, what will it cost to board 3 men and 6 women the same time, the women being boarded at half the price of the men?

23. If 7 yards of cloth cost 21 dollars, what will be the cost of 5 pieces, each containing 13 yards?

24. If 16 lb. of beef cost two dollars, what will 30 pounds cost?

25. If 9 bushels of wheat are of the same value as 3 yards of cloth worth 4 dollars a yard, what will be the cost of 360 bushels?

26. If 9 sheep cost $27, and a lamb is worth one-third as much as a sheep, what will be the cost of 54 lambs?

27. If 1500 men require 45000 rations of food for a month, how many rations will a garrison of 2400 men require?

28. What is the cost of 27 yards of velvet, if five-eighths of a yard cost 5 dollars?

29. If I travel at the rate of 58 miles in two hours, in a railroad car, how far will I travel in 36 hours?

30. What will be the cost of 60 yards of broadcloth, at the rate of $29 for 15 yards?

31. If 39 horses consume 400 bushels of oats in a month, how many bushels would serve 195 horses the same time?

32. If a man travels at the rate of 125 miles in 7 days, in how many days will he travel 1125 miles?

33. If 140 bushels of oats cost $56, what will be the cost of 900 bushels?

34. What distance will a man travel in a railroad car in 26 hours, if in 7 hours he travels 287 miles?

35. If a man travels at the rate of 420 miles in 12 days, how far will he travel in a leap year, supposing him to rest on Sundays?

36. If 1 vest costs $2.50, 1 pair of pantaloons twice as much as a vest, and 3 coats 8 times as much as a pair of pantaloons, what will be the cost of 9 coats?

Promiscuous Questions.

1. If 569 be subtracted from a certain number, the remainder will be 479: what is the number?

2. If 461 be added to a certain number, the sum will be 709: what is the number?

3. A gentleman, by mistake, paid $6714.75, which was $126.87½ more than he owed: what was the debt?

4. A merchant bought a quantity of flour for $672: he paid $60 for freight and cartage, and $27 for storage; he then sold the flour, and gained $150: how much did he receive for it?

5. A father was born in 1809; his oldest son, who was 18 years older than the youngest, was born in 1835: how old was the father when the youngest son was born?

6. A merchant bought 225 barrels of flour, at $6 per barrel; he sold 75 barrels at $6.50 per barrel, 112 barrels at $6.81¼ per barrel, and the remainder at $7 per barrel: what did he gain?

7. If a person has a yearly salary of $1625, and spends $965; in what time can he pay, by annual instalments, for a house, valued at $3960?

8. If a father earns $12 per week, and his son $5 per week, how long a time must they work, that they may earn, together, $306?

9. If a person receives $1.75 per day, in how many weeks, of 6 days each, would he have received $966.00?

10. If a person receives a yearly salary of $1903.20, what are his daily wages, allowing 52 weeks to the year, and 6 days to the week?

11. A merchant bought 25 hogsheads of sugar, at $149 apiece; he paid $2 each for storage, $1.50 a piece for cartage, and $1 each for other expenses; he sold the sugar for $4000: how much was gained?

12. John earns $15 per week, Thomas earns $17, and Samuel $18: in what time can they together earn $750?

13. How many days were there between January 1st. and July 16th, both inclusive?

14. A farmer bought a horse and cow for $165.50; he paid $36 more for the horse than for the cow: what did he pay for each?

15. The divisor is 475, and the quotient is 362: what is the dividend?

16. The divisor is 650, the quotient is 437, and the remainder is 212: what is the dividend?

17. A scholar in performing an example, obtained a quotient 512; but before dividing, he ought to have multiplied the dividend by 24, which he omitted: what was the true quotient?

18. Find the amount of the following bill:

BROOKLYN, March 20th, 1863.

Mr. James Richmond

Bought of Samuel Jones:

12 lb. of butter, at 24 cents per lb. . . . . $
28 lb. of sugar, at 9½ cents per lb. . . . . .
7 lb. of cheese, at 11 cents per lb. . . . . .
½ lb. of tea, at 33 cents per lb. . . . . .
2 lb. of coffee, at 32 cents per lb. . . . . .
3 gal. of syrup, at 56 cents per gal. . . . . .

Amount . $_____

Received Payment.

19. At the rate of $1620 for 12 horses, what would be the cost of 475 horses?

20. How many pounds of tea, at 95 cents per pound, can be bought for 15 barrels of flour, at $6.25 per barrel?

21. What is the cost of 720 eggs, at 18 cents per dozen?

22. What is the cost of 24648 steel pens, at the rate of 45 cents per gross?

23. How many barrels of flour would be required to fill 75 bags, each bag to contain 25 lb.?

24. The grounds of a country residence have a front of 250 feet, which is to be fenced with panels 10 feet long: how many posts are required?

25. Suppose that, in the before-mentioned fence, 31 posts were to be used, what would be the length of a panel?

26. How many posts would be required to fence the 4 sides of a lot, that is 260 feet long and 180 feet wide, if each panel is 10 feet long?

27. A gentleman bought an equal number of oranges and lemons for $1.50; he paid 3 cents apiece for oranges, and two cents for lemons: how many of each kind did he buy?

28. A manufacturer employed an equal number of men and boys; each man received $10 per week, and each boy $4; and the weekly wages of men and boys amounted to $168: how many of each class were employed?

29. What is the greatest common divisor of 1728 and 276?

30. What is the largest number that will exactly divide, at the same time, 576 and 1016?

31. What is the greatest common divisor of 296 and 1760?

32. A drover has two flocks of sheep—one containing 275 head, and the other 475 head; he wished to divide each flock into an exact number of smaller flocks of the same size, so as to have as few flocks as possible: what must be the number in each flock?

33. What will be the cost of 8 lb. of tea, at 75 cents per half-pound?

34. How many eighths of a dollar are there in 9\frac{3}{8}$?

35. How many sixteenths are there in the sum of 7 and $9\frac{5}{16}$?

36. A gentleman wishes to change a double-eagle for quarter-dollars: how many should he receive?

37. A person has 125 quarter-dollars: how many dollars has he?

38. In $\frac{5967}{16}$ of a mile, how many miles are there?

39. Change $\frac{144}{1250}$ to an equivalent fraction, in its lowest terms.

40. What divisor will reduce $\frac{75}{180}$ to its lowest terms?

41. What fraction, in its lowest terms, is equal to $\frac{144}{1728}$?

42. A merchant owning $\frac{7}{8}$ of a vessel, sold $\frac{1}{5}$ of his share to the captain: what part of the vessel did the captain buy?

43. If $\frac{1}{16}$ of a vessel was sold for $4500, what was the value of the vessel?

44. How much is $\frac{3}{5}$ of $56915?

45. If a ship is valued at $69475, what is the value of $\frac{5}{16}$?

46. Change $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{5}{6}$ into equivalent fractions, that have a like denominator.

47. Show which of the two fractions, $\frac{7}{9}$ or $\frac{8}{11}$, is the larger.

48. A fortune was thus divided: $\frac{1}{3}$ of it to a daughter; $\frac{1}{4}$ of it to a son; $\frac{1}{3}$ of it to the widow, and the remainder to benevolent institutions: what part of it did the institutions receive?

49. What is that number, from which, if you take $\frac{11}{12}$, the remainder will be $\frac{5}{6}$?

50. A person spent $\$\frac{7}{8}$ for a handkerchief, $\$\frac{9}{10}$ for a pair of gloves, and had $\$1\frac{2}{5}$ remaining: how much had he at first?

51. How much must be taken from $\frac{5}{6}$, that the remainder may be $\frac{2}{3}$?

52. If a mechanic receive $\$1\frac{7}{8}$ per day, and spend $\$1\frac{2}{5}$, how much will he save in 9 days?

53. A person, having $\$7\frac{3}{4}$, spent $\frac{2}{5}$ of it for a pair of shoes, and $\$2\frac{1}{5}$ for a hat: what had he left?

54. A person bought 12 lb. of butter, at $\$\frac{6}{25}$ per lb., and gave, in payment, a $3 bill: how much change did he receive?

55. What is the cost of 75 eggs, at 15 cents per dozen?

56. By what must $4\frac{1}{3}$ be increased, that the result may be $6\frac{1}{2}$?

57. A merchant offers to sell me a hogshead of sugar, containing 1675 lb., for $100.50: at this rate, what is the value of 7 lb.?

58. If $9\frac{1}{8}$ yards of cloth cost $\$16\frac{3}{4}$, what will be the cost of $15\frac{1}{4}$ yd. at the same rate?

59. If $\frac{7}{8}$ yd. of cloth cost $\$.3\frac{1}{2}$, how much cloth can be purchased for $3?

60. If $\frac{7}{9}$ hogshead of wine cost $147, how many gallons can be bought for $\$62\frac{4}{5}$?

61. How much must be paid for $\frac{15}{16}$ cord of wood, at the rate of $\$6\frac{1}{2}$ per cord?

62. If 25 bbl. of flour, valued at $\$5\frac{3}{4}$ per bbl., be given for 24 cords of hickory, what is the value of 1 cord?

63. If a laborer receives $\$9\frac{1}{4}$ per week, and spends $\$7\frac{3}{5}$, how much will he save in 6.5 weeks?

64. What will be the cost of paving 314.75 yd. of street, at the rate of $2.05 per running yard?

65. A gentleman having a farm of 242 acres, retains 150 acres, and sells the remainder at $96.25 per acre: what did he receive for the acres sold?

66. A gentleman expends $15423 in purchasing a farm, at the rate of $145.50 per acre: how many acres did he buy?

67. What will be the cost of 5645 feet of lumber, at the rate of $3.25 per 100 feet?

68. How much must be paid for $12650 feet of lumber, at the rate of $2.87½ per 100 feet?

69. If a barrel holds 3 bushels of potatoes, how many barrels can be bought for $176.2, at the rate of $.62½ per bushel?

70. If a person is 36 years of age, how many seconds has he lived?

71. How many more minutes has a person lived, who is 59 years old, than one who is 47 years old?

72. Suppose a man to sleep exactly 6 hours every day, how many weeks is passed in sleep, in a common year?

73. A beam, after being squared, is 15 inches by 15 inches, and 56 feet long: what length of plank, 3 inches thick, can be cut from the beam, if nothing is lost in sawing?

74. What will be the cost of 4 pieces of cloth, each containing 47 yd. 3 qr. at $.56¼ per quarter?

75. If 7 lb. of sugar cost 91 cents, what must be paid for 1 cwt. 3 qr. 15 lb. at the same rate?

76. How much sugar can be bought for $10, if 7 lb. cost 84 cents?

77. If 3 cwt. 2 qr. 20 lb. of sugar cost $40.70, what is the cost of one pound?

78. If a single rail, of a railroad, be 9 feet in length, how many rails would be required for a road, whose length is 12 mi. 5 fur. 3 rd. 4 yd.?

79. The circumference of a wheel is 14 ft. 9 in.: how many times will it turn in going a distance of 16 miles?

80. A piece of muslin was found to be 37 yd. 1 qr. 3 na. long; after being wet, it was found to be 36 yd. 3 qr. 1 na. long: how much had it shrunk?

ANSWERS.

PAGE.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
19.	1	1182	2	543	3	1907	4	1305	5	728
19.	6	2312	7	1197	8	2257	9	1384	10	2007
20.	11	12702	12	25128	13	123159	14	244973		
20.	15	71112	16	185259	17	97205	18	122016		
20.	19	855578	20	809345	21	1046788				
20.	22	4805017	23	2430592	24	148388				
20.	25	2815452	26	20503107	21.	27	13714			
21.	28	7439	29	3217111	30	85812115				
21.	31	72469550	32	46122955	33	165934426				
21.	34	677287179630	35	1440020006598786 76						
22.	1	12	2	2 and 10, &c.	3	177	4	1861		
22.	5	3681	6	35570	7	16	8	28		
22.	9	10	10	22	11	20	12	17	13	40
22.	14	5282	23.	15	8818	16	1049	17	6353	
23.	18	320	19	8510	20	990	21	1859		
23.	22	1859	23	4966	24	1178—6603	25	527		
24.	26	6946	27	122201	28	44853	29	794		
24.	30	459	28.	2	8	3	5	4	9	5	14	6	35
28.	7	10	8	41	9	24	29.	10	311	11	34		

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21.	34	677287179630	35	1440020006598786 76						
22.	1	12	2	2 and 10, &c.	3	177	4	1861		
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28.	7	10	8	41	9	24	29.	10	311	11	34		

29. ‖ 12 | 103 ‖ 13 | 1 ‖ 14 | 108 ‖ 15 | 360 ‖ 16 | 120

29. ‖ 17 | 331 ‖ 18 | 104 ‖ 19 | 6102 ‖ 20 | 8210 ‖ 21 | 101

29. ‖ 22 | 7001 ‖ 30. ‖ 1 | 4937 ‖ 2 | 2145 ‖ 3 | 13662

30. ‖ 4 | 29663 ‖ 5 | 56129 ‖ 6 | 23332 ‖ 7 | 6999

30. ‖ 8 | 9999 ‖ 9 | 12962 ‖ 10 | 22285 ‖ 11 | 60204

30. ‖ 12 | 25943 ‖ 13 | 832 ‖ 14 | 94967 ‖ 15 | 369547

30. ‖ 16 | 819905 ‖ 17 | 888 ‖ 18 | 809 ‖ 19 | 6868

30. ‖ 20 | 6945 ‖ 21 | 671483 ‖ 22 | 2925579 ‖ 23 | 9607861

30. ‖ 24 | 474046 ‖ 25 | 450158 ‖ 26 | 126301 ‖ 27 | 89001

30. ‖ 28 | 6606661 ‖ 29 | 89031 ‖ 31. ‖ 1 | 2475 ‖ 2 | 99556

31. ‖ 3 | 595596 ‖ 4 | 9999 ‖ 5 | 2165558 ‖ 6 | 8 ‖ 7 | 35

31. ‖ 8 | 652 ‖ 9 | 746 ‖ 10 | 658 ‖ 11 | 687 ‖ 12 | 2505

31. ‖ 13 | 2782 ‖ 14 | 1786 ‖ 15 | 45 ‖ 16 | 283 ‖ 17 | 1801

31. ‖ 18 | 86 ‖ 32. ‖ 1 | 38 ‖ 2 | 42 ‖ 3 | 186 ‖ 4 | 22

32. ‖ 5 | 3061 ‖ 6 | 72 ‖ 7 | 19553068 ‖ 8 | 643 ‖ 9 | 236

32. ‖ 10 | 146 ‖ 11 | 37 ‖ 33. ‖ 12 | 18 ‖ 13 | 26 ‖ 14 | 100

33. ‖ 15 | 4271 ‖ 16 | 2999813 ‖ 17 | 998994 ‖ 18 | 5999795

33. ‖ 19 | 519 ‖ 20 | 1204 ‖ 21 | 55 ‖ 22 | 4000 ‖ 23 | 8642

36. ‖ 1 | 18 ‖ 2 | 40 ‖ 3 | 42 ‖ 4 | 24 ‖ 5 | 72 ‖ 6 | 20

36. ‖ 7 | 24 ‖ 8 | 63 ‖ 9 | 56 ‖ 10 | 135 ‖ 11 | 28

36. ‖ 12 | 72 ‖ 13 | 60 ‖ 14 | 12 ‖ 15 | 42 ‖ 16 | 45

38. ‖ 5 | 90294 ‖ 6 | 212891 ‖ 7 | 196832 ‖ 8 | 377730

38. ‖ 9 | 732900 ‖ 10 | 7763368 ‖ 11 | 7874181

38. ‖ 12 | 8279145 ‖ 13 | 954020 ‖ 14 | 7901376

38. ‖ 15 | 4866001 ‖ 16 | 7147521 ‖ 17 | 245968

38. ‖ 18 | 891891 ‖ 19 | 5363976 ‖ 20 | 1893472

38. ‖ 21 | 2453835 ‖ 22 | 3786944 ‖ 23 | 7111104

38. ‖ 24 | 8999991 ‖ 25 | 32103576 ‖ 26 | 412848

38. ‖ 27 | 525184 ‖ 28 | 3934975 ‖ 29 | 113580

38. ‖ 30 | 12384 ‖ 31 | 128445 ‖ 32 | 1050 ‖ 33 | 1024

38. ‖ 34 | 26313 ‖ 35 | 157605480 ‖ 40. ‖ 4 | 12972

40. ‖ 5 | 161028 ‖ 6 | 1284304 ‖ 7 | 146148 ‖ 8 | 1564175

40. ‖ 9 | 1523808 ‖ 10 | 32543875 ‖ 11 | 75132634

40. ‖ 12 | 54584008 ‖ 13 | 46550590 ‖ 14 | 117571625

40. ‖ 15 | 15893814 ‖ 16 | 13381368 ‖ 17 | 38109392

40. ‖ 18 | 631747382 ‖ 19 | 2715093675 ‖ 20 | 353744824

40. ‖ 21 | 9685689325 ‖ 22 | 160190606212 ‖ 41. ‖ 23 |

41. ‖ 2745980776 ‖ 24 | 12350043948 ‖ 25 | 283695552

41. ‖ 26 | 8892180 ‖ 27 | 19413388 ‖ 28 | 1975857372

41. ‖ 29 | 549032047252 ‖ 30 | 2264620 ‖ 31 | 27777750

41. ‖ 32 | 1568700 ‖ 33 | 1018081 ‖ 34 | 256036

41. ‖ 35 | 140955000 ‖ 36 | 60401664286588 ‖ 42. ‖ 2 | 9000

42. ‖ 3 | 298655 ‖ 4 | 216180 ‖ 5 | 429030 ‖ 6 | 1106560

42. ‖ 7 | 1502720 ‖ 8 | 709450 ‖ 9 | 2724120

43. ‖ 1 | 27560; 275600 ‖ 2 | 5680000 ‖ 3 | 76904300000

43. ‖ 4 | 40630; 406300 ‖ 6 | 3700600; 37006000

43. ‖ 7 | 5704680 ; 57046800 ; 570468000 ; 5704680000

43. ‖ 1 | 135000 ‖ 2 | 4824000 ‖ 3 | 681800000 ‖ 4 | 2070000

43. ‖ 5 | 42325000 ‖ 6 | 195000000 ‖ 7 | 9000000

43. ‖ 8 | 6250000 ‖ 9 | 21049000 ‖ 10 | 20881000

44. ‖ 1 | 207 ‖ 2 | 3000 ‖ 3 | 3920 ‖ 4 | 19355 ‖ 5 | 31360

44. ‖ 6 | 1037400000 ‖ 7 | 1148 ‖ 8 | 8575 ‖ 9 | 288

44. ‖ 10 | 12600 ‖ 11 | 400 ‖ 12 | 1200 ‖ 13 | 365000

44 ‖ 14 | 138600 ‖ 15 | 209250 ‖ 45. ‖ 1 | 495 ‖ 2 | 58890

45. ‖ 3 | 9625 ‖ 4 | 27456 ‖ 5 | 43 ‖ 6 | 2250 ‖ 7 | 824

45. ‖ 8 | 12960 ‖ 9 | 6125 ‖ 10 | 36750 ‖ 11 | 374

46. ‖ 12 | 855 ‖ 13 | 5760 ‖ 14 | 6 ‖ 15 | 1385 ‖ 16 | 42250

46. ‖ 17 | 4950 ‖ 52. ‖ 4 | $4790\frac{3}{6}$ ‖ 5 | $139251\frac{1}{7}$ ‖ 6 | $933844\frac{4}{9}$

52. ‖ 7 | $64\frac{1}{2}$ ‖ 8 | 2464 ‖ 9 | $9950\frac{4}{8}$ ‖ 10 | $1079361\frac{3}{6}$

52. ‖ 11 | $1639292\frac{3}{5}$ ‖ 12 | 130588 ‖ 13 | $152102\frac{6}{7}$

52. ‖ 14 | $16776833\frac{3}{6}$ ‖ 15 | 8217710 ‖ 16 | $450052771\frac{1}{2}$

52. ‖ 17 | $16410718\frac{8}{9}$ ‖ 18 | $98750113\frac{3}{4}$ ‖ 19 | 4174450

52. ‖ 20 | $27867191\frac{1}{7}$ ‖ 21 | $32078004\frac{4}{5}$ ‖ 22 | 91001100

52. ‖ 23 | $46673337\frac{7}{9}$ ‖ 24 | 2750275 ‖ 25 | $6118327\frac{5}{6}$

52. ‖ 26 | 47000047 ‖ 27 | $70876196\frac{6}{8}$ ‖ 28 | $98149942\frac{2}{7}$

52. ‖ 29 | $156980943\frac{3}{5}$ ‖ 30 | $76174841\frac{1}{6}$ ‖ 31 | $124880884\frac{4}{8}$

52. ‖ 32 | 10219999 ‖ 1 | 3 ‖ 2 | 4 ‖ 3 | 8 ‖ 53. ‖ 4 | 35

53. ‖ 5 | 66 ‖ 6 | 1416 ‖ 7 | 96 ‖ 8 | 121 ‖ 9 | 8 ‖ 10 | 144

53. ‖ 11 | 168 ‖ 12 | 6 ‖ 13 | 142 ‖ 14 | 44 ‖ 15 | 1202

53. ‖ 16 | 2616 ‖ 17 | 952 ‖ 18 | 437 ‖ 19 | 7749

56. ‖ 4 | $58\frac{5}{15}$ ‖ 5 | $594\frac{2}{16}$ ‖ 6 | 44720 ‖ 7 | $63645\frac{3}{15}$

56. ‖ 8 | $25242\frac{6}{19}$ ‖ 9 | $15575\frac{9}{21}$ ‖ 10 | $49789\frac{1}{14}$

56. ‖ 11 | $45513\frac{9}{13}$ ‖ 12 | $84276\frac{5}{11}$ ‖ 13 | $50037\frac{6}{18}$

56. ‖ 14 | $15403\frac{42}{627}$ ‖ 15 | $6571\frac{25}{201}$ ‖ 16 | $13767\frac{83}{156}$

56. ‖ 17 | $6199\frac{374}{544}$ ‖ 18 | $752795\frac{14}{119}$ ‖ 19 | $503310\frac{10}{18}$

56. ‖ 20 | $27665\frac{20}{25}$ ‖ 21 | $45363\frac{22}{31}$ ‖ 22 | $733511\frac{31}{35}$

56. ‖ 23 | $479196\frac{26}{41}$ ‖ 24 | $39377\frac{204}{245}$ ‖ 25 | $128259\frac{92}{296}$

56. ‖ 26 | $1218698\frac{94}{384}$ ‖ 27 | $405096\frac{684}{1352}$ ‖ 28 | $227008\frac{2635}{3045}$

56. ‖ 29 | $364446\frac{7}{240}$ ‖ 30 | $139780\frac{203}{328}$ ‖ 31 | $166524\frac{232}{416}$

56. ‖ 32 | $120081\frac{1347}{5943}$ ‖ 33 | $217761\frac{1182}{3742}$ ‖ 57. ‖ 1 | 266

57. ‖ 2 | 315 ‖ 3 | 402 ‖ 4 | 1935 ‖ 5 | 774 ‖ 6 | 12864

57. ‖ 8 | 318 ‖ 9 | $598\frac{4}{15}$ ‖ 10 | $358\frac{2}{18}$ ‖ 11 | $13007\frac{8}{24}$

57. ‖ 12 | $42986\frac{19}{21}$ ‖ 13 | $8791\frac{12}{33}$ ‖ 14 | $91436\frac{26}{81}$

57. ‖ 15 | $210420\frac{33}{45}$ ‖ 16 | $36670\frac{32}{64}$ ‖ 17 | $13888\frac{64}{72}$

57. ‖ 18 | $263321\frac{8}{36}$ ‖ 19 | $470906\frac{23}{42}$ ‖ 58. ‖ 1 | $674\frac{7}{10}$

58. ‖ 2 | $2704\frac{60}{100}$ ‖ 3 | 927 ‖ 4 | $704\frac{963}{1000}$ ‖ 5 | $239\frac{4704}{10000}$

58. ‖ 6 | $97\frac{469}{1000}$ ‖ 7 | $96\frac{7000}{10000}$ ‖ 8 | $490\frac{400}{1000}$ ‖ 9 | $6972\frac{4}{1000}$

58. ‖ 10 | $4970\frac{906}{1000}$ ‖ 59. ‖ 1 | $28087\frac{14}{20}$ ‖ 2 | $346737811\frac{56}{200}$

59. ‖ 3 | $1162\frac{636}{5000}$ ‖ 4 | $7\frac{20275}{47000}$ ‖ 5 | $111\frac{292}{6400}$ ‖ 6 | $8005\frac{39300}{125000}$

59. ‖ 1 | 150 ‖ 2 | 507 ‖ 3 | 29 ‖ 4 | 615 ‖ 5 | 36 ‖ 6 | 124

59. ‖ 7 | 191 ‖ 8 | 167 ‖ 9 | $13\frac{152}{216}$ ‖ 10 | 4 ‖ 60. ‖ 11 | 31

60. ‖ 12 | 32 ‖ 13 | 22 ‖ 14 | $144\frac{6}{26}$ ‖ 15 | 49 ‖ 16 | 720

60. ‖ 17 | 230—92 ‖ 18 | $754\frac{2}{7}$ ‖ 19 | 127 ‖ 20 | 19

60. ‖ 21 | 67 | 22 | 252 ‖ 23 | 4560 ‖ 24 | $55\frac{5}{25}$

61. ‖ 1 | 24 ‖ 2 | 11000 ‖ 3 | 184 ‖ 4 | he loses 26 dollars

61. ‖ 5 | 12 ‖ 6 | 60 ‖ 7 | 504 ‖ 8 | 11 ‖ 9 | 13 ‖ 10 | $7\frac{100}{600}$

61. ‖ 11 | 53 ‖ 62. ‖ 12 | 40 ‖ 13 | 436 ‖ 14 | 6124

62. ‖ 15 | 40 ‖ 16 | 17 ‖ 17 | 437 ‖ 18 | 73 ‖ 19 | 174

62. ‖ 20 | 20 years ‖ 21 | 314 ‖ 22 | 13570 ‖ 64. ‖ 2 | 72

64. ‖ 3 | $25\frac{1}{3}$ ‖ 4 | $6\frac{2}{3}$ ‖ 5 | $20\frac{4}{7}$ ‖ 6 | $1\frac{7}{9}$ ‖ 7 | 350 ‖ 8 | 45

64. ‖ 9 | 10 ‖ 65. ‖ 10 | 21 ‖ 11 | 160 ‖ 12 | 78 ‖ 13 | 180

65. ‖ 14 | 294 ‖ 15 | 120 ‖ 16 | 864 ‖ 17 | 34 ‖ 18 | 24

65. ‖ 19 | 3 ‖ 20 | 66 ‖ 21 | 150 ‖ 22 | 555 ‖ 66. ‖ 2 | 25

66. ‖ 3 | 30 ‖ 4 | 4 ‖ 5 | 12 ‖ 6 | 3 ‖ 7 | 8 ‖ 8 | 5 ‖ 9 | 960

66. ‖ 10 | none ‖ 11 | 24 ‖ 12 | none ‖ 75. ‖ 1 | $\frac{4}{2}$

75. ‖ 2 | $\frac{10}{2}$ ‖ 3 | $\frac{21}{3}$ ‖ 4 | $\frac{18}{6}$ ‖ 5 | $\frac{48}{8}$ ‖ 6 | $\frac{40}{20}$

75. ‖ 7 | $\frac{7}{3}$ ‖ 8 | $\frac{15}{4}$ ‖ 9 | $\frac{17}{6}$ ‖ 10 | $\frac{31}{9}$ ‖ 1 | $\frac{300}{20}$ ‖ 2 | $\frac{400}{16}$

75. ‖ 3 | $\frac{1410}{30}$ ‖ 4 | $\frac{3000}{40}$ ‖ 5 | $\frac{522}{18}$ ‖ 6 | $\frac{7056}{63}$ ‖ 76. ‖ 1 | $\frac{36}{7}$

76. ‖ 2 | $\frac{20}{3}$ ‖ 3 | $\frac{54}{5}$ ‖ 4 | $\frac{151}{9}$ ‖ 5 | $\frac{151}{8}$ ‖ 6 | $\frac{155}{6}$ ‖ 7 | $\frac{301}{8}$

76. ‖ 8 | $\frac{637}{10}$ ‖ 9 | $\frac{499}{11}$; $\frac{289}{10}$ ‖ 10 | $\frac{2104}{7}$; $\frac{8007}{20}$ ‖ 11 | $\frac{103}{4}$; $\frac{83}{5}$

76. ‖ 12 | $\frac{547}{9}$; $\frac{424}{7}$ ‖ 77. ‖ 1 | 2; 2 ‖ 2 | 2; 2 ‖ 3 | 8; 2

77. ‖ 4 | $4\frac{1}{6}$ ‖ 5 | $5\frac{3}{7}$ ‖ 6 | $6\frac{6}{8}$ ‖ 7 | $4\frac{9}{16}$ ‖ 8 | $6\frac{4}{18}$ ‖ 9 | $28\frac{22}{26}$

77. ‖ 10 | $6\frac{30}{160}$ ‖ 11 | $8\frac{4}{24}$ ‖ 12 | $40\frac{5}{18}$ ‖ 13 | $136\frac{4}{7}$

77. ‖ 14 | $2\frac{400}{430}$ ‖ 15 | $9\frac{60}{84}$ ‖ 16 | $14\frac{228}{262}$ ‖ 17 | $111\frac{675}{800}$

77. ‖ 18 | 22 ‖ 78. ‖ 1 | $\frac{3}{4}$ ‖ 2 | $\frac{4}{5}$ ‖ 3 | $\frac{7}{8}$ ‖ 4 | $\frac{1}{3}$ ‖ 5 | $\frac{4}{7}$

78. ‖ 6 | $\frac{7}{9}$ ‖ 7 | $\frac{16}{95}$ ‖ 8 | $\frac{1}{12}$ ‖ 9 | $\frac{32}{81}$ ‖ 10 | $\frac{1}{11}$ ‖ 11 | $\frac{3}{40}$

78. ‖ 12 | $\frac{4}{225}$ ‖ 13 | $\frac{25}{116}$ ‖ 14 | $\frac{48}{175}$ ‖ 15 | $\frac{419}{637}$ ‖ 16 | $\frac{3}{5}$

78. ‖ 1 | $\frac{1}{4}$ ‖ 2 | $\frac{9}{56}$ ‖ 3 | $\frac{1}{36}$ ‖ 4 | $1\frac{1}{4}$ ‖ 5 | $3\frac{2}{21}$ ‖ 6 | $1\frac{11}{27}$

78. ‖ 7 | $\frac{10}{27}$ ‖ 8 | $\frac{9}{16}$ ‖ 79. ‖ 9 | $\frac{5}{12}$ ‖ 10 | $\frac{16}{45}$ ‖ 11 | $\frac{5}{6}$

79. ‖ 1 | $\frac{72}{126}$, $\frac{105}{126}$, $\frac{42}{126}$ ‖ 2 | $\frac{12}{30}$, $\frac{45}{30}$, $\frac{20}{30}$ ‖ 3 | $\frac{16}{112}$, $\frac{42}{112}$, $\frac{56}{112}$

79. ‖ 4 | $\frac{20}{8}$, $\frac{34}{8}$ ‖ 5 | $\frac{64}{12}$, $\frac{33}{12}$ ‖ 6 | $\frac{5}{15}$, $\frac{72}{15}$ ‖ 7 | $\frac{48}{8}$, $\frac{22}{8}$

79. ‖ 8 | $\frac{12}{7}$, $\frac{35}{7}$ ‖ 9 | $\frac{44}{3}$, $\frac{18}{3}$ ‖ 10 | $\frac{252}{126}$, $\frac{42}{126}$, $\frac{72}{126}$

79. ‖ 11 | $\frac{153}{6}$, $\frac{32}{6}$ ‖ 12 | $\frac{60}{90}$, $\frac{75}{90}$, $\frac{72}{90}$ ‖ 13 | $\frac{180}{315}$, $\frac{210}{315}$, $\frac{63}{315}$

79. ‖ 14 | $\frac{192}{384}$, $\frac{288}{384}$, $\frac{288}{384}$ ‖ 80. ‖ 1 | $3\frac{3}{4}$ ‖ 2 | $6\frac{1}{2}$ ‖ 3 | $4\frac{3}{5}$

80. ‖ 4 | $4\frac{8}{9}$ ‖ 5 | $2\frac{214}{15}$ ‖ 6 | $3\frac{2}{13}$ ‖ 7 | 7 ‖ 8 | $3\frac{5}{11}$ ‖ 9 | $4\frac{3}{16}$

80. ‖ 10 | $2\frac{19}{20}$ ‖ 81. ‖ 1 | $\frac{342}{144} = 2\frac{3}{8}$ ‖ 2 | $1\frac{23}{126}$ ‖ 3 | $1\frac{11}{18}$

81. ‖ 4 | $1\frac{23}{30}$ ‖ 5 | $2\frac{104}{105}$ ‖ 6 | $\frac{3}{4}$ ‖ 7 | 2 ‖ 8 | $3\frac{1}{4}$ ‖ 9 | $1\frac{121}{168}$

81. ‖ 10 | $2\frac{115}{126}$ ‖ 11 | $3\frac{1}{14}$ ‖ 12 | $3\frac{8}{9}$ ‖ 13 | $\frac{4}{5}$ ‖ 14 | $1\frac{23}{63}$

81. ‖ 1 | $10\frac{1}{12}$ ‖ 2 | $12\frac{178}{315}$ ‖ 3 | $12\frac{25}{36}$ ‖ 4 | $10\frac{304}{315}$

81. ‖ 5 | $13\frac{149}{280}$ ‖ 6 | $12\frac{13}{21}$ ‖ 7 | $8\frac{29}{56}$ ‖ 8 | $10\frac{9}{14}$ ‖ 9 | $15\frac{7}{40}$

81. ‖ 10 | $13\frac{23}{126}$ ‖ 1 | $\frac{45}{56}$ ‖ 2 | $1\frac{3}{8}$ ‖ 3 | 3 ‖ 82. ‖ 4 | $\frac{14}{15}$

82. ‖ 5 | $2\frac{29}{30}$ ‖ 6 | 12 ‖ 7 | $1\frac{5}{9}$ ‖ 8 | $1\frac{59}{80}$ ‖ 9 | $21\frac{101}{120}$

82. ‖ 10 | $18\frac{51}{56}$ ‖ 11 | $23\frac{7}{10}$ ‖ 12 | $16\frac{31}{140}$ ‖ 13 | $44\frac{31}{120}$

82. ‖ 14 | $4\frac{11}{15}$ ‖ 15 | $11\frac{93}{40}$ ‖ 83. ‖ 1 | $\frac{8}{8} = 1$ ‖ 2 | $\frac{4}{9}$

83. ‖ 3 | $\frac{4}{30} = \frac{2}{15}$ ‖ 4 | $\frac{18}{50} = \frac{9}{25}$ ‖ 5 | $\frac{1}{27}$ ‖ 6 | $\frac{2}{45}$ ‖ 7 | $\frac{13}{27}$

83. ‖ 8 | $\frac{1}{30}$ ‖ 84. ‖ 1 | $\frac{1}{12}$ ‖ 2 | $\frac{8}{45}$ ‖ 3 | $\frac{28}{99}$ ‖ 4 | $\frac{5}{6}$ ‖ 5 | $\frac{15}{16}$

84. ‖ 6 | $\frac{4}{95}$ ‖ 7 | $\frac{3}{28}$ ‖ 8 | $1\frac{1}{15}$ ‖ 9 | $\frac{17}{300}$ ‖ 10 | $3\frac{959}{1188}$

84. ‖ 11 | $1\frac{2}{5}$ ‖ 12 | $1\frac{16}{21}$ ‖ 13 | $\frac{59}{117}$ ‖ 14 | $\frac{67}{240}$ ‖ 15 | $\frac{7}{20}$

84. ‖ 16 | $3\frac{43}{300}$ ‖ 1 | $\frac{13}{24}$ ‖ 2 | $\frac{11}{40}$ ‖ 3 | $2\frac{283}{300}$ ‖ 4 | $2\frac{11}{40}$

84. ‖ 5 | $\frac{131}{420}$ ‖ 6 | $\frac{23}{30}$ ‖ 7 | $2\frac{1}{9}$ ‖ 8 | $15\frac{7}{8}$ ‖ 9 | $3\frac{9}{70}$ ‖ 10 | $\frac{17}{40}$

85. ‖ 1 | $3\frac{1}{2}$ ‖ 2 | $5\frac{1}{3}$ ‖ 3 | 3 ‖ 4 | $13\frac{13}{24}$ ‖ 5 | 14 ‖ 6 | $3\frac{3}{35}$

85. ‖ 7 | $3\frac{51}{55}$ ‖ 8 | $3\frac{3}{14}$ ‖ 9 | $1\frac{13}{21}$ ‖ 10 | 12 ‖ 86. ‖ 1 | 3

86. ‖ 2 | $7\frac{7}{9}$ ‖ 3 | $12\frac{4}{5}$ ‖ 4 | $14\frac{2}{3}$ ‖ 5 | 75 ‖ 6 | $30\frac{8}{17}$ ‖ 7 | 75

86. ‖ 8 | $110\frac{16}{49}$ ‖ 9 | 27 ‖ 10 | $109\frac{3}{8}$ ‖ 11 | $25\frac{7}{8}$ ‖ 1 | $86\frac{1}{4}$

86. ‖ 2 | 98 ‖ 3 | 396 ‖ 4 | 316 ‖ 5 | 275 ‖ 6 | 637

86. ‖ 7 | 408 ‖ 8 | 408 ‖ 9 | $44\frac{4}{5}$ ‖ 10 | $58\frac{1}{2}$ ‖ 11 | 354

87. ‖ 1 | $\frac{14}{27}$ ‖ 2 | $\frac{10}{21}$ ‖ 3 | $\frac{21}{40}$ ‖ 4 | $1\frac{17}{48}$ ‖ 5 | $10\frac{1}{8}$ ‖ 6 | $\frac{3}{70}$

87. ‖ 7 | $\frac{8}{35}$ ‖ 8 | $\frac{1}{8}$ ‖ 1 | $1\frac{17}{32}$ ‖ 2 | $2\frac{2}{9}$ ‖ 3 | $28\frac{1}{3}$ ‖ 4 | $87\frac{3}{4}$

87. ‖ 5 | $38\frac{1}{3}$ ‖ 6 | 7 ‖ 7 | $5\frac{29}{32}$ ‖ 8 | 9 ‖ 88. ‖ 9 | $\frac{7}{10}$

88. ‖ 10 | $8\frac{71}{98}$ ‖ 11 | $\frac{21}{32}$ ‖ 12 | $\frac{36}{715}$ ‖ 13 | $\frac{1}{2}$ ‖ 14 | $\frac{21}{80}$

88. ‖ 15 | $2\frac{24}{25}$ ‖ 16 | $18\frac{35}{48}$ ‖ 17 | $\frac{91}{96}$ ‖ 18 | $2\frac{1}{7}$ ‖ 19 | $4\frac{3}{8}$

88. ‖ 20 | $1\frac{17}{28}$ ‖ 21 | $19\frac{1}{2}$ ‖ 22 | $13\frac{13}{20}$ ‖ 23 | $79\frac{7}{20}$ ‖ 24 | $15\frac{13}{16}$

89. ‖ 1 | $\frac{4}{17}$ ‖ 2 | $\frac{2}{11}$ ‖ 3 | $\frac{9}{70}$ ‖ 4 | $\frac{2}{15}$ or $\frac{12}{90}$ ‖ 5 | $\frac{7}{80}$

89. ‖ 6 | $\frac{2}{19}$ ‖ 7 | $\frac{3}{16}$ ‖ 8 | $\frac{2}{200} = \frac{1}{100}$ ‖ 9 | $\frac{3}{140}$ ‖ 10 | $\frac{5}{48}$

89. ‖ 11 | $\frac{12}{49}$ ‖ 12 | $\frac{7}{96}$ ‖ 13 | $\frac{4}{27}$ ‖ 14 | $\frac{9}{224}$ ‖ 15 | $\frac{1}{20}$

89. ‖ 16 | $\frac{5}{112}$ ‖ 17 | $\frac{1}{25}$ ‖ 18 | $\frac{3}{7}$ ‖ 19 | ‖ 20 | $\frac{2}{15}$

90. ‖ 22 | $\frac{17}{21}$ ‖ 23 | $1\frac{9}{25}$ ‖ 24 | $1\frac{5}{18}$ ‖ 25 | $\frac{22}{35}$ ‖ 26 | 1

90. ‖ 27 | $\frac{25}{63}$ ‖ 28 | $2\frac{22}{35}$ ‖ 29 | $\frac{31}{160}$ ‖ 30 | $\frac{61}{132}$ ‖ 31 | $2\frac{2}{5}$

90. ‖ 32 | $\frac{32}{45}$ ‖ 33 | $\frac{11}{20}$ ‖ 34 | $10\frac{1}{8}$ ‖ 35 | $7\frac{1}{20}$ ‖ 36 | $6\frac{1}{3}$

90. ‖ 37 | $4\frac{1}{6}$ ‖ 91. ‖ 1 | $6\frac{2}{3}$ ‖ 2 | $13\frac{1}{2}$ ‖ 3 | 72 ‖ 4 | $18\frac{2}{3}$

91. ‖ 5 | $15\frac{3}{4}$ ‖ 6 | $15\frac{5}{9}$ ‖ 7 | $12\frac{3}{8}$ ‖ 8 | 24 ‖ 9 | $26\frac{1}{4}$ ‖ 10 | 36

91. ‖ 11 | 81 ‖ 12 | $23\frac{1}{3}$ ‖ 13 | $51\frac{3}{7}$ ‖ 14 | 160 ‖ 15 | 45

91. ‖ 16 | $28\frac{1}{5}$ ‖ 17 | $18\frac{2}{3}$ ‖ 18 | $13\frac{1}{3}$ ‖ 19 | 60 ‖ 20 | $\frac{20}{29}$

91. ‖ 21 | $\frac{49}{64}$ ‖ 22 | $\frac{15}{22}$ ‖ 23 | $1\frac{11}{34}$ ‖ 24 | $4\frac{5}{19}$ ‖ 25 | $1\frac{3}{4}$

91. ‖ 26 | $1\frac{5}{23}$ ‖ 27 | $3\frac{6}{19}$ ‖ 28 | $6\frac{2}{5}$ ‖ 29 | $11\frac{1}{13}$

92. ‖ 1 | $\frac{25}{27}$ ‖ 2 | $\frac{54}{77}$ ‖ 3 | $\frac{16}{33}$ ‖ 4 | 1 ‖ 5 | $\frac{16}{21}$ ‖ 6 | $\frac{50}{63}$

92. ‖ 7 | $1\frac{1}{35}$ ‖ 8 | $\frac{28}{39}$ ‖ 9 | $\frac{28}{75}$ ‖ 10 | $\frac{5}{98}$ ‖ 11 | $11\frac{1}{9}$

92. ‖ 12 | $7\frac{5}{21}$ ‖ 13 | $\frac{7}{30}$ ‖ 14 | $\frac{2}{49}$ ‖ 15 | $1\frac{1}{2}$ ‖ 16 | $\frac{16}{25}$

92. ‖ 17 | $\frac{20}{33}$ ‖ 18 | $\frac{77}{120}$ ‖ 19 | $2\frac{2}{3}$ ‖ 20 | $1\frac{11}{39}$ ‖ 21 | $7\frac{7}{15}$

92. ‖ 22 | $16\frac{4}{5}$ ‖ 23 | $6\frac{17}{18}$ ‖ 24 | $5\frac{31}{45}$ ‖ 25 | $5\frac{4}{33}$ ‖ 26 | $1\frac{1}{5}$

92. ‖ 27 | $\frac{55}{72}$ ‖ 28 | $\frac{125}{832}$ ‖ 29 | $\frac{8}{13}$ ‖ 93. ‖ 1 | $16\frac{107}{168}$

93. ‖ 2 | $14\frac{17}{132}$ ‖ 3 | $1\frac{5}{12}$ ‖ 4 | $1\frac{13}{28}$ ‖ 5 | $2\frac{7}{16}$ ‖ 6 | $1\frac{98}{99}$

93. ‖ 7 | $75\frac{61}{165}$ ‖ 8 | $5\frac{1}{7}$ ‖ 9 | $2\frac{1}{4}$ ‖ 10 | $21\frac{3}{7}$ ‖ 11 | $\frac{1}{14}$

93. ‖ 12 | $1\frac{2}{9}$ ‖ 13 | $\frac{3}{10}$ ‖ 14 | $2\frac{14}{45}$ ‖ 15 | $42\frac{31}{45}$ ‖ 16 | $\frac{3}{28}$

94. ‖ 17 | $1\frac{19}{35}$ ‖ 18 | 25 ‖ 19 | $\frac{75}{304}$ ‖ 20 | $1\frac{3}{203}$ ‖ 21 | $19\frac{5}{7}$

94. ‖ 22 | $\frac{9}{20}$ ‖ 23 | $45\frac{1}{2}$ ‖ 24 | $\frac{1}{12}$ ‖ 25 | $9\frac{3}{8}$ ‖ 26 | $\frac{11}{56}$

94. ‖ 27 | $\frac{194}{945}$ ‖ 28 | $9\frac{5}{8}$ ‖ 29 | $24\frac{7}{8}$ ‖ 30 | $12\frac{7}{8}$

97. ‖ 1 | .6 ‖ 2 | .41 ‖ 3 | .059 ‖ 4 | .0047 ‖ 5 | .095

97. ‖ 6 | .0080 ‖ 7 | 300.027 ‖ 8 | .000049 ‖ 9 | .0019

97. ‖ 10 | .0060 ‖ 11 | .000041 ‖ 22 | 4.25 ‖ 23 | 21.47

97. ‖ 24 | 60.001 ‖ 25 | 300.049 ‖ 26 | 600.06 ‖ 27 | 29.041

98. ‖ 1 | .3 ‖ 2 | .05 ‖ 3 | .006 ‖ 4 | .14 ‖ 5 | .015

98. ‖ 6 | 14.06 ‖ 7 | 12.5 ‖ 8 | 15.07 ‖ 9 | 20.015

98. ‖ 10 | 19.56 ‖ 11 | 9.105 ‖ 12 | 10.016 ‖ 13 | 9.019

98. ‖ 14 | .0150 ‖ 15 | 4.50 ‖ 16 | 6.540 ‖ 99. ‖ 37 | 59.3
99. ‖ 38 | 45.16 ‖ 39 | 64.004 ‖ 40 | 69.010 ‖ 41 | .00054
99. ‖ 42 | 400.029 ‖ 43 | 5.000007 ‖ 44 | 4006.000049
99. ‖ 45 | 56.0006 ‖ 46 | 1500.0000015 ‖ 47 | 39.640
99. ‖ 48 | 5000.005 ‖ 49 | 36000000.000036 ‖ 50 | 31000-
99. ‖ .000049 ‖ 51 | .00075 ‖ 52 | 51.000051 ‖ 53 | 60000.060
100. ‖ 1 | 13.3397 ‖ 2 | 45.3739 ‖ 3 | 67.64049 ‖ 4 | 347.6997
100. ‖ 5 | 510.5797 ‖ 6 | 53.82085 ‖ 7 | 4.82125 ‖ 8 | 156.012
101. ‖ 9 | 303.1625 ‖ 10 | 520.70125 ‖ 11 | 414.962
101. ‖ 12 | 725.604024 ‖ 13 | 242.046 ‖ 14 | 626.75
101. ‖ 15 | 10.715 ‖ 16 | 17900.8125 ‖ 17 | 4726.94
101. ‖ 18 | 1606.69 ‖ 102. ‖ 1 | 23.900 ‖ 2 | 39.0543
102. ‖ 3 | 148.39841 ‖ 4 | 37.328 ‖ 5 | 2868.1375
102. ‖ 6 | 494.8125 ‖ 7 | 17.9375 ‖ 8 | 217.275 ‖ 9 | 16.065
103. ‖ 10 | 73.58 ‖ 11 | 27.625 ‖ 12 | 47.9375 ‖ 13 | 5.875
103. ‖ 14 | 17.9375 ‖ 15 | 31.625 ‖ 16 | 15.4875
103. ‖ 17 | 2948.46035 ‖ 18 | 1193.4725 ‖ 19 | 3.5111
103. ‖ 20 | 24.550 ‖ 21 | 266.807 ‖ 22 | 24.93796
104. ‖ 1 | .099 ‖ 2 | 5.0681 ‖ 3 | 474.6112 ‖ 4 | 195.763
104. ‖ 5 | 43.82 ‖ 6 | 13.6885 ‖ 7 | 651.75 ‖ 8 | 514.625
104. ‖ 9 | 290.2875 ‖ 10 | 1037.345 ‖ 11 | 520.935
105. ‖ 1 | 2.68172 ‖ 2 | .9893359 ‖ 3 | 30.8204286
105. ‖ 4 | .03587869 ‖ 5 | 100.78 ‖ 6 | 54906.3 ‖ 7 | 47.04

105. ‖ 8 | 6974.3 ‖ 106. ‖ 9 | 1.0016 ‖ 10 | .001852
106. ‖ 11 | 295.993871 ‖ 12 | .03324688 ‖ 13 | .00375
106. ‖ 14 | .04352 ‖ 15 | .006501 ‖ 16 | 450.49 ‖ 17 | 4.5
106. ‖ 18 | 2490.3 ‖ 19 | 5336.44875 ‖ 20 | 3831.263550
106. ‖ 21 | 4.53876 ‖ 22 | 41.4958332 ‖ 23 | 17.514640
106. ‖ 24 | 3708.1803645 ‖ 25 | .03121875 ‖ 26 | .01078125
106. ‖ 27 | .25; .0049 ‖ 28 | 7.20759375 ‖ 29 | 694683.25
106. ‖ 30 | 9386.250000 ‖ 31 | 591.3884 ‖ 32 | 1914.2375
106. ‖ 33 | 5337.3375 ‖ 34 | 1250.000 ‖ 35 | 3109.6875
106. ‖ 36 | 4687.50 ‖ 37 | 3.28125 ‖ 107. ‖ 38 | 1267.70896875
107. ‖ 39 | 105.46875 ‖ 40 | 55034.75 ‖ 41 | 28350.000
108. ‖ 2 | 6.12 ‖ 3 | 6.4 ‖ 4 | 12263 ‖ 5 | 2.7046 ‖ 6 | .61046
108. ‖ 7 | .47043 ‖ 8 | 17.54755+ ‖ 9 | 36.7 ‖ 10 | 62.80866+
108. ‖ 11 | 10932.8125 ‖ 12 | 62607.213 ‖ 13 | 4076.39635
108. ‖ 14 | .096147 ‖ 15 | 10001 ‖ 16 | 19929.6 ‖ 17 | .001
108. ‖ 18 | 12.5 ‖ 19 | 1.55263+ ‖ 20 | 147.490268
108. ‖ 21 | 10000 ‖ 22 | .02567 ‖ 23 | 100 ‖ 24 | 11.56329+
108. ‖ 25 | 30.511904+ ‖ 26 | 7.270833⅓ ‖ 27 | 15
108. ‖ 28 | 48.94769+ ‖ 29 | 38 ‖ 109. ‖ 1 | .11 ‖ 2 | .45
109. ‖ 3 | 5999.994 ‖ 4 | .0025 ‖ 5 | 10 ‖ 6 | 1000
109. ‖ 7 | .099999 ‖ 8 | 8.1325 ‖ 9 | 78.525 ‖ 10 | .14
109. ‖ 11 | 50.96 ‖ 12 | 16.3 ‖ 13 | 14 ‖ 14 | 15696.267
109. ‖ 15 | 11.30625 ‖ 16 | 151.3788 ‖ 17 | 3000.46

109. ‖ 18 | 1000000 ‖ 19 | 93.75 ‖ 110. ‖ 1 | .5 ‖ 2 | .625

110. ‖ 3 | .4; .3125; .6 ‖ 4 | .64 ‖ 5 | .875 ‖ 6 | .34375

110. ‖ 7 | .175; .66666+ ‖ 8 | .34; .2 ‖ 9 | .4; .375; .1125

110. ‖ 10 | .192; 3.875 ‖ 11 | .0164102+; .08 ‖ 12 | .03125;

110. ‖ .177777$\frac{7}{9}$; .085 ‖ 1 | $\frac{47}{100}$ ‖ 2 | $4\frac{69}{100}$ ‖ 3 | $3\frac{1}{250}$

110. ‖ 4 | $64\frac{49}{10000}$ ‖ 5 | $87\frac{49}{100}$ ‖ 112. ‖ 1 | 5.43 ‖ 2 | 27.67

112. ‖ 3 | 40.829 ‖ 4 | 30.432 ‖ 5 | 105.644 ‖ 6 | .378 ‖ 7 | .657

112. ‖ 8 | 1.011 ‖ 9 | .25 ‖ 10 | 3.75 ‖ 11 | .088 ‖ 12 | 12.129

112. ‖ 13 | .009 ‖ 14 | .022 ‖ 15 | 60.05 ‖ 16 | 49.406 ‖ 17 | 200.83

113. ‖ 1 | 1525 ‖ 2 | 47375 ‖ 3 | 960 ‖ 4 | 8750 ‖ 5 | 700

113. ‖ 6 | 1600; 16000; 160000 ‖ 7 | 30.3; 303; 3030

113. ‖ 8 | 1600; 16000 ‖ 9 | 101.09; 1010.9; 10109

113. ‖ 10 | 4007.54; 40075.4; 400754 ‖ 11 | 70.46; 704.6;

113. ‖ 7046 ‖ 12 | 124.19; 1241.9; 12419 ‖ 114. ‖ 1 | 56.72

114. ‖ 2 | 72.356 ‖ 3 | 3.674 ‖ 4 | 127.45 ‖ 5 | 269.45 ‖ 6 | 64.9

114. ‖ 7 | 24.69 ‖ 8 | 47.6 ‖ 9 | .57 ‖ 10 | 1.57; .75; 1.27

114. ‖ 11 | 6 dollars 74 cents 9 mills; 37 dol. 4 cents 9 mills

114. ‖ 12 | 40.40 9; 904.60 7 ‖ 13 | 10.46 0; 270.46 0

114. ‖ 14 | 874.90 4; 47.04 9 ‖ 115. ‖ 1 | 51.411 ‖ 2 | 117.906

115. ‖ 3 | 323.913 ‖ 4 | 278.0545 ‖ 5 | 2780.131 ‖ 6 | 62.99

115. ‖ 7 | 2.598 ‖ 8 | 17.625 ‖ 116. ‖ 9 | 3.49 ‖ 10 | 22.18

116. ‖ 11 | 28.671 ‖ 12 | 23.75 ‖ 13 | 501.775 ‖ 14 | 15.22

116. ‖ 15 | 3.63 ‖ 117. ‖ 1 | 114.866 ‖ 2 | 37.827 ‖ 3 | 131.875

117. ‖ 4 | 5.225 ‖ 5 | .43 ‖ 6 | 3.625 ‖ 7 | 4.81 ‖ 8 | 3.62

118. ‖ 9 | 13.655 ‖ 10 | .67 ‖ 11 | 2.87 ‖ 12 | 5.52

118. ‖ 13 | 6.77 ‖ 14 | 9.12 ‖ 1 | 2.51 ‖ 2 | 99.23 ‖ 3 | 206.26

119. ‖ 4 | 15.21 ‖ 5 | 17.50 lost ‖ 6 | 23.14 ‖ 1 | 3144.125

119. ‖ 2 | 5372.294 ‖ 120. ‖ 3 | 140522.071 ‖ 4 | 10.773

120. ‖ 5 | 3577.3285 ‖ 6 | 75 ‖ 7 | 37.375 ‖ 8 | 49.50

120. ‖ 9 | 70.3125 ‖ 10 | 60.125 ‖ 11 | 1261.00 ‖ 12 | 348

120. ‖ 13 | 1810599.85 ‖ 14 | 10368.75 ‖ 15 | 34787.5

120. ‖ 16 | 93.69 ‖ 17 | 38.00 ‖ 18 | 1124.90 ‖ 19 | 125.625

121. ‖ 1 | 96 ‖ 2 | $1 ‖ 3 | 1679.125 ‖ 4 | 1679.125

121. ‖ 5 | 72071.50 ‖ 6 | 13.54 ‖ 7 | 65.84 ‖ 8 | 6.005

122. ‖ 9 | 8.93 ‖ 10 | 26.86 ‖ 123. ‖ 11 | 225.625 ‖ 12 | 23.03

123. ‖ 13 | 28.745 ‖ 124. ‖ 1 | $16.863\frac{1}{28}$ ‖ 2 | $31.726\frac{15}{37}$

124. ‖ 3 | $12.60\frac{29}{37}$ ‖ 4 | $535\frac{57}{516}$ ‖ 5 | $147\frac{2360}{4175}$ ‖ 6 | 13.01285+

124. ‖ 7 | 22.4 ‖ 8 | .27 ‖ 9 | $1.91\frac{2}{3}$ ‖ 125. ‖ 10 | $129\frac{23}{35}$

125. ‖ 11 | .4074 ‖ 12 | 75 ‖ 13 | 376 ‖ 14 | $2504\frac{1}{6}$

125. ‖ 15 | 8550 wid.'s; 3420 child's ‖ 1 | $105\frac{5}{19}$ ‖ 2 | $60\frac{5}{46}$

125. ‖ 3 | 821 ‖ 4 | 226.60 ‖ 126. ‖ 5 | 27.50 ‖ 6 | $4\frac{26}{125}$

126. ‖ 7 | 241.92 ‖ 8 | 95 ‖ 9 | 594.2965 ‖ 10 | 120.48

126. ‖ 11 | 51711.55 ‖ 12 | $4\frac{167}{287}$ ‖ 13 | 9 ‖ 14 | 7000 entire

126. ‖ gain; 43.75 gain per A ‖ 15 | 1200 ‖ 142. ‖ 3 | 77

142. ‖ 4 | $757\frac{1}{2}$ ‖ 5 | 11942 ‖ 6 | 662 ‖ 7 | 8800

143. ‖ 8 | 19140 ‖ 9 | 2196 ‖ 10 | 294 ‖ 11 | 4262 ‖ 12 | 59

143. ‖ 13 | 668 ‖ 14 | 217728 ‖ 15 | 6525 ‖ 16 | 36340
143. ‖ 17 | 31069½ ‖ 18 | 4554 ‖ 19 | 11847 ‖ 20 | 8576
143. ‖ 21 | 27360 ‖ 22 | 1100 ‖ 23 | 6336 ‖ 24 | 5904
143. ‖ 25 | 99000 ‖ 26 | 1588752 ‖ 27 | 2160 ‖ 28 | 132480
143. ‖ 29 | 1882 ‖ 30 | 12579 ‖ 31 | 50765 ‖ 32 | 164367
143. ‖ 33 | 986765 ‖ 34 | 2674445 ‖ 35 | 279814/4 ‖ 36 | 2633760
145. ‖ 1 | £35 3s. 9d. ‖ 2 | £51 16s. 3d. 2far. ‖ 3 | 439 rd.
145. ‖ 3½ yd. 1 in. ‖ 4 | 891 mi. 0 fur. 7 rd. 4 yd. 1 ft. 6 in.
145. ‖ 5 | 2428 ‖ 6 | 17 P. 17¾ sq. yd. 7 sq. ft. 12 sq. in.
145. ‖ 7 | 19 A. 3 R. 37 P. 11¾ sq. yd. 7 sq. ft. ‖ 8 | 5 sq. mi.
145. ‖ 481 A. 7 P. ‖ 9 | 13 sq. ch. 2 P. 25½ sq. yd. 7 sq. ft.
145. ‖ 10 | 14 cu. yd. 14 cu. ft. 1193 cu. in. ‖ 11 | 10937
145. ‖ 12 | 52 T. 16 cu. ft. 1176 cu. in. ‖ 13 | 312 bbl.
145. ‖ 16 gal. 2 qt. 1 pt. ‖ 14 | 1735 pi. 1 hhd. 53 gal.
145. ‖ 15 | 6819 tuns 1 hhd. 61 gal. 1 qt. ‖ 16 | 6047 bu.
145. ‖ 2 pk. 4 qt. 1 pt. ‖ 17 | 27706 bu. 1 pk. 4 qt.
145. ‖ 18 | 2011 ch. 15 bu. ‖ 19 | 4 qr. 7 lb. 1 oz. 8 dr.
146. ‖ 20 | 29 cwt. 2 qr. 13 lb. 1 oz. ‖ 21 | 27 T. 7 cwt. 1 qr. 4 oz.
146. ‖ 22 | 2352483 T. 11 cwt. 9 lb. ‖ 23 | 85 oz. 10 pwt. 9 gr.
146. ‖ 24 | 392 lb. 19 pwt. ‖ 25 | 105 ℔ 11 ♃ 6 ♉ 1 ☉ 16 gr.
146. ‖ 26 | 156 ℔ 4 ♃ 7 ♉ 1 ☉ ‖ 27 | 875 ℔ 6 ♃ 1 ♉
146. ‖ 28 | 102 ℔ 3 ♃ 7 ♉ 1 ☉ 5 gr. ‖ 29 | 181 hr. 50 m. 4 sec.
146. ‖ 30 | 603 da. 19 hr. 44 m. ‖ 31 | 1 mo. 14 da. 19 hr.
146. ‖ 7 m. 49 sec. ‖ 32 | 12 yr. 167 da. 22 hr. 47 m
146. ‖ 33 | 990 yr. 365 da. 6 hr. ‖ 34 | 1 yr. 311 da. 19 hr. 25 m.
146. ‖ 35 | 1 mo. 29 da. 10 hr. 49 m. ‖ 36 | 241° 48′ 13″

146. ‖ 37 | 849 hr. an. 13° 24′ ‖ 38 | 150 s. 8° 13′
146. ‖ 39 | 19 c. 311° 12′ ‖ 40 | 27° 43′ 24″ ‖ 41 | 62 c. 39° 11′ 3″
146. ‖ 1 | 63329 ‖ 2 | 45327600 ‖ 3 | 1238449 ‖ 4 | 24656
146. ‖ 5 | 1520640 ‖ 147. ‖ 6 | $282.75 ‖ 7 | 234 ‖ 8 | 1809420
147. ‖ 9 | 6576 ‖ 10 | 34 wk. 2 da. 20 hr. ‖ 11 | 24719040
147. ‖ 12 | 2980 A. 3 R. 29 P. ‖ 13 | 3 C. 6 cu. ft. 1049 cu. in.
147. ‖ 14 | 2218914 ‖ 15 | 6552 ‖ 16 | 65538 ‖ 17 | 608
147. ‖ 18 | 35848 ‖ 19 | 256 ‖ 20 | $1329.74 ‖ 21 | 128960
147. ‖ 22 | 15229 ‖ 23 | $1311.50 ‖ 24 | $630 ‖ 148. ‖ 25 | 136000
148. ‖ 26 | 90720—544320 ‖ 27 | 47680 ‖ 28 | 4938
148. ‖ 29 | 110 T. 16 cwt. 2 qr. 17 lb. 9 oz. ‖ 30 | $282.06
148. ‖ 31 | 514 T. 13 cwt. 3 qr. ‖ 32 | 619 re. 15 qu. 20 sh.
148. ‖ 33 | 422 yd.—$949.50 cost ‖ 34 | 102¼ ‖ 35 | 15823 yd.
148. ‖ 2 qr. ‖ 36 | 25704 ‖ 37 | 1355260½ ‖ 38 | 3991680
148. ‖ 39 | 27⅐ ‖ 40 | 15 ‖ 41 | 16 ‖ 42 | 11697 9/39
149. ‖ 1 | £10 1s. 4d. ‖ 2 | 26 yd. 2 ft. 6 in. ‖ 3 | £18 1s.
149. ‖ 9d. 3far. ‖ 4 | 42 bu. 2 pk. 2 qt. ‖ 5 | 45 lb. 1 oz. 8 dr.
149. ‖ 6 | 39° 37′ 47″ ‖ 150. ‖ 7 | £15 2s. 7d. ‖ 8 | £184
150. ‖ 19s. 6½d. ‖ 9 | 208 mi. 7 fur. 27 rd. ‖ 10 | 240 T.
150. ‖ 5 cwt. 3 qr. 13 lb. ‖ 11 | 170 cwt. 2 qr. 13 lb. 4 oz.
150. ‖ 12 | 195 lb. 8 oz. 17 pwt. 1 gr. ‖ 13 | 168 mi. 5 fur.
150. ‖ 12 rd. 4 yd. ‖ 14 | 2 A. 2 R. 39 P. 15¾ sq. yd. 6 sq. ft.
150. ‖ 68 sq. in. ‖ 15 | 3 da. 15 hr. 19 m. ‖ 16 | 3 T. 8 cwt. 3 qr.
150. ‖ 21 lb. ‖ 17 | 136 bu. 2 pk. 3 qt. ‖ 18 | 180 yd. 3 na.
151. ‖ 19 | 2 yr. 231 da. 3 hr. 21 m. 5 sec. ‖ 20 | 188 A. 37 P.
151. ‖ 21 | 1 lb. 9 oz. 3 pwt. 10 gr. ‖ 22 | 1301 mi. 5 fur. 18 rd.

151. ‖ 23 | 2 tuns 3 hhd. 58 gal. 1 qt. 1 pt. ‖ 24 | $272.76

151. ‖ 25 | 194 bu. 2 pk. 5 qt. ‖ 26 | £522 1s. 3d.

152. ‖ 1 | £37 16s. 8d. ‖ 2 | £122 18s. 4d. 2far. ‖ 3 | 9s. 7¾d.

152. ‖ 4 | 64 T. 17 cwt. 2 qr. 17 lb. ‖ 5 | 1 cwt. 12 lb. 12 oz.

152. ‖ 6 | 181 yd. 2 qr. 2 na. ‖ 7 | 81 mi. 2 fur. 31 rd. 2½ yd.

152. ‖ 2 ft., or 81 mi. 2 fur. 31 rd. 3 yd. 6 in. ‖ 8 | 46 tuns

152. ‖ 1 pi. 1 hhd. 44 gal. 3 qt. 1 gi. ‖ 153. ‖ 9 | £1 14s. 7d.

153. ‖ 10 | 62 lb. 3 oz. 15 pwt. 19 gr. ‖ 11 | 54 A. 1 R.

153. ‖ 29 P. 25¼ sq. yd. 7 sq. ft. ‖ 12 | 17 yd. 3 qr. 2 na.

153. ‖ 13 | 38 gal. 0 qt. 1 pt. ‖ 14 | 17 yd. 2 ft. 9 in.

153. ‖ 15 | 6 cwt. 2 qr. 20 lb. ‖ 16 | 26 ch. 22 bu. 3 pk.

153. ‖ 17 | 3 ℔ 6 ℥ 3 ʒ 2 ℈ 9 gr. ‖ 18 | 21 lb. 2 oz. 16 pwt. 6 gr.

153. ‖ 19 | 1 T. 17 cwt. 2 qr. 9 lb. ‖ 20 | £7 9s. 11d.

153. ‖ 21 | 1 hhd. 18 gal. 2 qt. 1 pt. ‖ 22 | 3 yd. 1 qr. 2 na.

154. ‖ 1 | £12 19s. ‖ 2 | 34 bu. 2 pk. 6 qt. ‖ 3 | 20 cwt.

154. ‖ 2 qr. 15 lb. ‖ 4 | 79 yd. 4 in. ‖ 5 | 160 mi. 4 fur. 30 rd.

154. ‖ 6 | 95 wk. 5 da. 18 hr. ‖ 7 | 26 A. 0 R. 28 P.

154. ‖ 8 | 1 hhd. 29 gal. 2 qt. 1 pt. ‖ 155. ‖ 9 | 195 bu. 7 qt. 1 pt.

155. ‖ 10 | 51 ℔ 6 ℥ 7 ʒ 2 ℈ 8 gr. ‖ 11 | 39 wk. 3 da. 21 hr. 50 m.

155. ‖ 12 | £11 8s. 4d. ‖ 13 | 13 hhd. 30 gal. 1 qt. 1 pt.

155. ‖ 14 | 30 A. 2 R. 25 P. ‖ 16 | 4533 yd. ‖ 17 | 146 da. 11 hr.

155. ‖ 18 | £48 2s. 6d. ‖ 19 | 457 da. 15 hr. ‖ 20 | 442 gal. 3 qt.

155. ‖ 21 | 132 re. 18 qu. 8 sh. ‖ 22 | 1225 T. 13 cwt. 2 qr. 1 lb.

156. ‖ 23 | 607 A. 1 R. 15 P. ‖ 24 | 6 da. 0 hr. 12 m.

156. ‖ 25 | 19 mi. 2 fur. 14 rd. ‖ 26 | 39 bu. ‖ 27 | 10 gal. 2 qt.

156. ‖ 28 | 19 A. 0 R. 29 P. ‖ 29 | 862 gal. 3 qt.

156. ‖ 30 | 452 L. 1 mi. 7 fur. 7 rd. ‖ 31 | 152 hr. 20 m. 15 sec.

156. ‖ 32 | 999 yd. ‖ 33 | 1 lb. 6 oz. 18 pwt. ‖ 34 | 15°

156. ‖ 35 | 11 lb. 11 oz. ‖ 36 | 137 bu. 3 pk. 2 qt.

156. ‖ 37 | 156 cwt. 2 qr. 19 lb. ‖ 38 | 341 A. 1 R. 6 P.

156. ‖ 39 | 71 T. 10 cwt. 1 qr. ‖ 157. ‖ 1 | £3 4s. 9d.

157. ‖ 2 | 6 bu. 3 pk. 6 qt. ‖ 3 | 2 cwt. 3 qr. 20 lb.

157. ‖ 4 | 9 mi. 3 fur. 20⅘ rd. ‖ 5 | 9 s. 9° 27′ 35″

158. ‖ 6 | 6 yr. 5 mo. 15 da. 18 hr. ‖ 7 | 6 T. 12 cwt. 2 qr.

158. ‖ 16 lb. 4 oz. 8 dr. ‖ 8 | 1 C. 2 C. ft. ‖ 9 | 5 cwt. 2 qr.

158. ‖ 16 lb. ‖ 10 | 3 ℔ 4 ℥ 6 ʒ 1 ℈ 16 gr. ‖ 11 | 24 mi. 4 fur.

158. ‖ 4$\frac{15}{20}$ rd. ‖ 12 | 12 A. 1 R. 25 P. ‖ 13 | 29 yd. 2 qr. 3 na.

158. ‖ 14 | 5 L. 2 mi. 6 fur. 36 rd. ‖ 15 | 2 fur. 2 rd.

158. ‖ 16 | 1 C. 21$\frac{76}{100}$ cu. ft. ‖ 17 | 9 hr. 36 m.

158. ‖ 18 | 10 mi. 2 fur. 20 rd. ‖ 19 | 9 ‖ 20 | 114

159. ‖ 1 | $50 ‖ 2 | $37.50 ‖ 3 | $9.375 ‖ 4 | $25 ‖ 5 | $50

160. ‖ 1 | 318.75 ‖ 2 | 21.60 ‖ 3 | 14188.50 ‖ 4 | 10186.00

160. ‖ 5 | 1215.00 ‖ 161. ‖ 1 | 273.52 ‖ 2 | 30.3075

161. ‖ 3 | 167.34996 ‖ 4 | 399.322 ‖ 5 | 184.80 ‖ 6 | 1081.92

161. ‖ 1 | 121.220625 ‖ 2 | 305.9525 ‖ 3 | 290.814875

161. ‖ 4 | 1060.6599 ‖ 162. ‖ 1 | 75.385 ‖ 2 | 4.947$\frac{174}{758}$

162. ‖ 3 | 66.6666⅔ ‖ 4 | 6.25 ‖ 1 | 350 ‖ 2 | 96 ‖ 3 | 32

163. ‖ 1 | 4 ‖ 2 | 3 ‖ 3 | 3 ‖ 4 | 3 ‖ 5 | 4 ‖ 6 | 15 ‖ 7 | 10

163. ‖ 8 | 2 ‖ 9 | 2 ‖ 10 | 8 ‖ 11 | 2 ‖ 164. ‖ 1 | 5⅓ ‖ 2 | 18

164. ‖ 3 | 8 ‖ 165. ‖ 4 | 4 ‖ 5 | 72 ‖ 6 | 20 ‖ 166. ‖ 2 | 60

166. ‖ 3 | 160 ‖ 4 | 450 ‖ 5 | 480 ‖ 6 | 960 ‖ 7 | 54 ‖ 8 | 128

167. ‖ 9 | 6.00 ‖ 10 | 2.00 ‖ 11 | 567 ‖ 12 | 120 ‖ 13 | 54

167. | 14 | 407 | 15 | 451.20 | 16 | 201.75 | 17 | 162.50

167. | 18 | 24 | 19 | 36 | 20 | $442\frac{2}{19}$ | 21 | 650 | 22 | 19.79

167. | 23 | 195 | 168. | 24 | $3\frac{2}{4}$ | 25 | 480 | 26 | 54

168. | 27 | 72000 | 28 | 216 | 29 | 1044 | 30 | 116

168. | 31 | 2000 | 32 | 63 | 33 | 360 | 34 | 1066

168. | 35 | 10990 | 36 | 120 | 169. | 1 | 1048 | 2 | 248

169. | 3 | 6587.875 | 4 | 909 | 5 | 44 | 6 | 166.50 | 7 | 6

169. | 8 | 18 wk. | 9 | 92 | 10 | 6.10 | 11 | 162.50

170. | 12 | 15 | 13 | 197 | 14 | 100.75; 64.75 | 15 | 171950

170. | 16 | 284262 | 17 | 12288 | 18 | $8.79\frac{1}{2}$ | 19 | 64125

170. | 20 | $98\frac{13}{19}$ | 21 | 10.80 | 171. | 22 | $77.02\frac{1}{2}$ | 23 | $91\frac{111}{196}$

171. | 24 | 26 | 25 | 8 ft. 4 in. | 26 | 89 | 27 | 30 | 28 | 12

171. | 29 | 12 | 30 | 8 | 31 | 8 | 32 | 25 | 33 | 12.00 | 34 | $\frac{75}{8}$

172. | 35 | $\frac{261}{16}$ | 36 | 80 | 37 | 31.25 | 38 | $372\frac{15}{16}$ | 39 | $\frac{72}{625}$

172. | 40 | $\frac{5}{12}$ | 41 | $\frac{1}{12}$ | 42 | $\frac{7}{40}$ | 43 | 72000 | 44 | 34149

172. | 45 | 21710.9375 | 46 | $\frac{60}{90}$, $\frac{72}{90}$, $\frac{75}{90}$ | 47 | $\frac{77}{99}$, $\frac{72}{99}$

172. | 48 | $\frac{13}{60}$ | 49 | $1\frac{3}{4}$ | 50 | $3\frac{7}{40}$ | 51 | $\frac{11}{18}$ | 173. | 52 | $4\frac{111}{40}$

173. | 53 | $2\frac{9}{20}$ | 54 | .12 | 55 | $.93\frac{3}{4}$ | 56 | $2\frac{21}{6}$ | 57 | .42

173. | 58 | $27\frac{145}{146}$ | 59 | $\frac{3}{4}$ | 60 | $20\frac{14}{15}$ | 61 | $6\frac{3}{32}$ | 62 | $5.98\frac{23}{24}$

173. | 63 | 10.725 | 64 | 645.2375 | 65 | 8855.00 | 66 | 106

174. | 67 | 18346.25 | 68 | 363.6875 | 69 | 94 | 70 | 1136-

174. | 073600 | 71 | 6311520 | 72 | 13 wk. 6 hr. | 73 | 280 ft.

174. | 74 | 429.75 | 75 | 24.70 | 76 | $83\frac{1}{8}$ lb. | 77 | .11

174. | 78 | $7413\frac{7}{18}$ | 79 | $5727\frac{27}{59}$ | 80 | 2 qr. 2 na.

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DAVIES' NEW INTELLECTUAL ARITHMETIC.
DAVIES' ELEMENTS OF WRITTEN ARITHMETIC.
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