

DECIMAL FRACTIONS.		PAGE
Notation and Numeration		95
Addition		100
Subtraction		102
Multiplication		105
Division		107

UNITED STATES CURRENCY.		PAGE
Coins, &c.		111
Reduction		113
Addition		115
Subtraction		117
Multiplication		119
Division		124

DENOMINATE NUMBERS.		PAGE
Tables		127-143
Reduction		143
Addition		149
Subtraction		152
Multiplication		154
Division		157

ANALYSIS.		PAGE
General Principles		159-162

RATIO AND PROPORTION.		PAGE
General Principles		163
Rule of Three		165
Promiscuous Questions		169

ELEMENTS OF ARITHMETIC.

Definitions.

1. A **UNIT** is a single thing, or one.
2. A **NUMBER** is a unit, or a collection of units.
3. **ARITHMETIC** is the science of numbers and the art of computation.
4. An **OPERATION** is something done with numbers.
5. An **ANSWER** is the result of a correct operation.
6. A **RULE** is the direction for performing an operation.

Operations of Arithmetic.

7. There are five fundamental operations of Arithmetic: Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

NOTATION AND NUMERATION.

8. **NOTATION** is the method of expressing numbers, either by letters or figures.

NUMERATION is the art of reading, correctly, any number expressed by letters or figures.

There are two methods of Notation: one by letters, and one by figures. The method by letters is called, the *Roman Notation*; the method by figures is called, the *Arabic Notation*.

Roman Notation.

9. The Roman notation employs seven capital letters. They express the following values :

I	V	X	L	C	D	M
One,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

All other numbers are expressed by combining these letters:

1. Every time a letter is repeated, the number which it denotes is repeated.
2. If a letter denoting a less number be written on the right of one denoting a greater, the number expressed will be denoted by the sum of the numbers.
3. If a letter denoting a less number be written on the left of one denoting a greater, the number expressed will be the difference of the numbers.
4. A dash (—), placed over a letter, increases the number for which it stands, a thousand times.

Roman Table.

I	. . .	One.	LXXX	. . .	Eighty.
II	. . .	Two.	XC	. . .	Ninety.
III	. . .	Three.	C	. . .	One hundred.
IV	. . .	Four.	CC	. . .	Two hundred.
V	. . .	Five.	CCC	. . .	Three hundred.
VI	. . .	Six.	CCCC	. . .	Four hundred.
VII	. . .	Seven.	D	. . .	Five hundred.
VIII	. . .	Eight.	DC	. . .	Six hundred.
IX	. . .	Nine.	DCC	. . .	Seven hundred.
X	. . .	Ten.	DCCC	. . .	Eight hundred.
XX	. . .	Twenty.	DCCCC	. . .	Nine hundred.
XXX	. . .	Thirty.	M	. . .	One thousand.
XL	. . .	Forty.	MD	. . .	Fifteen hundred.
L	. . .	Fifty.	MM	. . .	Two thousand.
LX	. . .	Sixty.	V̄	. . .	Five thousand.
LXX	. . .	Seventy.	X̄	. . .	Ten thousand.

Arabic Notation.

10. ARABIC NOTATION is the method of expressing numbers by figures. Ten figures are used. They are,

0	1	2	3	4	5	6	7	8	9
Naught,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

All numbers are expressed by these figures, employed singly, or in combination. When in combination, the value of each is determined by the following principles :

1. That the same figure may express different values ;
2. That it expresses its least value, when placed in the first place on the right ;
3. That it expresses, when placed in the 2d place from the right, a value ten times as great as when in the 1st place ; when put in the 3d place, a value 10 times as great as in the 2d place, and so on for every place to the left.
4. The names of the places are the following :

Billions.	Hundreds of Millions,	Tens of Millions,	Millions,	Hundreds of Thousands,	Tens of Thousands,	Thousands,	Hundreds,	Tens,	Units,
1	1	1	1	1	1	1	1	1	1

5. The value of 1 in each place is shown by the Table :

10 Units	. . .	make	1 Ten.
10 Tens	. . .	"	1 Hundred.
10 Hundreds	. . .	"	1 Thousand.
10 Thousands	. . .	"	1 Hundred thousand.
10 Hundred thousands	. . .	"	1 Million.
10 Millions	. . .	"	1 Ten million.
	&c.,		&c.

Express the following numbers by figures :

1. Forty-six. Also, ninety-nine. Also, five hundred.
2. Write four thousand. Also, six thousand and eleven.
3. Write nine thousand, eight hundred and seventy-six.

11. We have seen that, when a number is expressed by a single figure, as 8, it is read by its name, eight.

When a number is expressed by two figures, as 26, it is read from the right:

Tens.	Units.
2	6

that is, 6 units and 2 tens; and from the left, twenty-six.

When a number is expressed by three figures, as 375, it is read from the right:

Hundreds.	Tens.	Units.
3	7	5

that is, 5 units, 7 tens, and 3 hundreds; and from the left, three hundred and seventy-five.

When a number is expressed by four figures, as 6037, it is read:

Thousands.	Hundreds.	Tens.	Units.
6	0	3	7

that is, 7 units, 3 tens, no hundreds, and 6 thousands, or six thousand and thirty-seven.

1. Write all the numbers from three hundred and fifty-five, to three hundred and fifty-six, and read them.
2. Write all the numbers from six hundred and seventy, to six hundred and seventy-five, and read them.
3. Write all the numbers from one thousand and five, to one thousand and ten, and read them.
4. Write all the numbers from six thousand and eleven, to six thousand and twenty, and read them.
5. Write all the numbers from seven thousand and forty, to seven thousand and fifty, and read them.

Numeration Table.

6th Period. Quadrillions.	5th Period. Trillions.	4th Period. Billions.	3d Period. Millions.	2d Period. Thousands.	1st Period. Units.
Hundreds of Quadrillions	Hundreds of Trillions	Hundreds of Billions	Hundreds of Millions	Hundreds of Thousands	Hundreds
Tens of Quadrillions	Tens of Trillions	Tens of Billions	Tens of Millions	Tens of Thousands	Tens
Quadrillions	Trillions	Billions	Millions	Thousands	Units
9 2 0,	3 2 3,	8 4 2,	7 6 8,	3 1 9,	6 7 5.
5 7,	2 8 9,	6 7 8,	5 4 1,	2 9 7,	3 1 3.
6,	4 0 7,	2 1 2,	9 3 6,	8 7 6,	5 4 1.
.	9 1 2,	7 6 1,	2 5 7,	3 2 7,	8 2 6.
.	8 4,	9 1 2,	8 7 6,	4 1 9,	2 8 5.
.	.	6 4 1,	0 0 0,	9 0 7,	4 5 6.
.	.	8 9 4,	6 0 2,	0 4 3,	2 8 8.
.	.	7 2,	5 4 9,	1 3 6,	8 2 2.
.	.	6,	2 4 5,	2 8 9,	4 2 1.
.	.	.	5 4 8,	0 0 0,	0 8 7.
.	.	.	.	2 1 0,	4 6 0.
.	.	.	.	0 0 1,	4 4 3.
.	.	.	.	4 2 4,	0 8 7.
.	.	.	.	8 7,	4 2 1.
.	.	.	.	6,	0 2 3.
.	3 0 7.
.	4 9.

1. The first line, directly under units, is read, five; the second, forty-nine; the third, three hundred and seven; the fourth, six thousand and twenty-three, and so on.
2. Numbers expressed by more than three figures, are separated, by a comma, into periods of three figures each, beginning at the right.
3. Each period contains three figures, except the one at the left, which may contain one, two, or three figures.
4. The pupil should be required to commit, thoroughly, the names of the periods, and to read the figures fluently.

Exercises in Notation and Numeration.

1. Write 1 in each place of the first period, and read the number.
2. Write 3 in the 3d place of the first period, and 0's in the other places, and read the number.
3. Write 8 units, 5 tens, and 6 hundreds, and read the number.
4. Write 9 hundreds, 6 tens, and 7 units, and read.
5. Write 4 in each place of the first two periods, and read.
6. Write 3 in each place of the second period, and a 0 in each place of the first, and read.
7. Write 7 thousand, 6 hundred and fifty-five.
8. Write 5 in each place of the 3d period, 2 in each place of the 2d, and 1 in each place of the 1st, and read.
9. Write six hundred and four millions, ninety-five thousand, three hundred and forty-two.
10. Write 421 billions, a 0 in each place of the 3d period, and four thousand and sixteen, and read.
11. Write 0 in each place of the first three periods, and 5 in the first place of the 4th, and read the number.

Orders of Units.

1 is called	a unit of the	1st order.
10, or 1 ten, is called	" "	2d order.
100, or 1 hundred, is	" "	3d order.
1000, or 1 thousand, is	" "	4th order.

And so on for the higher numbers: hence,

Units of the first order are written in the first place, at the right;

Units of the second order, in the second place;

Units of the third order, in the third place;

Units of the fourth order, in the fourth place; and so on for places to the left.

Examples in Writing and Reading.

1. Write 6 units of the first order.
2. Write four units of the first order with five of the second.
3. Write nine units of the 3d order with 3 of the second and 1 of the first, and read.
4. Write 8 units of the 3d order with none of the second and six of the first.
5. Write 7 units of the 5th order with 2 of the second.
6. Write 5 units of the 6th order with 4 of the 4th, 3 of the 3d, and 1 of the 1st, and read.
7. Write 4 units of the 6th order with 5 units of the 1st order, and read.
8. Write 3 units of each order to the 6th, and read the number.
9. Write 9 units of each order to the 9th, and read the number.

Analysis of Numbers.

Let the pupil point off and read the following numbers; then write them in words.

1. 85	7. 50482	13. 275047078
2. 137	8. 602142	14. 4127043047
3. 6704	9. 8969797	15. 9730417071
4. 3678	10. 71462108	16. 10470621048
5. 30421	11. 104000009	17. 27049632101
6. 200410	12. 570478010	18. 31047021412

NOTE.—Let each of the above examples, after having been written on the blackboard, be analyzed as a class exercise; thus,

Ex. 1. How many tens in 85? How many units over?

2. In 137, how many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place? How many tens in the number? How many units?

3. In 6704, how many thousands in the thousands place? How many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place?