CASE I.

32. When the multiplier is any composite number.

Rule.

I. *Separate the composite number into its factors:*

II. *Multiply the multiplicand by one factor, and the product by a second factor; and so on, till all the factors have been used; the last product will be the product required.*

Examples.

1. Multiply 321 by $16 = 4 \times 4 = 8 \times 2$.

321		321		321
4		8		16
1284	or	2568		1926
4		2		321
5136		5136		5136

2. Multiply 375 by 24.
3. Multiply 6095 by 49.
4. Multiply 7206 by 30.
5. Multiply 6810 by 63.
6. Multiply 19760 by 56.
7. Multiply 37568 by 40.
8. Multiply 20270 by 35.
9. Multiply 75670 by 36.

CASE II.

33. When the multiplier is 1, with any number of ciphers annexed; as, 10, 100, 1000, &c.

1. Multiply 376 by 10.

Analysis.—The operation is performed by simply annexing the 0's of the multiplier to the multiplicand.

OPERATION.

$376 \times 10 = 3760.$

Rule.

Annex to the multiplicand as many ciphers as there are in the multiplier, and the number so formed will be the required product.

Examples.

1. Multiply 2756 by 10; by 100.
2. Multiply 5680 by 1000.
3. Multiply 7690430 by 10000.
4. Multiply 4063 by 10; by 100.
5. Prove the last example by multiplying by the factors of 10, and of 100.
6. Multiply 37006 by 100; by 1000.
7. Multiply 570468 by 10, by 100, by 1000, and by 10000.

CASE III.

34. When there are ciphers on the right of one or both of the factors.

1. Multiply 520 by 360.

Multiply 52 by 36 and add two 0's to the product.

$$520 \times 360 = \left\{\begin{array}{r} 520 \\ 360 \\ \hline 312 \\ 156 \\ \hline 187200 \end{array}\right.$$

Rule.

I. *Omit the ciphers and multiply the significant figures:*

II. *Then place as many ciphers at the right hand of the product as there are in both factors.*

Examples.

1. Multiply 6750 by 20.
2. Multiply 80400 by 60.
3. Multiply 974000 by 700.
4. Multiply 230 by 9000.
5. Multiply 16930 by 2500.
6. Multiply 3750 by 52000.
7. Multiply 3000 by 3000.
8. Multiply 2500 by 2500.
9. Multiply 3007 by 7000.
10. Multiply 190 by 109900.

Practical Questions.

1. If one pound of butter costs 23 cents, what will be the cost of 9 pounds at the same rate?

2. If one horse costs 125 dollars, what will be the cost of 24 horses at the same rate?

3. How many miles will a ship sail in 16 days, if it sails 245 miles a day?

4. If a gentleman's annual expenses amount to 2765 dollars, to what will they amount in 7 years?

5. In a barrel of flour there are 196 pounds: how many pounds are there in 160 barrels?

6. What is the product, when the multiplier is 3500 and the multiplicand 296400?

7. How much money is required to buy 164 barrels of flour at 7 dollars a barrel?

8. What amount of money will enable me to give 35 dollars to each of 245 laborers?

9. If a horse eats 12 bushels of oats in a month, how many bushels would 6 horses eat in 4 months?

10. How much hay would be required to feed 8 horses for 9 weeks, if one horse ate 175 pounds in one week?

11. In how many days could one man dig a trench, if 25 men can dig it in 16 days?

12. If 40 men could build a wall in 30 days, how many men would be required to do it in one day?

13. How many days are there in 1000 common years of 365 days each?

14. Suppose a book to have 360 pages, each page to contain 35 lines, and each line to contain 11 words: how many words are there in the book?

15. If a chest of tea contains 75 pounds, and is worth 62 cents per pound, what would be the value of 45 chests of the same size and at the same price?

Examples, combining the previous Rules.

1. The cost of a horse is 145 dollars; of a carriage 275 dollars; and of a set of harness 75 dollars: what is the entire cost?

2. The income of a person is 4725 dollars per year, and his expenses are 2460 dollars for the same time: how much could he, at this rate, save in 26 years?

3. A person bought 275 horses at 100 dollars per head, and sold them at 135 dollars per head: what did he gain?

4. A farmer has 3 flocks of sheep, numbering respectively 50, 96, and 140 head. If, at shearing, each yields 4 pounds of wool, what will be its value at 24 cents per pound?

5. If a person, out of his monthly salary, spends 30 dollars for board, 8 dollars for washing, and saves 5 dollars, what is his monthly salary?

6. In a factory, 45 hands receive each 30 dollars per month; 24 receive each 25 dollars, and 15 receive each 20 dollars: what is the amount of the monthly payment?

7. A merchant sold 156 yards of cloth, which cost 4 dollars a yard, at 6 dollars per yard; and 256 barrels of flour, which cost 5 dollars, at 7 dollars per barrel: how much did he gain?

8. Suppose an orchard to contain 16 rows of apple-trees, and each row to have 27 trees in it: how many bushels will the orchard produce, if 30 bushels are gathered from each tree?

9. A farmer has a farm of 175 acres, the whole of which was sown with wheat: what will be the yield, if each acre produces 35 bushels?

10. If a lot of ground, that cost 375 dollars, were sold for 250 dollars, what would be the loss in selling 294 lots at the same rate?

11. Two men are 950 miles apart. If they travel towards

each other, one at the rate of 30 miles, and the other of 42 miles per day, how many miles will they be apart at the end of 8 days?

12. If two men, at the same time, leave the same place, and travel in opposite directions, one at the rate of 26 miles, and the other of 19 miles per day, how far apart will they be at the end of 19 days?

13. A drover bought 180 head of cattle, in Illinois, at 25 dollars a head, and sent them to New York where he sold them at a profit of 7 dollars a head: what did he receive for the drove?

14. A lady purchased, at a dry goods store, 9 yards of cloth at 3 dollars per yard; 15 yards of satin at 2 dollars; 7 yards of merino at 1 dollar per yard; she paid for the above articles 3 twenty-dollar bills and 1 ten-dollar bill: what balance was due her?

15. A person contracted to do a piece of work for 5000 dollars. He hired 5 cartmen for 60 days at 5 dollars each, per day; 25 laborers for 75 days at 1 dollar each, per day, and two overseers for 80 days at 3 dollars each, per day: what amount did the contractor save for himself?

16. In one year, a farmer sold the produce of his farm, as follows: 300 bushels of wheat at 60 cents a bushel; 500 bushels of corn at 35 cents a bushel; 150 bushels of oats at 45 cents a bushel: what amount did he receive?

17. A fortune of 25000 dollars is so divided that each of 4 sons is to receive 3725 dollars, and each of two daughters 2575 dollars, and the widow the remainder: what is the widow's share?

DIVISION.

1. When a number is divided into 2 equal parts, each part is called, *one-half of the number.*

What is one-half of 4 apples? What is one-half of 4? How many times is 2 contained in 4?

2. When a number is divided into 3 equal parts, each part is called, *one-third of the number.*

What is one-third of 9 apples? What is one-third of 9? How many times is 3 contained in 9?

3. When a number is divided into 4 equal parts, each part is called, *one-fourth of the number.*

What is one-fourth of 12 pears? What is one-fourth of 12?

4. When a number is divided into 5 equal parts, each part is called, *one-fifth of the number.*

What is one-fifth of 10 marbles? What is one-fifth of 10?

5. When a number is divided into 6 equal parts, each part is called, *one-sixth of the number.*

35. Division is the operation of dividing a number into equal parts; or, of finding how many times one number is contained in another.

36. The Dividend is the number to be divided.

37. The Divisor is the number by which we divide.

38. The Quotient is the result of the division.

39. The Remainder is what is left after the operation.

40. There are three signs used to denote Division:

$18 \div 4$, expresses that 18 is to be divided by 4.

$\frac{18}{4}$, expresses that 18 is to be divided by 4.

4) 18, expresses that 18 is to be divided by 4.

Division Table.

1 in 1 1 time	2 in 2 1 time	3 in 3 1 time
1 in 2 2 times	2 in 4 2 times	3 in 6 2 times
1 in 3 3 times	2 in 6 3 times	3 in 9 3 times
1 in 4 4 times	2 in 8 4 times	3 in 12 4 times
1 in 5 5 times	2 in 10 5 times	3 in 15 5 times
1 in 6 6 times	2 in 12 6 times	3 in 18 6 times
1 in 7 7 times	2 in 14 7 times	3 in 21 7 times
1 in 8 8 times	2 in 16 8 times	3 in 24 8 times
1 in 9 9 times	2 in 18 9 times	3 in 27 9 times
4 in 4 1 time	5 in 5 1 time	6 in 6 1 time
4 in 8 2 times	5 in 10 2 times	6 in 12 2 times
4 in 12 3 times	5 in 15 3 times	6 in 18 3 times
4 in 16 4 times	5 in 20 4 times	6 in 24 4 times
4 in 20 5 times	5 in 25 5 times	6 in 30 5 times
4 in 24 6 times	5 in 30 6 times	6 in 36 6 times
4 in 28 7 times	5 in 35 7 times	6 in 42 7 times
4 in 32 8 times	5 in 40 8 times	6 in 48 8 times
4 in 36 9 times	5 in 45 9 times	6 in 54 9 times
7 in 7 1 time	8 in 8 1 time	9 in 9 1 time
7 in 14 2 times	8 in 16 2 times	9 in 18 2 times
7 in 21 3 times	8 in 24 3 times	9 in 27 3 times
7 in 28 4 times	8 in 32 4 times	9 in 36 4 times
7 in 35 5 times	8 in 40 5 times	9 in 45 5 times
7 in 42 6 times	8 in 48 6 times	9 in 54 6 times
7 in 49 7 times	8 in 56 7 times	9 in 63 7 times
7 in 56 8 times	8 in 64 8 times	9 in 72 8 times
7 in 63 9 times	8 in 72 9 times	9 in 81 9 times
10 in 10 1 time	11 in 11 1 time	12 in 12 1 time
10 in 20 2 times	11 in 22 2 times	12 in 24 2 times
10 in 30 3 times	11 in 33 3 times	12 in 36 3 times
10 in 40 4 times	11 in 44 4 times	12 in 48 4 times
10 in 50 5 times	11 in 55 5 times	12 in 60 5 times
10 in 60 6 times	11 in 66 6 times	12 in 72 6 times
10 in 70 7 times	11 in 77 7 times	12 in 84 7 times
10 in 80 8 times	11 in 88 8 times	12 in 96 8 times
10 in 90 9 times	11 in 99 9 times	12 in 108 9 times

5 in 10 how many times?	6 in 48 how many times?
8 in 16 how many times?	8 in 96 how many times?
9 in 27 how many times?	9 in 36 how many times?
6 in 54 how many times?	8 in 48 how many times?

Practical Questions.

1. In how many days, at 2 dollars a day, will a man earn 16 dollars?

2. How many hats, at 3 dollars each, may be bought for 27 dollars?

3. If it takes 6 yards of cloth for a suit, how many suits may be made from 42 yards?

4. How many boxes, each holding 3 pounds, would be filled by 24 pounds of sugar?

5. A fathom is 6 feet: how many fathoms of depth are there in a river that is 24 feet deep?

6. In how many days could a man walk 63 miles, if he walked 9 miles per day?

7. In what time could a mechanic earn 42 dollars, if he earned 7 dollars per week?

8. In how many days will Daniel and Samuel together earn 72 cents, if Daniel earns 5 cents, and Samuel 3 cents per day?

9. If two men, who are 63 miles apart, should walk towards each other, the one at the rate of 3 miles, and the other of 4 miles per hour, in how many hours would they meet?

10. How many yards of cloth, at 4 dollars a yard, will pay for 6 barrels of flour at 6 dollars a barrel?

11. How many weeks' labor, at 9 dollars a week, will pay for 6 barrels of flour at 6 dollars a barrel?

12. Paid 24 dollars for 4 barrels of flour: what was the cost of one barrel?

13. If 8 men gained 40 dollars, how much did each gain?

14. If 54 dollars will buy 9 barrels of flour, how much will buy one barrel?

15. A person received 30 dollars for 6 days' labor: at what rate per day was he paid?

SHORT DIVISION.

41. When the divisor does not exceed 12.

Let the pupil divide, in each of the following examples, and *read* the result in each case.

1)	1	4	3	2	5	8	7	6	9	12	11	10
2)	2	8	6	4	10	16	14	12	18	24	22	20
3)	3	12	9	6	15	24	21	18	27	36	33	30
4)	4	16	12	8	20	32	28	24	36	48	44	40
5)	5	20	15	10	25	40	35	30	45	60	55	50
6)	6	24	18	12	30	48	42	36	54	72	66	60
7)	7	28	21	14	35	56	49	42	63	84	77	70
8)	8	32	24	16	40	64	56	48	72	96	88	80
9)	9	36	27	18	45	72	63	54	81	108	99	90
10)	10	40	30	20	50	80	70	60	90	120	110	100
11)	11	44	33	22	55	88	77	66	99	132	121	110
12)	12	48	36	24	60	96	84	72	108	144	132	120

1. Divide 48 into 2 equal parts.

OPERATION.

Divide each figure separately, by the divisor.

Divisor. Dividend.

2) 4 8

2 4 quotient.

2. Divide 31254 by 6.

We first say, 6 in 3 we can't; then 6 in 31, 5 times and 1 over; then 6 in 12, twice; then 6 in 5, 0 times; then 6 in 54, 9 times.

OPERATION.

6) 3 1 2 5 4

5 2 0 9

3. Divide 327 by 8.

We divide as before, and find a remainder of 7. We write 8 under 7, and the quotient is 40 and 7 divided by 8.

OPERATION.

8) 3 2 7

4 0—7 rem.

4 0 $\frac{7}{8}$ quotient.

Rule.

I. *Write the divisor on the left of the dividend. Begin at the left hand and divide each figure of the dividend by the divisor, and set each quotient figure under its dividend.*

II. *If there is a remainder after any division, annex to it the next figure of the dividend and divide as before.*

III. *If any dividend is less than the divisor, write* 0 *for the quotient figure, and annex the next figure of the dividend for a new dividend.*

IV. *If there is a remainder, after dividing the last figure, set the divisor under it, and annex the result to the quotient.*

Proof.

Multiply the quotient by the divisor, and to the product add the remainder, if any: if the work is right, the result will be equal to the dividend.

Examples.

(1.)	(2.)	(3.)
4) 3276	6) 4167	8) 27458
Answer, 819	$694\frac{3}{6}$	3432—2 rem.
4	6	8
Proof, 3276	4167	27458

(4.)	(5.)	(6.)
6) 28743	7) 97476	9) 840460

7. Divide	129	by 2.	20. Divide	19507034	by 7.
8. Divide	9856	by 4.	21. Divide	16039004	by 5.
9. Divide	79604	by 8.	22. Divide	91001100	by 9.
10. Divide	647617	by 6.	23. Divide	42006004	by 9.
11. Divide	819647	by 5.	24. Divide	22002200	by 8.
12. Divide	391764	by 3.	25. Divide	36709967	by 6.
13. Divide	1064720	by 7.	26. Divide	47000047	by 3.
14. Divide	10066100	by 6.	27. Divide	56700958	by 8.
15. Divide	16435420	by 2.	28. Divide	68704960	by 7.
16. Divide	90010555	by 2.	29. Divide	78490473	by 5.
17. Divide	14769647	by 9.	30. Divide	45704905	by 6.
18. Divide	39500047	by 4.	31. Divide	99904708	by 8.
19. Divide	25046700	by 6.	32. Divide	91979998	by 9.

Practical Questions.

1. If 12 apples be equally divided among 4 boys, how many will each have?

2. If 24 peaches be equally divided among 6 boys, how many will each have?

3. A man has 32 miles to walk, and can travel 4 miles an hour: how many hours will it take him?

4. A farmer receives 245 dollars for 7 cows: how much is that apiece?

5. How many lead pencils could you buy for 396 cents, if they cost 6 cents apiece?

6. How many oranges could you buy for 8496 cents, if they cost 6 cents apiece?

7. A trader wishes to pack 768 hats in boxes, and can put but 8 hats in a box: how many boxes does he need?

8. If a man could build 7 rods of fence in a day, how long will it take him to build 847 rods?

9. If a man pays 56 dollars for seven yards of cloth, how much is that a yard?

10. Nine men receive 1296 dollars for doing a piece of of work: how much does each one receive?

11. A merchant has 1344 dollars with which he is going to buy cloth at 8 dollars a yard: how many yards can he purchase?

12. James is to learn forty-two verses of Scripture in a week: how much must he learn each day?

13. A man has 994 pounds of butter, and wishes to put 7 pounds in a box: how many boxes does he need?

14. James goes to school for 6 weeks, and receives 264 credit marks: how many does he get each week?

15. An estate, worth 7212 dollars, is to be equally divided among 4 sons and 2 daughters: what will be the part of each?

16. If 5 bushels of wheat make 1 barrel of flour, how many barrels will 13080 bushels make?

17. If 8568 marbles are divided equally among 9 boys, how many marbles will each boy have?

18. How many barrels of flour, at 8 dollars a barrel, can be bought for 3496 dollars?

19. If an estate, worth 69741 dollars, is to be equally divided among 7 sons and 2 daughters, what is the portion of each?

LONG DIVISION.

42. When the divisor exceeds 9.

1. Divide the number 4564 into 14 equal parts.

The operation in this, and in all similar examples, is performed according to the following Rule.

NOTE.—The numbers 45, 36, and 84 are called *partial dividends.*

OPERATION.

	Thous.	Hunds.	Tens.	Units.	Hunds.	Tens.	Units.
14)	4	5	6	4	(3	2	6
		4	2				
		3	6				
		2	8				
			8	4			
			8	4			

Rule.

I. *Write the divisor on the left of the dividend.*

II. *Note the fewest figures of the dividend, at the left, that will contain the divisor, and set the quotient figure at the right of the dividend.*

III. *Multiply the divisor by the quotient figure, subtract the product from the first partial dividend, and to the remainder annex the next figure of the dividend, forming a second partial dividend.*

IV. *Find, in the same manner, the second and succeeding figures of the quotient, till all the figures of the dividend are brought down.*

Proof.

Multiply the divisor by the quotient, and to the product add the remainder. If the work is right, the sum will be the same as the dividend.

NOTE 1.—There are five operations in Division: 1st, To write down the numbers; 2d, Divide, or find how many times; 3d, Multiply; 4th, Subtract; 5th, Bring down, to form the partial dividend.

2. The product of a quotient figure by the divisor must never be larger than the corresponding partial dividend; if it is, the quotient figure is too large, and must be diminished.

3. When any one of the remainders is greater than the divisor, the quotient figure is too small, and must be increased.

4. The unit of any quotient figure is the same as that of the partial dividend from which it is obtained. The pupil should always name the unit of every quotient figure.

5. The unit of a remainder is the same as that of the dividend.

Examples.

1. Divide 32641 dollars into 24 equal parts.

OPERATION.

Divisor.	Dividend.	Quotient.
24)	32641	(1360 $\frac{1}{24}$
	24	
	86	
	72	
	144	
	144	
	01 Rem.	

PROOF.

	1360
	24
	5440
	2720
	32640
	1 Rem.
	32641

2. Divide 874614 by 75.

OPERATION.

75)	874614	(11661 $\frac{39}{75}$
	75	
	124	
	75	
	496	
	450	
	461	
	450	
	114	
	75	
	39 Rem.	

3. Divide 409059 by 96.

OPERATION.

96)	409059	(4261 $\frac{3}{96}$
	384	
	250	
	192	
	585	
	576	
	99	
	96	
	3 Rem.	

4. Divide 875 by 15.
5. Divide 9506 by 16.
6. Divide 804967 by 18.
7. Divide 954678 by 15.
8. Divide 479604 by 19.

9. Divide 327084 by 21.
10. Divide 697047 by 14.
11. Divide 591678 by 13.
12. Divide 927041 by 11.
13. Divide 900672 by 18.

14. Divide 9657723 by 627.
15. Divide 1320796 by 201.
16. Divide 2147735 by 156.
17. Divide 3372630 by 544.
18. Divide 89582619 by 119.

19. Divide 90604 by 18.
20. Divide 691645 by 25.
21. Divide 1406275 by 31.
22. Divide 25672916 by 35.
23. Divide 19647062 by 41.

24. 9647569 ÷ 245 =
25. 37964756 ÷ 296 =
26. 467980126 ÷ 384 =
27. 547690476 ÷ 1352 =
28. 691464280 ÷ 3045 =

29. 87467047 ÷ 240 =
30. 45848043 ÷ 328 =
31. 69274216 ÷ 416 =
32. 71364891 ÷ 5943 =
33. 81486974 ÷ 3742 =

CONTRACTIONS.

43. Contractions, in Division, are short methods of finding the quotient when the divisor is a composite number.

CASE I.

44. When the divisor is any composite number.

1. Divide 18576 dollars equally among 24 men.

$24 = 2 \times 3 \times 4$.

Rule.

Divide by the factors of the divisor, in succession, and the last quotient will be the quotient required.

OPERATION.

2) 1 8 5 7 6
3) 9 2 8 8
4) 3 0 9 6
7 7 4

Examples.

1. Divide the number 19152 by $72 = 8 \times 9$.
2. Divide the number 3780 by $12 = 3 \times 4$.
3. Divide the number 19296 by $48 = 3 \times 4 \times 4$.
4. Divide the number 92880 by $48 = 2 \times 2 \times 2 \times 2 \times 3$.
5. Divide the number 111456 by $144 = 4 \times 4 \times 9$.
6. Divide the number 308736 by $24 = 3 \times 2 \times 2 \times 2$.

To find the true remainder, if any.

7. Divide 3274 by the number $24 = 2 \times 3 \times 4$.

Rule.

Multiply each remainder by all the preceding divisors except its own, and add the products to the first remainder.

OPERATION.

2) 3 2 7 5
3) 1 6 3 7 . . 1
4) 5 4 5 . . 2
1 3 6 . . 1

1
$2 \times 2 = 4$
$1 \times 3 \times 2 = 6$
True remainder, 11

8. Divide the number 5088 by $16 = 2 \times 2 \times 2 \times 2$.
9. Divide the number 8974 by $15 =$
10. Divide the number 6446 by $18 = 2 \times 3 \times 3$.
11. Divide the number 312176 by $24 =$
12. Divide the number 902725 by $21 = 3 \times 7$.
13. Divide the number 307697 by $35 =$
14. Divide the number 7406342 by $81 = 9 \times 9$.
15. Divide the number 9468933 by $45 =$
16. Divide the number 2346912 by $64 = 2 \times 4 \times 8$.
17. Divide the number 1000000 by $72 =$
18. Divide the number 9479564 by $36 = 6 \times 6$.
19. Divide the number 4709063 by $42 = 2 \times 3 \times 7$.

CASE II.

45. When the divisor is 10, 100, 1000, &c.

1. From 6272 marbles, how many piles can be formed, of 100 each?

Rule.

I. *From the right hand, cut off by a line, as many figures as there are ciphers in the divisor.*

II. *The figures at the left will be the quotient, and those at the right, the remainder.*

OPERATION.

$1|00\,)\,62|72$

62—72 rem.

$62\frac{72}{100}$, quotient.

Examples.

1. Divide 6747 by 10.
2. Divide 270460 by 100.
3. Divide 927000 by 1000.
4. Divide 704963 by 1000.
5. Divide 2394704 by 10000.
6. Divide 97469 by 1000.
7. Divide 967000 by 10000.
8. Divide 490400 by 1000.
9. Divide 6972004 by 1000.
10. Divide 4970906 by 1000.

CASE III.

46. When the divisor contains significant figures, with ciphers on the right of them.

1. Divide 77256 by 3700.

Rule.

I. *Cut off the ciphers by a line, and cut off the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor; and if there is no remainder, the figures cut off will be the true remainder. But if there is a remainder, annex to it the figures cut off, and the result will be the true remainder.*

OPERATION.

$37|00\,)\,772|56\,(20\frac{3256}{3700}$

74

3256 rem.

$20\frac{3256}{3700}$, quotient.

Examples.

1. Divide the number 561754 by $20 = 2 \times 10$.
2. Divide the number 6934756356 by $200 =$
3. Divide the number 5810636 by $5000 = 5 \times 1000$.
4. Divide the number 349275 by $47000 =$
5. Divide the number 71692 by $6400 = 64 \times 100$.
6. Divide 1000664300 by $125000 = 125 \times 1000$.

Practical Questions.

1. If an individual earns 3 dollars per day, in what time will he earn 450 dollars?
2. If a person pays, per week, to his laborers, 4563 dollars, at the rate of 9 dollars each, how many does he employ?
3. How many cattle, at 35 dollars per head, can be bought for 1015 dollars?
4. If 45 men earn in one year 27675 dollars, how much does each earn?
5. What is the price of one boat, if 25 boats cost 900 dollars?
6. A farmer has 6944 pounds of butter, which he wishes to put into firkins, each to contain 56 pounds: how many firkins does he require?
7. A barrel of flour contains 196 pounds: how many barrels are required to hold 37436 pounds?
8. If a vessel sails 5678 miles in 34 days, how many miles does it sail per day?
9. In how many days will a vessel reach her destined port, which is 2960 miles distant from the place of sailing, if she sails 216 miles per day?
10. If one man can earn a certain sum of money in 28 days, how long would it take 7 men to earn the same amount of money?

11. If one man can dig a trench in 279 days, in how many days could 9 men have dug the same?

12. A man has sufficient feed to keep one horse 224 days: how many days would the feed last 7 horses?

13. A contractor engages to do a piece of work in 60 days. Supposing that one man can do it in 1320 days, how many men must be employed to finish the work in time?

14. Suppose that, in a month, a mechanic labors 26 days, and receives 3750 cents: how much does he receive a day?

15. How many times would a measure, 18 feet long, be laid down in measuring a distance of 882 feet.

16. How many times would a surveyor lay down a chain, 66 feet long, in measuring a distance of 47520 feet?

17. How many dozen eggs are there in a barrel, containing 2760 eggs; and how many baskets would contain them, if each basket held 30?

18. A farmer wishes to fence, with rails, his fields to the length of 5280 feet. Suppose a panel to be 7 feet long, how many panels will there be?

19. Massachusetts, by the census of 1850, had a population of about 990600 on an area of 7800 square miles: what was the population to the square mile?

20. By the census of 1850, Maine had a population of about 570000, and an area of 30000 square miles: what was the population to the square mile?

21. New York had a population of about 3087695 upon an area of 46085 square miles: what was the population to the square mile.

22. If the divisor be 3675 and the dividend 926100, what is the quotient?

23. What is the divisor when the dividend is 1732800, and the quotient 380?

24. A grocer has 1380 pounds of buckwheat, which he wishes to put into bags, each containing 25 pounds: how many bags would be required?

Examples in the preceding Rules.

1. If one person receives 36 dollars per month, and another 45 dollars per month, in what time will they together earn 1944 dollars?

2. A gentleman died possessed of a fortune of 50000 dollars; he directed that after paying his debts, amounting to 6000 dollars, the remainder should be equally divided among his four children: what was the portion of each?

3. How many pounds of coffee, worth 18 cents a pound, must be given for 368 pounds of sugar, worth 9 cents a pound?

4. James has 14 calves, worth 4 dollars each; 40 sheep, worth 3 dollars each; he gives them all for a horse worth 150 dollars: does he make or lose, and how much?

5. How many barrels of flour, at 5 dollars per barrel, will pay for 20 barrels of potatoes, at 3 dollars per barrel?

6. If a suit of clothes requires 9 yards of cloth, how many suits may be made from 12 pieces of cloth, each containing 45 yards?

7. A young man having engaged his services at the rate of 756 dollars a year, left his employer at the end of 8 months: what should he receive, at the rate agreed on?

8. A person buys a house and lot for 3500 dollars, of which sum he pays 1850 dollars cash, and agrees to pay the remainder in annual instalments of 150 dollars each: in how many years will the debt be paid?

9. A person bought 65 cattle for 1950 dollars, and sold them for 2795 dollars, what was the gain on each?

10. I bought 600 barrels of flour for 3500 dollars, and wish to gain 700 dollars, after paying 100 dollars for charges: at what price per barrel must they be sold?

11. If a drover asks 1125 dollars for a flock of 375 sheep, how many head would 159 dollars buy at that rate?

12. A person bought a farm of 192 acres for 11520 dollars, and sold 60 acres for 6000 dollars: what did he gain on each acre sold?

13. The product of two numbers is 6843020, and one of them is 15695: what is the other?

14. If the dividend is 2884476, the quotient 471, and the remainder 72, what is the divisor?

15. A merchant sold 45 barrels of flour at 6 dollars per barrel, and 25 barrels of potatoes at 2 dollars per barrel; and took in payment, butter at 8 dollars per firkin: how many firkins did he receive?

16. A person paid, at a hotel, 12 dollars for the first week's board, 15 dollars for the second, 18 dollars for the third, and 23 dollars for the fourth: what was the equal or average price per week?

17. A grocer has five boxes of sugar, containing respectively 175 pounds, 250 pounds, 523 pounds, 365 pounds, and 872 pounds: had each box contained an equal quantity of the sugar, what would have been the weight in each?

18. A shopkeeper sold on Monday to the amount of 37 dollars; on Tuesday, 49 dollars; Wednesday, 105 dollars; Thursday, 66 dollars; Friday, 56 dollars; and on Saturday, 125 dollars: what was the average sale of each day?

19. In a certain school, the attendance was, on Monday, 154; on Tuesday, 170; on Wednesday, 190; on Thursday, 184; on Friday, 172: what was the average attendance?

20. If a man's salary is 1600 dollars a year, and his expenses 850 dollars, in what time will he be worth 20000 dollars, if he is worth 5000 when his salary begins?

21. Two persons counting their money, found that together they had 684 dollars; but one had 28 dollars more than one-half of it: how much had the other?

22. What is the cost of 32568 oranges, at the rate of 25 cents per dozen? and how many pineapples, at the rate of 3 for 15 cents, will pay for the oranges?

CANCELLATION.

47. Cancellation is a process of shortening Arithmetical operations in Division, by omitting, or *cancelling*, factors common to the dividend and divisor.

It depends on the principle that,

If the dividend and divisor be both divided by the same number, the quotient will not be changed.

1. Divide 42 by 14.

Analysis.—Resolve the dividend and divisor into factors, then cancel those which are common, and mark the cancelled figures.

OPERATION.

$$\frac{42}{14} = \frac{6 \times \cancel{7}}{2 \times \cancel{7}} = \frac{6}{2} = 3.$$

2. In 5 times 35 how many times 7?

Analysis.—Resolve 35 into the two factors 7 and 5, then cancel the 7.

OPERATION.

$$\frac{35 \times 5}{\cancel{7}} = \frac{\cancel{7} \times 5 \times 5}{\cancel{7}} = 25.$$

3. In 18 times 15 how many times 45?

Analysis.—We see that 15 is a factor of 45. Divide by this factor, write the quotient 3 under 45, and cancel 15 and 45. Again, 3 is a factor of 18: divide by it, and write the quotient 6 above 18, and cancel 18 and 3: hence, the quotient is equal to 6.

OPERATION.

$$\frac{\overset{6}{\cancel{15} \times \cancel{18}}}{\underset{3}{\cancel{45}}} = \frac{6}{1} = 6.$$

Rule.

I. *Write the dividend above a horizontal line, and the divisor below it.*

II. *Cancel all the factors that are common to the dividend and divisor, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTES.—1. If one of the numbers contains a factor equal to the product of two or more factors of the other, they may all be cancelled.

2. If the product of two or more factors of the dividend is equal to the product of two or more factors of the divisor, they may all be cancelled.

3. If all the factors of the dividend are cancelled, the quotient 1 must be put for the factor last cancelled.

Examples.

1. Divide the product of $6\times8\times12\times15$ by $3\times4\times9$.

We cancel the factor 3, in 3 and 6; then 4, in 4 and 8; then 3, in 9 and 12; then 3, in 3 and 15.

OPERATION.

$$\frac{\overset{2}{\cancel{6}}\times\overset{2}{\cancel{8}}\times\overset{4}{\cancel{12}}\times\overset{5}{\cancel{15}}}{\underset{3}{\cancel{3}}\times4\times\cancel{9}}=80.$$

2. Divide the product of $8\times9\times5\times6$ by $3\times5\times2$.

3. Divide the product of $9\times8\times7\times6$ by $2\times3\times4\times5$.

4. Divide the product of $6\times5\times3\times7\times8$ by $4\times3\times7\times9$.

5. Divide the product of $4\times5\times6\times8\times9$ by $7\times5\times4\times3$.

6. Divide the product of $10\times5\times4\times3$ by $4\times9\times12$.

7. Multiply $8\times7\times5$ by $3\times6\times5$, and divide the product by $3\times4\times6$.

8. Divide the product of $15\times20\times24$ by $8\times5\times4$.

9. If the product of 20 by 12, be divided by the product of 1, 2, 3 and 4, what will be the quotient?

10. How many loads of wood, at 4 dollars a load, must be given for 14 yards of broadcloth at 6 dollars a yard?

11. How many bushels of buckwheat, at 45 cents a bushel, must be given for 120 bushels of corn at 60 cents a bushel?

12. How many bushels of oats, at 48 cents a bushel, must be given for 6 boxes of raisins, each containing 52 pounds, at 12 cents a pound?

13. How many pounds of butter, at 25 cents a pound, will pay for 60 pounds of tea at 75 cents a pound?

14. How many sheep, at 6 dollars a piece, must be given for a drove of 36 cows at 49 dollars a piece?

15. How many lambs, at 3 dollars each, must be given for 20 tons of hay at 18 dollars a ton?

16. How much butter, at 26 cents a pound, will pay for 8 boxes of sugar, each weighing 216 pounds, if the sugar is valued at 13 cents per pound.

17. A man bought 136 yards of cloth, at 4 dollars a yard, and paid for it in hay, at 16 dollars a ton: how many tons did he give?

18. A worked for B, 36 days at 6 shillings a day, and took his pay in wheat at 9 shillings a bushel: how many bushels did he receive?

19. How many boxes of tea, each containing 48 pounds, worth 75 cents a pound, must be given for 6 barrels of sugar, each containing 200 pounds, at 9 cents a pound?

20. If sugar can be bought for 11 cents a pound, how many bushels of oats, at 66 cents a bushel, must be given for 396 pounds?

21. A person worked 125 days for 6 shillings a day, and received in payment corn at 5 shillings a bushel: how much corn did he receive?

22. How many chickens, at 3 shillings a piece, must be given for 185 turkeys, worth 9 shillings a piece?

COMMON DIVISOR.

48. An EXACT DIVISOR of a number, is a divisor which will divide it without a remainder.

49. A COMMON DIVISOR of two or more numbers, is a divisor which will divide each, separately, without a remainder.

50. The GREATEST COMMON DIVISOR of two or more numbers, is the greatest number that will divide each, separately, without a remainder.

To find the greatest common Divisor.

1. What is the greatest common divisor of 112 and 144?

Rule.

Divide the greater number by the less; and then divide the divisor by the remainder; and continue the operation till nothing remains. The last divisor will be the greatest common divisor of the two numbers.

OPERATION.

112) 144 (1
112
32) 112 (3
96
16) 32 (2
32

2. What is the greatest common divisor of 75 and 275?
3. What is the greatest common divisor of 420 and 510?
4. Find the greatest common divisor of 216 and 316.
5. Find the greatest common divisor of 24 and 1956.
6. Find the greatest common divisor of 39 and 192.
7. What is the greatest common divisor of 1728 and 5000?
8. What is the greatest common divisor of 3750 and 5495?
9. What is the greatest common divisor of 960 and 1920?
10. What is the greatest common divisor of 376 and 495?
11. What is the greatest common divisor of 96 and 360?
12. What is the greatest common divisor of 113 and 7650?

COMMON FRACTIONS.

51. A UNIT is a single thing; as, 1 apple, 1 chair, 1 pound of tea; and is denoted by 1.

If a unit be divided into two equal parts, each part is called, *one-half*.

If a unit be divided into three equal parts, each part is called, *one-third*.

If a unit be divided into four equal parts, each part is called, *one-fourth*.

If a unit be divided into twelve equal parts, each part is called, *one-twelfth;* and if it be divided into *any number* of equal parts, we have a like expression for each part.

The parts are thus written:

$\frac{1}{2}$	is read,	one-half.	$\frac{1}{7}$	is read,	one-seventh.
$\frac{1}{3}$	. .	one-third.	$\frac{1}{8}$	. .	one-eighth.
$\frac{1}{4}$	. .	one-fourth.	$\frac{1}{10}$	. .	one-tenth.
$\frac{1}{5}$	. .	one-fifth.	$\frac{1}{15}$	. .	one-fifteenth.
$\frac{1}{6}$	. .	one-sixth.	$\frac{1}{50}$	. .	one-fiftieth.

52. The UNIT OF A FRACTION is the single thing that is divided into equal parts.

53. A FRACTIONAL UNIT is one of the equal parts of the unit that is divided.

54. A FRACTION is a fractional unit, or a collection of fractional units.

1. If an apple be divided into 30 equal parts, write the fractional unit.

2. If a pear be divided into 29 equal parts, write the fractional unit.