till the whole of the ice is turned into water. To effect this transformation 144 units of heat must be supplied, equivalent to 144 × 772 = 111,168 foot-pounds of work. In other words, it would require as much heat to raise a pound of ice at 32° through a height of *about* 21 *miles*, as it does to convert it into water of 32°. As the temperature remains stationary during the melting process, the question arises—what becomes of the heat which has been expended. The early discoverers of this phenomenon, being unable to account for the heat thus apparently lost, invented the theory that it had become *latent*, or concealed in the water, and, in accordance with this theory, it was said that the *latent heat* of water was 144°. In accordance with the mechanical theory, it is recognised that the heat thus expended is spent in doing internal work on the particles of the ice, which results in their cohesion being overcome so that the condition of the ice is changed from the solid to the liquid state. We should say, therefore, that 144 units of internal work, or of latent work, are done upon the ice in order to transform it into water. The term, internal or latent work, is used in contradistinction to the external work, which, as we have seen before, a body can perform when increasing in volume under the influence of heat.

It should here be noticed how enormous are the internal forces which have to be overcome in changing the molecular condition of the ice from the solid to the liquid state. The 111,168 foot-pounds of work necessary to effect the transformation in a pound of the substance being equivalent to about 595 tons raised one inch high. At the same time, the volume of the water is slightly less than that of the ice from which it was formed, being in the ratio of 1·0908 to 1·000127, thus showing that there is no great change in the distance apart of the molecules.

If we continue to apply heat to the water, its effect is to raise the temperature till the boiling-point is reached. This point being 180° above the temperature of melting ice, the number of units of heat required to accomplish this rise in temperature would, if the specific heat were unity throughout the whole process, be 180. As, however, the specific heat is only unity at 39·1°, and after that increases slightly with the temperature, the actual number of heat units required has been found to be 180·9. During this rise in temperature the volume of the water decreases slightly to 39·1°, the point at which it is reckoned unity, and after that increases with the temperature till it reaches 1·04315 at 212°.

The further application of heat to the water will not increase its temperature, so long as the pressure is that of the atmosphere, but will only result in the formation of steam of atmospheric pressure. In other words, the water will boil and will continue to do so till it is all turned into steam. To effect this change from water of 212° into steam of the same temperature, 965·7 units of heat are required, equal to 745,520 foot-pounds. That is to say, to turn a pound of water having the temperature of 212° into steam of atmospheric pressure, heat has to be supplied to it, equivalent to the work involved in raising the weight of water up to a height of about 146 miles, or, in raising about 346 tons a foot high. When the whole of the water is turned into steam its volume is about 1650 times that of the water from which it was formed, and in this state it is called dry saturated steam, and in many of its qualities it resembles a gas. Its temperature is the same as that of the water from which it was formed, viz. 212°.

If more heat be added, the pressure remaining that of the atmosphere, the temperature of the steam will rise, and it will become what is called superheated, which means that it is of higher temperature than the water from which it was formed. The specific heat of steam is only 0·4805, so that for every unit of heat now supplied to it the temperature will rise 2°·08, and the volume will also increase directly as the absolute temperature.

Just as in the case of the liquefaction of ice, so with the vaporisation of boiling water, the 965·7 units of heat which have been supplied for this purpose, and which produce no effect on the thermometer, have all been expended in doing work. Part of the work so done is internal or latent work expended in overcoming the molecular resistances of the water, and part is expended in doing external work against the pressure of the atmosphere. To make this point clear, and to show how much heat is spent in each of these sorts of work, suppose a cubic foot of water at the temperature of 212° to be inclosed in a cylinder of indefinite length and of one square foot in section. Suppose the water to be covered in by a piston without weight, and free to move. The pressure of the air on this piston will be 14·7 lbs. × 144=2116·8. The cubic foot of water weighs 62·42 lbs. To turn it into steam requires, therefore, 62·42 × 965·7=60279 units of heat, equivalent to 46,535,388 foot-pounds. At the end of the process the piston is lifted 1,650 feet from the bottom of the cylinder, or 1,649 feet from its original position. The external work done is, therefore, this height multiplied by the pressure on the piston, or 1649 × 2116·8=3,490,603 foot-pounds; while the internal or latent work is equal to the total minus the external work = 46,535,388 – 3,490,603 =43,044,785 foot-pounds. The external is to the internal work, therefore, in the ratio of 1 to 12·33 nearly.

Suppose now the piston in the above experiment to be loaded with some other weight in addition to that of the atmosphere, say with another 14·7 lbs. to the square inch, and that heat be applied as before. The result would be that the water would rise in temperature to nearly 249° before steam began to form. When it did form, the steam would have the same temperature as the water, viz., 249°, and the same pressure as the piston sustains, viz. 29·4 lbs. per square inch. When the water was all turned into steam, the piston would have risen to a height of 858 feet above the cylinder bottom, i.e. to 857 feet above its original position. The quantity of heat requisite to effect this transformation is found by experiments to be 1157·85 units for each pound of water, whereas in the last example only 965·7 + 180·9 = 1146·6 units were required, thus showing an increase of 11·25 units. Also the way in which the heat is expended is different. For instance, in the second example the final temperature of the water is 249° instead of 212°; consequently, the heat expended in raising it from 32° is about 218·4 units against 180·9. The heat expended in merely vaporising the water is 1157·85 – 218·4=939·45 in the second example against 965·7 in the first, showing, therefore, a decrease of 26·45 units. Now, of these 939·45 units, a certain quantity is spent in doing external work, against the load on the piston. The whole heat thus expended on the cubic foot of water=144 × 29·4 × 857=3,628,195 foot-pounds=4699·7 thermal units. Consequently, the internal or latent work done per pound of water $= 939·45 - \frac{4699·7}{62·42}$ =864·15 units; and the ratio of the external to the internal work is 1 to 11·47 instead of 1 to 12·33, as in the first example.

We see, therefore, that by increasing the load on the piston we have changed everything, viz. the temperature at which the water boils; the temperature, pressure, volume, and consequently density of the steam; the total heat necessary to effect the change, and also the proportions of the heat which are expended in raising the temperature of the water, in vaporising it and in doing external and internal work.

The laws which regulate many of these changes are not yet perfectly understood, and consequently at present only empirical formulæ are available to express them. The formulæ are founded upon the results of experiments which have been carried out in the most exhaustive manner. The results of these experiments are recorded in tables, so that the student is, except for the purpose of analytical calculation, rendered independent of the formulæ.

Connection between pressure and temperature of steam.—The connection between the pressure and temperature of steam was determined by Regnault, and the numerical results are given in the Table, page 489, transformed into English measure for every degree between 100° and 401° Fahrenheit. Regnault's experiments were made at pressures varying from 3 lbs. to 200 lbs. per square inch. It will be seen by studying the table that the pressure increases with the temperature, but not in a uniform manner as in the case of gas. For instance, starting at 212°, the pressure is 14·7 lbs. per square inch and the increment of pressure per degree of rise of temperature is 0·29 lb. At 300°, however, the pressure is 67·22 lbs. and the increment of pressure per degree is 1 lb.; while at 408° the pressure is 270·99 lbs., and the increment of pressure per degree is 3 lbs.

Total heat of steam.—The connection between the temperature of the steam and the total quantity of heat required to raise the water from 32° and vaporise it was also determined by Regnault, and the numerical results are given both in thermal units and in foot-pounds in col. 4 of the Table. It will be seen that the total heat increases with the temperature, the rate of increase being about 0·305 of a thermal unit for each degree above 212°, so that if 1146·6 units is the total heat of one pound of steam at 212°, and if we want to know the total heat at any other temperature $t°$, it will be given by the expression

$$\text{Total heat} = 1146{\cdot}6 + {\cdot}305\,(t° - 212°).$$

Heat of vaporisation of steam.—The heat of vaporisation, as distinguished from the total heat, is easily calculated, if we know the total heat, by subtracting from this latter the number of units of heat required to raise the water from 32° to the boiling-point; see col. 3 of the Table. The Table shows that the heat of vaporisation diminishes as the temperature of the steam increases, but not by a constant rate. The rate of diminution increases with the temperature.

When a table is not at hand, very approximate results can be obtained by assuming that the rate of diminution is 0·71 of one thermal unit for every degree above 212°. Thus for steam of $t°$ temperature,

$$\text{Heat of vaporisation} = 965{\cdot}7 - {\cdot}71\,(t° - 212°),$$

965·7 units being the heat of vaporisation of steam of 212°.

Volume of steam.—The heat spent in external work depends of course on the volume which the steam occupies when formed. Experiments have also been made upon this point. The volume in cubic feet occupied by a pound of water when turned into steam is called the *specific volume* of the steam. The term *relative volume* is used to denote the comparison between the volume occupied by the steam, and that occupied by the water from which it is formed.

Connection between pressure and volume of steam.—The weight in pounds of a cubic foot of steam is called its density. In the case of a gas the connection between the pressure, the volume, and the density is, as we have seen, extremely simple. The equation pv = constant, giving the connection between pressure and volume, while the density is exactly proportional to the pressure. In the case of steam, the relationship is not so simple. No rational formula has ever been devised to express the relationship, but experiments have been made for each separate case, the results of which are given in col. 5 of the Table. An empirical formula has been given by Rankine, which very nearly gives the results of the experiments on pressure and volume, and is of the same form as pv = constant. Rankine's formula, connecting the pressure and volume of steam, is as follows,

$$pv^{\frac{17}{16}} = \text{constant}$$

where p is the pressure in pounds per square inch, and v is the volume in cubic feet, the value of the constant being 475.

External work done during vaporisation of water.—This formula enables the external work done during the vaporisation of water to be calculated, but except where it is necessary to use a formula in analytical investigations, the figures are best taken from the Table by multiplying the volume as given in col. 5 by the pressure per square foot. It will be noticed on studying the results that the external work done increases slowly with the temperature, but not by a uniform rate of increase. The rate diminishes as the temperature rises.

Internal work done during vaporisation.—The heat expended in doing internal or latent work during vaporisation, in altering the molecular constitution of the water, is the difference between the heat of vaporisation and the heat expended in doing external work. It may be deduced from the Table by subtracting the external work, plus the heat expended in raising the temperature of the water as given in col. 3 from the total heat as given in col. 4. It diminishes with the temperature by about 0·792 unit for every degree.

The heat necessary for turning water of 32° into steam, at constant pressure, is expended in the three following ways, which must be kept distinct from one another.

1. In raising the temperature of the water from 32° to the temperature of the boiling-point, which last depends upon the pressure.

2. In changing the physical constitution of the water from the liquid state to the condition of steam. This is what has been called above internal or latent work.

3. In doing external work, by overcoming the resistance of the atmosphere, or other external resistance through a certain space, corresponding to the volume which the steam occupies at the particular pressure.

It should here be noticed that when steam is formed in a boiler, in connection with a non-expansive engine at work, it is generated under the condition of nearly constant pressure; the piston which is constantly moving backwards

and forwards in a cylinder which is in communication with the boiler, corresponding to the piston, in the example given above, while the forces which in a steam engine oppose the motion of the piston correspond to the weights placed upon the piston in the example. The case of steam formed in a close vessel is different, for here no heat has to be expended in doing external work, for by the nature of the case none can be done.

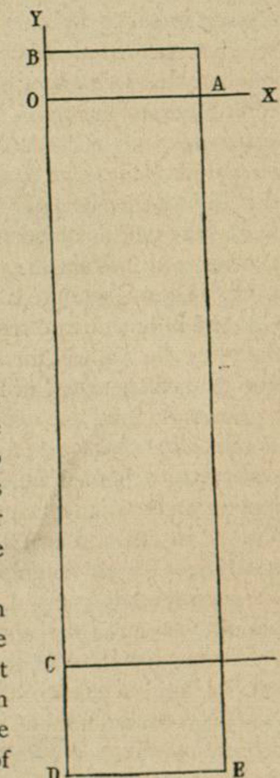

Fig. 13.

The relative proportions of the three separate quantities of heat necessary to raise a pound of water from 32° to boiling temperature, and then to evaporate it, may perhaps best be exhibited by a graphical diagram. Draw a line OX (fig. 13) along which to measure the volume of one pound of water when turned into steam. Suppose the water as before contained in a cylinder having a section of one square foot.

Draw a line OY along which to measure the total pressure in pounds on the piston. Let us take for the first illustration steam of 30 lbs. pressure to the square inch, the temperature of which, according to the Table, is about 250°, and the specific volume 13·49 feet. Now the original volume occupied by the pound of water is 0·016 cubic foot, therefore the space through which the steam lifts the piston when doing work is 13·49 − ·016 = 13·474 feet. Measure off a length OA to scale along OX to represent 13·474 feet, and a length OB along OY to represent to scale the pressure on the piston $=144 \times 30=4320$ lbs. Then the exterior work done by the steam when formed is $=4320 \times 13·474=58207·68$ foot-pounds, and this is represented on the diagram by completing the rectangle AB, the area of which is of course OA × OB, and which therefore represents the above number of foot-pounds. Similarly the heat spent in internal work and in raising the temperature of the water may be represented by the areas of rectangles. For the sake of comparison these rectangles should have the same base OA as the original rectangle, and should be drawn below OX. Now the heat spent in internal work may be calculated from the Table to be 680,021 foot-pounds, which is about 11·6 times as much as the external work. Draw therefore OC downwards at right angles to OA and in length 11·6 times as much as OB. The area of the rectangle AC will then represent the heat calculated in foot-pounds expended in internal work.

Similarly the heat expended in raising the temperature of the water from 32° to 250° can be represented. This heat is 219·5 thermal units=169,454 foot-pounds, which is about 2·91 times as much as the heat spent in external work. From C therefore draw CD downwards, in length equal to 2·91 times OB, and complete the rectangle. Its area will then represent the amount of heat calculated in foot-pounds required to raise the temperature of the water from 32° to 250°.

An inspection of this diagram will show what a very wasteful kind of steam engine such a cylinder and piston would constitute; for the whole of the work done by the heat expended is represented by the rectangle AB, while the whole heat supplied is represented by the big rectangle BE, which is 15·51 times the area of AB. Therefore for every 15·51 units of heat supplied to such an engine only one unit can possibly be expended in doing useful work.

By constructing, with the aid of the tables, similar diagrams for every pressure of steam, we should be able to

see at a glance the proportion between heat supplied and useful work done.

Expansion of Gas and Steam.

Isothermal lines.—Boyle's law, connecting the volume and pressure of gas, viz. pv=constant, assumes that during the variation of pressure and volume the temperature remains constant. Suppose a portion of air or gas inclosed in a cylinder, provided as before with a movable weighted piston. The inclosed gas would attain a certain definite volume, pressure, and temperature, the pressure being, of course, in equilibrium with the weight of the loaded piston. If now the load on the piston be diminished exactly as in the example on page 29, the gas will expand, raising the remaining weights through a certain space, and consequently doing external work. This work is done at the expense of the heat contained in the gas below the piston. The result will be that the temperature of the gas will fall by an amount which can be easily calculated when we know the quantity of external work done and the specific heat and the weight of the gas. Such expansion, then, does not fulfil the condition laid down in Boyle's law, that the temperature should remain constant.

In order that this latter condition should hold, heat must be supplied to the gas from some external source. It was shown before, that when the pressure and volume vary according to Boyle's law, the different states of the substance as regards pressure and volume for any given temperature may be represented graphically by the ordinates of a rectangular hyperbola. Such a line is said to be an *isothermal curve of expansion*, or, shortly, an *isothermal*; so called from two Greek words which signify equal and temperature, because the temperature is supposed to remain the same throughout all the changes in pressure and volume indicated by the co-ordinates of the curve. There is, of course, a separate isothermal for every temperature: for, with a given mass of gas, the variations in pressure and volume are different for different temperatures, though following the same law. For instance, if, at any given volume, the temperature is in one case 32°, and in another 100°, the pressure will be greater in the second than in the first instance, in accordance with Gay Lussac's law.

If we have a series of isothermal lines drawn to scale, as in fig. 14, for a portion of any gas, such as air, we can

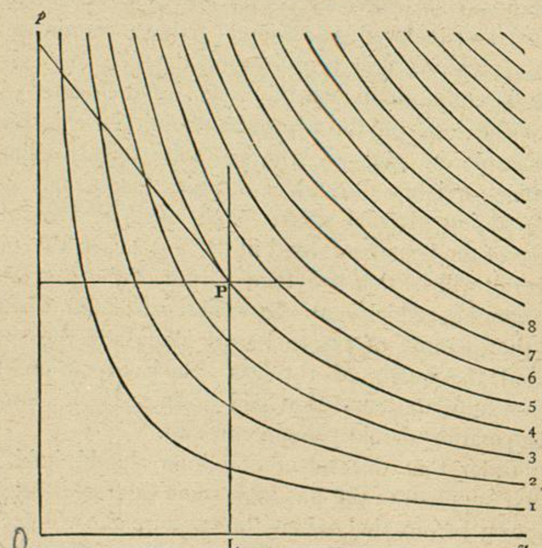

Fig. 14.

immediately find out by simple measurement either the temperature, the pressure, or the volume, when any two of these quantities are given. Each isothermal should be marked with the degree of temperature for which it is drawn. Suppose in the figure that there is a separate line drawn for each degree, and suppose that lines measured parallel to O*p* represent pressures, and those parallel to O*v* volumes. Let

[1] Figs. 14 and 15 are taken from Clerk Maxwell's *Theory of Heat.*

the temperature 4°, and volume equal to OL, be given, and let it be required to find the pressure, we have simply to draw an ordinate LP vertically upward, till it meets the isothermal for 4°, then LP will be the required pressure.

The isothermals of dry saturated steam are very different to those of a gas. Suppose that a pound of water in a cylinder closed by a piston be turned into steam of atmospheric pressure, and that the piston be then pressed down, while the temperature is *maintained at* 212°, the pressure will not rise at all, while the volume will diminish,

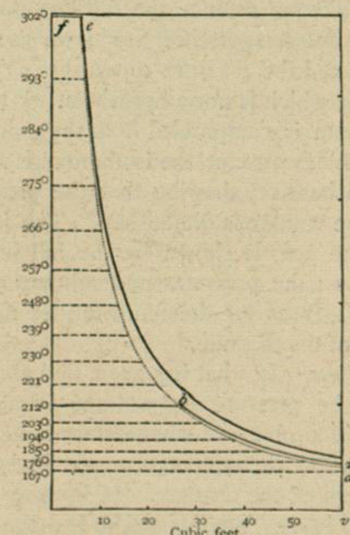

Fig. 15.

and, to permit of this taking place, part of the steam will be converted back into water. The reason of this is that dry saturated steam at a given temperature, say 212°, can exist at no *higher* pressure than the natural pressure of formation *at that temperature*, as shown by Regnault's tables. Take, for instance, steam of 341° This is the temperature at which the steam forms when the pressure is 120 pounds on the square inch, and it can exist at no higher pressure so long as the temperature remains the same. If, however, instead of pressing the piston down, it were raised up, the volume of the steam would increase, and if the temperature were maintained constant the pressure would diminish as the volume increased, but not strictly in accordance with Boyle's law; that is, as has been before explained, the product pv would not be quite constant, though nearly so; and the isothermal curve would consequently not be a perfect hyperbola. Fig. 15 illustrates the isothermal lines of steam and water. Take for instance the full line *cba* which is the isothermal for the temperature 212°; we see that as the volume is enlarged the pressure diminishes as the ordinates of the curve *ba*, which is not a hyperbola. When, however, the mass of steam is compressed from the point *b*, the temperature remaining constant the isothermal is a straight line parallel to the base *ov*, showing that the pressure remains unaltered as the volume is diminished. The isothermal for the temperature 302° is shown by the full line *fed*. The dotted line shows the pressures and volumes at which condensation commences for the temperatures marked on the left hand side of the diagram.

Adiabatic lines.—In what has been said above regarding variations in the pressure and volume of gas and steam, the only condition observed when determining the shape of the curves was that the temperature was to be maintained constant throughout the changes. If, however, a portion of gas be inclosed in a cylinder as before and the cylinder and piston be supposed to be absolute non-conductors of heat, and to be also incapable of communicating any heat of their own to the gas, and if the pressure be then made to vary, the temperature will fall when the volume increases; for external work will be done, and, as this work can only be done at the expense of the heat contained in the gas, its temperature must fall. Now the pressure of the gas, other things being equal, depends on the temperature; conse-

quently, when a gas expands in a non-conducting cylinder doing external work, its pressure will be less for a given volume than when it expands isothermally to the same volume.

Let us now, instead of allowing the gas to expand in the cylinder, compress it by adding weights to the piston. The piston and weights, when closing in upon the gas, do work upon it, and consequently raise its temperature. As the cylinder is non-conducting, none of the heat represented by this increased temperature can escape, and its effect consequently is to increase the pressure of the gas to a greater degree than would be the case if the compression took place isothermally.

The effect of this difference in the conditions can easily be exhibited graphically. Take two cylinders of equal size and inclose in each of them an equal portion of gas, so that in their initial state the volumes, pressures, and temperatures shall be the same. Let the ordinates of the point of intersection of the two curves on fig. 16 represent the initial pressure and volume of each portion of gas. Let one of the cylinders be a non-conductor and in the other let the gas change its volume isothermally. In the latter case the variations in its pressure and volume will be represented by the isothermal curve or hyperbola. If, however, the gas be prevented from receiving or parting with heat by the non-conducting cylinder, as it expands from the point of intersection, the fall in the temperature will cause the pressure ordinates at each point to be less than the corresponding ordinates of the isothermal, and the new curve will consequently fall below the hyperbola. When, on the other hand, the volume is diminished, this can only be accomplished by doing work

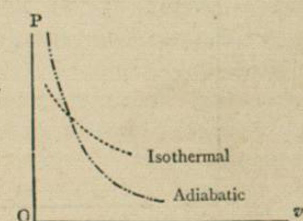

Fig. 16.

upon the gas, and, as no heat can escape, the temperature rises, and the pressure ordinates will consequently be greater than in the first case, and the curve will lie above the hyperbola. The second curve is represented on the diagram by the second dotted line. It is called an *adiabatic* line in contradistinction to the isothermal. The term *adiabatic* is derived from two Greek words signifying not passing through, and the line is so called because, in the operation above described, no heat passes through the cylinder either in or out.

Adiabatic lines of steam.—The adiabatic lines of dry saturated steam differ essentially from the corresponding lines for gases. Take a given mass of steam at the pres-

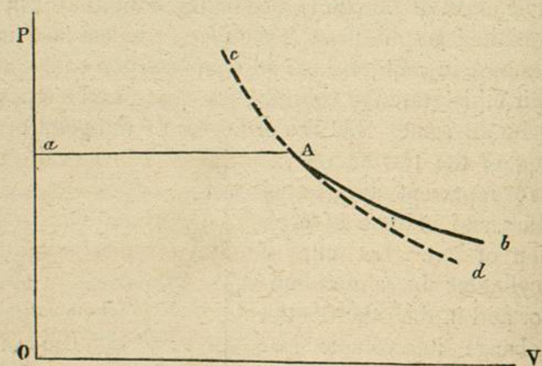

Fig. 17.

sure of the atmosphere, and the corresponding temperature 212°. Then, as we have seen, if the volume of the steam be diminished and the temperature preserved at 212°, the pressure will remain constant, and part of the steam will be changed back into water, because steam at the temperature 212° can exist at no higher pressure than that of the atmosphere. The isothermal will be then, as before explained, a horizontal straight line A*a*, fig. 17, while the volume is being diminished from the point A; and in order

to effect such a diminution and keep the temperature constant, heat must be abstracted from the steam. If, however, the compression be effected adiabatically, so that no heat can escape, the work done upon the steam by the act of compressing it will raise its temperature above 212°, and consequently enable it to exist at a higher pressure; and the more it is compressed, the greater will be the increase in the temperature, and consequently in the pressure. The adiabatic line therefore, during the diminution of volume of dry saturated steam, will be a curved line A*c*, resembling very closely the corresponding line for a gas. It has been shown by Rankine and Clausius that if the cylinder, in addition to containing the steam, held also some water at the same temperature, the heat generated by the compression is sufficient to cause some of this water to become steam.

If we now return to the point A, and allow the volume to increase, so that the steam does external work, without gaining or losing any additional heat by conduction from outside bodies, the temperature will fall, for the external work is done at the expense of the heat contained in the steam. The heat will diminish so much that when the pressure is reduced to any given extent there will no longer be the quantity of heat present in the mass of steam, necessary, according to Regnault's experiments, to maintain it in the dry saturated condition (see p. 60 and the Table, p. 489), and consequently part of the steam will condense back into water, and in so doing will part with that portion of its heat which effected its transformation from water into steam. The heat thus liberated will maintain the remainder of the steam in the dry and saturated condition. Consequently, during expansion from the point A, the volume for a given pressure will be less—being reduced by the condensation of so much steam—than if the whole of the steam were maintained throughout in the dry and saturated condition, and less still than if the temperature were maintained uniform throughout.

The above example is applied to steam of the atmospheric pressure and corresponding temperature, but any other temperature might have been chosen, and the same reasoning would have applied. Just as in the case of the isothermals, so with the adiabatic lines, a separate one can be drawn for every separate degree.

We have now examined into the nature of the expansion lines of gases and steam for two separate cases, viz. first, the case of the temperature being maintained constant throughout the change, and second, the case of no heat being allowed to escape from or reach the gas and steam by conduction, radiation, &c., from or to other matter.

It is evident, however, that these are not the only possible cases, for we might, had we wished, have supplied or abstracted any quantity of heat we chose, to or from the gas, during the process of alteration of volume and pressure, and thus have made the shapes of the curves of expansion anything we pleased. The two cases above described are, however, the most important.

CHAPTER III.

THEORETICALLY PERFECT HEAT ENGINES.

Application of Boyle's and Charles' laws to gases—Specific heat of gases at constant pressure and at constant volume—Cycle of operations—Ratio of heat expended to work done—Graphic representation of external work done during the expansion of a gas—Nature of the curves of expansion of gases as influenced by the supply of heat—Heat supplies: (1) when the curve of expansion is a rectangular hyperbola, the equation for which is $pv=c$; (2) when the equation of the curve takes the form $pv^n=c$, where n has any value except unity—Nature of the curve when no heat is supplied or abstracted—The ideally perfect heat engine—Calculation of the efficiency of such engines—The reversed action of the ideal heat engine—Proof that no other engine has a greater efficiency than the ideal heat engine—Practical limits of efficiency in the ideal heat engine—Laws connecting the pressure, temperature, and volume of dry saturated steam—Specific heats of water and steam—Law connecting the pressure, volume, and temperature of superheated steam—Total heat expended in converting water into steam—Proportion of total heat expended in doing external and internal work—Expenditure of heat in a steam engine when the steam is not used expansively—Method of representing heat by an 'equivalent pressure'—Expenditure of heat in a steam engine when the steam is used expansively, 1st, when the curve of expansion is a rectangular hyperbola, 2nd, when the steam remains dry and saturated throughout whole stroke—To realise latter condition steam jacket is necessary—Rankine's formulæ for the expenditure of heat in a steam engine—Theory of the ideally perfect heat engine applied to steam—How actual steam engines differ from the ideal heat engine—Summary of laws and formulæ.

The last chapter contained a sketch of the principles of the science of heat and an account of the effects of heat upon gases and water. The present chapter will deal with the conversion of heat into mechanical work through the instrumentality of heat engines, and will contain an account of an ideal heat engine which is perfect in theory; that is to say, no other conceivable engine can get more work out of the heat supplied to it than the one about to be described. Practical difficulties render the realisation of such an engine