The above example is applied to steam of the atmospheric pressure and corresponding temperature, but any other temperature might have been chosen, and the same reasoning would have applied. Just as in the case of the isothermals, so with the adiabatic lines, a separate one can be drawn for every separate degree.

We have now examined into the nature of the expansion lines of gases and steam for two separate cases, viz. first, the case of the temperature being maintained constant throughout the change, and second, the case of no heat being allowed to escape from or reach the gas and steam by conduction, radiation, &c., from or to other matter.

It is evident, however, that these are not the only possible cases, for we might, had we wished, have supplied or abstracted any quantity of heat we chose, to or from the gas, during the process of alteration of volume and pressure, and thus have made the shapes of the curves of expansion anything we pleased. The two cases above described are, however, the most important.

CHAPTER III.

THEORETICALLY PERFECT HEAT ENGINES.

Application of Boyle's and Charles' laws to gases—Specific heat of gases at constant pressure and at constant volume—Cycle of operations—Ratio of heat expended to work done—Graphic representation of external work done during the expansion of a gas—Nature of the curves of expansion of gases as influenced by the supply of heat—Heat supplies: (1) when the curve of expansion is a rectangular hyperbola, the equation for which is $pv=c$; (2) when the equation of the curve takes the form $pv^n=c$, where n has any value except unity—Nature of the curve when no heat is supplied or abstracted—The ideally perfect heat engine—Calculation of the efficiency of such engines—The reversed action of the ideal heat engine—Proof that no other engine has a greater efficiency than the ideal heat engine—Practical limits of efficiency in the ideal heat engine—Laws connecting the pressure, temperature, and volume of dry saturated steam—Specific heats of water and steam—Law connecting the pressure, volume, and temperature of superheated steam—Total heat expended in converting water into steam—Proportion of total heat expended in doing external and internal work—Expenditure of heat in a steam engine when the steam is not used expansively—Method of representing heat by an 'equivalent pressure'—Expenditure of heat in a steam engine when the steam is used expansively, 1st, when the curve of expansion is a rectangular hyperbola, 2nd, when the steam remains dry and saturated throughout whole stroke—To realise latter condition steam jacket is necessary—Rankine's formulæ for the expenditure of heat in a steam engine—Theory of the ideally perfect heat engine applied to steam—How actual steam engines differ from the ideal heat engine—Summary of laws and formulæ.

THE last chapter contained a sketch of the principles of the science of heat and an account of the effects of heat upon gases and water. The present chapter will deal with the conversion of heat into mechanical work through the instrumentality of heat engines, and will contain an account of an ideal heat engine which is perfect in theory; that is to say, no other conceivable engine can get more work out of the heat supplied to it than the one about to be described. Practical difficulties render the realisation of such an engine impossible, but the study of it is nevertheless of the greatest importance, as enabling us to find out the deficiencies of existing engines, and to ascribe to each of these deficiencies its due share in causing waste of heat.

On account of the greater simplicity of gas, it will be found convenient, first to describe the mode of operation of the ideal engine when worked with gas or air, and afterwards to apply the results obtained to the case of steam. Before doing so, however, it will be necessary to recapitulate the laws affecting gases which were explained in the last chapter, but with greater numerical exactitude, and then, from these laws, to make certain deductions, which, as they refer to the power of doing work through the medium of gases, are commonly classed under the head of the Thermodynamics of Gases.

Numerical application of Boyle's law to gases —The first of the laws referred to is Boyle's law, connecting the pressure and the volume of the gas when the temperature is maintained constant. The algebraical expression for this law was shown to be $pv=c$.

If one pound's weight of air be taken, at the pressure p_0 of the atmosphere, equal to 2116·8 lbs. on the square foot, at the temperature 32°, then the volume of this pound of air, or v_0, multiplied by the pressure on the square foot has been proved by Regnault's experiments to be

$$p_0v_0=26{,}214 \text{ foot-pounds.}$$

This quantity, 26,214 foot-pounds, is therefore the value of the constant c, so long as the temperature remains 32°.

If the temperature be changed, the value of the constant is changed also. This leads us to Gay Lussac's law (see page 46) connecting the pressure and volume with the temperature. This law states that the product pv is increased when the temperature is raised from 32° to 212°, in the ratio of 1 to 1·3654; and that for each of the 180 degrees intermediate between 32° and 212° the increase is

$\frac{1}{180}$th part of the increase at 212°. If then p_0v_0 be the pressure and volume at 32°, and pv be the pressure and volume at any other temperature $t°$, then if $t°=212°$,

$$pv=p_0v_0+\cdot3654\,p_0v_0,$$

and if $t°$ be any other temperature then

$$pv=p_0v_0+\frac{\cdot3654}{180}(t°-32°)\,p_0v_0.$$

This rate of increase of course applies also when the temperature is raised above 212°.

It was also shown (see page 51) that if the temperature be reckoned from the bottom of the tube of the air thermometer, which was shown to be 492° below 32° Fahrenheit, the above law could be greatly simplified.

For, the product of the pressure and volume of a portion of gas is proportional to the absolute temperature, so that if $\tau°$ be the absolute temperature corresponding to $t°$; then, remembering that 492·6° absolute, corresponds to 32° on the ordinary scale, and attaching the same values as before to all the other symbols, we have—

$$pv : \tau :: p_0v_0 : 492\cdot6$$

$$\therefore pv=\tau\frac{p_0v_0}{492\cdot6}.$$

Now p_0v_0 as stated above $=26{,}214$

$$\therefore \frac{p_0v_0}{493}=53\cdot2.$$

Hence we get $\quad pv=53\cdot2.\tau,$

which is a very simple expression, connecting the pressure, the volume, and the absolute temperature.

Specific heat of gases at constant volume, and at constant pressure.—The next law, which is now mentioned for the first time, relates to the specific heat of gases, and asserts that, if a gas be heated at constant pressure, it requires the same quantity of heat to raise its temperature from any point, say 212° to 213°, as it does from any other point, say 32° to 33°. In other words, the specific heat of a gas at constant pressure does not change with the temperature, as is the case with water.

The capacity of air for heat, that is, the amount of heat required to raise one pound of it through 1° of temperature, the pressure being maintained constant, is, according to Regnault's experiments, 0·2375 thermal units, equal to 183·35 foot-pounds. This quantity of heat is, as has been shown before, not all expended in merely raising the temperature of the air; for, the heating having been accomplished at constant pressure, part of the heat has been spent in doing external work.

Let v_1, τ_1, v_2, τ_2 be the original and final volumes and absolute temperatures of a pound of air; and let p be the pressure which remains constant. Then the external work is measured by the increase in the volume, viz. $v_2 - v_1$ multiplied by the pressure p; therefore

$$\text{External work} = (v_2 - v_1)p\,;$$

and, as we have seen, $vp = 53{\cdot}2 . \tau$; therefore

$$\text{The external work} = 53{\cdot}2\,(\tau_2 - \tau_1).$$

Also the total heat expended equals the specific heat multiplied by the number of foot-pounds in one thermal unit multiplied by the number of degrees of rise of temperature. The usual symbol for the specific heat at constant pressure multiplied by the number of foot-pounds in one thermal unit is K_p;[1] and as the rise in temperature is $\tau_2 - \tau_1$, we have

$$\text{Total heat expended} = K_p(\tau_2 - \tau_1).$$

Hence the heat expended in doing internal work—that is, in merely raising the temperature of the air—is the difference between the total heat expended and that part which is spent in doing external work

[1] K_p and K_v are spoken of hereafter for the sake of brevity and in accordance with a usual custom as specific heats; but in reality a specific heat is only a ratio, whereas K_p and K_v are absolute quantities.

Therefore the internal work $= (\tau_2 - \tau_1)\ (K_p - 53{\cdot}2)$.

Now this latter quantity within the right-hand bracket is also the value of the specific heat of air when heated at constant volume; because, as we know by Joule's experiment (see page 54), the mere separation of the particles of air requires no heat to effect it when no external work is done, and as the heat is only expended in doing external and internal work, and as, moreover, when the air is heated at constant *volume* no external work is done, therefore the specific heat of air heated at constant volume is the same as the internal specific heat at constant pressure, and, calling the specific heat at constant volume K_v, we have

$$K_v = K_p - 53{\cdot}2 = 130{\cdot}25 \text{ foot-pounds.}$$

Consequently the heat required to raise the temperature of one pound of air at constant volume from $\tau_1°$ to $\tau_2°$ is

$$K_v(\tau_2 - \tau_1).$$

From the equation $K_v = K_p - 53{\cdot}2$ we get, by simply transposing, $K_p - K_v = 53{\cdot}2$. That is to say, the difference in the two specific heats of air is equal to the constant quantity 53·2, which, as we have seen before, when multiplied by the absolute temperature, equals the product pv.

From the result given above for the value of the heat expended in internal work, when the air was heated at constant pressure, viz. $(K_p - 53{\cdot}2)\,(\tau_2 - \tau_1) = K_v(\tau_2 - \tau_1)$, we see that the internal work is proportional to the change of temperature, and is equal to the change of temperature multiplied by the specific heat at constant volume.

This result is true whether the air be heated at constant pressure, or at constant volume, or partly in the one way and partly in the other, or in fact in any way we can conceive of. For, as an example, first change the air from volume v_1, and temperature τ_1, to volume v_2, keeping the pressure constant at p_1; let the new temperature be τ; then by the above the heat expended in internal work is $K_v(\tau - \tau_1)$.

Next change the pressure from p to p_2, the volume being kept constant at v_2. To do this we must add heat to the gas, raising its temperature to τ_2; the heat spent is $K_v(\tau_2 - \tau)$, which is all internal work; adding to this the heat spent in doing internal work during the first part of the operation, we get

$$\text{Total heat spent in internal work} = K_v(\tau - \tau_1 + \tau_2 - \tau)$$
$$= K_v(\tau_2 - \tau_1).$$

This result might be proved to be true for any other case which might arise, by similar reasoning to the above, but it may also be shown to be generally true from the following considerations.

Cycle of Operations.—If a substance such as gas or water be subjected to the action of heat, and be thus brought through a series of changes of state, and eventually brought back to its original condition, it is said to have undergone a *Cycle of Operations.* During these changes of state heat has been expended in doing two things only, viz. external work, and internal work of various sorts, such as altering the temperature or the molecular condition of the substance. When, however, the body is brought back to its original condition, the sum of all the quantities of heat expended in doing internal work must be nil, because if during one part of the operation heat has been thus expended, when the substance is brought back to its original condition this heat is again liberated or rejected.

Now when the state of gas or air is changed by the action of heat in any way whatever we can analyse the operation into three distinct sets of processes, viz.,

1st. Heating at constant pressure, the volume being changed;

2nd. Heating at constant volume, the pressure being changed; and

3rd. One or more cyclical processes.

Now during the latter processes no heat is spent in

internal work, and during the two former the heat thus spent is as we have seen $= K_v(\tau_2 - \tau_1)$. Hence the proposition is universally true that when the state of a gas is changed by the action of heat, the quantity of heat spent in doing internal work depends only on the difference of temperatures of the two states, and is equal to the specific heat at constant volume multiplied by this difference of temperatures.

From the above fundamental laws we are enabled to reason on all the questions which may arise regarding the thermodynamics of gases. All that we require to know is, how much heat is expended in doing external, and how much in doing internal work. The total heat expended is equal to the sum of these two quantities. When we possess this information we can deduce all that it is requisite to know regarding the pressure and temperature at every stage of the process. Conversely if we know the pressure and temperature we can calculate the external and internal work done, and the expenditure of heat. The internal work is, as has been proved above, always given by the expression $K_v(\tau_2 - \tau_1)$. The external work is different in different cases. For instance, if during the changes of volume and pressure sufficient heat be supplied to keep the temperature uniform, we get a certain quantity of external work. If on the contrary no heat be supplied we get quite another quantity, and if more than enough or less than enough heat be supplied to keep the temperature uniform, we get still different quantities of external work done in each case.

The quantity of external work done is perhaps best calculated and exhibited by means of diagrams. We have seen (see page 44) how the varying pressure and volume of a portion of gas can be represented by the ordinates of a line GD, fig. 11. We also saw (see page 63) how work done could be represented by the area of a rectangle. An extension of these methods will now be explained.

Let *ac*, *c*O, fig. 18, represent the initial, and *bd*, *d*O the final pressures and volumes of a portion of gas. Let the intermediate co-ordinates of the curve *ab* represent the intermediate pressures and volumes while the gas is expanding. Draw a line *ef*, indefinitely near and parallel to the line *ac*. While the volume of the gas has been increasing from O*c* to O*f*, the pressure has been falling from *ac* to *ef*. Now the external work done is represented by the increase in volume *cf*, multiplied by the pressure. The pressure in this case is not uniform, but decreases as the ordinates of the curved line *ae*. We must therefore multiply the increase of volume by the average pressure. It is difficult to find the average pressure when the line *ae* is curved; but if we take *ef* as being very near indeed to *ac*, we may regard *ae* as being to

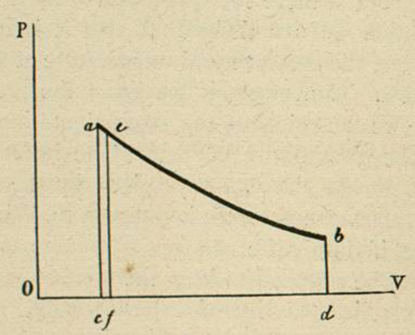

Fig. 18

all intents and purposes a straight line, and in this case the average pressure will be represented by the mean between the two lines *ac* and *ef*, viz. $\frac{ac+ef}{2}$. The external work done is therefore represented by the expression $cf \times \frac{ac+ef}{2}$.

But this expression also represents the area of the strip *aefc*, therefore the external work done while the volume of the gas is increasing from O*c* to O*f* is represented by the area *aefc*. We can divide the whole figure *abdc* up into a series of such strips, and the above reasoning would hold good for each of them. Now the sum of the areas of these strips equals the area of the figure *abdc*; therefore when the volume of a portion of gas increases from O*c* to O*d*, the pressure at the same time varying as the vertical ordinates of the line *ab*, the external work done during the process is represented by the area inclosed by the base line *cd* representing the increase of volume, the vertical lines *ac* and *bd* representing the initial and final pressures, and the line *ab*, which represents the way in which the pressure varies.

The line *ab* may be anything that we please. For instance, if during the expansion of the gas enough heat were supplied to it to keep the pressure uniform throughout, it would be a straight line drawn through *a*, parallel to OV. If enough heat were supplied to keep the temperature uniform, the line would, as has been proved, be an isothermal, which for gases is a common rectangular hyperbola denoted by the equation $pv =$ constant.

If no heat were supplied and none allowed to escape, the line would be an adiabatic, the equation for which will be shown to be $pv^{\gamma} =$ constant. The symbol γ which is of very constant occurrence denotes the ratio $\frac{K_p}{K_v}$.

Most of the lines occurring in the theory of the heat engine are denoted by the equation $pv^n =$ constant, where the index n varies according to the supply of heat. For instance, the two preceding cases are special instances of this equation in the first of which $n=1$, and in the second $n=\gamma$.

The area of the figure *abdc* depends, of course, upon the special form of the line *ab*, and can be readily calculated for each case by those who are familiar with the processes of analytical geometry.

We can now examine into the most important cases that arise.

1. Heat expended in changing the state of a gas when the temperature remains constant throughout the change. The total heat expended = the internal work done + the

external work done. The internal work in this case is nil, because the temperature does not change, and consequently the expression K_v $(\tau_2 - \tau_1)$ vanishes.

The external work is obtained by calculating the area of the diagram *abdc* (see fig. 18). We assume that the initial pressure and volume are represented respectively by the line *ac*, drawn to scale so as to represent pounds on the square foot, and O*c* drawn to represent cubic feet. Similarly the lines *bd* and O*d* represent the final pressure and volume to the same scale. As the temperature is uniform the line *ab* is a rectangular hyperbola, having for its asymptots OV and OP;

$$\text{and the area}^1 \text{ of the figure } abdc = ac \times Oc \times \log_e \frac{Od}{Oc}.$$

Also, as by the principle of the hyperbola $ac \times Oc = bd \times Od$

$$\therefore \text{ the area is also} = bd \times Od \times \log_e \frac{Od}{Oc}.$$

It will be noted that the logarithms used are hyperbolic. A table of the hyperbolic logarithms of the most useful numbers will be found at the end of the book (see page 498).

[1] The area is calculated in the following manner:

Let *a b*, fig. 18, be a curve of the equation.

$$pv = \text{constant}.$$

The area *a b d c*, is the sum of a number of small strips such as *a f*. By making these strips indefinitely narrow they may each be represented by the expression $p \times dv$, where p represents the height, and dv the indefinitely small width.

Let $ac = p_1$; $Oc = v_1$ and $bd = p_2$; $Od = v_2$.

$$\text{Then the area} = \int_{v_1}^{v_2} p.dv.$$

$$\text{Now as } pv = p_1 v_1$$

$$\therefore p = \frac{p_1 v_1}{v};$$

and substituting

$$\text{Area} = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v}$$

$$= p_1 v_1 (\log_e v_2 - \log_e v_1)$$

$$= p_1 v_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e r.$$

The ratio of the final volume O*d* to the initial volume O*c* is called the ratio of expansion, and is generally denoted by the symbol *r*. Calling the initial and final volumes v_1 and v_2, and the initial and final pressures p_1 and p_2 respectively, the expression for the area becomes $p_1 v_1 \log_e r$, or $p_2 v_2 \log_e r$ foot-pounds. Also if τ be the absolute temperature of the gas, then, as we have seen, $p_1 v_1 = p_2 v_2 = c\tau$,

$$\therefore \text{ the area} = c\tau \log_e r \text{ foot-pounds.}$$

This quantity is, therefore, the expenditure of heat in foot-pounds when a gas expands isothermally from volume v_1 to v_2.

2. Let the curve *ab*, instead of being an hyperbola of the equation pv=constant, be a curve of the form

$$pv^n = \text{constant},$$

where n may have any value we like to assign to it except unity. In this case the area[1] of the figure *abdc*

$$= \frac{p_1 v_1 - p_2 v_2}{n - 1}$$

[1] The area is calculated in the following manner:

Let *a b*, fig. 18, be a curve of the equation

$$pv^n = \text{constant}.$$

To find the arc *a b d c*

Let $ac = p_1$; $Oc = v_1$ and $bd = p_2$; $Od = v_2$;

also let p be any pressure ordinate, and v the corresponding volume.

$$\text{Then the area} = \int_{v_1}^{v_2} p.dv.$$

$$\text{Now as } pv^n = p_1 v_1^n$$

$$\therefore p = \frac{p_1 v_1^n}{v^n};$$

and substituting

$$\text{Area} = p_1 v_1^n \int_{v_1}^{v_2} \frac{dv}{v^n}$$

$$= p_1 v_1^n \frac{v_2^{1-n} - v_1^{1-n}}{1 - n}$$

$$= \frac{p_1 v_1 - p_1 v_1^n v_2^{1-n}}{n - 1}$$

$$= \frac{p_1 v_1 - p_2 v_2}{n - 1}.$$

Let τ_1 be the initial, and τ_2 the final absolute temperatures. The expression for the area becomes then

$$\frac{c\tau_1 - c\tau_2}{n-1} = \frac{c}{n-1}(\tau_1 - \tau_2).$$

As before, the heat expended = the internal work + the external work

$$= K_v(\tau_2 - \tau_1) + \frac{c}{n-1}(\tau_1 - \tau_2).$$

Also, as has been proved before, $c = K_p - K_v$;

$$\therefore \frac{c}{n-1} = \frac{K_p - K_v}{n-1} \text{ and } \frac{c}{n-1}(\tau_1 - \tau_2) = \frac{K_v - K_p}{n-1}(\tau_2 - \tau_1);$$

$\therefore$ substituting this value of $\frac{c}{n-1}(\tau_1 - \tau_2)$ in the above equation, we get heat expended

$$= (\tau_2 - \tau_1)\left(K_v + \frac{K_v - K_p}{n-1}\right) = (\tau_2 - \tau_1)\left(\frac{nK_v - K_p}{n-1}\right).$$

3. Let no heat be communicated to or taken from the gas during the expansion. In other words, let the line *ab* be an adiabatic curve.

The last expression for heat expended, viz. $(\tau_2 - \tau_1)\left(\frac{nK_v - K_p}{n-1}\right)$ must, when no heat is expended, equal zero. Hence one or other of the terms within the brackets must equal zero. We know, however, that $(\tau_2 - \tau_1)$ cannot equal zero; because during the expansion the temperature falls, and therefore τ_2 is less than τ_1. If, therefore, we make the other term, viz. $\frac{nK_v - K_p}{n-1} = 0$, we get

$$nK_v - K_p = 0 \quad \therefore n = \frac{K_p}{K_v} = \gamma.$$

The equation pv^n = constant, becomes therefore pv = constant for the case of adiabatic expansion.

As no heat is supplied to the gas, the external work

must be done at the expense of the heat already existing in the gas, consequently the temperature falls, and the internal work done is of a negative character

The temperature at the end of the expansion may be found in the following manner, using the same symbols as before. We have $c\tau_1 = p_1v_1$, and $c\tau_2 = p_2v_2$.

$$\text{Now } p_1v_1^{\gamma} = p_2v_2^{\gamma} \qquad \therefore p_2v_2(v_2^{\gamma-1}) = p_1v_1(v_1^{\gamma-1})$$

$$\therefore p_2v_2 = p_1v_1\left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$\therefore c\tau_2 = c\tau_1\left(\frac{v_1}{v_2}\right)^{\gamma-1} \quad \therefore \tau_2 = \tau_1\left(\frac{1}{r}\right)^{\gamma-1}$$

which, as $\gamma = 1{\cdot}408$ becomes $\tau_2 = \tau_1\left(\frac{1}{r}\right)^{{\cdot}408}$

an expression which, when the initial pressure and volume and the ratio of expansion are given, enables us to find the final temperature.

Supposing in all the above examples that the gas were compressed back to its original condition, the varying pressures and volumes could be represented graphically, just as in the case of expansion. If the conditions of compression were the same as those of expansion, the same curve would represent each operation. For instance, if the temperature were maintained constant the curve would be an hyperbola. If no heat were added or abstracted, the compression curve would be an adiabatic line, and the temperature would rise as the compression continued. If the conditions of the compression were different to those of the expansion, the curve would also be different.

The Ideally Perfect Heat Engine.

We are now in a position to examine into the theory and conditions of working of the ideal heat engine referred to at the beginning of the chapter. This engine requires to be made of materials which do not exist in practice; the only object in discussing it is therefore to separate the action of the heat on the gas from the accidents of its surroundings, so that we may be enabled to ascribe to the surroundings in actual engines their proper influence. The efficiency of the action of this engine does not depend in any way upon the mechanism by which its motion may be converted, but only on the manner in which it receives and rejects heat. We will therefore, for the sake of simplicity, suppose it to consist of

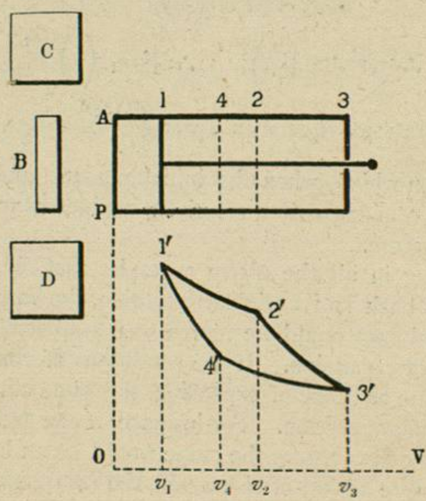

Fig. 19.

a working cylinder, connected by means of a piston and connecting rods with a crank.

Let AP, fig. 19, be the working cylinder which contains between the end AP and the piston when at its initial position 1 a certain quantity, say one pound, of gas at the temperature τ_1. The space in front of the piston is supposed to be a perfect vacuum, so that the engine is single-acting. The sides of the cylinder are supposed to be made of an ideal substance which can neither give heat to nor

receive it from the gas. The bottom of the cylinder is, on the contrary, supposed to be made of a substance which though it has no capacity for heat itself, is nevertheless a perfect conductor of heat. C is a source of heat having the absolute temperature τ_1. D is similarly a body having a temperature τ_2 less than τ_1 which is used for the reception of the heat rejected by the gas. B is a cover made of the same material as the sides of the cylinder which, when applied to the bottom AP, renders it perfectly non-conducting like the rest of the cylinder.

Let the piston commence to move forwards from the position 1. To prevent the temperature of the gas from falling, apply the source of heat C to the end of the cylinder. As this end is a perfect conductor, heat will flow into the gas and maintain the temperature constant. Let the co-ordinates of the point 1′ with reference to the lines OV, OP denote the initial volume and pressure of the gas. So long as the body τ_1 is kept in contact with the cylinder end, the gas will expand isothermally, and the variations in its pressure and volume will be denoted by the co-ordinates of the hyperbolic curve 1′2′. When the piston has reached the point 2, the exact position of which will be presently determined, withdraw the body τ_1 and apply the non-conducting cover B. The gas will now continue to expand adiabatically, as represented by the curve 2′3′. During this part of the expansion the temperature will fall, and the point 2′ must be so chosen that the temperature will, when the piston has reached the end of its stroke, have fallen to that of the cold body, viz. τ_2. The piston must now be caused to return in the opposite direction. As the engine is single-acting, this can only be done by the application of forces external to the engine. The piston when returning will compress the gas and do work upon it. To prevent the temperature from rising apply the cold body D to the end of the cylinder. The compression will then take place isothermally, at the temperature τ_2, along the hyperbolic curve 3′4′. When the piston has reached the point 4—the position of which will be presently determined—the cold body must be withdrawn, and the non-conducting cover reapplied to the end of the cylinder. The compression of the gas must then be continued, and as no heat can escape it will take place adiabatically along the line 4′1′. During this part of the compression the temperature will rise, and, if the point 4 has been rightly chosen, it will reach τ_1 when the piston has returned to its original position.

During the first operation the gas has been receiving heat, and doing external work, which is represented by the area of the figure 1′ 2′ v_2 v_1. During the second operation the gas has received no heat from without, but, at the expense of the heat which it already possessed, it has done external work, represented by the area 2′ 3′ v_3 v_2. During the third operation, work has been done upon the gas, measured by the area 3′ 4′ v_4 v_3, and the gas has rejected heat into the body D; and during the fourth and last operation work has been done upon the gas represented by the area 4′ 1′ v_1 v_4, with the result of restoring it to the original pressure, volume, and temperature. We see therefore that the work done by the gas exceeds the work done upon the gas, by the difference between the sum of the two first, and the sum of the two last-mentioned areas. This difference is equal to the area 1′ 2′ 3′ 4′, which therefore represents the effective work done by the engine.

Calculation of the efficiency of perfect heat engines.—We must now calculate the heat expended, and the work done. During the first operation, the heat supplied to the gas is all expended in doing external work: for no internal work is done, as the temperature of the gas is not raised. The heat supplied therefore in foot-pounds is equal to the area 1′ 2′ v_2 $v_1 = p'$ v' $\log_e r = c\tau_1 \log_e r$, where r is as before the ratio $\frac{v_2}{v_1}$. During the second operation no heat is supplied to the gas. During the third operation, the gas

rejects heat equal in amount to the work done upon the gas $= c\tau_2 \log_e r$, where r is the ratio $\frac{v_3}{v_4}$ which will be presently proved $= \frac{v_2}{v_1}$. During the fourth operation the gas rejects no heat. The total heat supplied therefore $= c\tau_1 \log_e r$, and the total heat rejected $= c\tau_2 \log_e r$.

The work done can be calculated in two ways. We may either compute the area 1′ 2′ 3′ 4′, or we may make use of the principle of the cycle of operations. For since the gas returns to its original condition no heat is spent in doing internal work upon the gas and the heat expended must therefore equal the external work done, plus the heat rejected.

Consequently the external work

$$= c\tau_1 \log_e r - c\tau_2 \log_e r = (\tau_1 - \tau_2)\, c \log_e r.$$

The efficiency of the engine is the ratio of the work done to the heat expended

$$= \frac{(\tau_1 - \tau_2)\ c \log_e r}{\tau_1\, c \log_e r} = \frac{\tau_1 - \tau_2}{\tau_1}$$

That is to say the efficiency of the engine is the ratio of the difference of temperatures of the sources of heat and of cold to the temperature of the source of heat; the temperatures being reckoned in absolute measure.

The efficiency of the engine can only become equal to unity, i.e. the engine can only turn the whole of the heat supplied to it into mechanical work, when the temperature $\tau_2 = 0$; that is to say, when the cold body has the absolute zero of temperature; a result which is of course unattainable.

On the other hand, the nearer to unity the fraction $\frac{\tau_1 - \tau_2}{\tau_1}$ becomes, the greater is the efficiency of the engine.

This result can only be attained by making $\tau_1 - \tau_2$ as nearly as possible equal to τ_1. To do this we must make τ_1 as large and τ_2 as small as possible.