the pound of steam is p, the final pressure p_f, and that the steam is dry and saturated at the end of the stroke. Now, the heat rejected is the amount of heat in the steam at the end of the expansion, together with the work done upon the exhausting steam, by the piston overcoming the back pressure p_b. The heat in the steam at the end of the expansion is the total heat of formation of dry saturated steam of the pressure p_f, minus the amount due to doing external work $= p_f v_f$; for, of course, the heat spent in doing external work disappears from the steam, having been transmuted into the work done.

$$\text{Consequently heat rejected} = H_f - p_f v_f + p_b v_f,$$

where $p_b v_f$ is the work done in overcoming the back pressure p_b, through the space v_f, and H_f is the total heat of formation of dry saturated steam of the pressure p_f.

Now, the heat expended equals the heat rejected, plus the external work done by the steam during admission and expansion. If p_m be the mean or average pressure throughout the stroke, then $(p_m - p_b)v_f$ is the external work done, and consequently

$$\text{Heat expended} = H_f - p_f v_f + p_b v_f + (p_m - p_b)v_f$$
$$= H_f + p_m v_f - p_f v_f$$

If we wish to express these results per cubic foot swept through by the piston, we have only to divide by v_f, the number of feet occupied by the steam at the end of the stroke, and we get

$$\text{Heat expended} = \frac{H_f}{v_f} + p_m - p_f.$$

Steam Jackets.—We must now ascertain whether the heat contained in the steam, as supplied by the boiler, is as much as the above quantity $\frac{H_f}{v_f} + p_m - p_f$, for if not, one of two things must happen, viz. either more heat must be supplied to the steam while it is in the cylinder from some

external source, or else at the end of the stroke it will not be dry and saturated, but a certain proportion will be condensed into water.

Let H_i be the total heat contained in a pound of the steam, in its initial condition, as supplied by the boiler, then, as v_f equals the contents of the cylinder, or the number of cubic feet swept through by the piston in one stroke, therefore $\frac{H_i}{v_f}$ is the expenditure of heat per cubic foot swept through, provided that no heat is obtained from any other source than from the boiler. But the heat actually expended per cubic foot swept through is not $\frac{H_i}{v_f}$ but $\frac{H_f}{v_f} + p_m - p_f$. Subtracting, therefore, the first from the last of these quantities, we get a difference $= p_m - p_f + \frac{H_f - H_i}{v_f}$. Now, the numerical value of the two quantities H_i and H may be taken from Table I., and as H_i is always greater than H_f, the quantity $\frac{H_f - H_i}{v_f}$ will always be negative, and for every particular case the difference $p_m - p_f + \frac{H_f - H_i}{v_f}$ will be found to be a positive quantity; therefore, the heat actually wanted for the steam in order that it may remain dry and saturated is greater than the quantity present in the steam as it is supplied by the boiler.

The difference must therefore be supplied to the steam while in the cylinder, and this is usually effected by surrounding the latter with a casing always kept full of boiler steam or hot air. The temperature of the steam in the jacket is evidently greater than the mean temperature of the steam in the cylinder, and consequently heat will flow from the former to the latter, and will either check or wholly prevent condensation, according to the quantity of heat thus supplied.

The question, whether or no it is desirable to prevent condensation during expansion, is a rather complicated one, and will be discussed in Chapter XI.

In the foregoing we were only concerned with proving that if the condition be given that the steam must be dry and saturated at the end of the expansion, in order to fulfil this condition, heat must be supplied to the steam from a hot casing, which is generally called a steam jacket.

Rankine's empirical formulæ for the expenditure of heat in a steam engine.—From the above formula for the expenditure of heat, viz. : $\frac{H_f}{v_f}+p_m-p_f$ it would be easy to construct a numerical formula involving only the mean and final pressures, and the temperature of the steam and feed water and certain constants. It has, however, been found by Rankine that the results are equally well given by a very simple empirical formula which for condensing engines is :

$$\text{Heat expended} = p_m + 15p_f;$$

and for non-condensing engines :

$$\text{Heat expended} = p_m + 14p_f:$$

the results being expressed in foot-pounds per cubic foot swept through by the piston.

Theory of the Perfect Heat Engine applied to the Case of Steam.

We can now proceed to apply the principles laid down with regard to perfect heat engines to the case where steam is employed instead of a gas. The amount of steam and fuel necessary for a perfect steam engine under given circumstances will first be considered. The nature of the diagram indicating the varying states of the steam in such an engine will then be examined, and finally the question will be discussed how far actual steam engines of the best

construction comply with, and how far they depart from, the conditions of maximum efficiency.

The efficiency of a perfect heat engine has been shown (see page 89) to be $\frac{\tau_1-\tau_2}{\tau_1}$, where τ_1 and τ_2 are the absolute temperatures of the sources of heat and cold. Hence, in such an engine, if H be the quantity of heat supplied, and W the exterior work done, we obtain the relation

$$H.\frac{\tau_1-\tau_2}{\tau_1}=W \text{ or } H=W.\frac{\tau_1}{\tau_1-\tau_2}.$$

Hence if we require to know the least amount of heat necessary in order to obtain one horse-power per hour when the limits of temperature within which the engine works are known, we have $W=33{,}000\times 60=1{,}980{,}000$ foot-pounds,

$$\text{and } H=1{,}980{,}000\times\frac{\tau_1}{\tau_1-\tau_2}\text{ foot-pounds.}$$

In a steam engine the limits of temperature ought to be the temperature of the hot gases in the furnace of the boiler on the one hand, and the temperature of the condenser on the other, or in the case of non-condensing engines, the lower limit is the temperature due to the pressure of the atmosphere, i.e. $212°+461°$ absolute. We possess at present, however, no means of utilising the temperature of the furnace gases, and consequently the higher limit in a steam engine must be taken to be the temperature of the steam in the boiler.

Let us consider the case of a perfect engine working with steam of 60 pounds pressure, as before; the temperature of the condenser being $104°+461°$ absolute. The absolute temperature of steam of 60 pounds pressure is $293°+461°=754°$. The quantity of work obtained per pound of steam is the total heat contained in the steam, multiplied by the fraction $\frac{754-565}{754}=\frac{1}{4}$ very nearly. Now, a perfect engine, as will be seen presently, always uses the same water over and over again, and always evaporates it from the temperature of the steam; consequently the heat supplied to the water in order to turn it into steam of 293° is less than the quantity given in Table I., by the amount necessary to heat the water from 32° to 293°; that is to say, the quantity of heat in question is 904,106 – 202,444 = 701,662 foot-pounds. Therefore the work obtainable per pound of steam is $701{,}622 \times \frac{1}{4} = 175{,}413$ foot-pounds.

In order to obtain from this engine one horse-power per hour we should require to expend therefore $\frac{1{,}980{,}000}{175{,}413} = 11 \cdot 3$ pounds of steam. If we wish to find out at what expenditure of fuel this power is attained, we must know what heat can be developed by the combustion of a given weight of fuel. This subject will be fully dealt with in the chapter on boilers; but at present it may be stated that one pound of good average coal properly burnt should give 12,000,000 foot-pounds of heat. Now as one pound of steam of 60 pounds pressure requires for its formation, from water of 293°, 701,662 foot-pounds, the pound of coal should theoretically be able to evaporate $\frac{12{,}000{,}000}{701{,}662} = 17$ pounds of water, and consequently we ought to require $\frac{11 \cdot 3}{17} = \cdot 66$ pound of coal per horse-power per hour.

The actual amount of water which a pound of fuel can evaporate in a good boiler is, however, much less than the above; in fact, as will be seen hereafter, it seldom exceeds eleven, and is more often from seven to eight pounds. If, for the sake of simplicity, we suppose that 11·3 pounds of water are evaporated by a pound of coal, then, in the case of the engine under discussion, we should require to burn one pound of fuel per horse-power per hour. In the best constructed modern steam engines working with steam of the pressure under discussion, viz. sixty pounds absolute,

or about forty-five pounds above the atmosphere, the amount of fuel burnt per horse-power per hour is far greater than one pound. It is, in fact, never less than two pounds, while in engines of inferior construction, from eight to ten pounds, and even larger quantities, are consumed. We see, therefore, plainly, that, in addition to the loss of heat which takes place in the boiler, there are other sources of waste. It becomes, then, necessary to compare the working of an actual engine with that of the theoretical engine, step by step, in order to ascertain all the causes of inefficiency.

Causes of loss of efficiency in steam engines.—In accordance with the principles laid down, the water should receive all its heat at the fixed higher temperature; in other words, it should be turned into steam at constant pressure. The steam should then be allowed to expand adiabatically, till the temperature falls to that of the condenser. It should next reject heat into the condenser at this fixed temperature, though none of the steam itself is supposed to enter the condenser. Finally, at a given point, the cooled steam, or mixture of steam and water as it would probably be, should be compressed till it returns to its first condition of water, of the original temperature of the steam.

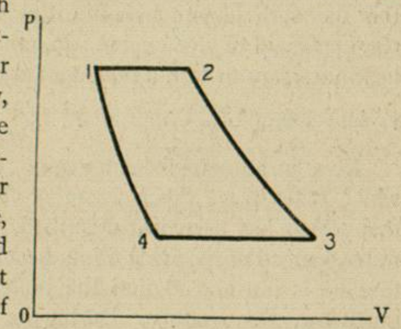

Fig. 22.

These changes are indicated by fig. 22.

Let the point 1 represent the volume and pressure of one pound of water, the pressure being that at which the steam is formed. In strict theory, the steam ought to be formed at the temperature of the furnace gases. This, however, as has been before stated, is practically impossible, and we will suppose for the moment that the water receives its heat at the constant temperature, due to the pressure at which the steam is formed. During evaporation the pressure remains constant and the volume increases, till the whole of the water is converted into steam. This state of things is indicated by the point 2. The steam is now allowed to expand adiabatically along the curve 2 3 till the temperature has fallen to that of the source of cold. The volume is then reduced at the constant pressure, corresponding to the temperature of the source of cold, till the point 4 is reached, when the mixture of steam and water is compressed adiabatically along the line 4 1, till the whole is re-converted into water of the original temperature, pressure, and volume. The area of this diagram, as before, represents the external work done during the cycle of operations, and it may either be computed analytically, if we know the equations of the adiabatic curves, or, calculated more simply on the principle that the work done equals the heat supplied, multiplied by the efficiency of the engine; in other words it equals the heat necessary to turn a pound of water of the temperature τ_1 into steam of τ_1, multiplied by $\frac{\tau_1 - \tau_2}{\tau_1}$.

Now, in an actual steam-engine, the series of operations which takes place differs more or less at every step from that which has been just described. In the first place, the water, instead of receiving all its heat at the higher temperature τ_1, is introduced into the boiler as feed water at a much lower temperature. During the process of evaporation, the condition of receiving heat at constant pressure is fulfilled, so long as the pressure in the boiler is kept constantly the same, which it never can be when the steam is worked expansively.

During the expansion, the condition that heat shall not be supplied to, or abstracted from, the steam is not fulfilled, because the metals of which cylinders are constructed render the fulfilment of this condition impossible. Cylinders

are of three sorts; the first sort is made of metal directly exposed to the outer air. In this case, the metal being a good conductor is rapidly heated by the steam, and parts with its heat to surrounding bodies by radiation and conduction, thus causing the steam to be cooled during its passage through the cylinder, so that the expansion line falls below the proper adiabatic curve.

The second class of cylinder is clothed with some non-conducting substance, so as to prevent the escape of heat to outside bodies. For the sake of simplicity, we will suppose the substance to be a perfect non-conductor. When first the steam enters such a cylinder it finds the metal cool, and parts with some of its heat; after a few strokes, however, the cylinder gets warmed, and if the temperature of the steam remained uniform throughout the entire stroke, no further loss would ensue from this cause. But the temperature of the steam is only uniform while the line 1, 2 is being described; after that point, and during expansion, the temperature drops from τ_1 to τ_2; consequently, when the steam enters, it finds the sides and end of the cylinder cooled down, they having just been in contact with steam of the temperature τ_2. Part of the heat of the steam, therefore, is spent in re-heating the metal of the cylinder. This causes part of the steam to condense, if it be originally in the dry and saturated condition. When, however, the expansion begins, the temperature of the steam rapidly drops below the temperature of the walls of the cylinder. These latter consequently give up part of their heat again to the steam, and partially re-evaporate the condensed portions. This re-evaporation is facilitated by the circumstance that a portion of the condensed steam has the temperature τ_1, and consequently, when the expansion commences and the pressure falls, it is too hot to remain any longer in the condition of water, and its surplus heat helps in re-evaporating it. Thus, though no heat is lost to external objects during the stroke, when the cylinder is perfectly clothed, still, during one period, heat is taken from, and during another period given back to the steam, by the cylinder, and consequently, the curve of expansion is not, strictly speaking, adiabatic. The exact effect of this peculiar action on the shape of the expansion curve is difficult to ascertain, because the rapidity with which the metal of the cylinder can take up and give off heat is not accurately known. The subject will, however, be further examined in Chapter XI.

The third description of cylinder is surrounded by a jacket or casing filled with steam from the boiler, and which is itself covered with non-conducting substances so as to prevent the escape of heat to the outside. It is evident that in this case the temperature in the jacket is higher than the average temperature in the cylinder, and consequently heat will flow from the former to the latter throughout a great part of the stroke, and will thus tend to raise the curve of expansion above the adiabatic line. The effect of the action of the jacket upon the working of the steam engine will be more particularly considered in Chapter XI.

During the third operation in an actual steam engine, viz. the rejection of heat, the condition of maximum efficiency is not fulfilled, for the heat is not all rejected at the temperature of the condenser. If the expansion were carried so far that the temperature of the steam were reduced to the temperature of the condenser, this condition could be fulfilled, but in practice it is not found possible to carry the expansion so far, and consequently, when condensation commences, there is a sudden drop from the temperature due to the terminal pressure of the steam, to that of the condenser. This is illustrated by the diagram fig. 23, where the point 3 represents the pressure of the steam at the end of the expansion, and the vertical ordinate of the point 4 represents the back pressure due to the temperature of the condenser. When the steam commences to reject its heat, the temperature suddenly falls from that due to the pressure of the point 3 to that due to point 4. In strict theory the

expansion should have been continued to the point 3′, where the curve intersects the horizontal line of back, or condenser pressure.

During the remainder of the period of heat-rejection the condition of maximum efficiency is fulfilled very approximately. At this part of the process, however, another evil arises, for the metal of the cylinder was heated up to a certain temperature during the admission and expansion of the steam; when, however, the steam is being condensed, its temperature is much lower, and consequently the cylinder parts with some of its heat to the condensing steam, thus retarding condensation, and cooling the cylinder down;

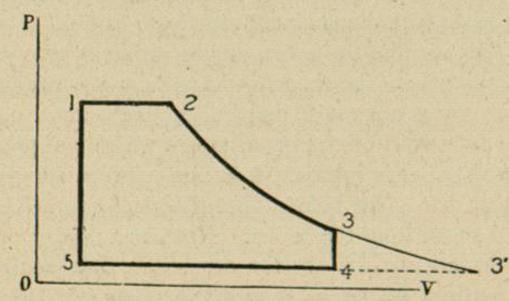

Fig. 23.

so that, as has been before stated, some of the fresh steam on entering is condensed. If the cylinder be provided with a steam jacket the first of these evils may be increased, for, during half the time such an engine is running, the steam jacket is employed in wasting heat on the condensing steam.

The fourth condition of maximum efficiency, viz. that at a certain point the rejection of heat should be stopped, and the mixture of steam and water in the cylinder should be compressed into water of the original pressure and temperature, which water should be used over again in the boiler, is not fulfilled at all in the ordinary steam engine. If the engine be of the condensing type, the condensation is carried out completely, and all the heat in the steam is spent in warming up from 20 to 30 times its own weight of water employed in the condensation to a temperature about one third of that of the water in the boiler. Of this water, only one twentieth to one thirtieth can be used over again to feed the boiler, and must, when in the boiler, be suddenly raised from the temperature of the condenser to that of the steam, thus infringing the first condition of efficiency.

If, on the other hand, the engine be a non-condenser, the steam all escapes into the open air, and is there condensed, and the boiler is fed with cold water, unless some special provision is made for heating the latter with waste steam, or furnace gases, which arrangement has, of course, nothing to do with the engine, properly so called.

We thus see that the actual engine differs at every stage of its working from the theoretically perfect heat engine, and these differences are multiplied and rendered more complicated by numerous other circumstances which will presently be referred to. For instance, it has been taken for granted, in all that has gone before, that the engine receives dry saturated steam from the boiler. Now, as a matter of fact, boilers do not usually deliver dry steam, but send over large quantities of hot water with the steam into the cylinders. When this takes place, the calculations for heat expended and work done have to be materially modified; for it is evident that a large quantity of heat has been spent in warming this water up, from the temperature of the feed to that of the steam; which heat is wholly or in greater part wasted, as no work can be done by this water unless it evaporates in the cylinder. Under the most favourable circumstances the water can only be partially evaporated, viz. when a jacket supplies heat to the steam in the cylinder, and when by expansion the pressure of the steam is so far lowered that the water is too hot to remain water at the lower pressure, and consequently expends its surplus heat in partially evaporating itself. This subject of wet steam is chiefly of interest in so far as it affects the subject of jacketing; and will consequently be referred to again in Chapter XI.

Again, in all that has gone before, it has been assumed that the action of the valves which admitted the steam from the boiler to the cylinder, and from the cylinder to the condenser, was perfect; that is to say, that they opened and closed fully and quickly, precisely at the proper moment, and in no way by their slowness of motion or imperfect design strangled the steam on its passage to or from the cylinder. In actual steam engines the valves not infrequently fall short of this ideal perfection.

The results usually attained in practice fall short of what they should do owing to the three following sets of causes :—

1. The boiler is imperfect, inasmuch as it wastes heat, and delivers water along with the steam to the cylinder.

2. The engine, considered merely as a heat engine, is imperfect for the following reasons :—

a. The limits of temperature within which it is possible to work it in practice are narrow, so that the numerical value of the fraction $\frac{\tau_1 - \tau_2}{\tau_1}$ is very small.

b. The series of operations does not comply with the conditions of maximum efficiency, for the heat is neither received nor rejected at constant temperatures, and the metal of which the cylinder is made is capable of absorbing, transmitting, and radiating heat, so that adiabatic expansion is impossible.

3. The engine, considered as a piece of mechanism, is defective for the following reasons:—

a. Work is lost in friction of the different moving parts.

b. The passages conveying the steam from the boiler to the cylinder, and from the latter to the open air or condenser, always impede somewhat the motion of the steam, thus diminishing the useful pressure on the piston and increasing the back pressure.

c. It is impossible to avoid leaving, and is even necessary to allow a certain space between the piston, when at its extreme positions and the face of the cylinder cover, which space, together with the cubic contents of the steam port, is called *clearance.* Now, it is evident that this clearance space has to be filled with fresh steam at every stroke, which does no work except when expanding, and consequently causes a loss of efficiency.

The losses due to the imperfections of the boiler and of the mechanism will be duly considered in the chapters devoted to these subjects. The losses due to the defects of the engine proper considered as a heat engine have been considered so far as space and the scope of this work will allow in the present chapter. Some of them will, however, be referred to again in Chapter XI., which deals principally with the refinements of the engine, contrived to neutralise the deficiencies.

Summary of the Laws and Formulæ of Thermodynamics.

It may be useful and convenient to sum up here the principal laws and formulæ of the science of heat, as explained in this and the preceding chapter.

First Law of Thermodynamics.—Heat and mechanical energy are mutually convertible. A unit of heat requires for its production, and produces by its disappearance, a fixed amount of mechanical energy.

British unit of heat.—The unit of heat used in this country is the quantity of heat required to raise one pound of water of the temperature 39·3° through one degree Fahrenheit.

Mechanical equivalent of heat.—One British thermal unit is equivalent to 772 foot-pounds of mechanical work.

Second Law of Thermodynamics.—Heat cannot pass from a cold to a hot body by a self-acting process unaided by external agency.

Boyle's law applied to gases.—The product of the pressure and volume of a portion of gas is a constant quantity so long as the temperature remains constant.

For air at 32° the constant quantity is 26,214 foot-pounds. Hence the expression of the law for air is :

$$pv = 26{,}214 \text{ foot-pounds.}$$

Law of Charles and Gay Lussac applied to gases.—When the pressure is constant all gases expand alike for the same increase of temperature. The amount of the expansion between 32° and 212° is ·3654 of the original volume; and for each degree between 32° and 212° it equals $\frac{\cdot 3654}{180} =$ ·00203.

Similarly, when the volume remains constant the pressure varies in the above proportion.

Combination of the two foregoing laws.—The product of the pressure and volume of a portion of gas is proportional to the absolute temperature. Thus :

$$\frac{p_1 v_1}{\tau_1} = \frac{pv}{\tau} \text{ and } \therefore p_1 v_1 \tau = pv\tau_1.$$

N.B. The absolute temperature is equal to the ordinary temperature on Fahrenheit's scale plus 461°.

Hence, remembering that the absolute temperature of 32° is 493, and that the value of pv for 32° is 26,214, we get the very important law

$$pv = 53{\cdot}2\,\tau.$$

The specific heat of gas at constant pressure is the same at all temperatures.

The mechanical equivalent of the heat required to raise one pound of air, one degree, at constant pressure is :

$$K_p = {\cdot}2375 \text{ thermal unit} = 183{\cdot}35 \text{ foot-pounds.}$$

If a gas expand without doing external work, its temperature is unchanged.

The mechanical equivalent of the heat required to raise one pound of air, one degree, at constant volume is :

$$K_v = \cdot 1686 \text{ thermal unit} = 130 \cdot 2 \text{ foot-pounds.}$$

The ratio of the two above numbers is :

$$\gamma = \frac{183 \cdot 25}{130.2} = 1 \cdot 408.$$

Expenditure of heat during isothermal expansion :

For isothermal expansion Boyle's law applies.

$$\therefore pv = \text{constant,}$$

and the external work done during the expansion

$$= pv \log_e r = c\tau \log_e r \text{ foot-pounds.}$$

As the temperature of the gas does not alter during the expansion, there is no internal work done, and, consequently, the above expression represents the total heat supply.

Expenditure of heat when expansion takes place according to the formula :

$$pv^n = \text{constant.}$$

The external work done during expansion

$$= \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{c}{n-1}(\tau_1 - \tau_2) \text{ foot-pounds.}$$

The internal work done in changing the temperature from τ_1 to τ_2

$$= K_v (\tau_2 - \tau_1)$$

Therefore, the total heat expended is the sum of the two above quantities :

$$= (\tau_2 - \tau_1)\left(\frac{n K_v - K_p}{n-1}\right)$$

Expenditure of heat during adiabatic expansion :

The results in this case are got by substituting $\gamma = 1 \cdot 408$ for n in the above formula. Hence heat expended in doing external work during expansion

$$= \frac{c}{\cdot 408}(\tau_1 - \tau_2) \text{ foot-pounds ;}$$

the internal work

$$= K_v (\tau_2 - \tau_1)$$

and the total heat supplied

$$(\tau_2 - \tau_1)\left(\frac{1 \cdot 408 K_v - K_p}{\cdot 408}\right)$$

The final temperature in adiabatic expansion is

$$\tau_2 = \tau_1 \left(\frac{1}{r}\right)^{\cdot 408}$$

The efficiency of a perfect heat engine is the ratio of the difference of the absolute temperatures of the sources of heat and cold, to the absolute temperature of the source of heat

$$= \frac{\tau_1 - \tau_2}{\tau_1}$$

Law connecting the pressure and volume of dry saturated steam.

$$p v^{1 \cdot 0646} = 475$$

where the pressure is expressed in pounds per square inch and the volume in cubic feet.

$$\therefore \log. v = 2 \cdot 516 - \cdot 939 \log. p.$$

The specific heat of water is constantly varying. It has the value unity = 772 foot-pounds only at the temperature 39·3°. At 400° the specific heat = 802·88 foot-pounds.

Superheated steam.

The law connecting the pressure, volume, and absolute temperature of superheated steam is

$$p v = 85 \cdot 5 \tau$$

as against $p v = 53 \cdot 2 \tau$ in the case of air.

The mechanical equivalent of the heat required to raise one pound of superheated steam, one degree, is

At constant pressure, 370·56 foot-pounds.
At constant volume, 285·03 foot-pounds.

The ratio of the two numbers, or $\gamma = 1 \cdot 3$.

Total heat required to change one pound of water of 32° *into steam of* $t°=885{,}200 + 235{\cdot}46\ (t°-212°)$ approximately.

If the water were originally hotter than 32°, say $t_1°$.

Total heat required $= 885{,}200 + 235{\cdot}46\ (t°-212°) - 772\ (t_1° - 32°)$ approximately.

Of the above the quantity required to do external work

$$= p\,v \text{ foot-pounds.}$$

where $p\,v$ is calculated from the equation $p\,v^{1{\cdot}0625} = 475$.

Zeuner's empirical equation for the heat expended in doing external work.

$$\text{External work} = 15{,}450 + 846\ t - h \text{ foot-pounds.}$$

where h is the quantity of heat required to raise the water from 32° to the temperature of the steam.

Expenditure of heat per pound of steam expressed by an equivalent pressure.

$$P = \frac{H}{v}$$

where P is the equivalent pressure required. H is the heat of formation of one pound of steam in foot-pounds, and v the corresponding volume of the steam.

Expenditure of heat per cubic foot swept through by the piston.

$$\text{Heat expended} = \frac{H}{v},$$

$$\text{and heat rejected} = \frac{H}{v} - p + p_b$$

where p = pressure of the steam per square foot, and p_b = the back pressure per square foot.

Expansive working of steam.

To find expenditure of heat when steam at end of stroke is dry and saturated.

Let H_f be the total heat of formation of one pound of steam having the pressure p_f per square foot at some point just before the end of the stroke and the corresponding volume v_f. Let p_m be the mean pressure, and let the other symbols have the same meanings as before.

$$\text{Heat rejected} = H_f - p_f\,v_f + p_b\,v_f.$$

Heat expended $= H_f - p_f\,v_f + p_b\,v_f + (p_m - p_b)\,v_f = H_f + p_m\,v_f - p_f\,v_f$.

Heat expended per cubic foot swept through by the piston

$$= \frac{H_f}{v_f} + p_m - p_f.$$

Rankine's empirical formula for the expenditure of heat in a steam engine.

Heat expended $= p_m + 15\,p_f$ for condensing engines.
$= p_m + 14\,p_f$ for non-condensing engines.