cylinders, the rate of expansion, the weight of engine, tender, and train, and the varying resistances, to be solved; but such an equation would be too complicated, and, when used for finding the speed, of too high dimensions for ordinary use. The analysis of the resistances given above will, however, be useful to students, as it will often facilitate the solution of individual problems.

CHAPTER V.

THE MECHANICS OF THE STEAM ENGINE.

Elementary principles of dynamics—Definitions of mass, weight, velocity, motion, force—Units employed in their measurement—The laws of motion and examples of their application—Work and energy—Motion of bodies in circles—Application to fly-wheels—Centrifugal force—Conversion of work done in the cylinder into work done on the crank—1st case, when pressure of steam is uniform, connecting rod supposed to be of infinite length and moving parts without weight—2nd case, when steam is allowed to expand, the other conditions remaining unchanged—Curve of effort on crank pin—3rd case, when the length of the connecting rod is taken into account—4th case, when the weights and velocities of the reciprocating parts are taken into account—Power absorbed in accelerating these parts—Power restored by their retardation—The consequent modification of the pressures shown by indicator diagrams necessary for calculating effort on crank pin—Effect of steam distribution on the action of the moving parts—Means of equalising the tangential effort on crank-pin—Fly-wheels—Theory of their action—Graphic diagrams illustrating their action.

Before discussing the questions of applied mechanics which arise in the study of the moving parts of the steam engine, it will be useful for the sake of accuracy, to recapitulate briefly the elementary principles of dynamics, a previous acquaintance with the principles of the composition of resolution of forces and velocities, on the part of the reader being, however, assumed. We will start with the following definitions :—

1. Mass. This word denotes the quantity of matter contained in a body.

2. Weight is the attraction which the earth exercises on a mass.

3. Velocity is the speed at which a body moves, i.e. the space which it traverses in a given time.

4. Motion. This word is employed in dynamics, not

merely to denote movement on the part of a body, but also takes account of the mass of the body moved. Thus, if two bodies have each the same velocity, but the mass of one be double that of the other, then the *motion*, or *quantity of motion*, or *momentum*, as it is variously termed, of the body having the larger mass is double the motion of the smaller one. If the bodies had each the same mass, but the velocity of one were double that of the other, then the motion of the body having the greater velocity would be likewise double that of the other. When the velocity remains constant, the motion varies as the mass moved. When the mass is constant, the motion varies as the velocity. Therefore, generally, the motion varies as the mass × the velocity.

5. Force is any cause which produces, or tends to produce, motion in a body, or which changes, or tends to change, the motion of a body.

Units adopted in measuring Mass, Weight, Velocity, and Force.

The only means we have of measuring the masses of different bodies, i.e. the quantities of matter in them, is by weighing them; that is to say, by comparing the attractions of the earth on them relatively to some standard substance. Consequently, the measure of mass is dependent upon the unit chosen to measure weight.

The unit of weight adopted in this country is the weight in London of a certain piece of platinum, kept in the office of the Exchequer, and called a pound avoirdupois. The weight of this piece of platinum varies in different parts of the globe. Its weight depends on the attraction exercised by the earth upon the matter contained in it. This force of attraction, called gravitation, was discovered by Newton to depend on the distance between the centre of the earth and the object attracted. In consequence of the flattening

of the earth towards the poles, and its bulging out towards the equator, the surface of the earth is nearer to the centre in London than in more southern places, and consequently the weight of the standard piece of platinum is greater in London than it is at the equator.

The mass of a body is then measured by its weight at a given place. There are two units of mass made use of in dynamics. The so-called *gravitation unit of mass* is the quantity of matter contained in a body weighing 32·2 pounds.

The so-called *absolute unit of mass is* the quantity of matter in a body weighing one pound.

Let M denote the mass of a body measured by the gravitation unit, then its weight by the definition is $W=M\times 32\cdot2$ lbs. The symbol g is used to denote the number 32·2. Hence we have

$$W=M.g \text{ or } M=\frac{W}{g}.$$

The velocity of a body is measured in different ways according as it is a linear velocity, i.e. due to a motion of translation of the body from one point to another; or an angular velocity, i.e. due to the rotation of the body round an axis.

Velocity is uniform when the body traverses equal spaces in equal times. When uniform, linear velocity is always measured by the number of feet of linear space, traversed in one second of time. Thus, for instance, we speak of a body having a velocity of two thousand feet a second.

Let v denote the velocity of a body moving uniformly. Let s denote total the number of feet which it passes over, and t denote the number of seconds occupied in describing the s feet.

$$\text{Then } v=\frac{s}{t} \text{ or } s=t.v.$$

Further, the centrifugal force (see page 159) which in all cases tends to diminish the weight is greater at the equator than at the poles.

Force is also, like mass, measured in two ways. According to the *gravitation* system, a force is measured by the weight which it can support. Thus a string is said to exercise a force of ten pounds when the tension in the string is sufficient to prevent a force of ten pounds from falling to the earth. The unit of force in gravitation measure is the force which can support a weight of one pound.

The second or *absolute* system of measuring force is more in harmony with the definition. By this system a force is measured by the velocity which it can impart to a given mass in a given time, when acting continuously on the mass for that time. Thus, for instance, the force of gravity is measured by the velocity which it can generate in a given mass when acting on it for a second of time. The unit of force in absolute measure is the force which can generate a velocity of one foot per second, when acting on a mass weighing a pound, during one second of time.

The Laws of Motion.

First Law.—Every body continues in a state of rest or of uniform motion in a straight line, unless compelled by impressed forces to change that state.

This law lays down that matter has of itself no power to change its own condition of rest or of uniform motion. In other words, it possesses what is called inertia. Consequently, when we note that a body is not moving with a uniform velocity, we know that it is being acted upon by external force.

Second Law.—Change of motion is proportional to the impressed force, and takes place in the straight line in which the force is impressed.

In the above statement the word motion has the meaning already explained, viz. Mass multiplied by velocity.

Thus if two equal forces act on two unequal masses, the quantity of motion generated in each case will be the same, but the greater mass will have the least velocity, and the

product of the mass multiplied by the velocity will be the same in each case. If the velocities of the two masses are to be equal, then the force acting on the greater mass must be greater than the other in the same ratio that the mass itself is greater.

This law enables us to compare the relative magnitudes of forces, for we have only to observe the velocities generated by the various forces in the same mass, when acting for the same time. The standard for comparison is the velocity generated in a mass by the force of gravity, i.e. by its own weight when acting for a second of time. This velocity is g or 32·2 feet per second; that is to say, if the force of gravity act on a free body for one second, it will at the end of the second have imparted to the body a velocity of 32·2 feet per second. Thus a force F acting on a body weighing 20 lbs. for a second generates a velocity of 50 feet per second; what is the magnitude of the force in gravitation measure?

The force of gravity, i.e. the weight of the body, or 20 lbs., would generate a velocity in the body of 32·2 feet per second. Therefore we have

$$F : 20 :: 50 : 32{\cdot}2.$$

$$\therefore F = \frac{50 \times 20}{32{\cdot}2} = 31 \text{ lbs.}$$

In the general case, if the velocity v be generated in one second by a force F in a body weighing W lbs., then

$$F : W :: v : g.$$

$$\therefore F = \frac{Wv}{g} \text{ lbs.}$$

The statement in the above law that the change of motion takes place in the direction in which the impressed force acts may be illustrated by the following example :—

A ball is projected from a rifle in a perfectly horizontal direction, AB (fig. 25), from a height above the ground, AH. Directly it leaves the muzzle of the gun it is acted on by two forces, viz. the momentum acquired while in the barrel, and which would in a given number of seconds carry it to, say, B; and the force of gravity which acting alone would in the same time carry it to, say, H. According to the law, each force generates motion in the ball in the direction in which each acts, and the consequence is, that at the end of the given time, the ball

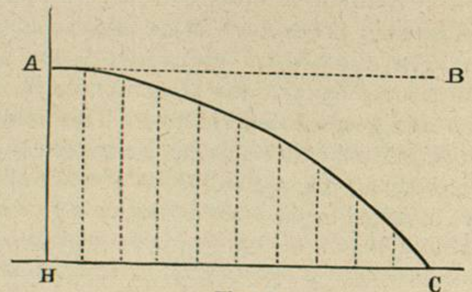

Fig. 25.

will have travelled as far forward as B, and as far downwards as H, and hence its resultant position will be C. The law in effect states that dynamical forces may be compounded and resolved in the same manner as statical forces.

The motion of bodies moving under the action of a constant force.—Let us take, for example, a body falling freely from a state of rest under the action of gravity. At the end of the first second, its velocity, having been zero to start with, will have increased up to 32·2 or g feet per second. As the force continues to act uniformly it will have produced precisely the same effect by the end of another second; that is to say, the velocity will have been gradually increased by another 32·2 feet per second. Hence the velocity at the end of the second second will be 64·4 or $2g$ feet per second. At the end of 3 seconds it will be $3g$ and so on, and generally at the end of t seconds $v=gt$.

Next, as to the space traversed at the end of t seconds. If it were travelling all the time with its final velocity gt we should, using the formula $s=vt$, have for the space

$s=gt.t=gt^2$. As, however, the velocity is constantly increasing, we can only take the average, which, in consequence of the perfectly uniform nature of the rate of increase, or acceleration as it is called, is very easy to calculate, and is in fact half the sum of the initial and final velocities, and in this particular case $=\frac{o+gt}{2}=\frac{gt}{2}$.

Hence, instead of $s=gt^2$ we have

$$s=\frac{gt^2}{2}=\tfrac{1}{2}tv,$$

where v is the final velocity. These formulæ connect the space, time, and velocity.

If we wish to connect the space traversed with the final velocity, without taking account of the time we can eliminate t by combining the formulæ,

$$s=\tfrac{1}{2}gt^2, \text{ and } v=gt.$$

$$\text{Hence } s=\tfrac{1}{2}g.\frac{v^2}{g^2}=\frac{v^2}{2g}$$

$$\text{Therefore } v^2=2g.s.$$

$$\text{or } s=\frac{v^2}{2g}.$$

Third Law.—To every action there is always an equal and contrary reaction.

Examples of the Application of the Laws of Motion.

Example (1).

A body falls from rest under the influence of gravity; what will be its velocity at the end of 10 seconds, what will be the total space traversed, and what the space traversed during the tenth second?

1. $v=gt=32·2\times 10=322$ feet per second.
2. $s=\frac{1}{2}gt^2=16·1\times 100=1{,}610$ feet.
3. Space described in nine seconds.
   $s=\frac{1}{2}gt^2=16·1\times 81=1304·1$ feet.

∴ Space described in tenth second $=1610-1304·1$
$=305·9$ feet.

Example (2).

A rifle bullet is shot vertically upwards with a velocity of 1,000 feet per second; find the maximum height to which it would reach if it experienced no resistance from the air. How long a time will it take to reach this height? The height to which the bullet will reach is equal to the height down which a body must fall from rest in order to acquire a velocity of 1,000 feet per second. For, from the moment it commences to rise, its velocity is being diminished at the rate of 32·2 feet per second, till the initial velocity is all expended and the bullet come to rest at the top of its flight. If from this point it commenced to fall it would attain a velocity of 1,000 feet a second on reaching the starting point. Using, therefore, the formula

$$s=\frac{v^2}{2g}=\frac{1{,}000{,}000}{2\times 32{\cdot}2},$$

we have $s=15{,}527$ feet.

To find the time occupied. As a velocity of 32·2 feet is generated in each second, a velocity of 1,000 feet per second will be generated in $\frac{1{,}000}{32{\cdot}2}$

$$=31{\cdot}05 \text{ seconds.}$$

Example (3).

A weight of 12 lbs. rests on a perfectly smooth surface and is connected by a string passing over a smooth pulley with a weight of 6 lbs. hanging vertically downwards. What velocity per second will the weights have at the end of the first second from rest?

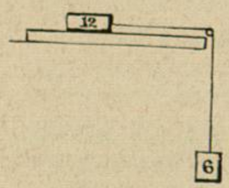

Fig. 26.

The tension, T, in the string is uniform throughout the string. Also, the weight 12 is caused to move by the tension of the string, while the weight 6 is caused to move by its own weight acting downwards, minus the tension in the string acting upwards.

Let v be the velocity generated in the mass of 12 lbs. by T; then $T : 12 : \text{mass}\times v : \text{mass}\times g$

$$\therefore \frac{T}{12}=\frac{v}{g}.$$

As the velocity of the mass of 6 lbs. is the same $=v$, and as this velocity is generated by the force $6-T$, we have

$$6-T : 6 : \text{mass}\times v : \text{mass}\times g$$

$$\therefore \frac{6-T}{6}=\frac{v}{g},$$

$$\therefore \frac{T}{12}=\frac{6-T}{6} \therefore 6\,T=72-12\,T,$$

$$\therefore 18\,T=72 \quad \therefore T=4 \text{ lbs.}$$

To find the velocity generated at the end of one second.

Gravity, or the weight 12, acting on the mass of 12 lbs. for one second, would generate a velocity of 32·2 feet per second, therefore the tension $T=4$ lbs. will generate a velocity

$$=\frac{4\times 32{\cdot}2}{12}=10{\cdot}73 \text{ feet per second.}$$

Example (4).

A mass of 200 lbs. is moved from rest by a constant force F, and passes over a space of 60 feet in the first second; what is the measure of the force?

As the space passed over from rest is 60 feet, the velocity at the end of the second is $2\times 60=120$ feet per second. The force of gravity, or 200 lbs., would in the same time generate a velocity of 32·2.

$$\therefore F : 200 :: 120 : 32{\cdot}2.$$

$$\therefore F=\frac{200\times 120}{32{\cdot}2}=745{\cdot}3 \text{ lbs.}$$

The following example illustrates the application of the laws of motion to the moving parts of a steam engine.

Example (5).

The piston, piston rod, and connecting rod of an engine weigh 400 lbs., the stroke $1\frac{2}{3}$ ft., and the number of revolutions 200 per minute. During each revolution this mass starts from a state of rest; its velocity is gradually increased up to a maximum, and from this point it diminishes till it comes to rest again at the end of the stroke. During the first part of the stroke a certain proportion of the total steam pressure is required to generate this velocity in the moving parts, and whatever pressure remains over is all that is available for transmission to the crank. During the latter part of the stroke, after the point of maximum velocity is reached, the speed of the moving parts has to be reduced to nothing by the end of the stroke, and while this reduction is taking place the moving parts press upon the crank-pin with more or less severity depending on their weight, their maximum velocity, and the space in which the velocity is reduced from the maximum to zero. Consequently, while during the first part of the stroke the crank-pin has only a portion of the steam pressure transmitted to it, during the latter portion, on the contrary, it is subjected not only to the full steam pressure acting on the piston, but also to the extra pressure due to the pulling up of the moving parts.

In the present case the length of stroke and speed of rotation of the crank are such, that at the commencement of the stroke the piston moves through ·000183 ft. in the thousandth part of a second; what is the total pressure, P, required to move the mass of 400 lbs. from rest over this space in a given time?

The velocity at the end of the time $= 2 \times {\cdot}000183$ ft. per $\frac{1}{1000}''$,

$\therefore$ the velocity per second at the end of one second

$$= 2 \times {\cdot}000183 \times 1000^2 = 365{\cdot}4.$$

Now, gravity, or the weight of the moving parts, i.e. 400 lbs., is capable of generating in them a velocity of 32·2 feet per second.

$$\therefore 400 : 32{\cdot}2 :: P : 365{\cdot}4.$$

$$\therefore P = \frac{400 \times 365{\cdot}4}{32{\cdot}2} = 4539 \text{ lbs.}$$

The diameter of the cylinder is 10 inches, its area $\therefore = 78{\cdot}5$ sq. inches, and the pressure of steam at the commencement of the stroke necessary to impart the required velocity to the moving parts

$$= \frac{4539}{78{\cdot}5} = 57{\cdot}8 \text{ lbs. per square inch.}$$

At the extreme end of the stroke the motion of the moving parts is arrested at the same rate as it is imparted at the commencement,[1] and consequently a pressure at the rate of 57·8 lbs. per square inch of piston area is transmitted to the crank in addition to whatever pressure of steam may happen to be acting on the piston at that moment.

Energy.—The term energy, or capacity for doing work, has been already explained (see page 25). The matter is

[1] For the sake of simplicity the influence of the length of the connecting rod relatively to the length of the crank in retarding the motion of the piston, &c., is here neglected.

now referred to again for the purpose of showing the effect on the working of steam engines, of the energy stored up in masses in motion. A body in motion possesses energy; for, if the motion be, for instance, vertically upwards, it will carry the body up to a certain height before it is brought to rest, i.e. it will overcome the attraction of the earth through a certain space. The height to which the body will rise is, as explained in Ex. (2), p. 152, equal to the height down which the body must fall in order to acquire the same velocity.

In questions concerning the steam engine we are chiefly concerned with the energy of bodies in motion. Very frequently work is said to be stored up in a body in motion, or in a raised weight. What is really meant is that energy or the capacity of doing work is stored.

Take as an example the case of a cannon-ball weighing 100 lbs. and having a velocity of 2,000 feet a second; what is its capacity for doing work? The velocity of 2,000 feet a second would be acquired by falling down a height S, calculated by the formula

$$S = \frac{v^2}{2g} = \frac{4{,}000{,}000}{64{\cdot}4} = 62{,}111 \text{ ft.}$$

Therefore the velocity of 2,000 ft. per second is capable of raising the body to a height of 62,111 feet, and the work which would be done=the height multiplied by the weight

$$= \frac{v^2}{2g} \times w = 62{,}111 \times 100 \text{ foot-pounds.}$$

Vice versâ, in order to impart this velocity to the cannon-ball, 6,211,100 foot-pounds of work would have to be done upon it before it left the bore of the gun. If the bore of the latter were six inches in diameter, and ten feet long from the front of the powder-cartridge to the muzzle, what would be the average pressure of the powder gases per square inch? As 6,211,100 foot-pounds of work have to be done on the shot while it traverses the space of ten feet, the total average pressure on the shot must be